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Effect of Tire Pressure on Efficiency

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Effect of Tire Pressure on Efficiency

Abstract
Many people ignore tire pressure in their day-to-day lives. In a country striving for maximum efficiency, neglecting to maintain correct tire pressure can noticeably affect the amount of horsepower required to overcome the drag forces due to tire/roadway friction. In order to quantify these horsepower changes, 21 test trials were conducted by allowing a test vehicle to coast from 50 mph down to 20 mph at pressures ranging from 50 psi to 20 psi. After compiling the results, it was shown that lower tire pressures increased the horsepower necessary to propel the automobile.

Keywords
Tire Pressure, Velocty, Horsepower

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PROBLEM STATEMENT

More than ever before, the automotive world is focusing on ways to obtain higher efficiency. Many new technological advancements target power consumption, but one of the easiest ways to increase efficiency often goes ignored: the vehicle's tire pressure. The only connection between a vehicle and its driving surface is the four contact patches where the tires meet the roadway. Monitoring tire pressure and attempting to minimize the amount of rolling friction may combat energy waste. From the data collected at different tire pressures, determine if there is a difference in the efficiency of over and under inflated tires.

MOTIVATION

Modern advertisements for new cars showcase their fuel efficiency. This paradigm shift highlights the fact that automobile manufacturers are spending additional resources to engineer a more efficient vehicle. While the new technological advancements are effective, it is important implement simple techniques such as maintaining proper tire pressure to avoid wasting energy/fuel. As a tire pressure drops, the walls of the tire become soft and flexes when the wheel rotates. This deformation increases the drag and causes the vehicle to use more energy to maintain forward motion. The objective of this project is to quantify the change in the horsepower required to maintain a constant velocity based on the changes in tire pressure. We pursued this goal by collecting field data and analyzed it using calculus principles.

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MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

In this experiment, we measured the time required to slow a vehicle from $50\ mph$ to $20\ mph$ through wind resistance and tire friction. Changing only the pressure in the tires allowed us to determine the overall influence that tire pressure plays in efficiency.

There are several sources which cause drag on a car. Wind can push or pull on a car affecting the power requirements to maintain a constant velocity. The shape of a vehicle controls how well it resists drag as it passes through the air. Internally, drive train friction can differ from vehicle to vehicle. The drive train friction determines how much energy from the engine is absorbed before transferring its power to the wheel or the ground.

The tire-roadway interaction also affects the drag on a vehicle. Driving surfaces can range from dirt roads and mud to paved highways. Each driving surface can produce a very different amount of friction. Some tires are made of compounds tailored to different aspects of driving. If maximum grip is preferred, a tire manufactured from a softer compound can be used

Figure 1: 2004 Saturn Ion used for testing.

to increase grip. However, an increased grip also increases friction which in turn diminishes the efficiency of the power transfer from the tire to the ground.

TESTING CONDITIONS

Wind speed and roadway variations were controlled by conducting all tests consecutively on the same roadway (Figure 2), in the same direction, and on the same day. Also, the same vehicle (Figure 1) was used for all trials so that the vehicle aerodynamics remained constant. To exclude drive train friction, all testing and timing was done while the car was in neutral so that the transmission was not actively creating drag. These restrictions were employed to isolate the tire pressure as the sole variable governing the power required to maintain a constant velocity.

Figure 2: Roadway used for testing.

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DATA COLLECTION

The recorded data was collected after pumping the tires up to a specific pressure starting at a high of 50 psi and going down to a low of 20 psi in 5 psi increments. The test was conducted by bring the test vehicle to a velocity of 50 mph. Then at a specific location the test car was set to coast. A stopwatch with a lap button (Figure 3) was used to record how long it took the coasting vehicle to drop each 5 mile/hour increment. The results of 3 test runs (Appendix A - Table 1) were averaged for each of the tire pressures.

ANALYSIS

The velocities acquired by our tests can be used to calculate acceleration of the test vehicle (R.C. Hibbler, 19) using

$$a = \frac{\Delta v}{\Delta t} \tag{1}$$

and the displacement (R.C. Hibbler, 20) using

$$\Delta s = \int v\, dt. \tag{2}$$

Figure 3: Stopwatch and analog speedometer used to determine the rate of deceleration.

The force on the vehicle (R.C. Hibbler, 110), was calculated using

$$F = m\,a\,. \tag{3}$$

The mass in (3) was determined by adding the mass of the test driver to the mass of the vehicle which was given by the Saturn owner's manual (Saturn). Thus, the total mass of car and driver was

$$m_{total} = m_{vechicle} + m_{occupant} = 2{,}750\ lbs + 180\ lbs = 2{,}930\ lbs.\,. \tag{4}$$

From the force given in (3) and the test velocities, the power of the test car (Serway Jewett, 214) was found through the relationship

$$P = F\,v\,. \tag{5}$$

For automobiles, the most common unit of power is horsepower (What is Horsepower?). Note that one unit of horsepower is approximately

$$1\ hp \approx 17{,}695.7\ \frac{ft^2\ lbs}{s^3}\,. \tag{6}$$

Putting everything together, we determined the horsepower over time for the test vehicle under various tire pressures.

DISCUSSION

The 'Velocity vs Time' graph for each tire pressure (Appendix B - Chart 1) shows the steadily decreasing amount of time it took to deccelerate given lower tire pressures. Notice the relatively flatter slope for all the lines in the 45-40 mph velocity zone. This can be attributed to a slight downhill grade going onto a bridge on the roadway used for testing. The momentary downhill slope is also reflected by the flucations in the chart of the 'Horsepower vs Velocity'

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(Chart 2 – Appendix B). Accounting for changes in the test track, we interpolate the average trial velocities for each tire pressure using a 5^{th} order polynomial. For instance, the average velocities over three trials using $20\ psi$ (Appendix A – Table 1) were found to be:

	Mean Measured Times at Set Velocities - 20 psi						
Velocity (mph)	50	45	40	35	30	25	20
Velocity (ft/s)	77.3	66	58.7	51.3	44	36.7	29.3
Time (s)	0.00	7.80	17.67	28.40	37.70	49.93	63.20

Note that all units were first standardized in terms of feet and seconds. Microsoft Excel interpoloated the converted velocities as

$$v(t) = 73.3585 - 1.2204\,t + 0.0462\,t^2 - 0.0018\,t^3 + 3.2\,E^{-05}t^4 - 1.9\,E^{-07}t^5. \qquad (7)$$

Using equation (7) with (1) we found the interpolated acceleration to be

$$a(t) = \frac{dv}{dt} = -1.2204 + 0.0924\,t - 0.0054\,t^2 + 1.28\,E^{-04}\,t^3 - 9.6\,E^{-07}t^4 \qquad (8)$$

and from (2) the interpolated displacement was

$$d(t) = \int_0^t v(s)\,ds = 73.3585\,t - 0.6102\,t^2 + 0.0154\,t^3 - 0.00046\,t^4 + 6.3\,E^{-06}t^5 - 3.19\,E^{-08}t^6. \qquad (9)$$

Using the total test time for the $20\ psi$ test, we can estitmate the total distance traveled during the $20\ psi$ test as

$$d(63.2) = 3{,}120.8\ ft \qquad (10)$$

which corresponds with Table 8 in Appendix A. The remaining trials were similarly analysized and their values are summarized in Tables 2 – 8 found in Appendix A.

The 'Horsepower vs Velocity' chart (Appendix B – Chart 2) shows the amount of horsepower acting against the coasting car. Viewed another way, this chart indicates the amount of horsepower needed to maintain the indicated velocity. The negative horsepower is a reflection of wind resistence on the car and friction between the tires and the pavement. Comparing the horsepower values for each tire pressure (Appendix B – Chart 3), it is clear that the lower pressures require more horsepower to maintain a constant velocity than the higher pressures.

CONCLUSION AND RECOMMENDATIONS

All of the results from this project pointed to the fact that there is a relationship between lower tire pressures and lower efficiency. The increased horsepower consumption will be most noticable at higher speeds as is indicated in Chart 3 of Appendix B. For instance, at $50\ mph$ the $50\ psi$ tires required $11.37\ hp$ to maintain a constant velocity while the $20\ psi$ set required $14.82\ hp$ which was a suprising 30% increase in required horsepower. At $20\ mph$, all the horsepower discrepencies were negligable as all he values fell between $3.405 \pm 0.705\ hp$.

Variances in the slower velocities can also be attributed to the road condions of the test track. At $50\ psi$ the test car breached the $20\ mph$ threshold $4{,}108.8\ feet$ from the starting line while breaching the same threshold $3{,}120.8\ feet$ from the starting line under $20\ psi$. The grade of the road was influencing the calculated horsepowers differently at these two positions. The problem may be avoided in future experiments by recording the velocities at fixed road markers to ensure all measurements are taken under identical road conditions. Also, it is recommend that future research incorporate the use of a more accurate digital speedometer. Using an analog speedometer meant trusting the eye to accurately guage the moment when the needle passed the specified limits.

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NOMENCLATURE

Symbol	Description	Unit
F	Force	$ft \cdot lbs/s^2$
m	Mass	lbs
mph	Miles per hour	$miles/hour$
P	Power	$ft \cdot lbs/s$ and Hp
psi	Pounds per square inch	lbs/in^2
s	Position	ft
v or Vel	Velocity	ft/s and mph
a or Acc	Acceleration	ft/s^2
Δv	Change in velocity	ft/s
Δs	Displacement	ft
Δt	Change in time	s

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APPENDIX A - TABLES

Pressure	Trial	50 mph	45 mph	40 mph	35 mph	30 mph	25 mph	20 mph
50 psi	1	0.0	10.3	24.8	35.6	48.8	66.7	87.9
	2	0.0	9.2	23.5	33.2	46.2	62.7	81.8
	3	0.0	9.5	23.5	33.0	47.5	62.6	84.1
	Mean	0.00	9.67	23.93	33.93	47.50	64.00	84.60
45 psi	1	0.0	9.8	25.3	35.9	48.5	66.6	86.1
	2	0.0	10.2	24.8	35.9	50.2	68.8	86.2
	3	0.0	9.6	23.4	32.7	46.0	62.2	80.4
	Mean	0.00	9.87	24.50	34.83	48.23	65.87	84.23
40 psi	1	0.0	9.0	22.6	34.3	46.0	61.3	83.3
	2	0.0	9.2	21.2	33.1	44.4	58.1	78.8
	3	0.0	9.1	22.0	31.5	44.4	58.1	78.0
	Mean	0.00	9.10	21.93	32.97	44.93	59.17	80.03
35 psi	1	0.0	9.0	21.6	30.1	41.5	55.7	75.5
	2	0.0	8.7	21.3	31.0	41.7	55.7	74.3
	3	0.0	9.1	21.6	31.9	43.2	58.3	77.3
	Mean	0.00	8.93	21.50	31.00	42.13	56.57	75.70
30 psi	1	0.0	8.0	20.9	30.8	41.6	55.2	72.5
	2	0.0	8.3	20.0	31.2	41.6	54.6	72.0
	3	0.0	8.7	21.6	31.3	41.7	57.2	75.7
	Mean	0.00	8.33	20.83	31.10	41.63	55.67	73.40
25 psi	1	0.0	8.3	18.9	30.1	38.8	51.3	65.8
	2	0.0	8.2	19.8	30.2	39.5	52.7	69.6
	3	0.0	8.1	19.5	31.5	41.9	55.6	72.6
	Mean	0.00	8.20	19.40	30.60	40.07	53.20	69.33
20 psi	1	0.0	7.9	17.7	29.0	37.4	49.1	61.9
	2	0.0	7.9	17.8	28.1	38.1	50.1	63.0
	3	0.0	7.6	17.5	28.1	37.6	50.6	64.7
	Mean	0.00	7.80	17.67	28.40	37.70	49.93	63.20

Table 1: Under various tire pressures, this table lists the number of seconds it took the test car to reach 5 mph speed thresholds after coasting from 50 mph.

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50 psi - Test Statistics								
Measured	Mean Time (s):	0.00	9.67	23.93	33.93	47.50	64.00	84.60
	Velocity (mph):	50.00	45.00	40.00	35.00	30.00	25.00	20.00
	Velocity (ft/s):	73.33	66.00	58.67	51.33	44.00	36.67	29.33
Interpolated	Velocity (ft/s):	73.28	66.21	58.12	51.97	43.69	36.75	29.32
	Acceleration (ft/s^2):	-0.9370	-0.5973	-0.5874	-0.6338	-0.5523	-0.2905	-0.7320
	Displacement (ft):	0.0	671.8	1558.3	2109.1	2756.9	3414.3	4108.8
Calculated	Force (ft lbs/s^2):	-2745.5	-1750.1	-1721.1	-1856.9	-1618.3	-851.2	-2144.6
	Power (ft^2 lbs/s^3):	-201179	-115877	-100023	-96499	-70708	-31278	-62887
	Power (HP):	-11.37	-6.55	-5.65	-5.45	-4.00	-1.77	-3.55

Table 2: Measured, Interpolated and Calculated values for the experiment with the tires inflated to 50 psi.

45 psi - Test Statistics								
Measured	Mean Time (s):	0.00	9.87	24.50	34.83	48.23	65.87	84.23
	Velocity (mph):	50.00	45.00	40.00	35.00	30.00	25.00	20.00
	Velocity (ft/s):	73.33	66.00	58.67	51.33	44.00	36.67	29.33
Interpolated	Velocity (ft/s):	73.29	66.18	58.20	51.88	43.73	36.73	29.33
	Acceleration (ft/s^2):	-0.9584	-0.5701	-0.5767	-0.6355	-0.5445	-0.2662	-0.8395
	Displacement (ft):	0.0	685.2	1595.3	2164.4	2803.7	3505.7	4127.1
Calculated	Force (ft lbs/s^2):	-2808.2	-1670.3	-1689.8	-1862.0	-1595.3	-779.9	-2459.6
	Power (ft^2 lbs/s^3):	-205811	-110531	-98352	-96603	-69761	-28647	-72128
	Power (HP):	-11.63	-6.25	-5.56	-5.46	-3.94	-1.62	-4.08

Table 3: Measured, Interpolated and Calculated values for the experiment with the tires inflated to 45 psi.

40 psi - Test Statistics								
Measured	Mean Time (s):	0.00	9.10	21.93	32.97	44.93	59.17	80.03
	Velocity (mph):	50.00	45.00	40.00	35.00	30.00	25.00	20.00
	Velocity (ft/s):	73.33	66.00	58.67	51.33	44.00	36.67	29.33
Interpolated	Velocity (ft/s):	73.31	66.08	58.48	51.56	43.87	36.70	29.33
	Acceleration (ft/s^2):	-1.0188	-0.6431	-0.5950	-0.6531	-0.6057	-0.3875	-0.5552
	Displacement (ft):	0.0	631.7	1430.2	2038.2	2608.3	3178.2	3871.4
Calculated	Force (ft lbs/s^2):	-2985.0	-1884.3	-1743.4	-1913.7	-1774.7	-1135.2	-1626.7
	Power (ft^2 lbs/s^3):	-218837	-124513	-101959	-98666	-77856	-41663	-47712
	Power (HP):	-12.37	-7.04	-5.76	-5.58	-4.40	-2.35	-2.70

Table 4: Measured, Interpolated and Calculated values for the experiment with the tires inflated to 40 psi.

35 psi - Test Statistics								
Measured	Mean Time (s):	0.00	8.93	21.50	31.00	42.13	56.57	75.70
	Velocity (mph):	50.00	45.00	40.00	35.00	30.00	25.00	20.00
	Velocity (ft/s):	73.33	66.00	58.67	51.33	44.00	36.67	29.33
Interpolated	Velocity (ft/s):	73.30	66.12	58.36	51.70	43.80	36.71	29.33
	Acceleration (ft/s^2):	-1.0703	-0.6391	-0.6553	-0.7325	-0.6499	-0.3248	-0.8440
	Displacement (ft):	0.0	619.7	1402.4	1925.9	2456.5	3032.0	3677.5
Calculated	Force (ft lbs/s^2):	-3136.1	-1872.7	-1920.1	-2146.1	-1904.3	-951.6	-2472.9
	Power (ft^2 lbs/s^3):	-229891	-123816	-112061	-110964	-83414	-34937	-72528
	Power (HP):	-12.99	-7.00	-6.33	-6.27	-4.71	-1.97	-4.10

Table 5: Measured, Interpolated and Calculated values for the experiment with the tires inflated to 35 psi.

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30 psi - Test Statistics								
Measured	Mean Time (s):	0.00	8.33	20.83	31.10	41.63	55.67	73.40
	Velocity (mph):	50.00	45.00	40.00	35.00	30.00	25.00	20.00
	Velocity (ft/s):	73.33	66.00	58.67	51.33	44.00	36.67	29.33
Interpolated	Velocity (ft/s):	73.32	66.05	58.55	51.48	43.91	36.69	29.33
	Acceleration (ft/s^2):	-1.1986	-0.6571	-0.6283	-0.7333	-0.6676	-0.3532	-0.8453
	Displacement (ft):	0.0	577.4	1356.0	1921.9	2423.6	2984.0	3580.6
Calculated	Force (ft lbs/s^2):	-3511.9	-1925.2	-1840.8	-2148.5	-1956.1	-1034.9	-2476.6
	Power (ft^2 lbs/s^3):	-257496	-127158	-107780	-110613	-85895	-37967	-72643
	Power (HP):	-14.55	-7.19	-6.09	-6.25	-4.85	-2.15	-4.11

Table 6: Measured, Interpolated and Calculated values for the experiment with the tires inflated to 30 psi.

25 psi - Test Statistics								
Measured	Mean Time (s):	0.00	8.20	19.40	30.60	40.07	53.20	69.33
	Velocity (mph):	50.00	45.00	40.00	35.00	30.00	25.00	20.00
	Velocity (ft/s):	73.33	66.00	58.67	51.33	44.00	36.67	29.33
Interpolated	Velocity (ft/s):	73.36	65.91	58.86	51.08	44.17	36.63	29.34
	Acceleration (ft/s^2):	-1.2235	-0.6960	-0.6372	-0.7391	-0.6940	-0.4430	-0.7075
	Displacement (ft):	0.0	568.1	1266.3	1883.0	2333.7	2860.4	3397.2
Calculated	Force (ft lbs/s^2):	-3584.8	-2039.2	-1867.1	-2165.5	-2033.5	-1298.0	-2072.9
	Power (ft^2 lbs/s^3):	-262967	-134407	-109886	-110616	-89815	-47541	-60814
	Power (HP):	-14.86	-7.60	-6.21	-6.25	-5.08	-2.69	-3.44

Table 7: Measured, Interpolated and Calculated values for the experiment with the tires inflated to 25 psi.

20 psi - Test Statistics								
Measured	Mean Time (s):	0.00	7.80	17.67	28.40	37.70	49.93	63.20
	Velocity (mph):	50.00	45.00	40.00	35.00	30.00	25.00	20.00
	Velocity (ft/s):	73.33	66.00	58.67	51.33	44.00	36.67	29.33
Interpolated	Velocity (ft/s):	73.36	65.89	58.88	51.07	44.18	36.62	29.34
	Acceleration (ft/s^2):	-1.2204	-0.7769	-0.6974	-0.7532	-0.7071	-0.5280	-0.7156
	Displacement (ft):	0.0	540.9	1156.0	1746.5	2189.0	2680.8	3120.8
Calculated	Force (ft lbs/s^2):	-3575.8	-2276.4	-2043.3	-2206.8	-2071.7	-1547.1	-2096.8
	Power (ft^2 lbs/s^3):	-262313	-150001	-120307	-112695	-91523	-56650	-61520
	Power (HP):	-14.82	-8.48	-6.80	-6.37	-5.17	-3.20	-3.48

Table 8: Measured, Interpolated and Calculated values for the experiment with the tires inflated to 20 psi.

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APPENDIX B – CHARTS

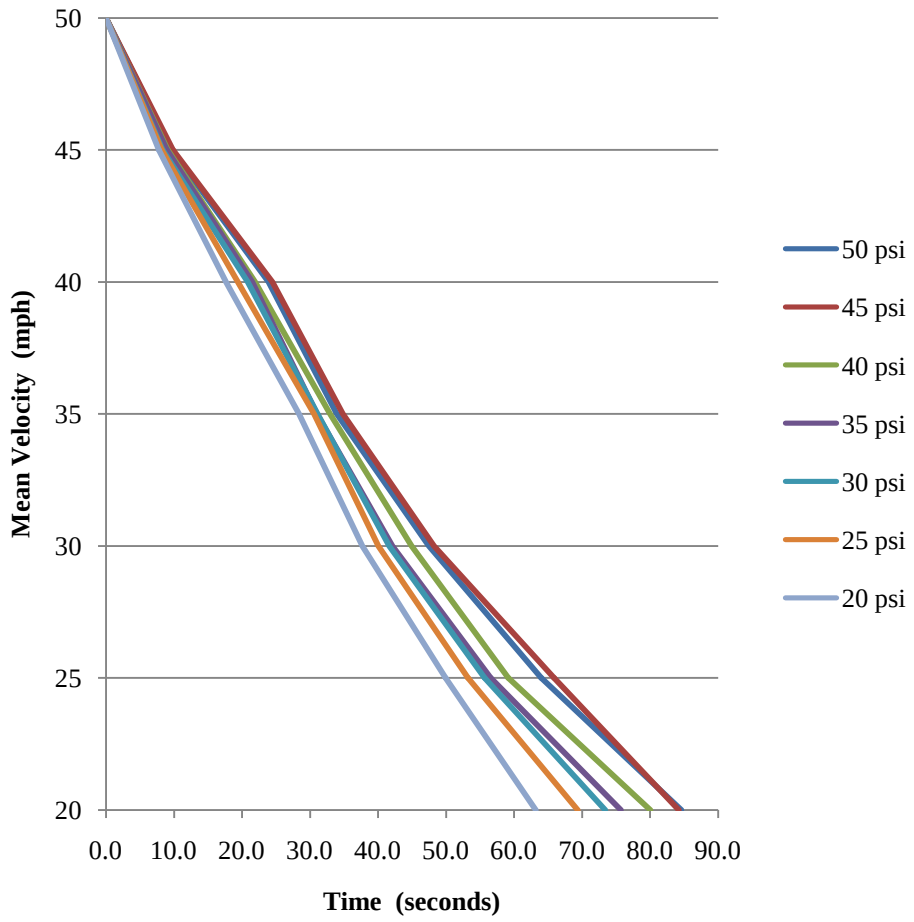

Chart 1: For various tire pressures, the test car was set to coast after reaching 50 mph. Shown above are the average velocities over three trials for each pressure tested.

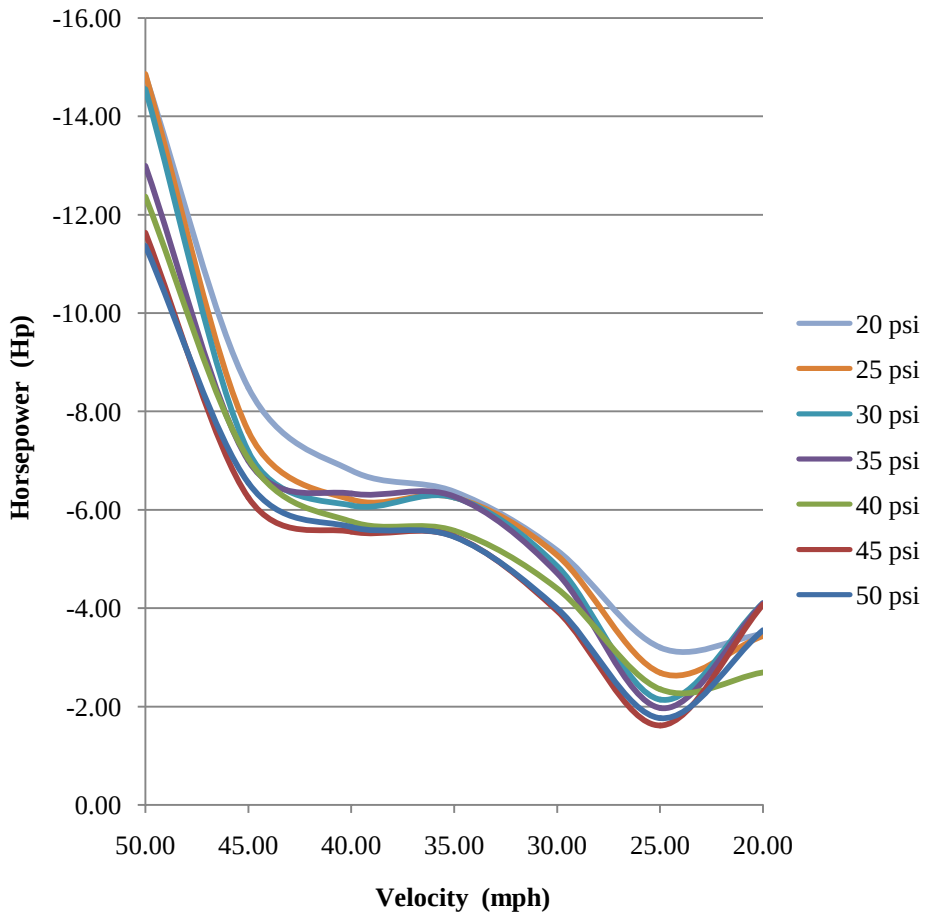

Chart 2: Plot of horsepower vs. velocity for tested tire pressures. Very negative horsepower suggests the test car is slowing quickly. The fluctuations in the downward slope were caused by inclines and declines in the test road.

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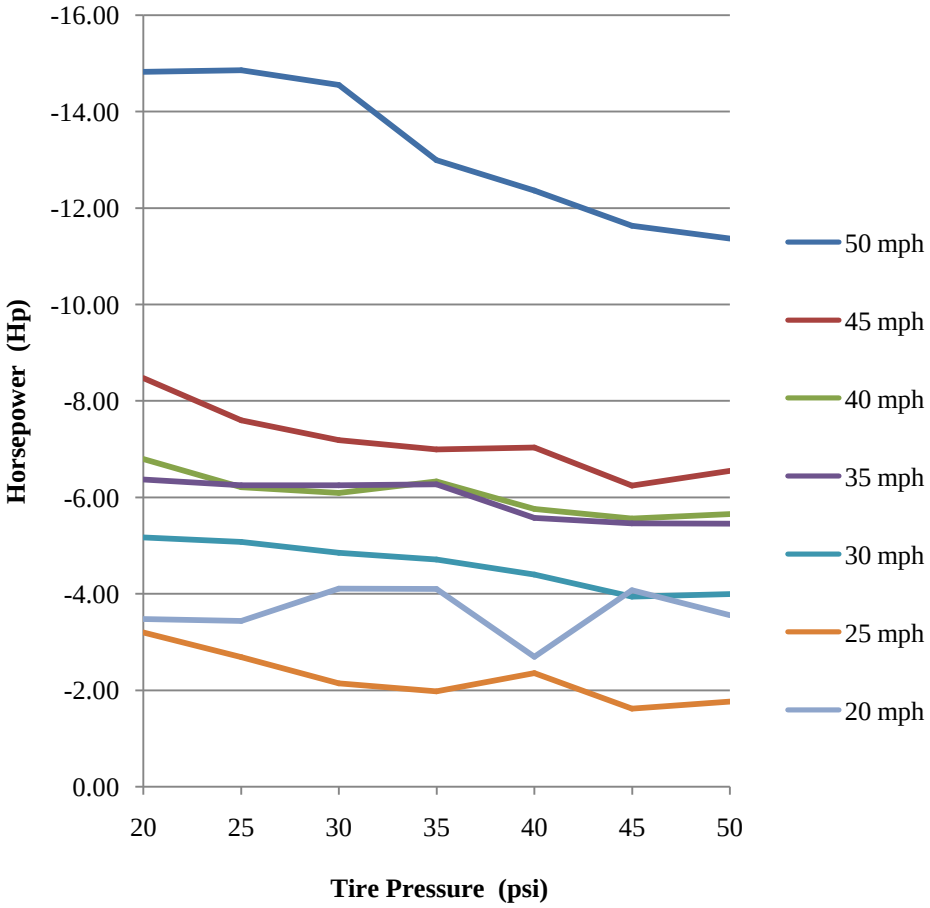

Chart 3: Plot of horsepower vs. tire pressure at each of the velocity thresholds. Very negative horsepower suggests the test car is slowing quickly. In general, faster slowing occurred for lower tire pressures.