11-14-2003

Vibrofluidized Bed Drying Of Citrus Processing Residue For Byproduct Recovery

Eric A. Roe
University of South Florida

Follow this and additional works at: https://scholarcommons.usf.edu/etd

Part of the American Studies Commons

Scholar Commons Citation
https://scholarcommons.usf.edu/etd/1460

This Dissertation is brought to you for free and open access by the Graduate School at Scholar Commons. It has been accepted for inclusion in Graduate Theses and Dissertations by an authorized administrator of Scholar Commons. For more information, please contact scholarcommons@usf.edu.
Vibrofluidized Bed Drying Of Citrus Processing Residue For Byproduct Recovery

by

Eric A. Roe

A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Department of Chemical Engineering
College of Engineering
University of South Florida

Major Professor: Richard A. Gilbert, Ph.D.
William E. Lee, Ph.D
William A. Miller, Ph.D.
Carl J. Biver, Ph.D.
Renee M. Goodrich, Ph.D.

Date of Approval:
November 14, 2003

Keywords: vibrating fluidized bed, fluidized bed, citrus byproducts, animal feed

© Copyright 2003, Eric A. Roe
Frustra fit per plura quod potest fieri per pauciora

“It is futile to do with more what can be done with fewer”

English philosopher and Franciscan monk
William of Ockham
ca.1285-1349

The ideal engineer is a composite … He is not a scientist, he is not a mathematician, he is not a sociologist or a writer; but he may use the knowledge and techniques of any or all of these disciplines in solving engineering problems.

N.W. Dougherty
1955
ACKNOWLEDGMENTS

I would like to express my sincere appreciation to my major professor and friend Dr. Richard Gilbert, for his encouragement, mentoring, support, and the opportunity to work on a project in which I am truly interested. His guidance, friendship, and sense of humor have allowed me to complete this significant milestone.

I would like to thank the members of my committee, Dr. Carl Biver, Dr. Renee Goodrich, Dr. William Lee, and Dr. William Miller, for their input into the project and their time, especially at the end of the project. I would like to sincerely thank Dr. Robert Braddock who chaired the dissertation defense. His level of interest and commitment to the project, and his valuable expertise in the citrus industry, helped to ensure that this research represents a valuable contribution. I would be remiss not to include special thanks to Dr. Scott Campbell who allowed me space in his laboratory during the early phases of this project.

Thank you to my friends and family. Having a great group of people like all of you in my life has made, and will continue to make, life that much richer.

Finally, I want to thank my wife Beth! Without her support and love, I could not have completed this work, and without her editing skills, this dissertation would have been a dozen or so run-on sentences.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vi</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>ix</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>xii</td>
</tr>
<tr>
<td>1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2 LITERATURE REVIEW</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Fluidization</td>
<td>3</td>
</tr>
<tr>
<td>2.1.1 Fundamental Concepts</td>
<td>4</td>
</tr>
<tr>
<td>2.1.2 Pressure Drop</td>
<td>7</td>
</tr>
<tr>
<td>2.1.3 Geldart Classifications</td>
<td>9</td>
</tr>
<tr>
<td>2.1.4 Particle Size Distribution</td>
<td>10</td>
</tr>
<tr>
<td>2.1.5 Agglomeration</td>
<td>12</td>
</tr>
<tr>
<td>2.1.6 Design</td>
<td>16</td>
</tr>
<tr>
<td>2.1.7 Modeling</td>
<td>17</td>
</tr>
<tr>
<td>2.2 Drying</td>
<td>21</td>
</tr>
<tr>
<td>2.2.1 Drying Principles</td>
<td>22</td>
</tr>
<tr>
<td>2.2.2 Drying Equipment</td>
<td>27</td>
</tr>
<tr>
<td>2.2.3 Drying of Foods (Dehydration)</td>
<td>28</td>
</tr>
<tr>
<td>2.3 Fluidized Bed Drying</td>
<td>29</td>
</tr>
<tr>
<td>2.3.1 Advantages and Disadvantages</td>
<td>31</td>
</tr>
<tr>
<td>2.3.2 Vibrofluidized Bed Dryers</td>
<td>32</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table 1 – Characteristics of Selected Dryers 28
Table 2 – Food Industry Applications for Selected Dryers 29
Table 3 – Estimated Annual Economic Impact of the Citrus Industry on Florida’s Economy, Avg. 1995-96 through 1999-00 41
Table 4 – FBD Energy Requirements 56
Table 5 – Thermocouple Calibration Descriptive Statistics 71
Table 6 – VFBD Minimum Vibrofluidization Results 98
Table 7 – Drying Parameters 98
Table 8 – Fluidization Parameters 107
Table 9 – Regression Statistics for Three-Phase Model Validation 109
Table 10 – Regression Data for Determination of Page’s Equation Parameters 110
Table 11 – Regression Statistics for Thin-Layer Model Validation 112
Table 12 – Sieving Data Tabulation 135
Table 13 – Regression Statistics for A/D Calibration 136
Table 14 – Regression Analysis of Variance for the A/D Calibration 136
Table 15 – Regression Statistics for Mass Flow Meter Calibration 137
Table 16 – Regression Statistics for the Differential Pressure Sensor Calibration 138
Table 17 – Inlet Thermocouple Calibration Descriptive Statistics 139
Table 18 – Outlet Thermocouple Calibration Descriptive Statistics 139
Table 19 – Regression Statistics for the Three-Phase Model at 144.7°C 145
Table 20 – Regression Statistics for the Three-Phase Model at 117.7°C 145
Table 21 – Regression Statistics for the Three-Phase Model at 137.7°C 145
Table 22 – Regression Statistics for the Three-Phase Model at 152.4°C 146
Table 23 – Regression Statistics for the Three-Phase Model at 104.1°C 146
Table 24 – Regression Statistics for the Thin-Layer Model at 144.7°C 146
Table 25 – Regression Statistics for the Thin-Layer Model at 117.7°C 146
Table 26 – Regression Statistics for the Thin-Layer Model at 137.7°C 147
Table 27 – Regression Statistics for the Thin-Layer Model at 152.4°C 147
Table 28 – Regression Statistics for the Thin-Layer Model at 104.1°C 147
LIST OF FIGURES

Figure 1 – Fluidized Bed Industrial Applications 3
Figure 2 – Fixed and Fluidized Bed Properties 5
Figure 3 – Contacting Regimes 7
Figure 4 – Ideal Pressure Drop – Velocity Curve 9
Figure 5 – Classification of Particles by Geldart 10
Figure 6 - Agglomeration Effects 15
Figure 7 – Transfers During Drying 23
Figure 8 – Transport Processes During Drying 24
Figure 9 – Typical Drying Curves 26
Figure 10 – Classification of Dryers Based on Heat Transfer Mechanism 27
Figure 11 – Standard Fluidized Bed Dryer 30
Figure 12 – Dependence of Drying Rate on Vibrational Acceleration, $A\omega^2/g$ 34
Figure 13 – Economic Structure of the Florida Citrus Industry 39
Figure 14 – Orange Juice Trends 40
Figure 15 – Grapefruit Juice Trends 41
Figure 16 – Citrus Processing Operation 42
Figure 17 – Processing Mass Balance 43
Figure 18 – Feed Mill Operations 46
Figure 19 – Three Types of Standard Citrus Dryers 47
Figure 20 – Fluidized Bed Balance Program 58
Figure 21 – Proposed FBD Feed Mill Payback Period 59
Figure 22 – Schematic of Lab Apparatus 61
Figure 23 – Close-up Photograph of Laboratory Apparatus 63
Figure 24 – Wide Shot of Laboratory Apparatus 64
Figure 25 – Digital Video Acquisition of VFBD Experiment 65
Figure 26 – LabVIEW Virtual Interface 67
Figure 27 – LabVIEW Graphic Representation of Data Acquisition Program 68
Figure 28 – Fluidized Bed Drying Schematic 79
Figure 29 – Logarithmic Plot of Final P.S.D. 92
Figure 30 – Bar Plot of Final P.S.D. 93
Figure 31 – Final P.S.D. Cumulative Undersize Distribution by Mass 94
Figure 32 – Fluidization Curve for VFBD Variant #1 96
Figure 33 – Fluidization Curve for VFBD Variant #2 96
Figure 34 – Fluidization Curve for VFBD Variant #3 97
Figure 35 – Fluidization Curve for VFBD Variant #4 97
Figure 36 – Experimental Drying Curves 99
Figure 37 – Drying Trial 1 101
Figure 38 – Drying Trial 2 102
Figure 39 – Drying Trial 3 103
Figure 40 – Drying Trial 4 104
Figure 41 – Drying Trial 5 105
Figure 42 – Three-Phase Model Predicted and Experimental Drying Curves 108
Figure 43 – Three-Phase Drying Model Validation 109
Figure 44 – Thin-Layer Model Predicted and Experimental Drying Curves 111
Figure 45 – Thin-Layer Drying Model Validation 112
Figure 46 – Schematic of Lab Apparatus 127
Figure 47 – Photograph of 1st Generation FBD 128
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\omega^2/g}$</td>
<td>Vibrational acceleration ratio</td>
<td>non-dim</td>
</tr>
<tr>
<td>A</td>
<td>Amplitude of the Vibration</td>
<td>mm</td>
</tr>
<tr>
<td>C</td>
<td>Moisture Content</td>
<td>%</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific Heat of the Fluid at Constant P</td>
<td>cal/g°C</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Nominal Sieve Aperture Size of the $i^{th}$ Sieve</td>
<td>mm</td>
</tr>
<tr>
<td>$d_{gw}$</td>
<td>Geometric Mean Diameter of Particles by Mass</td>
<td>mm</td>
</tr>
<tr>
<td>$d_p$</td>
<td>Particle Diameter</td>
<td>cm</td>
</tr>
<tr>
<td>D</td>
<td>Bed Diameter</td>
<td>cm</td>
</tr>
<tr>
<td>$D_{eff}$</td>
<td>Effective Diffusivity</td>
<td>cm²/s</td>
</tr>
<tr>
<td>$D_{ab}$</td>
<td>Binary Diffusivity for System AB</td>
<td>cm²/s</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational Acceleration</td>
<td>cm/s²</td>
</tr>
<tr>
<td>G</td>
<td>Mass Velocity of the Fluid (Air)</td>
<td>g/cm²/s</td>
</tr>
<tr>
<td>$h_o$</td>
<td>Heat Transfer Coefficient</td>
<td>W/cm²°C</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>H</td>
<td>Bed Height</td>
<td>cm</td>
</tr>
<tr>
<td>j_D</td>
<td>Chilton - Coburn j-factor</td>
<td>non-dim</td>
</tr>
<tr>
<td>k_c</td>
<td>Mass Transfer Coefficient</td>
<td>cm/s</td>
</tr>
<tr>
<td>k_f</td>
<td>Thermal Conductivity of Film at Mean Temperature</td>
<td>W/cm°C</td>
</tr>
<tr>
<td>MR</td>
<td>Moisture Ratio</td>
<td>non-dim</td>
</tr>
<tr>
<td>M*</td>
<td>Unaccomplished Moisture Change</td>
<td>non-dim</td>
</tr>
<tr>
<td>M</td>
<td>Moisture Content</td>
<td>%</td>
</tr>
<tr>
<td>n</td>
<td>Number of Sieves +1 (pan)</td>
<td>non-dim</td>
</tr>
<tr>
<td>P_i</td>
<td>Percentage by Mass of Particles on i^{th} sieve</td>
<td>%</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds Number</td>
<td>non-dim</td>
</tr>
<tr>
<td>S_{log}</td>
<td>Geometric Std. Dev. of Log-normal Dist. by Mass</td>
<td>non-dim</td>
</tr>
<tr>
<td>S_{gw}</td>
<td>Geometric Std. Dev. of Particle Diameter by Mass</td>
<td>mm</td>
</tr>
<tr>
<td>Sc</td>
<td>Schmidt’s Number</td>
<td>non-dim</td>
</tr>
<tr>
<td>Sh</td>
<td>Sherwood’s Number</td>
<td>non-dim</td>
</tr>
<tr>
<td>U</td>
<td>Velocity</td>
<td>cm/s</td>
</tr>
<tr>
<td>u_{t}</td>
<td>Terminal Velocity of Particles</td>
<td>cm/s</td>
</tr>
</tbody>
</table>
\begin{align*}
V & \quad \text{Volumetric Flow Rate} \quad \text{slpm} \\
V_{OM} & \quad \text{Minimum Superficial Velocity for Fluidization} \quad \text{slpm} \\
W_i & \quad \text{Mass on the } i^{th} \text{ Sieve} \quad \text{g} \\
Y & \quad \text{Humidity of the Air} \quad \frac{\text{g water}}{\text{g dry air}} \\
\Delta P_D & \quad \text{Distributor Pressure Drop} \quad \text{Pa} \\
\Delta P_B & \quad \text{Bed Pressure Drop} \quad \text{Pa} \\
\varepsilon_x & \quad \text{Fraction of Bed Occupied by } x \text{ phase} \quad \text{non-dim} \\
\varepsilon & \quad \text{Porosity} \quad \text{non-dim} \\
\phi_s & \quad \text{Spericity of Particle} \quad \text{non-dim} \\
\lambda & \quad \text{Latent Heat of Vaporization} \quad \text{cal/g} \\
\rho & \quad \text{Density of the Fluid (Air)} \quad \text{g/cm}^3 \\
\rho_p & \quad \text{Density of the Particle} \quad \text{g/cm}^3 \\
\sigma_c & \quad \text{Tensile Strength of the Particle} \quad \text{Pa} \\
\mu & \quad \text{Viscosity of the Fluid (Air)} \quad \text{g/cm s} \\
\omega & \quad \text{Angular Frequency} \quad \text{rad/s}
\end{align*}
Subscripts:

b  Bubble Phase

d  Dense Phase

e  Equilibrium conditions

f  Final conditions

g  Gas

i  Initial, Inlet

m  Mean Value

mf  Minimum Fluidization

mvf  Minimum Vibrofluidization

o  Outlet

s  Solid

sat  Saturation

t  Total

v  Vapor

w  Water
Approximately 44% of the citrus that is processed becomes processing residue. The residue consists of the non-juice components of a citrus fruit, primarily peel and pulp, and is recovered by conversion to animal feed. The material is hygroscopic, agglomerating, has a wide particle size distribution, and must be carefully dried to avoid thermal damage to nutrients and flavors. This dissertation evaluates the possibility of utilizing a vibrofluidized bed dryer for citrus processing residue. Results demonstrate that it is possible to overcome the agglomeration difficulties associated with this material, offering an economically viable alternative processing methodology.

To properly analyze this proposed system, a benchtop vibrofluidized bed dryer was designed, constructed and instrumented. Vibrofluidization and batch drying trials were conducted and analyzed. An economic evaluation of the proposed process was undertaken. Two mathematical models of the drying process were developed and validated.
Characteristics that describe the vibrofluidized bed drying of the residue were determined. The conditions that facilitated fluidization were: 1) A particle size distribution of the dried residue that was lognormal, had a geometric mean diameter, $d_{gw}$, of 3.829 mm, and a geometric standard deviation, $S_{gw}$, of $2.49 \times 10^{-07}$ mm. 2) A vibrational acceleration, $A_{\omega^2/g}$, of 2.54. 3) A minimum vibrofluidization velocity, $U_{mfv}$, of 4.2 cm/s. The controlling mechanism of the falling rate period was determined to be diffusion, with an effective diffusion coefficient, $D_{eff}$, of $2.85 \times 10^{-5}$ cm/s, and critical moisture content, $M_c$, of 30%. Economic evaluation of the proposed method has a payback period of 4.34 years, and an estimated processing cost of $33 per ton of dried material.

Models were developed based on bed hydrodynamics and three-phase drying kinetics, and thin-layer drying. Both models accurately predicted the drying curves. The three-phase kinetic drying model solved a series of simultaneous equations, and differential equations, based on moisture and enthalpy balances. This complex model successfully predicted the bed hydrodynamic properties and serves to facilitate scale-up, design, and bed configuration investigations. For the thin-layer drying model, the drying constants, $K$ & $N$, for Page’s equation were determined as a function of bed temperature. This computationally simple, single-parameter model would serve process control algorithms.
1 INTRODUCTION

Evidence of the presence of citrus in Florida dates back as early as 1579, in the region known as St. Augustine. It is believed that the earliest plantings are attributed to the Spaniards, and by 1800, numerous groves had been planted near and around St. Augustine, Tampa Bay, and along the St. Johns River. In 1813, the United States annexed Florida, and the state experienced rapid expansion of citrus cultivation for commercial purposes. By the late 1800’s, Florida was a well-established citrus producing state, bearing record crop sizes and shipping citrus to northern cities.

Present day figures reveal the 2002-03 Florida total orange crop forecast, released by the USDA Agricultural Statistics Board, was 200 million boxes. The two divisions of the forecast are early and midseason at 112 million boxes, and late type (Valencia) at 88 million boxes. In addition, the grapefruit forecast was 39 million boxes. Roughly 90 percent of the total orange crop is processed into juice and the remainder is shipped as fresh fruit. Citrus fruit is comprised of many parts; by weight, the juice only accounts for 50 – 60 percent. All other components must be disposed of; the peel (comprised of the flavedo and albedo), the segment membrane, juice vesicles, seeds, and central core. It is to the processor’s advantage to convert this material into valuable by-products. The cost of converting the processing residue into animal feed in a typical feed mill of
a citrus processing operation is approximately $40.00 to $65.00/ton of dried pellets and the market price is approximately $40.00/ton\textsuperscript{35,39}. This is the primary driving force for finding an alternative process that is more efficient and reduces production costs.

It is my hypothesis that this residue can be dried in a fluidized bed dryer more efficiently, at a lower cost per ton, and with less damage. This hypothesis is explored systematically in this dissertation. Initially, a literature review is provided. This is followed by a definition of the problem, a research plan, an economic evaluation, experimentation, and process modeling.

The model component of this dissertation begins with determination of the fluidized bed hydrodynamic parameters. This is followed by the development of two models for the fluidized bed drying of the citrus particles: The first is based upon moisture and energy balances, while the second is based upon thin-layer drying. The dissertation concludes with an experimental evaluation of the models followed by results, conclusions and recommendations sections.
2 LITERATURE REVIEW

2.1 Fluidization

In various operations, it is often necessary to contact granular material with a fluid (gas or liquid). The technique, that suspends or fluidizes the granular materials in a vertically rising fluid, is referred to as fluidization. It is a tool with many applications in the chemical, petroleum and food processing operations. Figure 1 presents many of the current industrial applications arranged by dominating mechanisms.

Figure 1 – Fluidized Bed Industrial Applications

Industrial Processes

Physical

- Heat and/or Mass Transfer between gas & particles
  - Solids drying
  - Absorption
  - Cooling
  - Freezing

- Heat and/or Mass Transfer between particle & particle or particle & surface
  - Plastics coating
  - Granulation
  - Mixing of solids
  - Dust filtration

- Heat Transfer between bed & surface
  - Heat treatment of textile fibers, wire, rubber, glass, and metal components
  - Constant temp. baths

Chemical

- Gas/gas reactions in which the solid acts as a catalyst or heat sink
  - Oil cracking, reforming
  - Manufacture of: acrylonitile, polyethylene, chlorinated hydrocarbons

- Gas/solids reactions in which the solids are transformed
  - Coal combustion & gasification
  - Roasting of Ni & Zn sulfides
  - Incineration of solid & liquid waste
  - Decomposition of limestone
2.1.1 Fundamental Concepts

Fluidization is the operation where a bed of particulate solids is made to behave like a liquid by the passage of a fluid (gas or liquid) at a flow rate above a critical value. If a fluid is passed upward through a bed of particles at a low rate, the fluid merely percolates through the void spaces between stationary particles; this is a fixed bed. With an increase in flow rate, the particles move apart and a few move in restricted regions; this is an expanded bed. At even a higher flow rate, a point is reached where all the particles are just suspended by the upward flowing fluid. At this point the frictional force between fluid and particles just counterbalances the weight of the particles, the vertical component of the compressive force due to adjacent particles disappears, and the pressure drop through any section of the bed nearly equals the weight of the fluid and particles in that section. A bed in this state is considered to be just fluidized or at minimum fluidization. At this point, the fluidized bed begins to exhibit liquid-like behavior.

At fluid velocities above the minimum fluidization velocity the bed characteristics vary according to the fluid properties. In liquid-solid systems, the bed expands in a smooth progressive manner. In gas-solid systems, instabilities arise due to bubbling and channeling. Because of these instabilities, the bed does not continue to expand and remains close to its volume at minimum fluidization. Both gas and liquid fluidized beds are considered to be dense-phase fluidized beds as long as the upper surface of the bed remains clearly defined. The liquid-like behavior of fluidized beds is illustrated in Figure 2.
Figure 2 – Fixed and Fluidized Bed Properties
For example, a light object will float on the surface and a heavy object will sink to the bottom of a fluidized bed. Solids will flow from a hole below the surface of the fluidized bed. The pressure difference between two points in a fluidized bed is approximately equal to the static head between these two points. When two fluidized beds are connected, their levels equalize.

These properties allow for various contacting schemes to be devised. These are typically countercurrent, crosscurrent, and solid circulation between multiple beds. In addition, the liquid-like behavior provides for rapid and easy transport, and intimate gas contact. These assets, and the flexibility for configuration, are commonly cited in the recommendation of fluidized beds in industrial applications.

Due to the effect of bed hydrodynamics on heat and mass transfer to and from the particles within a fluidized bed, the quality, or type of, fluidization is also important to consider when designing a fluidized bed process. There are several fluid/solids interaction regimes in fluidized bed processing. These are represented in Figure 3 (numbers correspond): 1) Fixed bed. 2) Fluidized bed at minimum fluidization. 3) Smooth fluidization. 4) Bubbling fluidization. 5) Slugging (axial slugs). 6) Slugging (flat slugs). 7) Turbulent fluidization. 8) Lean phase fluidization with pneumatic transport.
2.1.2 Pressure Drop

The force balance across a fluidized bed dictates that the pressure loss across the bed of particles is equal to the weight of the bed particles per unit area of the bed. Hence: \[ \Delta P = \frac{\text{weight of particles - upthrust on particles}}{\text{cross sectional area of bed}}. \] (1)

A typical way to evaluate a bed’s fluidization state is to track the pressure drop as a function of fluid velocity. An idealized representation of the pressure drop – velocity relationship is presented in Figure 4 22. A well-fluidized bed has a constant pressure drop when the fluid velocity is increased above the minimum fluidization velocity. However, identifying the minimum fluidization velocity may be difficult. Kunni & Levenspiel 42 describe the relationship between velocity and
pressure drop: As the fluid flow increases in a fixed bed, the pressure drop increases proportionally. As the fluid velocity increases further, a maximum pressure drop is observed. In the straight-line region of increasing velocity, below this maximum, the bed is considered to be a fixed bed. This region is described in general by the Ergun equation:

\[
\frac{-\Delta P}{H} = 150 \frac{(1-\varepsilon)^2 \mu U}{\varepsilon^3 \chi_{sv}^2} + 1.75 \frac{(1-\varepsilon) \rho_f U^2}{\varepsilon \chi_{sv}^3}.
\]  

(2)

As the fluid velocity increases above this maximum, the bed achieves fluidization. At this point, the bed expands and the pressure drop remains fairly constant with increasing fluid velocity. The fluidized bed region is described by the following equation:

\[
\Delta P = \frac{HA(1-\varepsilon)(\rho_p - \rho_f)g}{A}
\]  

(3)

where H is the bed height, \( \varepsilon \) is the bed voidage, \( \rho_p \) is the particle density, \( \rho_f \) is the fluid density, and A is the bed cross sectional area. When the pressure drop fluctuates with increasing fluid velocity, a slugging bed is created. A slugging bed is undesirable because the drying fluid does not uniformly contact the material to be dried.\(^{48} \)
2.1.3 Geldart Classifications

In much of the literature, investigations into how particle properties influence fluidization behavior have been undertaken. Geldart\textsuperscript{27,28} developed a way to classify particles and their fluidization behavior. This nomenclature is used throughout fluidization literature. Klinzing\textsuperscript{40} summarizes these classifications as follows: Group B particles contain materials such as sand and glass beads. They have a medium particle density and a size range of 75 to 600 microns. These particles fluidize easily, forming bubbles at, or slightly above, the minimum fluidization velocity. Group C particles are cohesive with strong interparticle forces. Generally, they have a diameter of less than 50 microns. They tend to form plugs or channels during fluidization. Group A particles are generally intermediate in size between groups B and C. They typically show an extended region non-bubbling above the minimum fluidization velocity. These
materials are rather ideal for processing in a fluidization mode. Group D particles are large and dense. They form permeable beds with high minimum fluidization velocities, and tend to spout and channel rather than fluidize. These classifications are represented graphically in the following figure, adapted from the Miyauchi et al.\textsuperscript{47}. The region A\textsuperscript{1} in the figure represents properties desired for well-behaved FCC catalyst.

![Figure 5 – Classification of Particles by Geldart](image)

2.1.4 Particle Size Distribution

A fluidizable particle is granular, and may be monodisperse (all particles of the same size) or polydisperse (a mixture of particle sizes)\textsuperscript{70}. The determination of the particle properties is essential for fully understanding the fluidization process. The particle sizes of a polydisperse material are typically determined by sieving. Sieving has been used since early Egyptian times for the preparation of
foods. It is a particularly useful technique since particles are classified on the basis of size alone, independent of their other properties (density, surface properties, etc.)\(^7\).

Typically, particle size data is presented in histograms, density distributions, and cumulative distributions. The size of particles is reported in terms of geometric mean diameter, \(d_{gw}\), and geometric standard deviation by mass, \(S_{gw}\). The American Society of Agricultural Engineering \(^8\) cites the following calculation formulas, for these descriptors, based upon derivations by Pfost and Headley and Sokhnansanj and Yang:

\[
d_{gw} = \log^{-1} \left[ \frac{\sum (W_i \log d_i)}{\sum W_i} \right] \tag{4}
\]

\[
S_{gw} = \frac{1}{2} d_{gw} \left[ \log^{-1} S_{log} - \left( \log^{-1} S_{log} \right)^{-1} \right] \tag{5}
\]

where,

\[
S_{log} = \left[ \frac{\sum W_i (\log d_i - \log d_{gw})^2}{\sum W_i} \right]^{1/2} \tag{6}
\]

For nonspherical particles, sieving can overestimate the particle diameter, \(d_p\), defined as the diameter of a sphere the same volume as the particle. To account for this, most fluidization work uses a product of particle sphericity and particle diameter, \(\phi_s\) and \(d_p\), for a complete size-shape description of a particle. Particle sphericity is defined as the surface area of a reference sphere having the same volume as the particle divided by the surface area of the particle. Sphericity equals one for sphere, and is between zero and one for other shapes\(^7\).
2.1.5 Agglomeration

Agglomeration is the amassing of particles during processing. It can present both a processing problem or an aid when attempting fluidization. In the former, sticky particles can agglomerate causing a significant increase in the minimum fluidization velocity of a forming bed, or defluidization of a stable bed. In the latter case, binders can be added to the fluidized bed in order to agglomerate fine materials and facilitate fluidization. The tendency to agglomerate depends on the stickiness of the particles (which can be a function of temperature), the available surface area, and the particle momentum. In general the agglomerating tendency, $A_g_p$, is:

\[ A_g_p = \frac{a \cdot b}{c}. \]  

(7)

At present time, there is not a quantitative relationship between these variables, and experiments are required to determine conditions for stable operation$^{29}$.

Passos and Muhumdar$^{52}$ present an investigation into the cohesive forces that develop in the drying of wet particles. Specifically, they discuss the drying of pasty materials. This system becomes complex due to the development of cohesive forces resulting from liquid bridges between particles. These forces affect gas and solids flow leading to uncontrollable agglomeration, defluidization and poor gas-solids contacting. To determine the effects of a viscous fluid on the fluidization of particles, they coated glass beads and plastic pellets with varying levels of glycerol, and observed the results. They performed their experiments in both a fluidized bed and a spouted bed. The experimental results are presented as both the pressure drop as a function of the air velocity, and the bed voidage
associated with different coating levels. This data demonstrates the difficulties associated with fluidizing sticky particles. At a lower liquid content, the bed of wet particles expands which hinders incipient fluidization; the minimum fluidization velocity is increased. At high liquid contents, the bed of wet particles contract and particles agglomerate even at high gas flows.

In a similar fashion, McLaughlin and Rhodes \(^{46}\) investigated agglomeration by studying the effects of the addition of non-volatile liquids with different viscosities and surface tension values to a gas-solid fluidized bed. Experimentally, the authors used Geldart group B particles at ambient temperature to avoid the effects of temperature on the particles and the liquids tested. These particles have low interparticle forces and typically bubbling of the fluidizing gas occurs at the minimum fluidization velocity. The authors also monitored bed behavior in order to classify the particle as either exhibiting Geldart group B, A, or C characteristics. Group B particles are described above, while group A particles can achieve non-bubbling fluidization, and group C particles exhibit cohesive powder characteristics with bed defluidization occurring as cracks and channels form in the bed.

Additionally, McLaughlin and Rhodes modeled their process and developed a total interparticle force term, \(F_{IP}\), that is a combination of the viscous force and the surface tension effects. They use this interparticle force, in a ratio with the drag force on the particle, to plot versus liquid addition and the particle’s group classification. This data is then evaluated to develop a criterion for bed
transition characterization from group B to A to C particles, effectively predicting the onset of bed defluidization. Figure 6 summarizes some of these results.

From the two plots provided, one can see that the transition from B to A, and A to C group classifications corresponds to ratios of interparticle force to fluid drag force of 0.06 and 1.07 respectively for the top figure, and 0.02 and 0.7 for the lower figure. The authors then state that the transitions from B to A occur at ratios between 0.02 and 0.06, and A to C occur between ratios of 0.7 and 1.07, and that the Geldart group transitions occur at fixed values in these ranges. Looking at the simplified force ratio plot in the lower portion of Figure 6, it may be of more use to fit equations to the data curves. This will allow the behavior transition information to be easily related to the ratio of free liquid to solids.
where • - Geldart group B, □ - group C, Δ - group A, ■ - group A/C

Figure 6 – Agglomeration Effects
2.1.6 Design

Critical information needed for the design of a fluidized bed for physical operations, such as heat and mass transfer and drying, includes\textsuperscript{42}: 1) The drying rate of the material. 2) The tendency of the solids to agglomerate, break or erode. 3) The tendency of the solids to coat the wall surfaces of the bed. 4) The effective particle diameter or particle size distribution. 5) The effective bubble diameter expected in the bed. 6) Properties of the exit gas stream relating to possible combustion.

Items 1 to 4 are usually determined in the laboratory utilizing benchtop experiments. Item 5 is most often approximated using an empirical relation, such as the one presented in the model development section. Item 6 must be considered due to the probable existence of fine solids in the exit gas stream. Where fines flow in a system, static charges may build and discharge causing a dust explosion. After considering the above items, design of the fluidized bed can begin.

The primary factors influencing the quality of fluidization are the distributor plate and the bed geometry. Whitehead \textsuperscript{72} indicates that the understanding of the effects resulting from the bed-distributor interactions is essential for design and operation. For good quality fluidization, the gas needs to be uniformly distributed across the entire bed cross section. The distributor must accomplish this and support the weight of the bed during start-up and shut-down, minimize the aeration of the bed material, not plug or foul during long periods of operation,
and prevent fine particles from falling into the plenum beneath the distributor.

Sufficient pressure drop across the distributor is required to achieve equal distribution of the gas flow over the entire distributor. Agarwal et al. believe, along with others, that the critical distributor design criteria is the ratio of the pressure drop across the distributor to the pressure drop across the bed. They recommend that the ratio of pressure drops should be 0.1 to 0.3, with agglomerating and hard to fluidize materials at the higher end of the range. This agrees with the ranges presented in Whitehead, which surveys the finding of several other investigators. Rather than use the low-end ratio, 0.3, for agglomerating materials, Qureshi et al. suggest a minimal value of 0.01 and offer an equation for determining an ideal ratio of pressures. This equation relates the bed diameter and height to the ratio of the distributor pressure drop, \( \Delta P_D \), to the bed pressure drop, \( \Delta P_B \), by the following equation:

\[
\frac{\Delta P_D}{\Delta P_B} = 0.01 + 0.2 \left[ 1 - \exp \left( \frac{-D}{2H} \right) \right]
\]  

where \( D \) is the bed diameter and \( H \) is the bed height. A direct relation exists between the bed pressure drop and the bed height. To predict the bed pressure drop, \( \Delta P_B \), Kuni and Levenspiel proposed the following relation:

\[
\Delta P_B = H(1 - \varepsilon_{mf})(\rho_p - \rho_f)(g)
\]  

2.1.7 Modeling

Essential velocity information, required for modeling fluidization, includes the terminal velocity, the minimum fluidization velocity, the bubble velocity and
the superficial velocity for the fluidized bed. For fluidization situations that include large and irregular particles, such as orange peel particles, these operation parameters are best determined by experimentation, but can be approximated with the following set of equations:

The terminal velocity, \( u_t \), is given by Newton’s law as

\[
    u_t = \frac{g d_p^2 (\rho_p - \rho)}{18 \mu}.
\]

(10)

The minimum fluidization velocity, \( V_{OM} \), can be obtained from

\[
    V_{OM} = \left[ \phi_s d_p g (\rho_p - \rho) \varepsilon^3 / 1.75 \rho \right]^{1/2}.
\]

(11)

However, it is often more convenient to use the terminal to minimum fluidization velocity ratio given as

\[
    \frac{u_t}{V_{mf}} = 1.75 \left[ g d_p (\rho_p - \rho) / \rho \right]^{1/2} \left[ 1.75 \rho / (g d_p (\rho_p - \rho) \varepsilon^3) \right]^{1/2}
\]

(12)

where \( u_t = 2.32 V_{mf} / \varepsilon^{1.5} \).

(13)

Determination of the remaining two velocity values depends on the operational conditions. If bubbling fluidization is assumed, then the expansion of the bed comes mainly from the space occupied by the gas bubbles. Under these conditions, the following approximations may be made to determine the bubble velocity, \( u_b \), and the superficial velocity, \( U \),

\[
    u_b \approx 0.7 (g d_b)^{1/2}
\]

(14)

\[
    U = \varepsilon_b u_b + (1 - \varepsilon_b) V_{mf}
\]

(15)

given the fraction of the bed occupied by bubbles, \( \varepsilon_b \).

If a force balance approach is taken in determining the minimum fluidization velocity, the agglomerating characteristics of the material must be incorporated into the fluidization force balance. To address agglomeration in a
force balance, Passos & Mujumdar\textsuperscript{52} account for the interparticle forces generated by the addition of the liquid binder. The authors apply a momentum balance to the bed structure and estimate the average tensile strength by developing a stress term, $\sigma_c$, associated with the interparticle cohesion, which is a function of the pressure drop across the bed, $\Delta P$, and the bed height at minimum fluidization, $H_{mf}$,

$$\sigma_c = K_{MAX} \frac{\Delta P'L}{2H_{mf}}$$  \hspace{1cm} (16)

where $K_{MAX} = \left[1 - \exp(-2\tan\phi_w D/H_{mf}/L)\right] \cdot \left(1 + \sin\phi_w \cdot (1 + D) \cdot \tan\phi_w \cdot D\right)$. \hspace{1cm} (17)

This stress term is a function of the particle properties, the interaction between the particle and the bed wall, the fluid flow, bed geometry, and the bed dimensions. It is then used to calculate an interparticle force, $F_H$,

$$F_H = \frac{8(\pi d_p^2 / \phi)}{9(1 - \varepsilon)N} \sigma_c$$  \hspace{1cm} (18)

which can be used in the particle force balance necessary for a fluidization model, where $N$ represents the average number of contact points between a particle and its neighbors, $\phi$ is the sphericity of the particle, $d_p$ is the particle diameter, and $\varepsilon$ is the bed void fraction. The authors compared their interparticle force equation with a published one for capillary binding force, $F_c$. They found that the $F_H$ equation produces a force value of the same magnitude as $F_c$ with fewer parameters.
Passos and Mujumdar’s interparticle force equation could help in the development of a model for the fluidization of the citrus particles, based upon force balances. The basic model would be developed by applying force balances on the particle, related to the fluid flow and gravitational effects. The inclusion of this interparticle force will add a factor to the model not previously considered, and will most likely assist in determining an accurate minimum fluidization velocity.

As mentioned in the section 2.1.5, McLaughlin and Rhodes\textsuperscript{46} address agglomeration by using a total interparticle force term, \( F_{IP} \), which is a combination of the viscous force and the surface tension force,

\[
F_{IP} = F_{V} + F_{S}. \tag{19}
\]

The viscous force term, \( F_{V} \), accounts for liquid bridging between the particles, and is developed by using the particle diameter, the liquid viscosity, the contact angle between particles and the characteristic frequency for particle oscillation,

\[
F_{V} = (3/8)\pi d^2 \mu \omega \sin^2 \theta. \tag{20}
\]

The surface tension term, \( F_{S} \), was estimated by

\[
F_{S} = \pi d \gamma \sin^2 \theta \tag{21}
\]

where the \( \gamma \) is the liquid surface tension, \( d \) is the particle diameter, and \( \theta \) is the particle contact angle. The total interparticle force is the sum of these two components and is represented as

\[
F_{IP} = (3/8)\pi d^2 \mu \omega \sin^2 \theta + \pi d \gamma \sin^2 \theta. \tag{22}
\]
The determination of the characteristic frequency and the contact angle are critical for this interparticle force approximation to be of use. In McLaughlin and Rhodes case, the contact angle was measured by using scanning electron microscopy on a sample of particles from the bed. The characteristic frequency of particle oscillation was not measured, but was approximated by measuring the oscillation frequency of bed pressure drop. The assumption is that the microscopic motions that make up the particle oscillation frequency can be approximated by the bulk particle motion effect on the bed pressure drop.

For large particulate fluidization, the determination of the interparticle force terms of the force components ($F_v$ & $F_s$), the approximations are too limited to be of value in the fluidization model. The contact angle measurement is taken at one point in time when the particles are at rest, and not being fluidized. The contact angle will change according to bed behavior, thus changing the interparticle force term. In addition, the approximation of the microscopic particle characteristic oscillation frequency, by the bed pressure drop frequency, for powders is weak. For large particles, van der Walls forces will most likely contribute strongly to the oscillation frequency, but will not be expressed in the pressure drop at the macroscopic scale of the bed.

2.2 Drying

Drying consists of a unit operation in which a liquid, typically water, is removed from a material in equipment termed dryers, it is traditionally defined as the unit operation that converts a liquid, solid, or semi-solid feed material into a
solid product of significantly lower moisture content \(^{11}\). The use of heat to remove liquid distinguishes drying from mechanical drying methods such as centrifugation, decantation, pressing, or sedimentation \(^{44}\). Drying is a process of simultaneous heat and mass transfer. Heat is supplied to the material to facilitate the evaporation of moisture; subsequently, the moisture is removed from the material into the drying medium. In dried citrus pulp production, the Florida citrus industry feed mill currently uses pressing and evaporation of the press liquid, followed by air-drying.

2.2.1 Drying Principles

For the majority of industrial drying processes, preheated air is used as the drying agent. This air-water vapor mixture transmits heat to the material surface via convection, and then by conduction to the interior of the material. In the opposite direction, moisture is simultaneously removed from the material. As a liquid, it moves from the inside of the material to the surface, and then evaporates by convection to the drying medium \(^{65}\). This process is represented in Figure 7.
The ability of air to remove moisture from a material depends upon the temperature and the amount of water vapor already contained in the air stream. The content of water vapor contained in the air stream is expressed as either absolute humidity (the mass of water vapor per unit mass of dry air), or relative humidity (the ratio of the partial pressure of water vapor in the air, to the partial pressure of saturated water vapor at the same temperature, multiplied by one-hundred)\textsuperscript{25}. The temperature of air can be determined using either a dry-b bulb or wet-b bulb thermometer. Dry-b bulb temperature is measured using a standard thermometer. A thermometer whose bulb is covered with a wet cloth measures wet-b bulb temperature. Heat is removed from the thermometer bulb as the water
in the cloth evaporates. The difference between these two temperatures is used to calculate the relative humidity of the air.

Typically, the formulation of a drying model is complex, as three types of transport exist for the liquid and vapor, presented in Figure 8 (numbers correspond): 1) Transport of liquid within the solid. 2) Evaporation of liquid from the surface of the solid. 3) Transport of vapor away from the solid. One, or a combination of the following mechanisms, controls the transport of liquid within the solid: Capillary flow, liquid diffusion, vapor diffusion, and/or viscous flow. The evaporation process is influenced by the particle surface area and local pressure environment. The transport of vapor away from the solid is affected by the gas flow and it’s path of travel. Determining the dominant mechanisms is critical for model development.
Drying data is typically presented as a pair of standard drying curves. The first curve plots moisture content versus drying time; the second curve plots drying rate versus moisture content. The data for determining representative curves is usually obtained under laboratory conditions by measuring the mass and temperature change of a material sample with time. The explanation of the shape of the drying curve is closely related to the mass and heat transfer operations within the system.

Typical representations of these convective drying curves are presented in Figure 9. Using these curves, the drying process can be described as a series of steps in which the drying rate plays a key role. The period from point A to B represents the warm up period for the product. Point B represents the equilibrium temperature of the product surface. After this warm up period, the curve takes on a linear characteristic; this period from point B to C is known as the constant rate period. During this period, the free water on the surface of the product is removed. This period is characterized by a constant drying rate, and lasts only as long as the water is supplied to surface as fast as it is evaporated away. The period from point C to D is known as the falling rate period. Here the drying rate starts to decrease, as the rate of drying is governed by the transport of water from the interior to the surface of the product. Finally, the period from D to E represents the second falling rate period, where the surface is completely dry and the plane of evaporation recedes from the surface. Point C is often referred to as the critical moisture content, $M_c$, and point E is often referred to as the equilibrium moisture content, $M_e$. 
Figure 9 – Typical Drying Curves
2.2.2 Drying Equipment

Dryers are often classified according to the method by which heat is transferred to the wet solid. The heat required for drying may be supplied by convection (direct), conduction (indirect), and/or radiation. Both direct and indirect dryers are used industrially. Direct dryers use hot gas, typically air, to contact the material. The vaporized liquid is transported away in the heating medium. In indirect dryers, heat is conducted into the material by the hot metal walls of the dryer, and through particle contact. The vaporized liquid is removed independently from the heating medium. Perry presents a classification system for industrial dryers based on the method of heat transfer, which is adapted in Figure 10.

![Figure 10 – Classification of Dryers Based on Heat Transfer Mechanism](image)

Engineering characteristics for the most common types of dryers are summarized in Table 1.
Table 1 - Characteristics of Selected Dryers

<table>
<thead>
<tr>
<th>Dryer Type</th>
<th>Evaporation Capacity (kgw/m²h) (kgw/m³h)</th>
<th>Energy Consumption (kJ/kgw)</th>
<th>Thermal Efficiency (%)</th>
<th>Residence Time (s, min, h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tray or Cabinet</td>
<td>0.1-1</td>
<td>3000-4500</td>
<td>50-80</td>
<td>2-24 h</td>
</tr>
<tr>
<td>Tunnel &amp; Conveyor</td>
<td>5-18</td>
<td>4000-6000</td>
<td>35-60</td>
<td>10-180 m</td>
</tr>
<tr>
<td>Rotary</td>
<td>30-120*</td>
<td>3500-6000</td>
<td>40-70</td>
<td>10-60 m</td>
</tr>
<tr>
<td>Fluidized Bed</td>
<td>30-90</td>
<td>3100-6000</td>
<td>40-80</td>
<td>5-30 m</td>
</tr>
<tr>
<td>Pneumatic</td>
<td>10-100*</td>
<td>3500-5000</td>
<td>50-75</td>
<td>2-15 s</td>
</tr>
<tr>
<td>Spray</td>
<td>1-30*</td>
<td>4000-5000</td>
<td>50-60</td>
<td>5-120 s</td>
</tr>
<tr>
<td>Drum</td>
<td>4-30</td>
<td>3000-3500</td>
<td>70-85</td>
<td>10-30 s</td>
</tr>
<tr>
<td>Vacuum &amp; Freeze</td>
<td>1-7</td>
<td>&gt;7500</td>
<td>-</td>
<td>1-24 h</td>
</tr>
</tbody>
</table>

Mujumdar and Menon 50 provide detailed classification schemes for industrial dryers with criteria necessary for appropriate selection. At the minimum, the following quantitative information is required to arrive at a suitable dryer selection41: 1) Dryer throughput and mode of production (batch/continuous). 2) Properties and variability of the wet feed and desired final product specifications. 3) Upstream and downstream processing operations. 4) Drying kinetics. 5) Quality parameters. 6) Safety aspects, such as fire and explosion hazards. 7) Value of the product. 8) Flexibility in capacity requirements. 9) Type and cost of fuel and electricity.

2.2.3 Drying of Foods (Dehydration)

The drying of foods is often referred to as dehydration. This differentiation is due to the fact that dehydration usually implies the removal of water, accompanied by a chemical change, which typically occurs in food drying 44. There are many reasons to dry foods and foremost among these is preservation. Dried foods can be stored for long periods of time due to their low water activity. The microorganisms that cause spoilage and decay are unable to grow and
multiply in the absence of sufficient water, and many of the enzymes that cause undesirable reactions to occur in foods cannot function without water. In addition to the increased stability of the food, drying causes a significant reduction in the weight and volume of the material. This contributes to reduced costs of packaging, handling, storing and distributing the foodstuffs.

Dryer selection is based upon the raw material properties, specifications for the final product, and dryer characteristics. Food product applications for the most common types of dryers are presented in Table 2.

Table 2 – Food Industry Applications for Selected Dryers

<table>
<thead>
<tr>
<th>Dryer Type</th>
<th>Product Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tray or Cabinet</td>
<td>Fruits, vegetables, meats, confectioneries</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Fruits, vegetables</td>
</tr>
<tr>
<td>Belt Conveyor</td>
<td>Grains, fruits, vegetables, cereals, nuts</td>
</tr>
<tr>
<td>Rotary</td>
<td>Seeds, grains, starch, sugar crystals</td>
</tr>
<tr>
<td>Pneumatic or Flash</td>
<td>Starch, pulps, crops, granules, powders</td>
</tr>
<tr>
<td>Fluidized Bed</td>
<td>Vegetables, granules, grains, peas</td>
</tr>
<tr>
<td>Spray</td>
<td>Milk, cream, coffee, tea, juices, eggs, extracts, syrups</td>
</tr>
<tr>
<td>Drum</td>
<td>Milk, soups, flakes, baby cereals, juices, purees</td>
</tr>
<tr>
<td>Foam Mat</td>
<td>Fruit juices and purees</td>
</tr>
<tr>
<td>Puffing</td>
<td>Fruits, vegetables</td>
</tr>
<tr>
<td>Freeze</td>
<td>Flakes, juices, meat, shrimp, coffee, vegetables, extracts.</td>
</tr>
</tbody>
</table>

2.3 Fluidized Bed Drying

Dryers, in which the drying gas fluidizes the solids, are known as fluidized bed, or fluid-bed, dryers (FBD). Figure 11 represents a typical continuous fluidized bed dryer. The process has been used industrially since 1948, and today is one of the most common types of dryers used in industry to produce dry particulate products such as polymers, fertilizers, pharmaceuticals, sand,
crushed minerals, and crystalline materials. The primary reasons for its popularity are due to its simple construction and low maintenance costs \(^{10}\).

The two main categories of fluidized bed dryers are batch and continuous. Batch FBDs are normally used when the production scale is small, and diverse products need to be run on the same production line \(^{59}\). They have superceded tray dryers as the most economic method of drying powders \(^{10}\). Continuous, or "well-mixed", FBDs facilitate drying of larger production volumes than batch dryers. They are considered well-mixed because the particle residence time approaches the perfect mixing law. Because of this near perfect mixing, the bed
has a uniform composition and temperature equal to the temperature of the outlet product and exhaust gas streams. Hence, the wet feed falls into a bed of almost dry particles, facilitating the processing of wetter feedstocks than possible in a batch FBD. The main drawback associated with a continuous FBD is that the wide particle residence time distribution leads to a wide range of moisture content in the final product.

2.3.1 Advantages and Disadvantages

Fluidized bed drying of granular fluidizable material gives several advantages over alternative processes: 1) Temperature is uniform throughout the bed so that even product dryness is obtained. 2) There is little chance that sensitive materials will suffer local overheating. 3) The excellent heat transfer coefficient from heating surfaces produces low-cost, minimum surface requirements. 4) The handling of particles is quite gentle compared to other types of dryers. 5) The lack of moving parts keeps reliability high with low maintenance costs. 6) A continuous process coupled with high throughput is possible. 7) The dryer is mounted vertically and saves space; this is especially important at plants where space is limited or land costs are high. 8) No skilled operator is required to operate the dryer.

The main disadvantage associated with fluidized bed dryers is that many materials are difficult to fluidize. Some potential feedstocks are too wet to fluidize, due to excessive surface moisture causing agglomeration complications. Another limitation is encountered when the feedstock has a very wide particle size distribution. In this case, the air velocity required to fluidize the large
particles can cause elutriation (undesired pneumatic transport out of the bed) of small particles \(^{59}\). Some other disadvantages associated with fluidized bed dryers are \(^{23}\): 1) Depending upon the particles, there may be erosion to the pipes and fluidization chamber due to particle abrasion. Should this be the case, more expensive, erosion resistant, materials would be required for equipment fabrication. 2) Elutriation of fines are inevitable. 3) The hydrodynamic features of the bed are complex, and hence modeling and scale-up are difficult. 4) Defluidization may occur if particle agglomeration arises during the drying process.

### 2.3.2 Vibrofluidized Bed Dryers

A vibrofluidized bed, or vibrating fluid bed dryer (VFBD), is typically a plug flow bed with a vibrating distributor plate, or a vibrating fluid bed conveyor. It offers several advantages over a standard FBD because any agglomerates arising in the feed will be kept moving by the vibrations of the distributor until they have dried sufficiently to breakup. Secondly, feeds with a wide particle size distribution can be processed successfully in this type of bed. The air velocity can be set low enough to avoid excessive elutriation of the smaller particles, while the largest particles are kept moving by the vibration. Finally, these beds are frequently used with feeds of entirely large particles, and with a minimum fluidization velocity greater than 1 m/s. Often beds of these types of particles must be operated with excessive air velocities, which are greater than required to satisfy mass and heat transfer considerations. The use of the VFBD allows the
air velocity to be kept in the vicinity of the minimum, with consequent savings in capital and operational costs\textsuperscript{29}.

The non-dimensional ratio, vibration acceleration, $A\omega^2/g$, where $A$ is the amplitude of vibration, $\omega$ is the angular frequency, and $g$ is acceleration due to gravity, is a key property for describing vibrated and vibrofluidized beds. This property serves as a representation of the mechanical input to the system, and most VFB properties are linked to it. For $A\omega^2/g < 1$, particles do not jump, they just slide against each other, reducing interparticle friction and bed voidage. In this range of $A\omega^2/g$, vibration is used for the compaction of powders. Upon increased vibration acceleration, a point is reached where the normal vertical force reaches zero and the bed loses contact with the supporting plane, but does not change its location. For greater values of $A\omega^2/g$, the bed separates from the plane at greater angles and the flight time is increased.

Many authors report an optimum range of vibration acceleration in which the bed structure is most suitable for drying and the drying rate is greatest. According to Mushtayev et al., Cheveilenko et al., and Osisnskii et al., the best results are obtained for $A\omega^2/g = 2-3$. It is recommended that, at the minimum, $A\omega^2/g$ should be near 1, while at the maximum, $A\omega^2/g$ should be near 6. At a constant air velocity, the influence of $A\omega^2/g$ on the drying rate can be represented as in Figure 12\textsuperscript{51}.
As in a non-vibrated fluidized bed, pressure drop across the vibrofluidized bed, $\Delta P_{vb}$, is a direct function of bed height. It is generally reported that pressure drop is reduced by vibration. The reduction is mainly ascribed to the increased bed voidage of a vibrofluidized bed, in comparison with a fluidized bed. This observation is seldom seen at low vibrational acceleration ($A\omega^2/g < 1$). Gupta and Mujumdar proposed the following correlation between fluidized bed pressure drop and vibrofluidized bed pressure drop as a function of the particle size, $d_p$, bed height, $H$, the vibrational acceleration, $A\omega^2/g$, and the particle sphericity, $\phi$:

$$\Delta P_{vb} = \Delta P_p \left[ 1 - 0.0935 \left( \frac{d_p}{H} \right)^{0.946} \left( \frac{A\omega^2}{g} \right)^{0.606} \phi^{1.837} \right].$$  \hspace{1cm} (23)
In this correlation, the vibrated fluidized bed pressure drop relates to the upper plateau of the vibrofluidization curve. The equation reportedly holds for frequencies higher than a threshold frequency of 4 to 6.4 hertz.

2.3.3 Modeling

Modeling a vibrofluidized bed dryer is a challenge that requires the combination of all the components presented thus far: 1) The fluidization equations developed earlier must be coupled with vibrational acceleration correlations. 2) Drying equations based on moisture and enthalpy balances, which correspond to the same parameters, must be derived. 3) Additional parameters must be determined, as needed, to satisfy the relations previously set forth. Principally these are the heat and mass transfer coefficients between the particles and the fluidizing medium.

The mass transfer coefficient, \( k_c \), that represents the mass transfer from the surface of a solid particle falling through a gas, may be determined from considering a single sphere and using the Chilton and Colburn j-factor analogy,

\[
\frac{j_D}{j} = \left( \frac{k_c}{Um} \right) \frac{Sc^{2/3}}{} \tag{24}
\]

in combination with the Dwivedi and Upahyad correlation for fluidized beds \(^{23}\),

\[
\varepsilon j_D = 0.765/Re^{0.82} + 0.365/Re^{0.386} \tag{25}
\]

which is valid over the range \( 0.01 \leq Re \leq 15,000 \), where the Reynolds number is expressed in terms of the superficial velocity, \( U \), the gas density, \( \rho \), the particle diameter, \( d_p \), and the gas viscosity, \( \mu \), \(^{33}\)

\[
Re = \rho Ud_p/\mu. \tag{26}
\]
The heat transfer coefficient, $h_o$, within a fluidized bed dryer can be estimated using heat transfer between a flowing fluid and the surface of a single sphere. The following equation is recommended:

$$h_o = \left[2.0 + 0.60\left(\frac{d_p G}{\mu}\right)^{0.50}\left(\frac{c_p \mu}{k_f}\right)^{1/3}\right]\left(\frac{k_f}{d_p}\right)$$

(27)

with $G = \rho u_t$  

(28)

where $u_t$ is the terminal velocity of the particles.

2.4 The Citrus Industry

The optimal application of the concepts associated with fluidization, drying, and fluidized bed drying to any industrial process requires an understanding of that industry. This section serves to provide that link to the citrus industry. The familiarization begins with a brief overview of history.

2.4.1 History

The original home of citrus fruits is the southeastern and eastern regions of Asia, China, Cochin China and the Malayan Archipelago. The introduction of citrus to Europe dates back to the third century B.C., when Alexander the Great conquered Western Asia. Explorers, soldiers, and crusaders all played a part in spreading citrus fruits throughout the Mediterranean world.

Citrus had its beginnings in Florida about 1579 near St. Augustine. Wherever Spanish settlements arose, citrus plantings were not too far behind. In Florida, citrus was further spread by traveling Indians. By 1800, there were numerous groves planted by the Spaniards and other settlers, around St.
Augustine, Tampa Bay, and along the St. Johns River. Shortly after the United States annexed Florida, settlers rapidly expanded the groves, and growers began shipping fruit commercially by boat to northern cities. The close of the Civil War marked the beginning of rapid development of commercial Florida Citrus. In 1886, the crop reached a volume of over one million 90-pound boxes for the first time.  

The next major development in the citrus industry was frozen concentrate orange juice. Born in wartime, this invention transformed Florida from a state known for its fresh fruit, to the second-largest seller of orange juice in the world. Louis Gardner MacDowell, Cedric Donald Atkins, and Edwin L. Moore were brought together in 1942 when the federal government asked the Florida Citrus Commission to develop a new orange juice product it could transport to troops starving for vitamin C on the battlefields of Europe. Operating from a tiny U.S. government-owned building in Winter Haven, the trio worked for three years to develop the product and ways to produce it. The result of their dedicated research was a process that involved evaporating the water from the juice at 80 degrees Fahrenheit, then returning a small, flavorful dose of fresh juice. The team then chilled the solution, canned it and froze it. By the time a U.S. patent was awarded on Nov. 9, 1948, the war was over and the process was being used successfully in commercial operations in Florida.  

Production of citrus in Florida today approximately equals the total production of all other major fruits in the United States. For the 1996-97 season,
the production of commercial citrus in Florida was approximately 295 million boxes. This figure is inclusive of oranges, grapefruit, tangelos, and limes\textsuperscript{2}.

The huge growth of the citrus industry in Florida cannot be attributed to any single factor. However, a major factor is the increase in technology that has enabled the processing, storage and shipping of fruit and juice to an ever-increasing market. In addition, the development of disease resistant varieties that flourish in the Florida climate, soil conditioned with ever improving agricultural practices, the increasing level of nutritional awareness among consumers, and its distinctive flavor, has ensured that demand for citrus and citrus juices is always high.

### 2.4.2 Economic Impact

Citrus fruits, including oranges, grapefruit, tangelos, tangerines, limes and specialty fruits, are Florida’s largest economic agricultural commodity. Florida is the world’s leading producer of grapefruit, and only second to Brazil in the production of oranges. The state produces over 80\% of the United States’ supply of citrus products. For the 1999-2000 season, the major economic factors associated with the citrus industry were estimated at $9.13 billion in industry output, $4.18 billion in value added, and 89,700 jobs\textsuperscript{34}. The economic structure of the Florida citrus industry is illustrated in Figure 13.

According to the Florida Department of Citrus, the estimated average FOB value of the Florida citrus crop for the 1995-96 thru 1999-2000 seasons was 3,989.5 million dollars. This is inclusive of both fresh and processed oranges, and grapefruit. This value only represents a good estimate due to the fact that
most Florida citrus companies are privately held or divisions within large multinational corporations.

![Economic Structure of the Florida Citrus Industry](image)

Figure 13 – Economic Structure of the Florida Citrus Industry

Estimates for the size of the crop are based on sampling and public information. The Florida Agricultural Statistics Service forecast, for the 2002-03 season, estimates the Florida crop (all oranges) at 197.0 million boxes (1997-98 actual: 244.0 million boxes). This drop of 14.3% in the crop size will drive prices up within the industry. A strong indicator of this is the orange juice (OJ) retail price. A.C. Neilsen reports that at $4.44 a gallon, retail prices are 18% higher than five years ago. Retail sales for both orange and grapefruit juice are relatively stable. Figures 14 & 15 present the juice sales volume and value data graphically. Recently there has been concern expressed about the future of
the Florida citrus industry based upon the recent reduced per capita consumption of orange juice, lower per box orange prices, and increased imports into North America from Brazil. In the same article, Tom Spreen, a University of Florida agricultural economist, is quoted as saying “Projected growth rates in both production and consumption over the next ten years are expected to be lower than those realized over the last ten years.” However, considering the long-term trends in a society where demand for citrus products has been steady or on the rise, the outlook for Florida Citrus growers and processors is still good, but may not be as positive as it was three years ago.

![Orange Juice Retail Sales, U.S.](image)

Figure 14 – Orange Juice Trends
In addition to direct sales for the processed citrus industry the Florida Department of Commerce, Bureau of Economic Analysis and the Florida Department of Citrus compiled the following data (Table 3) evaluating the economic impact that the entire citrus industry has on Florida’s economy.

Table 3 – Estimated Annual Economic Impact of the Citrus Industry on Florida’s Economy, Avg. 1995-96 thru 1999-00

<table>
<thead>
<tr>
<th>Item</th>
<th>Earnings</th>
<th>Output/Sales</th>
<th>Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Millions $</td>
<td></td>
<td>Equiv. Full Time Jobs</td>
</tr>
<tr>
<td>Fresh</td>
<td>337.0</td>
<td>1,064.2</td>
<td>16,705</td>
</tr>
<tr>
<td>Processed</td>
<td>1,923.4</td>
<td>6,821.5</td>
<td>57,201</td>
</tr>
<tr>
<td>TOTAL</td>
<td>2,260.4</td>
<td>7,885.7</td>
<td>73,905</td>
</tr>
</tbody>
</table>
2.4.3 Citrus Processing

In Florida, the majority of the citrus crop goes to processed products rather than for fresh fruit consumption. The typical Florida citrus processing system can be best summarized as a collection of several processing units that comprise the operation. These units are Fruit Receiving, Fruit Processing, Juice Handling, and By-Product Recovery (Figure 16).

In general, Fruit Receiving is responsible for the fruit from the time it arrives at the processing plant in bulk trailers, or field boxes, until it is stored in
the fruit bins with a completed State Inspection tag in place. Fruit Processing draws the fruit from the storage bins and is responsible for final grading and extraction. In Florida, there are two primary types of extractors used, the Brown system (Brown Citrus Systems, Winter Haven, FL) and the FMC system (FMC Citrus Division, Lakeland, FL). Though these two systems are different in their approach to juice extraction, their results are the same. Juice Handling is responsible for taking the freshly extracted juice, processing it and transporting it to storage. By-Product Recovery not only deals with the recovery of valuable by-products, but the conversion of processing residues into a sellable product, which can pose a serious disposal problem. The unit operations contained within, and the interconnectivity of these processing units, is presented in the flowchart (Figure 16). Since by-product recovery is integrated into the processing operations it is not presented as a separate sequence, but is represented as the grey boxes in the flowchart.

2.4.4 Feed Mill Expectations

In the state of Florida, every 1000 boxes (90,000 pounds) of oranges processed generates 40,000 pounds of peel, pulp and seeds (Figure 17).

---

**Figure 17 – Processing Mass Balance**

- 1,000 Boxes of Valencia Oranges (40.9 Metric Tons)
- 18.2 Metric Tons Peel, Pulp, & Seeds
- 22.7 Metric Tons 11.8 degree brix Juice (21,711 L)
- 113.6 Kg Cold Pressed Oil
- If evaporated: 3,130 L of 65 degree brix Concentrate
- 3.2 Kg Orange Essence Oil
- 45.5 Kg Orange Aroma

---
This juice-processing residue became a serious disposal problem as greater and greater quantities of citrus were processed. In order to dispose of this residue and turn it into a sellable product, the industry converts this residue into a by-product used as livestock feed. The current process employed utilizes presses, rotating dryers, and a waste heat evaporator that dries the feedstock from 80% to 10% moisture, creating dried citrus pulp (DCP). The majority of processors further process the dried peel and pulp by pelletizing the material to create citrus pulp pellets (CPP). The average seasonal production of dried pulp and pellets, from Florida citrus processors for the 2001-02 and 2002-03 seasons, was 937,368 tons. DCP and CPP are primarily utilized as livestock feed for cattle and sheep. It is second only to corn as a source of concentrated feed nutrients. It is a good source of calcium with a bulk carbohydrate concentrate that is fairly high in energy, but low in phosphorous and carotene. Today, roughly 90% of the DCP and CPP produced in the state of Florida is exported to Europe via the Netherlands. In 2001, a monitoring program was established for elevated levels of dioxin in DCP and CPP. The source for dioxin in the product has been traced to the addition of the wrong type of lime in the existing feed mill process.

Two main factors to consider in the drying of peel residue are the drying temperature and the final moisture content. Temperature is of importance because as it increases, it has a detrimental effect on the nutritional value of the final dried feed. If the temperature is too high, excessive dark coloration occurs and the feed becomes less palatable. The current industry standard for drying is an exit gas temperature of 150°C with a product temperature of between 75 -
80°C. The final moisture content should be in the range of 10 to 12%. High moisture content feed will have a tendency to mold in storage, and possibly generate sufficient heat for spontaneous combustion.

**2.4.5 Feed Mill Operations**

A feed mill is comprised of many more components than the dryer. Typically, the operation is comprised of the following unit operations: 1) Peel Storage. 2) Lime addition and reaction. 3) Hammer mill. 4) Pressing & molasses concentration. 5) Drying. 6) Pelletizing. 7) Finished product storage. A simplified process diagram of these unit operations is presented in Figure 18. This flow diagram primarily addresses the flow of materials from one unit operation to another. Not included in this diagram are several control schemes that must be incorporated into the process. These include a pH control loop for the lime addition, the furnace temperature, the dryer temperature (furnace exit and recirculated air control), and various pull out belts, screw conveyors, and blower motor systems. In Figure 18, the molasses produced by the evaporator is recycled into the feed stream. This is not always the case and molasses may be recovered from the process as a stand-alone by-product, typically sold as an alcohol fermentation substrate. As a result of the molasses concentration process, oil not removed in the processing operation can be recovered as d-limonene, also a stand-alone by-product.
In the unit operations diagramed in Figure 18, there are several equipment options for many of the steps. A shredder could replace the hammer mill used to reduce the particle size of the peel. The press utilized can be one of various types available. Kesterson \(^{38}\) discusses four types, typical in Florida feed mills: 1) Davenport press. 2) Louisville continuous press. 3) Vincent continuous press. 4) Zenith pulp press. Of these, only the Vincent and Zenith presses are currently in use. All operate on the same principle and mechanically reduce the material liquid content from ~80 to 70% water content. This expressed press liquor is sent to a waste heat evaporator that utilizes the exiting air from the dryer to concentrate it into approximately 40°Brix molasses. This operation helps to shift some of the energy load from the dryer. The three types of dryers used by the citrus industry as presented by Kesterson \(^{38}\) are direct fired rotary, triple pass parallel heat flow, and the rotary steam tube rotary. These are shown
schematically in Figure 19. Currently, the majority of feed mills utilize the rotary
direct fired dryer, very few use the triple pass parallel heat flow, and no steam
tube rotary dryers are in use. Disadvantages to these type of dryers include
excessive kiln temperatures, burning of fines, lower yield, and fire hazard from
burning particles. However, these have somewhat been eliminated by
lowering the temperature of the dryer or employing better process control.

Typically the dryer reduces the moisture content from approximately 70 to
18%. The material is not dried to its final target moisture content of 12%,
because the pellet mills require a slightly higher moisture content to form stable pellets, and the pelletizing and cooling operation further reduce the moisture to approximately 10 – 12%.

Based on conversations with industry researchers initiated in December 2000 \(^{35,39}\), the cost of converting the peel residue into animal feed, in the above-described operation, is either a breakeven or a loss situation. In the worst case, the manufacturing cost is approximately $65.00/ton of dried pellets, while the market price is $40.00/ton. This economic inefficiency was the primary driving force for finding an alternative process that is more efficient and reduces production costs.

### 2.4.6 Feed Mill Regulatory Aspects

The most significant regulations that affect the feed mill processes are environmental. The governing constraint is the Title V regulation adopted by the U.S. Environmental Protection Agency (EPA) in July 1992. This rule requires all “major” sources of regulated air pollutants to apply for, and obtain, a Title V operating permit. The citrus industry has determined that existing feed mills meet the criteria for major sources due to the volatile organic compounds (VOC’s) that exit with the dryer air. The primary VOC exiting feed mills is d-limonene, a volatile terpene hydrocarbon \(^{30}\). The source of this VOC is typically due to inefficient recovery of peel oil in the processing operation and d-limonene in the molasses process \(^{16}\). In addition to VOC’s, Title V dictates that pollutants must also be monitored and controlled in the feed mill operations. These
pollutants include particulate matter, sulfur dioxide, and emissions which include carbon monoxide and nitrogen due to fuel combustion\textsuperscript{19}.

In addition to the Title V program, another air regulatory program that affects citrus feed mills is the prevention of significant deterioration (PSD) new source review requirements. PSD new source review applies to major sources and major modifications. Under this regulation, the implementation of a fluidized bed design would be considered a major modification and would be subject to PSD review.
3 PROBLEM DEFINITION

Present techniques employed for the disposal of citrus processing residue are economically inefficient: They have operational costs higher than, or close to, the saleable value of the product. This research focuses on developing a viable alternative feed mill process. A vibrofluidized bed drying (VFBD) system would replace the presses, evaporator, and rotary kiln dryer currently being utilized. In this process, substantial savings should be realized in energy, operation and maintenance costs. The loss of the additional by-products, molasses and d-limonene, produced in the existing process, will offset some of these savings.

Before a pilot scale VFBD can be designed and demonstrated, drying and fluidization parameters will need to be determined. In earlier work, Roe determined the following drying parameters in a non-fluidized state; the mechanisms of both drying rate periods, and the drying rate, critical moisture content, and diffusion coefficient of the residue. However, the fluidization parameters still need to be determined and the drying parameters verified in the fluidized state. These values will be obtained with a benchtop unit developed for this research and described in section 6.1.1. The additional fluidization parameters are a particle size distribution that will allow fluidization to occur, the minimum fluidization velocity for this particle size distribution, and the exploration of vibrational energy input.
A mathematical understanding of the drying process will be necessary to develop a process that is energy efficient, scaleable, and meets the quality requirements of the product. The model, or models, will need to be rigorous enough to predict the bed hydrodynamic properties, as well as the drying process. Additionally, the models will need to be verified by comparison with experimental data.
4 RESEARCH PLAN

The research for this dissertation focuses on the feasibility, development, modeling, and model verification of a new and state of the art system for the drying of citrus processing residue in a vibrofluidized bed dryer. This research progresses through three phases. Phase 1 involves preliminary economic evaluation and modeling. In Phase 2, a benchtop unit is designed, built, and instrumented, with experiments conducted to verify the drying parameters and determine the fluidization design parameters. In Phase 3, the experimentally determined parameters are used to refine the model, and the model is verified.

The starting point for Phase 1 is the fluidized bed dryer model developed in earlier work by Roe. This three-phase drying model will be expanded to account for vibratory energy input required to overcome agglomeration issues. The economic evaluation will be initiated based upon energy balances and parameter predictions from the model.

In Phase 2, instrumentation is developed, installed, and calibrated, and experiments run with the bench top vibrofluidized bed to determine vibrofluidization and drying parameters. The model predicted fluidization velocity from Phase 1 is used to initiate this experimental phase. Here a particle size that fluidizes, the proper vibrational acceleration, minimum vibratory fluidization
velocity, the controlling mechanisms in the constant and falling rate periods, and the effective diffusion coefficient of the material are determined experimentally.

Finally, in Phase 3, the models are refined and verified, and the economic evaluation is finalized based on the successful operating conditions. The fluidization and drying data from Phase 2 is used to refine the kinetic three-phase model and predict the parameters of a second drying model based on thin-layer drying. Upon completion of model refinement and parameter prediction portion of Phase 3, both models are used to predict the drying rate in both the constant rate and falling rate periods. Drying curves are generated and verified by comparing experimental results with the predicted values. The economic analysis will also be updated based on the results from Phase 2, and used to predict both the payback period associated with the installation of a vibrofluidized bed drying system in a medium sized citrus processing plant and the cost associated with producing the final product.
5 ECONOMIC EVALUATION

5.1 Introduction

The current feed drying system incurs a high cost to the processor. The first requirement for the proposed system is a demonstration of improved economic performance. A good first pass economic comparison is an energy analysis of the two processes. This comparison will be based on the energy required to remove a standard unit of water from the material using rotary drying (the industry standard), and using fluidized bed air drying (the proposed system). Flink\textsuperscript{26} presents a system of equations and assumptions for this method. Energy consumption data on a pilot scale rotary system has been acquired, and will be compared to similar data collected on the laboratory bench top fluidized bed apparatus.

Since the feed mill contains many more unit operations than just the dryer, an economic analysis method that includes the entire process, and considers more than just energy consumption, will be conducted on the proposed system. Candidate methods that could be utilized to evaluate fixed and operating costs include: 1) Discounted Cash-Flow Rate of Return (DCFRR) - includes all of the cash flows over an entire project life and adjusts them to one fixed point in time. 2) Benefit-Cost Analysis - a courser method comparing the capital costs, operating costs, savings, and factors for benefits, such as reduced damage to
the surroundings and increased safety. 3) Payout time with and without interest - a quick cost comparison method that determines the time required to reduce an investment to zero. 4) Return on Original Investment (DuPont’s method) - calculates the percentage relationship of the average annual profit to the original investment without factoring in the time value of money. 5) Net Present Value (NPV) - compares the projects on the basis of present value allowing for the time value of money, since all cash flows are related to a base time before comparisons are made.

5.2 Analysis

Economic analysis was conducted using Benefit-Cost Analysis coupled with payback period (Payout time). The parameters used for the analysis were the Benefit-Cost profile and data collected from the bench top vibrofluidized bed, which was then scaled up for a medium sized citrus processing plant.

Key to this two-prong approach is the cost-savings and benefit-disbenefit profile development. Cost-savings consist of the capital outlay (equipment, design, fees, construction, engineering, working capital), annual expenses (operating costs, maintenance, depreciation), capital savings (salvage value) and annual saving (feed sales). Benefit-disbenefits are comprised of benefits (reduced operating cost, increased safety, Title V issues) and disbenefits (increase in production time, loss of production, increased hazards).

As an example of profile development, consider a medium sized plant that processes 80,000 boxes of fruit/day. A processing plant with that capacity would
generate $1.44 \times 10^6$ Kg of peel, pulp and seeds at 80% moisture to be dried per day. Based on the following experimental results from benchtop vibrofluidized bed trials, drying time of 30 minutes with an average drying temperature of 150˚C, fluidization velocity of 1 m/s in a 7.6 cm diameter bed, heat transfer coefficient of 0.2 W/cm$^2$ ˚C, and mass transfer coefficient of 7.1 cm/sec, the bed would need to accommodate approximately 40m$^3$ of product at one time. Assumptions were made for capital costing purposes which include bed diameter of 3m, air velocity of 2 times the minimum fluidization velocity, bed height of 5.66m, fluidized bed tower height of 18m, material of construction will be 304SS, energy costs $0.06 \text{ kW-hr}$, and additional equipment (in addition to the fluidized bed dryer) consisting of furnace, blower, ducting, cyclone, and controls.

<table>
<thead>
<tr>
<th>Table 4 – FBD Energy Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Plant Capacity (4,000 boxes/hr)</strong></td>
</tr>
<tr>
<td><strong>Unloading time</strong></td>
</tr>
<tr>
<td><strong>Incoming fruit (80,000 boxes/day)</strong></td>
</tr>
<tr>
<td><strong>Peel, rag &amp; seeds (80% moisture)</strong></td>
</tr>
<tr>
<td><strong>Operating Days (Nov. - June)</strong></td>
</tr>
<tr>
<td><strong>Drying Temperature (air exhaust)</strong></td>
</tr>
<tr>
<td><strong>Drying Time</strong></td>
</tr>
<tr>
<td><strong>Fluidization Velocity</strong></td>
</tr>
<tr>
<td><strong>Water removal req’s</strong></td>
</tr>
<tr>
<td><strong>Dry product out (10% moisture)</strong></td>
</tr>
<tr>
<td><strong>Enthalpy of the drying air</strong></td>
</tr>
<tr>
<td><strong>Enthalpy of the drying out</strong></td>
</tr>
<tr>
<td><strong>Enthalpy of the solid in</strong></td>
</tr>
<tr>
<td><strong>Enthalpy of the solid out</strong></td>
</tr>
<tr>
<td><strong>Humid Heat of the Air</strong></td>
</tr>
<tr>
<td><strong>Energy Req’s for water removal (from balances, see Figure 20)</strong></td>
</tr>
<tr>
<td><strong>Energy Req’s for fluidization</strong></td>
</tr>
<tr>
<td><strong>Energy for year</strong></td>
</tr>
<tr>
<td><strong>$/\text{yr for dryer}$</strong></td>
</tr>
<tr>
<td><strong>$/\text{ton of dried product (Dryer only)}$</strong></td>
</tr>
</tbody>
</table>
Cost estimates for energy were based on scaled up equipment and enthalpy balances on the dryer. Table 4 presents summarized assumptions and energy calculations. The energy requirements for water removal were calculated using the enthalpy balance program developed for this dissertation and presented in Figure 20. This program allows the user to specify the feed flow rate, moisture content and temperature, the drying air flow rate and temperature, the exiting product moisture content and temperature, and finally the exit air temperature. For ease in calculation, the heat capacities of the bed constituents and enthalpy of evaporation were taken as constants instead of as functions of temperature. These values change only about 10% over the temperature ranges in this scenario, which was determined to be acceptable. In addition, the balance was conducted such that exit air stream is not necessarily saturated. For example, at the conditions specified in Figure 20, based on physical observations in the laboratory, the predicted exit air is not saturated. This results in a process that is less efficient than it could be. This inefficiency only serves to overestimate the energy consumption, and subsequently the costs, making the economic analysis more conservative.

For the proposed fluidized bed feed mill, these assumptions and calculations result in a realistic payback period of 4.34 years, reducing the initial investment to zero. The annual expenses used to calculate this payback period are presented tabularly and graphically in the ten-year economic analysis presented in Figure 21. The payback period does not reflect the loss of income associated with the sale of molasses and d-limonene as separate by-products.
Figure 20 – Fluidized Bed Balance Program

Air exhaust:
- Temperature: 150°C
- Moisture content: 0.67 kg wtr/kg dry air
- Dry air out: 5992 kg/hr
- Water out: 809206 kg/hr
- Flow rate: 889136 kg/hr
- Enthalpy: 27026281.9 kJ/hr

Solid Product:
- Percent Moisture: 10%
- Solid out: 14370 kg/hr
- Water out: 1597 kg/hr
- Flow rate out: 15967 kg/hr
- Temperature: 80°C
- Enthalpy: 591405 kJ/hr

Wet Feed:
- Flow rate: 71860 kg/hr
- Percent solid: 20%
- Solid content: 14370 kg/hr
- Percent water: 80%
- Water content: 57480 kg/hr
- Temperature: 25°C
- Enthalpy: 6024623 kJ/hr

Air feed:
- Moisture content: 0.005 kg wtr/kg dry air
- Dry air: 809206 kg/hr
- Water in air: 4048 kg/hr
- Temperature: 300°C
- Flow rate: 813251 kg/hr
- Enthalpy of evap: 2114.6 kJ/kg

Physical constants:
- Cp, solid: 0.65 kJ/kgK
- Cp, water: 4.18 kJ/kgK
- Cp, vapor: 1.854 kJ/kgK
- Cp, dry air: 1.043 kJ/kgK

= Cell for Data to be entered into.  = Calculated Result
As an additional economic consideration, the cost per ton of dried feed was calculated from this cost estimate. This is a fairly common industry unit for feed cost. Based upon this scale up and estimates of the support equipment, the estimated product cost for the proposed fluidized bed drying system is $33/ton. The current price for feed is $40/ton, while the present cost associated with producing it, in existing feed mills, can be as much as $65/ton.
6 EXPERIMENTAL METHODS

6.1 Experimental Apparatus

Large portions of the personal energies associated with this study were directed at developing a fully instrumented, effective benchtop vibrofluidized bed dryer. The starting point for development was a fluidized bed previously used for studies related to the defluidization of viscous materials \(^{48}\), and preliminary investigations into fluidized bed drying of citrus pulp \(^{63}\). The fluidized bed was modified for the input of vibrational energy through attachment to a variable speed motor and cam, set-up with a connecting rod that facilitates stroke adjustment, suspension on rubber struts, flexible attach points for instrumentation and air supply, and reinforcement of the distributor plate and calming section of the bed. Two portable air compressors, coupled in parallel, provided the drying and fluidizing air supply. Because the pressure drop through the heat exchanger was too great, the heat exchange coils within the oven were reworked using larger diameter tubing. An isolated chamber for the mass flow meter was constructed upstream of the heat exchanger, to isolate the meter from vibration. It was designed so that the incoming air would not impinge on the sensor, thus, preventing a faulty reading.
6.1.1 Benchtop Vibrofluidized Bed Dryer

The experimental apparatus used for this project is shown schematically in Figure 22 and photographically in Figures 23, 24, and 25.

The main body of the fluidization apparatus contains two sections. The upper section is the freeboard, while the lower chamber is the actual fluidized bed. The dryer had a diameter of 7.6 centimeters and a height of 30.0 centimeters. The material to be fluidized is placed on a 70 mesh screen which is supported by a 10 mesh distributor plate. A calming section was installed under the distributor plate so that the fluidizing gas would be distributed uniformly to the bed. The entrance
of the calming section was filled with lead spheres to assist in air distribution prior to the distributor plate.

Figure 23 provides a good view of the fluidization chamber and the vibrational system. The key elements to observe here are the flexible connections for the instrumentation and the air supply, and the design of the vibratory system. The vibratory system consists of the black rubber suspension system, corner bracing on the bed frame, variable speed motor and adjustable stroke linkage.

In the wide shot presented in Figure 24, the instrument system can be seen. The data acquisition system, as well as any transducers that could be, were isolated from vibration by location off of the table with the VFBD, and in the cabinet to the right. The oven, with the heat exchanger for the drying air, is located under the table. The white tube in the lower right is a portion of the chamber constructed for the mass air flow meter, and located upstream of the oven. The video monitor, in the upper right corner of the picture, was connected to a second video camera located to the right of the VFBD and allowed for remote close-up monitoring of the VFBD during experimental trials.

The view provided in Figure 25 provides a good representation of the digital video acquisition set-up. The camera was a Sony digital video camera collecting video at 30 frames per second. Additionally, notice two changes to the system since Figure 23: 1) The drying air supply to the VFBD is now a braided stainless steel line, to accommodate the motion and high air temperature. 2) The relative humidity sensor has been added and is located at the top of the VFBD.
Figure 23 – Close-up Photograph of Laboratory Apparatus
Figure 24 – Wide Shot of Laboratory Apparatus
Figure 25 – Digital Video Acquisition of VFBD Experiment
The fluidizing gas used was compressed air. The 90 psi air supply was provided by portable air compressors joined in parallel. The air was passed through a de-oiling and drying filter, and then to a regulator to control the quantity delivered to the system. Prior to feeding into the apparatus, the air was passed through a heat exchanger. The heat exchanger was a coil of ⅜ inch tubing contained within a high temperature electric oven. The oven cycle time and element voltage could be adjusted to control the exit air temperature.

6.1.2 Data Acquisition

The Vibrofluidized Bed Dryer (VFBD) was fully instrumented utilizing the following: 1) Transducers - monitoring the variables of interest. 2) Signal conditioning accessories - filtering and optimizing the transducer signals for the input range of the data acquisition card. 3) Data acquisition collection box - the outputs of the signal conditioners are wired and routed to a ribbon cable connected to the data acquisition card. 4) Data acquisition card - an A/D card used to interface the measurement data and the computer. 5) LabVIEW Software - used to control the data acquisition and display.

Air temperature was monitored at the inlet and outlet of the FBD, using type T thermocouples. The differential pressure of the air stream, across the FBD, was measured using a Validyne DP15TL differential pressure transducer with a CD12 transducer indicator, and an Omega PX138 pressure sensor. The air flow was measured at the top of the fluidization chamber with a Kurtz series 410 insertion mass flow element, and a series 155 ADAM mass flow computer. A Dwyer pitot tube and manometer were also placed at the top of the chamber.
From this, a secondary non-interfaced airflow measurement was calculated by using the Air Velocity Calculator supplied with the pitot tube. The exit air relative humidity was measured using an Omega PX138 relative humidity sensor.

Data acquisition was accomplished by using a National Instruments DAQCard-700 PCMCIA card. This card allowed for 8 differential analog input channels, and has a 100 kS/s sample sampling rate, and 12 bit resolution. The card was connected to a Dell Inspiron 7000 laptop computer running LabVIEW 6i software on a Windows 98 operating system.

A LabVIEW virtual interface was written that allowed for the simultaneous collection of the temperature, pressure, relative humidity, and air mass flow data. These data streams were collected simultaneously while being displayed graphically on the computer, and being written to Excel files for later analysis. The LabVIEW virtual interface and visual programming are presented in Figures 26 and 27 respectively.
6.1.3 Instrument Calibration

Prior to experimentation, the data acquisition system was checked for reliability. This entailed calibrating not only the transducers used in data collection, but also the data acquisition system itself. Calibrations conducted in this investigation included the following: 1) Data acquisition system. 2) Mass air
flow meter. 3) Thermocouples. 4) Differential pressure sensors. 5) Relative humidity sensor.

The data acquisition system outlined above, N.I. DAQCard-700 and Dell Inspiron 7000 laptop computer running LabVIEW 6i on a Windows 98 operating system, was checked by sending known signals to the analog input channels and monitoring the recorded signals. The signal provided to the card was delivered by a Sun Equip. Co. D.C. power supply - model PS-303. This allowed for a stable voltage to be supplied to each channel of the card, at voltages in ranges similar to those from the instrumentation transducers. Additionally, an accurate multimeter, Fluke model 23, read the voltage going to the card to verify the signal voltage. A plot of the voltage supplied to the card and the average of the 51 recorded voltages, yielded a straight line with a slope of $1.00\pm0.00$. Regression statistics for the calibration can found in Appendix 4.

In order to ensure that the signal acquired by the data acquisition system was not aliased, trials were conducted to determine the Nyquist frequency of the system. A signal generator was connected to the five differential inputs on the DAQCard. A signal of a known amplitude and frequency was delivered to these inputs simultaneously. The sampling frequency of the system was varied and the resulting data was analyzed via Fourier transform to determine at what point the digital signal no longer represented the analog input. For this system the maximum signal frequency that could be reliably measured to avoid aliasing, the Nyquist frequency, was 50 hertz. The sampling frequency used for data collection in this research was below this maximum at 10 hertz and showed an
error of less than 1% from the Fourier transform analysis on simultaneous data acquisition on all input channels.

The Kurtz mass airflow sensor was calibrated using two different references simultaneously. A rotameter placed in line after the pressure regulator, and a pitot tube placed in the exit of the fluidization chamber, were the two references. The pitot tube was connected to a differential pressure transducer, whose output was referenced to a lookup table to provide the air velocity. To ensure that the calibration was accurate for the temperatures to be used in the dryer, the calibration was conducted at 100°C. The airflow was varied, via the pressure regulator, in intervals of 5 SCFM on the rotameter scale from 0 to 40 SLPM, and then to 8 FPM as indicated by the pitot tube. For each reference reading, the transducer output from the Kurtz flow meter was recorded. After the Kurtz flow meter was calibrated using it’s internal procedures, statistical analysis of the data showed strong agreement between the three devices. The value of the correlation coefficient, $R^2$, for the 3 trials was 0.994. A plot of the calibration data and the regression statistics can be found in Appendix 4.

The thermocouples, and their associated signal conditioners, were calibrated using a replicated two-point calibration. For each thermocouple, temperature readings, at ambient (21.11°C) and at water’s boiling point (100.0°C), were collected with the data acquisition system. Three replicates of each reading were conducted with each thermocouple. Statistical analysis of the combined data sets, presented in Table 5, revealed that the thermocouples were
reporting within acceptable limits. Descriptive statistics for the individual sets, and the combined set, can be found in Appendix 4.

Table 5 – Thermocouple Calibration Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Error</th>
<th>Std. Deviation</th>
<th>Sample Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Ambient</td>
<td>21.357</td>
<td>0.392</td>
<td>3.034</td>
<td>9.206</td>
</tr>
<tr>
<td>Inlet Boiling</td>
<td>100.379</td>
<td>0.287</td>
<td>2.223</td>
<td>4.942</td>
</tr>
<tr>
<td>Outlet Ambient</td>
<td>21.812</td>
<td>0.282</td>
<td>2.181</td>
<td>4.759</td>
</tr>
<tr>
<td>Outlet Boiling</td>
<td>100.785</td>
<td>0.306</td>
<td>2.368</td>
<td>5.605</td>
</tr>
</tbody>
</table>

The Omega model PX138 differential pressure sensor was calibrated by using a Barnart Vacuum Pressure station. This device allowed for the application of a vacuum, or positive pressure, to the ports of the sensor. For calibration, the input provided to the sensor was varied from –3 psi to 3 psi in 1 psi steps. The pressure reading from the data acquisition system was compared to the applied input. This procedure was repeated for both ports on the sensor. The value of the correlation coefficient, $R^2$, for the 3 trials was 0.9997. When plotted the data produced a linear graph with a slope of 1.003 and an intercept of 0.005. A plot of the calibration data and the regression statistics can be found in Appendix 4.

The manufacturers of the relative humidity sensor indicated that no calibration was required. In order to confirm the accuracy of the sensor it was tested by exposure to known humidity environments. When exposed to dry heated air from the system the output signal indicated the proper humidity.
When exposed to a saturated environment, a sealed headspace over distilled water, the output signal indicated the appropriate 100% relative humidity.

### 6.2 Particle Size Distribution

A particle size distribution that facilitates fluidization, and the repeatable procedure to create that distribution, needed to be determined. This was accomplished by the creation and characterization of a particle size distribution of feed material from peel cups and pomace from Brown extracted Valencia oranges. Initially, the peel material was acquired from the peel bins at a citrus processor. For the majority of the fluidization and drying trials the material was acquired from the Brown Citrus Systems pilot plant in Winter Haven, Florida. The optimization of a particle size distribution that facilitates fluidization is a practical exercise because industrial particle size preparation equipment is effective and economical.

For particle size preparation in the laboratory, the peels were quartered and placed into a Zyliss food chopper with the correct ratio of finisher pomace. A quantified number of plunges were made with the chopper to prepare the feed lot. This feed material was then tested in the VFB drying apparatus for quality of fluidization and its particle size distribution characterized using ASAE Standard: ANSI/ASAE S.319.3 Jul 97, “Method of Determining and Expressing Fineness of Feed Materials by Sieving”.

8.
6.3 Moisture Determination

Moisture must be monitored over drying time in order to create drying curves that facilitate the determination of the periods of drying, critical moisture content, and the controlling mechanism of the falling rate period. The standard methodology utilized to determine the moisture content was ASAE S358.2 Dec 99, “Moisture Measurement – Forages” \(^9\). During the drying process, the wet material being dried was removed from the fluidization chamber, placed in a tared glass petri dish, and weighed at periodic time intervals. After the material in the bed was dry, the sample was placed in the dish, placed in the drying oven, and allowed to dry for 24 hours. The oven was set at 87.8 °C. The oven also contained a desiccator to control the relative humidity inside the oven. At the end of this drying period, the sample was re-weighed to determine the mass of dry material. The percent moisture, dry basis, was determined as

\[
\text{\% Moisture}_{\text{db}} = \frac{\text{Mass of wet material} - \text{Mass of dry material}}{\text{Mass of dry material}} = M. \tag{29}
\]

Additionally, the percent moisture, wet basis, was determined by

\[
\text{\% Moisture}_{\text{wb}} = \frac{\text{Mass of wet material} - \text{Mass of dry material}}{\text{Mass of wet material}}. \tag{30}
\]

6.4 Vibrofluidization Data

Trials were conducted to determine the minimum vibrofluidization velocity, \(V_{\text{mvf}}\), of the peel material. The vibratory mechanism was modified to allow for 4 different stroke values, and set angular frequency value, measured using a photo-tachometer. These configurations created vibrational acceleration, \(A\omega^2/g\),
values of 1.0188, 1.5282, 2.5471, and 3.0565. Each of these variations was tested to determine quality of fluidization.

The procedure for data collection was methodical. The dryer was charged with a measured mass of wet material. The data acquisition system was started to collect mass flow and bed pressure drop data. The initial bed height was measured. The air supply was gradually increased to 50 PSI. The quality of fluidization was observed and noted. The vibration was started. The dynamic bed height was measured. The quality of fluidization was observed and noted. The air supply was slowly reduced to zero. This data was analyzed by plotting the pressure drop across the bed versus the air flow rate, to determine the minimum vibrofluidization velocity and quality of fluidization.

6.5 Drying Data

During the drying trials, real-time bed data was collected from the instrumentation described in section 6.1.2. Additionally, the bed was sampled, during the drying trials, to collect data on the solids moisture content. These samples were processed, as outlined in section 6.3, and the resulting moisture content versus time data was plotted and analyzed. These drying curves reveal the different periods of drying and the critical moisture content. To further facilitate the analysis of the drying data and model predictions, the moisture ratio, MR, was determined for each solids sample from the dryer, where

\[
MR = \frac{M - M_e}{M_o - M_e}. \tag{31}
\]
To verify the dominant moisture transport mechanism, the unaccomplished moisture change was plotted versus time. Unaccomplished moisture change, $M^*$, is defined as the ratio of free moisture in the solid at a specific time, $M$, to the total free moisture present at the start of the falling rate period, $M_c$. The slope of this semilogarithmic plot established whether a relationship exists in the falling rate period. If the result of the plot of $M^*$ versus time is a straight line, the controlling mechanism in the falling rate period is based on either diffusion, or capillary flow.

Assuming that the total drying time is a direct summation of the falling rate drying period and the constant drying rate period, the slope of the falling rate curve is related to the constant drying rate, and the time in the falling rate period, $t_f$, can be calculated from the energy transfer in the falling rate period and the inverse of the unaccomplished moisture change, by

$$t_f = \frac{\rho f d\lambda (M_c - M_e)}{h(T - T_s)} \ln \left( \frac{M_c - M_e}{M - M_e} \right). \quad (32)$$

If the slope from this equation agrees with the experimental data, the moisture movement is by capillary flow. If the slopes do not agree, the movement is by diffusion.

The diffusion coefficient is an important parameter to know when a Fickian model is used in the falling rate period. Since its physical meaning is questionable, especially in the drying of biological materials where moisture movement is complex, it is often referred to as the effective diffusivity, $D_{eff}$. The most practical method to determine the effective diffusivity is based on the solution of Fick’s second law. The solution for Fick’s diffusion equation with one-
dimensional moisture transfer and constant diffusivity for a sphere is given by Crank\textsuperscript{21} as

\[ M^* = \frac{M - M_e}{M_c - M_e} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left( -\frac{n^2 \pi^2}{r^2} D_{eff} t \right) \]  

(33)

where \( M^* \) represents the unaccomplished moisture change. The expansion of equation (32) for the three series is

\[ M^* = 0.608e^{-9.8N_{Fi}} + 0.152e^{-39.5N_{Fi}} + 0.06e^{-88.8N_{Fi}} \]  

(34)

where \( N_{Fi} \) is the Fick number equal to \( D_{eff}/L^2 \) and \( L \) is the characteristic dimension of the geometry. Crank further reduces this equation to

\[ \ln(M^*) = \text{Constant} t - s * t \]  

(35)

where \( s \) is the slope equal to the dehydration constant, \( c D_{eff}/L^2 \), and \( c \) is the constant in the first series, \( c = 9.8 \). This equation has the familiar linear form, and the effective diffusivity can be calculated from the slope of the natural logarithm of the unaccomplished moisture change plotted versus time, as shown by the following equation:

\[ D_{eff} = \frac{-s(L^2)}{c} . \]  

(36)
7 MODEL DEVELOPMENT

7.1 Introduction

The “secret for success” for fluidized bed drying is bubble action. The bubbling action, within a fluidized bed, promotes mixing and leads to uniformity in the bed temperature, which yields high rates of internal heat and mass transfer. The modeling of drying in such a bed requires that the parameters at the point of minimum fluidization be determined, the drying mechanisms in the constant and falling rates are understood, and relations for the appropriate constants are determined.

Drying kinetic models are typically based on the premise that the bed drying activity can be modeled using transport properties between phases within the dryer. Typically the dryer is represented as three phases; a solids phase, an interstitial gas phase, and a bubble phase, with heat and mass transfer occurring between the phases. A model based on this approach is presented in section 7.2.

Due to the excellent particle/gas mixing in a fluidized, or vibrofluidized bed, it has been suggested that drying can be modeled on thin-layer drying. The underlying assumption is that there is sufficient particle surface area exposed to the drying air, such that the system approximates thin-layer drying. In this approach, the drying constants, K and N, are used instead of the transport
properties. This approach has often been used for food, and especially for grains. A model based on this methodology is presented in section 7.3.

### 7.2 Three-Phase FBD Model

Combining the modeling concepts presented thus far, the Three-phase drying process model requires the following five step operational sequence: 1) Enter the experimentally determined operating conditions. 2) Choose a minimum fluidization correlation. 3) Simultaneously solve the equations for bed dynamics. 4) Use the fourth order Runge-Kutta method to evaluate the drying rate differential equations. 5) Plot the results of moisture content versus time.

The necessary constant and falling rate models developed herein are based upon the original work of Srinivasa Kannan et al.\textsuperscript{36}, and modified to account for the agglomerating characteristics of the feed material and the use of a vibrofluidized bed. The complexity inherent in this agglomerating fluidized system require the introduction of the following assumptions during the development of the model: 1) The system is operating in batch mode. 2) Particles are homogeneous in character, mostly spherical in shape, and do not shrink during drying. 3) All particles within the bed are the same temperature and have the same moisture content, at any point in the drying process. 4) The exiting drying fluid is in equilibrium with the particles. 5) Bubble size is uniform and does not depend upon location within the bed. 6) Intra-particle moisture movement can be characterized using Fick’s law, and effective diffusivity.
Figure 28 schematically represents the two-phase batch fluidized bed dryer used in this research. In the figure, $C$ represents the moisture content of the solids, $T$ is the temperature of that stream, $Y$ is the humidity of the stream, and $m$ is the stream mass flow. The dense phase consists of the solids phase and the interstitial gas phase. The bubble phase is the hot inlet air stream that fluidizes the solids, and serves as the drying medium.
The moisture and enthalpy balance equations, for this system, are used to develop the drying model, and are presented below. For convenience, the equations in each section of the Model Development chapter are labeled specifically for that section. The equations in this section follow the B# format.

The dense phase solids moisture balance is presented first as amount of solids in the bed multiplied by rate of removal of moisture in the bed per bed volume, \( \frac{dC}{dt} \), set equal to the bed average change in moisture content per change in time, \( \dot{W} \):\[
\rho_s (1 - \varepsilon_{mf})(1 - \varepsilon_b) \left( \frac{dC}{dt} \right) = -\dot{W} \quad (B1)
\]
where \( \varepsilon_{mf} \) and \( \varepsilon_b \) are the void fractions at minimum fluidization conditions and the bubble phase portion, respectively.

The dense phase interstitial gas phase moisture balance reflects the change in humidity, \( Y \), of the interstitial gas phase due to contributions from the solids and bubble phases, and is also related to the bed average change in moisture content per change in time:
\[
\rho_g \varepsilon_{mf} (1 - \varepsilon_b) \left( \frac{dY_g}{dt} \right) + \rho_g m_d (Y_d - Y_i) + \left( \frac{6k_{gb} \rho_g \varepsilon_b}{d_b} \right) (Y_d - Y_s) = \dot{W} \quad (B2)
\]

The dense phase solids enthalpy balance is the rise in sensible heat of the solids set equal to the heat input into the system, \( Q \), minus the loss of heat through evaporative cooling, \( \lambda \dot{W} \), represented as:
\[
\rho_s (1 - \varepsilon_{mf}) (c_{p,s} - c_{p,w}C) \left( \frac{dT_s}{dt} \right) = Q - \lambda \dot{W} \quad (B3)
\]
The dense phase interstitial gas phase enthalpy balance represents the enthalpy transfer from the drying medium, bubble phase, and the particles, solids phase, to the interstitial gas:

$$
\rho_s \varepsilon_{mf} \left(1 - \varepsilon_b\right) \left(c_{p,g} + c_{p,v} Y_i\right) \left(\frac{dT_g}{dt}\right) = \rho_g m_g \left(c_{p,g} + c_{p,v} Y_i\right) \left(T_i - T_d\right) + \left(\frac{6h_{rg} \varepsilon_{bg}}{d_b}\right) \left(T_b - T_d\right) + Q \quad (B4)
$$

Following similar derivations for the bubble phase, the following balances result: The bubble phase solids moisture balance

$$
\rho_g \varepsilon_b \left(\frac{dY_b}{dt}\right) + \rho_g m_b \left(Y_b - Y_i\right) = \left(\frac{6k_b \rho_g \varepsilon_b}{d_b}\right) \left(Y_d - Y_b\right). \quad (B5)
$$

Bubble phase interstitial gas phase enthalpy balance

$$
\rho_s \varepsilon_b \left(c_{p,g} + c_{p,v} Y_i\right) \left(\frac{dT_b}{dt}\right) + \rho_g m_g \left(c_{p,g} + c_{p,v} Y_i\right) \left(T_i - T_b\right) = \left(\frac{6h_{rg} \varepsilon_{bg}}{d_b}\right) \left(T_b - T_d\right). \quad (B6)
$$

The transport of mass and heat between these phases establishes the drying rates in both the constant and falling rate periods. The next two subsections present a derivation of these two drying rates.

### 7.2.1 Bed Parameters

The bed dynamics model starts by determining the minimum fluidization parameters based upon the empirical conditions. As cited by Pakowski in his chapter on vibrated bed dryers \(^{51}\), Jinescu developed the expression chosen for evaluating the minimum vibrofluidization velocity, \(U_{mvf}\), specifically for vibrofluidized beds of agglomerating materials as

$$
U_{mvf} = U_{mf} \left(1 - \frac{1 + k A_{\omega}^2}{2\pi j g}\right) \quad (F1)
$$
where \( j \) is the sum of lift and fall time divided by the vibration period, and \( k \) is the coefficient of collision elasticity, \( 0 < k < 1 \). The equation is adjusted for the minimum fluidization velocity, \( U_{mf} \), based on the assumption that at incipient vibrofluidization, the time that bed lifts up during the flight period is equal to the time of fall\(^{51}\). The minimum fluidization velocity was determined using the correlation for large particles proposed by Kunii and Levenspiel\(^{42}\) as

\[
U_{mf}^2 = \phi_s \cdot d_p \cdot g \cdot \frac{(\rho_p - \rho_g) \varepsilon_0^3}{(1.75 \rho_g)^5}
\]

(F2)

for \( \text{Re}_{p,mf} > 1000 \), where \( s \) is the particle sphericity, \( d_p \) is the mean particle diameter, \( \rho_p \) and \( \rho_g \) are the densities of the particle and the fluidizing gas, and \( \varepsilon_0 \) is the static bed voidage.

For comparison purposes, the equation for the minimum fluidization velocity for coarse particles equation, given by Chitester, et al.\(^{20}\), of

\[
U_{mf} = \left[ \frac{\mu_g}{\rho_g d_p^2} \right] \left( (28.7)^2 + 0.0494 \text{Ar} \right)^{0.5} - 28.7
\]

(F3)

was also used, where

\[
\text{Ar} = d_p^3 \rho_g (\rho_p - \rho_g) g / \mu_g^2.
\]

(F4)

Equation F3 can be used in place of equation F2 to fulfill the \( U_{mf} \) requirement in equation F1. Equations F2 and F3 are only two, of dozens, of semi-empirical relations for the minimum fluidization velocity available in literature.

For the bubble phase parameters, \( U_b \) and \( d_b \), equations by Werther and Mori and Wen cited by Kannan\(^{36}\) were used. The Bubble rise velocity equation, given by Werther as

\[
U_b = 1.6(D_t)^{0.4} (g d_b)^{0.5} + (U - U_{mf}).
\]

(F5)
Bubble diameter equation, given by Mori and Wen as
\[ d_b = 0.64[A_T(U - U_{mf})]^{2/5}. \tag{F6} \]

Additionally, relationships for the bed voidage, \( \varepsilon \), and the bed voidage at minimum fluidization conditions, \( \varepsilon_{mf} \), are required. For the purpose of estimating the bed voidage or porosity, the correlation of Dakshinamurthy et al., cited by Gupta \(^{31}\), was chosen. Their correlation is based on large volume of data obtained for various particles. The correlation is
\[ \varepsilon = (\varepsilon_g + \varepsilon_i) = \alpha \left( \frac{U_i}{U_l} \right)^b \left( \frac{U_g \mu_l}{\sigma} \right)^{0.08} \tag{F7} \]
where \( \alpha = 2.12 \) and \( b = 0.41 \). Additionally, the minimum fluidization bed voidage was determined using the correlation for large particles proposed by Kunii and Levenspiel \(^{42}\) as
\[ \varepsilon_{mf}^3 = U_{mf}^2 \frac{1.75 \rho_g}{d_{pg} \phi_s (\rho_s - \rho_g)}. \tag{F8} \]

### 7.2.2 Constant Rate Period

The starting point for the determination of the constant rate period drying rate is the moisture and enthalpy balances, for the solids and the interstitial gas in the dense phase. The moisture balance for the solids in the fluidized bed yields an expression for the rate of moisture removal per volume of the bed, \( \dot{W} \), as
\[ \dot{W} = -\rho_s (1-\varepsilon_i) dC/dt. \tag{CR1a} \]
If \( \varepsilon_v \) is the bubble void fraction and \( \varepsilon_b \) is the volume fraction of the bubbles in the bed, then the average bed voidage, \( \varepsilon_f \), is given by
\[ \varepsilon_f = \varepsilon_b \varepsilon_v + (1+\varepsilon_b) \varepsilon_e \tag{CR1b} \]
when there are no solids in the bubbles, \( \varepsilon_v = 1 \). An emulsion phase exists at minimum fluidization. The void fraction of the emulsion phase, \( \varepsilon_e \), equals the void fraction at minimum fluidization, \( \varepsilon_{mf} \), and

\[
(1 - \varepsilon_f) = (1 - \varepsilon_b)(1 - \varepsilon_{mf}). \tag{CR1c}
\]

The substitution of equation CR1c into CR1a gives the required moisture balance for the solids in the dense phase as a function of the change in the moisture content of the solids, \( C \),

\[
\dot{W} = -\rho_s(1 - \varepsilon_{mf})(1 - \varepsilon_b) \frac{dC}{dt}. \tag{CR2}
\]

This expression also indicates that \( W \) is proportional to the solids fraction in the bed. The moisture balance for the interstitial gas in the dense phase is a three-term expression given by

\[
\dot{W} = \rho_g\varepsilon_{mf}(1 - \varepsilon_b) \frac{dY_g}{dt} + \rho_g m_d(Y_d - Y_i) + 6K_b \rho_g \varepsilon_b(Y_d - Y_b) / \delta_b. \tag{CR3}
\]

Equating expressions CR2 and CR3, and rearranging, gives a relationship between the change in humidity of the air, \( Y_g \), and \( C \) as a function of time

\[
\rho_g\varepsilon_{mf}(1 - \varepsilon_b) \frac{dY_g}{dt} + \rho_g m_d(Y_d - Y_i) =
\]

\[
-\rho_s(1 - \varepsilon_{mf})(1 - \varepsilon_b) \frac{dC}{dt} + 6K_b \rho_g \varepsilon_b(Y_d - Y_b) / \delta_b. \tag{CR4}
\]

The enthalpy balance for the solids in the dense phase is given by

\[
\rho_s(1 - \varepsilon_{mf})(1 - \varepsilon_b)(c_{p,s} + c_{p,w} C) \frac{dT_s}{dt} = Q - \lambda \dot{W}. \tag{CR5}
\]

In this equation, \( \rho_s(1 - \varepsilon_{mf})(1 - \varepsilon_b) \) represents the solids fraction in the bed. The complete term on the left hand side of the equation represents the rise in the sensible heat of the solids. The first term on the right side of the equation, \( Q \),
represents the heat input, while the second term, $\lambda \dot{W}$, is the heat loss through evaporation of moisture from the solids.

The enthalpy balance for the interstitial gas in the dense phase is given by

$$\rho_g \varepsilon_{mf} (1 - \varepsilon_b) (c_{p,g} + Y_i c_{p,v}) \frac{dT_g}{dt} = \rho_g m_d (c_{p,g} + Y_i c_{p,v}) (T_i - T_d) +$$

$$6 h_b \varepsilon_b (T_b - T_d) \frac{dT_g}{dt} + \frac{\dot{Q}}{\rho_g}$$

In this equation, the first and second terms on the right hand side of equation represent the enthalpy transfer from the drying medium to the dense phase, and from the bubble phase to the dense phase. Combining the enthalpy balance equations, CR5 and CR6, and substituting in $W$ from CR3, we achieve the overall balance equation

$$\rho_s (1 - \varepsilon_{mf})(1 - \varepsilon_b) (c_{p,s} + c_{p,w} C) \frac{dT_s}{dt} = \rho_g m_d (c_{p,g} + Y_i c_{p,v}) (T_i - T_d) -$$

$$\rho_g \varepsilon_{mf} (1 - \varepsilon_b) (c_{p,g} + Y_i c_{p,v}) \frac{dT_g}{dt} + 6 h_b \varepsilon_b (T_b - T_d) \frac{dT_g}{dt} + \lambda$$

$$[\rho_g \varepsilon_{mf} (1 - \varepsilon_b) \frac{dY_g}{dt} + \rho_g m_d (Y_d - Y_i) + 6 K_b \rho_g \varepsilon_b (Y_d - Y_b) \frac{dT_g}{dt}]$$

If the terms involving the rate of change of humidity with respect to time, $dY_g/dt$, and the change in the temperature of the gas with time, $dT_g/dt$, are assumed small compared to the corresponding convective terms, equations CR3, CR4, and CR7 reduce to

$$W = \rho_g m_d (Y_d - Y_i) + 6 K_b \rho_g \varepsilon_b (Y_d - Y_b) \frac{dT_g}{dt}$$

Because the bed temperature is constant in the constant rate period, the left-hand side of equation CR10 should be set to zero. This leaves
\[ \rho_g m_d \left( c_{p,g} + Y_i c_{p,v} \right)(T_i - T_d) + 6h_b \varepsilon_b (T_b - T_d) / d_b + \lambda \left[ \rho_g m_d (Y_d - Y_i) + 6K_b \rho_g \varepsilon_b (Y_d - Y_b) / d_b \right] = 0 \]  
(CR11)

from which the dense-phase gas temperature, \( T_d \), and humidity, \( Y_d \) can be determined.

Following similar derivations for the bubble phase, neglecting the rate of change of the humidity and temperature when compared to the corresponding convective terms, the moisture balance becomes

\[ \rho_g m_b (Y_b - Y_i) = 6K_b \rho_g \varepsilon_b (Y_d - Y_b) / d_b \]  
(CR12)

or

\[ Y_b = \frac{(6K_b \varepsilon_b Y_d + d_b m_b Y_i) / (d_b m_b + 6K_b \varepsilon_b)}{6K_b \varepsilon_b + \rho_g m_b (c_{p,g} + Y_i c_{p,v})}. \]  
(CR13)

The enthalpy balance becomes

\[ \rho_g m_b (c_{p,g} + Y_i c_{p,v})(T_i - T_b) = 6h_b \varepsilon_b (T_b - T_d) \]  
(CR14)

or

\[ T_b = \frac{\rho_g m_b T_i d_b (c_{p,g} + Y_i c_{p,v}) + 6h_b \varepsilon_b T_d}{[6h_b \varepsilon_b + \rho_g m_b d_b (c_{p,g} + Y_i c_{p,v})]}. \]  
(CR15)

Since the resistance for mass transfer lies in the film surrounding the solid,

\[ W = (1 - \varepsilon_b) \rho_g a K_y (Y_{sat} - Y_d), \]  
(CR16)

substituting equation CR16 into CR8 gives

\[ \rho_g m_d (Y_d - Y_i) + 6K_b \rho_g \varepsilon_b (Y_d - Y_b) / d_b = (1 - \varepsilon_b) \rho_g a K_y (Y_{sat} - Y_d). \]  
(CR17)

The drying rate during the constant rate period is obtained by simultaneously solving equations CR9, CR11, CR13, CR15 and CR17. The differential equations, CR9 & CR10, are solved using fourth order Runge-Kutta algorithm to evaluate \( dC/dt \) at \( t + \Delta t \).
A number of semi-empirical equations are used to obtain required values for $U_{mf}$, $U_b$, $d_b$, $\varepsilon_{mf}$, $K_b$, $K_y$, $h_b$, and $Y_{sat}$. Beyond the relations provided in section 7.2, the follow equations for mass and heat transfer coefficients, cited by Kannan$^{36}$, were used: Mass transfer coefficient from the dense phase to the bubble phase equation, given by Sit and Grace as

$$K_b = \frac{U_{mf}}{3} + \left[ \frac{4D_m \varepsilon_{mf} U_b}{\pi d_b} \right]^{0.5}.$$  \hspace{1cm} (CR18)

Mass transfer coefficient from the particle surface to the bulk gas equation, given by Ranz as

$$K_y = \left( \frac{D_m}{d_p \psi \phi_s} \right) \left[ 2 + 1.8 \text{Re}_{mf}^{0.5} \text{Sc}^{0.33} \right]$$ \hspace{1cm} (CR19)

where,

$$\text{Re}_{mf} = \frac{\rho_g U_{mf} d_p}{\mu_g} \quad \text{and} \quad \text{Sc} = \frac{\mu_g}{\rho_g (D_A B)}.$$ \hspace{1cm} (CR20)

Heat transfer coefficient from the bubble phase to the dense phase prediction equation

$$h_b = \frac{U_{mf} \rho_g \lambda}{3} + \left[ \frac{4 \lambda \rho_g K_b \varepsilon_{mf} U_b}{\pi d_b} \right]^{0.5}.$$ \hspace{1cm} (CR21)

Finally, the saturation humidity equation from Treybal$^{68}$

$$Y_{sat} = 0.621 \frac{P_{water-sat}}{(1.0133 \times 10^5 - P_{water-sat})}$$ \hspace{1cm} (CR22)

where,

$$\ln (P_{water-sat}) = (-5800.2206/T) + 1.3915 - 0.0486 T + 0.4176 \times 10^{-4} T^2 - 0.1445 \times 10^{-7} T^3 + 6.546 \ln(T).$$ \hspace{1cm} (CR23)

### 7.2.3 Falling Rate Period

The falling rate period begins at the critical moisture content, $M_c$, where the constant rate period ends. Here, the drying rate is controlled by the moisture transport out of the solid. Having determined, in Phase 2 of the research, that
the controlling mechanism for the falling rate period is diffusion, the moisture movement can be defined by Fick’s diffusion equation

\[ \frac{\partial C}{\partial t} = D_{eff} \left( \frac{\partial^2 C}{\partial r^2} \right). \]  

(FR1)

The diffusion equation, for the falling rate period for a sphere, can be derived by assuming that the surface is at equilibrium with the moisture, and that the moisture distribution is uniform. For these conditions, the following equation is obtained

\[ \frac{(C - C_s)}{(C_o - C_s)} = \left(\frac{6}{\pi^2}\right) \sum \left[ \frac{1}{n^2} \exp\left(-\frac{n^2 D_{eff} t}{r^2}\right) \right] \]  

(FR2)

where \( C \) is the moisture content at time, \( t \), \( C_o \) is the initial moisture content, and \( C_s \) is the surface moisture content. This equation simplifies to a limiting form of the diffusion equation, represented as

\[ \frac{(C - C_s)}{(C_o - C_s)} = \left(\frac{6}{\pi^2}\right) \exp\left(-\frac{D_{eff} t}{r^2}\right). \]  

(FR3)

Equation FR3 may be differentiated to give the drying rate as

\[ \frac{dC}{dt} = -\left(\frac{\pi^2 D_{eff}}{6r^2}\right) (C - C_s). \]  

(FR4)

Simultaneous solution of this rate equation, along with the equations presented in the constant rate period, predicts the drying rate, temperature and humidity in the phases, and the moisture ratio, during the falling rate period. For purposes of numerical solution, the model employs a classical fourth-order Runge-Kutta algorithm to solve the series of first-order differential equations.
7.3 Thin-Layer Drying Model

Due to the thorough mixing achieved with fluidization and vibrofluidization, the drying process has been treated as thin-layer drying by other researchers, Ramesh and Rao\textsuperscript{58}, Shilton and Niranjan\textsuperscript{64}, Prasad et al.\textsuperscript{56}. The thin-layer model describes drying in a unified way, regardless of the controlling mechanism using two drying constants, K and N. This approach has proved to be a suitable model for the purposes of process design, optimization, and replacement of models where a large number of iterative calculations are required\textsuperscript{37}.

The starting point for a generally accepted model for thin-layer drying is Page's equation (1949) given as

\[
\frac{dM}{dt} = -K(M - M_e) \quad (\text{TL1})
\]

where \(M\) is the moisture content at time \(t\), and \(M_e\) is the equilibrium moisture content. The solution of equation TL1 results in

\[
MR = \frac{M - M_e}{M_o - M_e} = \exp(-Kt) \quad (\text{TL2})
\]

where \(MR\) is the moisture ratio at time \(t\), and \(M_0\) is the initial moisture content. The limitations of this equation in predicting the drying curves necessitated the introduction of a second drying parameter, \(N\)\textsuperscript{49}:

\[
MR = \frac{M - M_e}{M_o - M_e} = \exp(-Kt^N). \quad (\text{TL3})
\]
As suggested by Brooker et al.\textsuperscript{18}, and Weller and Bunn\textsuperscript{71}, for practical reasons, the equilibrium moisture content, $M_e$, of the dried product can be taken as the final moisture content, $M_f$, of the product. Hence, Page’s equation becomes

$$\frac{M - M_f}{M_o - M_f} = \exp(-Kt^N). \quad (\text{TL4})$$

The analysis of the experimental drying data will yield equations for the Page equation parameters, $K$ and $N$, in terms of a dryer property. Substitution of these equations into equation TL4, and then solving equation TL4 over time, will predict the drying time at the vibrofluidization velocity, assuming that the initial and final moisture content, and a dryer operating parameter used to define $K$ and $N$, are known.
8 RESULTS AND DISCUSSION

8.1 Particle Size Distribution

To determine the fluidizable particle size, the raw material was manually chopped into a size distribution that fluidized in the apparatus. With each particle size distribution tested the airflow rate was started at the value predicted by the model and increased manually until stable fluidization conditions were achieved, or the maximum available air velocity was met. The dried material from these trials was then characterized by ANSI/ASAE S.319.3 Jul 97, “Method of Determining and Expressing Fineness of Feed Materials by Sieving”, which is the standard method to determine the particle size distribution.

A particle size distribution that resembled normal, or lognormal, distribution was desired, with bimodal distributions avoided. These tests were used to develop a consistent feed material for the vibrofluidized bed dryer. The resulting material had a lognormal particle size distribution with a particle size between 1 and 7 mm with a tail of particles smaller than 1 mm, a geometric mean diameter, \(d_{gw}\), or median size of particles by mass, of 3.829 mm, with a geometric standard deviation of log-normal distribution by mass, \(S_{log}\), of 1.23E-08, and a geometric standard deviation of particle diameter by mass, \(S_{gw}\), of 2.49E-07 mm. This particle size distribution is similar to the one reported by Braddock and Miller\(^{15}\) for press cake and dried pulp. The main difference is that their reported
particle size distribution was bimodal with a peak near 1.5 mm and another fraction distributed in the 2.2 to 6 mm range. To facilitate fluidization and consistent product dryness, the particle size distribution achieved in this research is more uniform than standard press cake and dried pulp.

The data sheet for tabulation of the sieving data, and calculation of the lognormal particle size distribution parameters, is presented in Appendix 3. Graphical representations of the particle size distribution (P.S.D.) used in the drying experiments are presented in Figures 29, 30, and 31 as an exponential plot, linear bar chart, and the cumulative undersize distribution plot, respectively.

![Figure 29 – Logarithmic Plot of Final P.S.D.](image-url)
Figure 30 – Bar Plot of Final P.S.D.
Figure 31 – Final P.S.D. Cumulative Undersize Distribution by Mass
8.2 Vibrofluidization

Using processing residue with the particle size distribution described in section 8.1, the vibrofluidization velocity was determined as outlined in section 6.4. Tests were conducted at four different vibrational acceleration, \( A_\omega^2/g \), values of 1.0188, 1.5282, 2.5471, and 3.0565, and are represented below in Figures 32 - 35, respectively. Observe that the hysteresis between the increasing and decreasing velocity curves reduces, and the stability of the pressure drop above the minimum fluidization velocity increases, as the vibrational acceleration values approach the optimal conditions. This type of response to vibrational acceleration is typical. It follows the convention presented by Pakowski \(^{51}\), based upon analysis of VFBD literature, that there is an optimal range of vibrational acceleration where the bed structure is most suitable for drying, where \( 2 \leq A_\omega^2/g \leq 3 \). The optimal \( A_\omega^2/g \) value was determined to be 2.5471 in the 3\(^{rd}\) test configuration. Using the results from this configuration, the optimal minimum vibrofluidization velocity, \( U_{mfv} \), and the bed pressure drop, \( \Delta P \), were determined to be 10.9 cm/sec and 3900 Pa, respectively. These results, and the values from all 4 trials, are presented in Table 6.
Figure 32 - Fluidization Curve for VFBD Variant #1

Fluidization Curve 1
(Mass in bed = 235g, Vibrational Acceleration = 1.019)

Figure 33 – Fluidization Curve for VFBD Variant #2

Fluidization Curve 2
(Mass in bed = 234g, Vibrational Acceleration = 1.528)
Figure 34 - Fluidization Curve for VFBD Variant #3

Figure 35 - Fluidization Curve for VFBD Variant #4
Table 6 – VFBD Minimum Vibrofluidization Results

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Weight of Bed (g)</th>
<th>Vibrational Acceleration $A\omega^2/g$</th>
<th>Minimum Fluidization Velocity (SLPM)</th>
<th>Minimum Fluidization Velocity (cm/s)</th>
<th>Pressure Drop across Bed (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>234.77</td>
<td>1.019</td>
<td>256</td>
<td>23.7</td>
<td>2200</td>
</tr>
<tr>
<td>2</td>
<td>233.56</td>
<td>1.528</td>
<td>171</td>
<td>15.9</td>
<td>2700</td>
</tr>
<tr>
<td>3</td>
<td>233.27</td>
<td>2.547</td>
<td>118</td>
<td>10.9</td>
<td>3900</td>
</tr>
<tr>
<td>4</td>
<td>232.06</td>
<td>3.056</td>
<td>149</td>
<td>13.8</td>
<td>2900</td>
</tr>
</tbody>
</table>

8.3 Vibrofluidized Bed Drying

In earlier work $^{63}$, the drying parameters of the residue were determined in a non-fluidized state. These include the drying rate of the residue, the critical moisture content of the residue, $M_c$, the mechanisms of both drying rate periods, and the effective diffusion coefficient, $D_{eff}$. These parameters are compared to the values determined in the vibrofluidized bed dryer in Table 7. The values for the critical moisture content are in agreement with the results presented by Braddock and Miller $^{15}$ for the drying of press cake. At three different temperatures, they report a constant rate period from 73% to 25% moisture content, and a falling rate below that, indicating a critical moisture content of 25%.

Table 7 – Drying Parameters

<table>
<thead>
<tr>
<th>Bed Type</th>
<th>$M_c$ (%)</th>
<th>C.R. drying mechanism</th>
<th>F.R. drying mechanism</th>
<th>$D_{eff}$ (cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBD</td>
<td>25</td>
<td>Surface Evap.</td>
<td>Diffusion</td>
<td>8.28E-5</td>
</tr>
<tr>
<td>VFBD</td>
<td>30</td>
<td>Surface Evap.</td>
<td>Diffusion</td>
<td>2.85E-05</td>
</tr>
</tbody>
</table>
The drying curves for each of these trials are presented in Figure 36. These curves are also evident in Figures 42 and 44, since the model verification was anchored to the product dryness and drying time.

Figure 36 shows expected behavior for the drying trials as a function of average bed temperature. For all 5 temperatures, the three periods of drying can be observed. Initially, prior to a time of 5 minutes, the drying rate is slow during the warm up period. After that initial period, the drying enters the constant rate period. This can be observed in all the series by the straight-line region from 5
minutes to 13 – 15 minutes, or from the moisture ratio of 95 – 80 down to 36 – 24. This point of transition, from the constant rate period to the falling rate period at a moisture ratio of 30%, is the critical moisture content. Below this point, all the series can be seen to exhibit falling rate characteristics as the moisture ratio approached the equilibrium moisture content asymptotically. Additionally, it can be observed that the trial conducted at the lowest temperature, trial 5 (the purple diamond), has the slowest drying rate, as evidenced by the higher moisture ratio at each time step compared to the other trials. In the constant rate period, this convention also follows with the other temperature groupings. In the falling rate period, the drying rates are roughly equivalent, since the controlling mechanism is now the diffusion of moisture from the interior of the particle to the surface.

The data collected from the instrumentation during the drying trials are presented in Figures 37 – 41. These are provided to give the reader a more complete picture of the drying process. In these figures it can be observed: 1) The pressure drop across the bed increased until fluidization occurred, and then remained stable throughout the run, indicating good quality fluidization without bed destabilization. 2) The exit gas temperature was offset from the inlet gas temperature during the constant rate period, and began to approach it during the falling rate period when there was less evaporative cooling from the particle surfaces. 3) The exit gas humidity initially rose to the point of saturation, and then reduced as the moisture content in the bed decreased. 4) The pressure drop across the bed diminished, due to reduced bed mass from both water removal due to drying, and sample removal for moisture analysis.
Figure 37 – Drying Trial 1
Figure 39 – Drying Trial 3
Figure 40 – Drying Trial 4
Figure 41 – Drying Trial 5
8.4 Model Validation

The mathematical models of the drying process were verified by comparing experimentally measured fluidization parameters and drying curves with the model predictions. The three-phase vibrofluidized bed model predicted the bed hydrodynamic, or fluidization, parameters in the process of calculating the drying curves. The comparison of these parameters is presented in section 8.4.1 with the experimentally determined optimal vibrational acceleration. The validation of the model predicted drying curves, for the three-phase and thin-layer drying models, are presented in sections 8.4.2 and 8.4.3, respectively.

8.4.1 Fluidization Parameters

As indicated in section 2.3, FBD’s are categorized as either batch or continuous type processes. The necessary particle size for fluidization, and the minimum fluidization velocity, in a batch dryer, were determined by experimentation. This test also determined if the material is fluidizable. After the particle size distribution, vibrational acceleration and fluidization was achieved, the minimum fluidization velocity was compared to the model predicted value.

To determine the fluidization velocity, the predicted fluidization velocity from the steady state phase 1 model was used as a starting point. The airflow rate was started here, and then adjusted to reach good fluidization conditions. Since air velocity alone was not sufficient to achieve stable fluidization, the vibratory mechanism was added into the design matrix. Tests were conducted at four different vibrational acceleration, $A\omega^2/g$, values of 1.0188, 1.5282, 2.5471, and 3.0565. The optimal $A\omega^2/g$ value was determined to be 2.5471. The
experimental and predicted fluidization parameters are based on this level of vibrational input. In Table 8, the fluidization parameters predicted by the model are tabulated with the experimentally determined values.

<table>
<thead>
<tr>
<th>Table 8 – Fluidization Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>$U_{mf}$ (cm/s)</td>
</tr>
<tr>
<td>$U_{mvf}$ (cm/s)</td>
</tr>
<tr>
<td>$V_{mvf}$ (SLPM)</td>
</tr>
<tr>
<td>$\Delta P_{mvf}$ (Pa)</td>
</tr>
<tr>
<td>$\varepsilon$ (non.dim.)</td>
</tr>
<tr>
<td>Experimental:</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>10.9</td>
</tr>
<tr>
<td>118</td>
</tr>
<tr>
<td>3900</td>
</tr>
<tr>
<td>.52</td>
</tr>
<tr>
<td>Predicted:</td>
</tr>
<tr>
<td>51.68</td>
</tr>
<tr>
<td>11.72</td>
</tr>
<tr>
<td>126</td>
</tr>
<tr>
<td>3561</td>
</tr>
<tr>
<td>.61</td>
</tr>
</tbody>
</table>

The predicted minimum fluidization velocity, $U_{mf}$, of 51.68 cm/s was beyond the capability of the system. If the system had the ability to supply air at that flow rate, only the largest, densest particles would remain in the bed. When examining the remainder of the results, consider that these values all reflect a batch vibrofluidized bed dryer. If the system under consideration were continuous, these parameters would not change. The residence time distribution in a continuous VFBD would mainly affect the drying rate and final product moisture content.

8.4.2 Three-Phase VFBD Model

A plot of the experimental and predicted drying curves, for the experimental bed temperatures, is plotted in Figure 42. The data points represent moisture ratio values sampled during the drying trials. The curves represent the moisture ratio predicted by the model.
Performing a regression analysis, on the combined data sets, tested the statistical validity of the three-phase drying model. The combined data set is plotted in Figure 43. If the model perfectly predicted the experimental data, all the data points would fall directly on the 1:1 ratio line. The regression statistics for the combined data set is presented in Table 9, and the individual data sets are presented in Appendix 6.
Moisture Ratio
Predicted Using the Three-Phase Model

Figure 43 – Three-Phase Drying Model Validation

Table 9 – Regression Statistics for Three-Phase Model Validation

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ((R^2))</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>0.9940</td>
<td>0.0295</td>
<td>1.0302</td>
<td>0.9769</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of Square Regression</th>
<th>Sum of Square Residual</th>
<th>Mean Square of Regression</th>
<th>Mean Square of Residual</th>
<th>F-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.099</td>
<td>3.054E-2</td>
<td>5.099</td>
<td>8.720E-4</td>
<td>5.844E3</td>
</tr>
</tbody>
</table>

8.4.3 Thin-Layer Drying Model

As formulated in the model development section, Page’s equation for thin-layer drying was used to predict the drying curves for the vibrofluidized bed drying of citrus peel and pulp. The parameters of Page’s equation, \(N\) & \(K\), were determined by rearranging Page’s equation (TL4) into the form \(Y=mx + C\); the equation
\[
\ln(-\ln(MR)) = N \ln(t) + \ln(K)
\]  \hspace{1cm} (TL5)

of a straight line. The graph of \(\ln(-\ln(MR))\) on the Y axis and \(\ln(t)\) on the x-axis, will yield the slope as \(N\), and the y-intercept as \(\ln(K)\). These values were calculated for each temperature of the drying trials and are shown in Table 10 (see Figures 51 - 55 in Appendix 5 for more information).

Table 10 –Regression Data for Determination of Page’s Equation Parameters

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Avg. Bed Temperature, °C</th>
<th>(N)</th>
<th>(\ln(K))</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>144.65</td>
<td>2.0568</td>
<td>-5.0396</td>
<td>0.9958</td>
</tr>
<tr>
<td>2</td>
<td>117.65</td>
<td>2.3600</td>
<td>-6.0145</td>
<td>0.9974</td>
</tr>
<tr>
<td>3</td>
<td>137.73</td>
<td>1.9776</td>
<td>-4.2440</td>
<td>0.9890</td>
</tr>
<tr>
<td>4</td>
<td>152.40</td>
<td>1.9289</td>
<td>-4.5793</td>
<td>0.9855</td>
</tr>
<tr>
<td>5</td>
<td>104.13</td>
<td>2.9921</td>
<td>-7.7075</td>
<td>0.9940</td>
</tr>
</tbody>
</table>

Performing regression analysis and minimizing the standard error of deviation, the moisture ratio, time, and temperature were analyzed to determine the Page equation parameters as a function of the drying temperature. The best-fit polynomial for the data was determined by curvilinear regression as

\[
K = 1.565E-7T^3 - 5.724E-5T^2 + 7.068E-3T - 0.2917 \text{ with an } R^2 = 1.0 \hspace{1cm} (TL6)
\]

\[
N = -2.531E-5T^3 + 0.01016T^2 - 1.364T + 63.436 \text{ with an } R^2 = 0.9969. \hspace{1cm} (TL7)
\]

A similar third order polynomial was obtained by Ramesh and Srinivasa Rao \(^{58}\), for the vibrofluidized bed drying of rice. Similar second order polynomials were obtained by Pathak et al. \(^{53}\), and Prasad et al. \(^{56}\), for the thin-layer drying of rapeseed, and the fluidized bed drying of rough rice, respectively. A plot of the
experimental and predicted drying curves, for the experimental bed temperatures, is plotted in Figure 44.

Performing a regression analysis, on the combined data sets, tested the statistical validity of the thin-layer drying model. The combined data set is plotted in Figure 45. If the model perfectly predicted the experimental data, all the data points would fall directly on the 1:1 ratio line. The regression statistics for the
data set are presented in Table 11, and the individual data sets are presented in Appendix 6.

![ Moisture Ratio Predicted Using the Thin Layer-Model](image)

**Figure 45 – Thin-Layer Drying Model Validation**

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>0.9936</td>
<td>0.0305</td>
<td>1.0225</td>
<td>0.9675</td>
</tr>
<tr>
<td>Sum of Square Regression</td>
<td>Sum of Square Residual</td>
<td>Mean Square of Regression</td>
<td>Mean Square of Residual</td>
<td>F-Ratio</td>
</tr>
<tr>
<td>5.013</td>
<td>3.25E-2</td>
<td>5.013</td>
<td>9.27E-4</td>
<td>5.406E3</td>
</tr>
</tbody>
</table>
9 SUMMARY AND RECOMMENDATIONS

9.1 Summary

To study and evaluate the fluidized bed drying of citrus processing residue, a benchtop vibrofluidized bed drying system was successfully designed, executed, and modeled. The residue consists of the non-juice components of a citrus fruit; primarily peel and pulp. The material is hygroscopic, agglomerating, has a wide particle size distribution, and must be dried in a controlled environment to avoid thermal damage to nutrients and flavors. The driving force for this research was the economic constraints of the existing process.

Sieving was used to characterize the particle size distribution of the dried citrus residue that facilitated fluidization. The resulting material had a lognormal particle size distribution, a particle size between 1 and 7 mm with a tail of particles smaller than 1 mm, a geometric mean diameter, or median size of particles by mass, \(d_{gw}\), of 3.829 mm, a geometric standard deviation of lognormal distribution by mass, \(S_{log}\), of 1.23E-08, and a geometric standard deviation of particle diameter by mass, \(S_{gw}\), of 2.49E-07 mm.

Using the feed material described by the sieving analysis, vibrofluidization trials were undertaken. The test matrix allowed for varying air flow rates at four different vibrational acceleration levels. The configuration that best facilitated fluidization was a vibrational acceleration, \(A_{\omega^2/g}\), of 2.54, a minimum
vibrofluidization velocity, $U_{\text{mf}}$, of 4.2 cm/s, and a bed pressure drop, $\Delta P$, of 3900 Pa. All of these values were consistent with values predicted in the bed dynamics portion of the three-phase fluidized bed dryer model.

The drying parameters of the residue were determined in the vibrofluidized bed batch drying trials. These include: 1) The critical moisture content of the residue, $MC_c$, of 30%. 2) The mechanisms of both drying rate periods, surface evaporation in the constant rate period, and diffusion in the falling rate period. 3) The effective diffusion coefficient, $D_{\text{eff}}$, of the processing residue of $2.85 \times 10^{-5}$ cm/s.

Two models (three phase and thin-layer) were developed to predict vibrofluidized bed drying. Both were validated by comparing predictions versus experimental trials. The first, the three-phase model, solved a series of simultaneous equations to predict bed hydrodynamics, and then used a fourth-order Runge-Kutta algorithm to solve the moisture and enthalpy balance differential equations. The model successfully predicted the bed hydrodynamic properties and the drying curves. In the second model, the thin-layer drying model, based on Page’s equation, the drying constants, $K$ & $N$, were determined as a function of bed temperature to be $K = 1.565E-7T^3 - 5.724E-5T^2 + 7.068E-3T - 0.2917$, and $N = -2.531E-5T^3 + 0.01016T^2 - 1.364T + 63.436$. These equations, coupled with Page’s equation, successfully predict the drying curves and are consistent with other fluidized bed drying models for hygroscopic materials.
Based upon results of these trials, economic evaluation of the proposed process shows it to be advantageous when compared to the existing process that breaks even, or generates a loss. The proposed vibrofluidized bed drying method has an acceptable payback period of 4.34 years, and an estimated processing cost per ton of dried material of $33, which appears to make it a profitable enterprise.

In conclusion, this dissertation has demonstrated that this research, into a vibrofluidized bed drying operation represents state-of-the-art advancement in the citrus feed mill process. In addition, there are several valuable by-products of this research: 1) A spreadsheet based balance program to predict energy usage in a fluidized bed dryer when the user inputs the feed flow rate, moisture content and temperature, the air feed moisture content and temperature, the product moisture content and temperature, and the exhaust gas temperature. 2) A rigorous kinetic vibrofluidized bed drying model based on moisture and enthalpy balances and bed hydrodynamics, which predicts the fluidization parameters and drying curves. This complex model serves to facilitate scale-up and bed configuration investigations. 3) A simple single parameter model for the drying of citrus processing residue based on thin-layer drying. This model is computationally simple and would serve process control algorithms.

9.2 Recommendations

The results of this dissertation lead to several avenues for future research. Though it is suggested in literature by Liedy and Hilligardt 43, that scale up from
batchwise laboratory fluidized bed dryers to production continuous operation dryers is possible, it would be quite a leap of faith to scale up directly from the benchtop to the plant floor. Instead, the concept of vibrofluidized bed drying of citrus processing residue should be further studied using a medium scale pilot plant dryer. Hence, trials could be run in concert with pilot scale rotary kiln drying process to determine if the economic advantages presented in this dissertation are great enough to warrant replacement of aging citrus feed mill components.

As evidenced in the drying trial figures, the control of the inlet temperature was difficult in the benchtop unit. Though the system was allowed to reach steady state prior to the introduction of material into the bed, the drying temperature rose above the set point in every trial. Instrumenting a control loop into the apparatus should be explored. This better control would facilitate determination of the relationship between drying temperature and drying rates.

The relationship between the VFBD processing alternative and the emission of volatile organic compounds (VOCs) should be further studied. Although citrus processors attempt to recover most of the essential oils in the peel, the residual will be driven off in the drying process. The current process allows for at least some recovery of the oil left in the processing residue via the molasses process, recovering d-limonene. Citrus processing plants are increasingly coming under the scrutiny of the EPA in relation to Title V issues. Establishing the emission rates for this process will be essential for getting the vibrofluidized bed drying process permitted for use in citrus feed mills operating in Florida.
REFERENCE LIST


32. Heid, J.L. Drying Citrus Cannery Wastes and Disposing of Effluents in Food Ind. 1945, 17.


39. King, D. "Feed Mill Q.A. Programs and the European Market"; at 2003 Citrus Processing Short Course Clearwater Beach, FL.


BIBLIOGRAPHY

The Sunshine Feed [VHS], State of Florida, 1976.


Braddock, R.J. By-Products of Citrus Fruit. Food Technology, 1995; 49 (9), p74, 76-77.

Braddock, R.J.; Cadwallader, K.R. Citrus By-Products Manufacture for Food Use. Food Technology, 1992; 46 (2), p105-110.


Miller, W.M.; Braddock, R.J. Microwave Drying of Citrus Peel *Proceedings of the Florida State Horticultural Society*; 1982; p204-207.


van't Land, C.M. *Industrial Drying Equipment: Selection and Application*; Chemical Industries 45; Marcel Dekker, Inc.: New York, NY, 1991.


1) air compressors (drying air supply)
2) drying air exhaust
3) fluidization chamber (containing citrus product to be dried)
4) calming section (below distributor plate)
5) a.c. motor for vibrational energy input
6) drying filter and air regulator
7) mass airflow sensor
8) three way valve, for air diversion
9) heat exchanger for drying air temperature control
10) thermocouple (drying air - in)
11) thermocouple (drying air - out)
12) differential pressure transducer
13) humidity sensor (drying air - out)
14) pitot tube

Figure 46 - Schematic of Lab Apparatus
Figure 47 - Photograph of 1st Generation FBD
Appendix 1 (Continued)

Figure 48 – Photograph of Final VFBD
APPENDIX 2 – THREE-PHASE VFBD MODEL CODE

Rules

Bed Hydrodynamic Properties
\[ \text{Umf} = \left(\left( s \cdot dp \cdot g \cdot (\text{rhop} - \rho) \cdot \varepsilon^0 \right)^3 / (1.75 \cdot \rho) \right)^{0.5} \]
\[ \text{Vmf} = \text{Umf} \cdot 0.384 \]
\[ \text{uT} = (2.32 \cdot \text{Umf}) / (\varepsilon^0 \cdot 1.5) \]
\[ \text{Vt} = \text{uT} \cdot 0.384 \]
\[ \text{ub} = 0.7 \cdot (g^*db)^{0.5} \]
\[ \text{Vb} = \text{ub} \cdot 0.384 \]
\[ \text{U} = \varepsilon^b \cdot \text{ub} + (1 - \varepsilon^b) \cdot \text{Umf} \]
\[ \text{vib} = (A^*\omega^2) / (g / 100) \]
\[ \mu_g = (-1.1555 \cdot E^{-14} \cdot T^3) + (9.5728 \cdot E^{-11} \cdot T^2) + (3.7604 \cdot E^{-08} \cdot T) - 3.4484 \cdot E^{-06} \]
\[ \text{Umvf} = 5.238 \cdot \left( \frac{\text{rhop}}{\rho} \right)^{0.63} \cdot \left( \frac{1}{\mu_g} \right)^{0.33} \cdot \left( \frac{dp}{100} \right)^{0.88} \cdot (1 - c^*(vib)) \]
\[ \text{Vmvf} = \text{Umvf} \cdot 0.384 \]
\[ \text{Umvf1} = \text{Umf} \cdot (1 - ((1 + k) / (6.28^*)) \cdot (vib)) \]
\[ \text{Vmvf1} = \text{Umvf1} \cdot 0.384 \]
\[ (\text{Hd} - \text{H}) / \text{Hd} = (\varepsilon - \varepsilon^0) / (1 - \varepsilon) \]
\[ (\varepsilon^1 - \varepsilon^0) / (1 - \varepsilon^1) = 1 - \exp(-0.54/10/\text{Vmvf1-1}) \cdot (vib) \cdot (0.75 \cdot \text{Vmvf1}/10) \]
\[ \Delta P = \text{H} \cdot (1 - \varepsilon) \cdot \text{Hd} \cdot \text{rhop} \cdot (g) \]
\[ \Delta P_v = P \cdot (1 - 0.0935 \cdot (dp / \text{Hd})^0.946 \cdot (vib)^0.606 \cdot s^1.637) \]
\[ \text{db} = 0.474 \cdot \left( \frac{(U - \text{Umvf1}) / (1.6 \cdot \text{D}t^0.4 \cdot g^0.5)}{(\text{Hd} + 3.94 \cdot \text{Adp}^0.5)^0.8} \right) \]
\[ \text{db} = 0.64 \cdot (\text{Ab} \cdot (U - \text{Umvf1})^0.4) \]
\[ \text{Re} = U \cdot dp / g \]
\[ \text{Sc} = g / (\text{rho} - \text{Dab} / 1000) \]
\[ \text{BV} = \text{Ab} \cdot \text{Hd} / 28316.736 \]
\[ (1 - \varepsilon) = (1 - \varepsilon^b) / (1 - 0.4) \]
\[ \text{aif} = 6 \cdot ^{(1 - \varepsilon)} / (s \cdot dp) \]

Balance & Humidity Equations
\[ \text{mb} = \text{Vb} / \text{BV} \]
\[ \text{mt} = \text{V} / \text{BV} \]
\[ \text{md} = \text{mt} - \text{mb} \]
\[ \text{Vd} = \text{md} \cdot \text{BV} \]
\[ \ln(P_{\text{wsat}}) = (-5800.2206 / \text{Ti}) + 1.3915 - 0.0486 \cdot \text{Ti} + 0.4176 \cdot E^{-4} \cdot \text{Ti}^2 - 0.1445 \cdot E^{-7} \cdot \text{Ti}^3 + 6.546 \cdot \ln(\text{Ti}) \]
\[ \text{Ysat} = -0.621 \cdot P_{\text{wsat}} / ((1.0133 \cdot E^5) - P_{\text{wsat}}) \]
\[ \text{Yb} = (6 \cdot \text{Kb} \cdot \varepsilon^b \cdot \text{Yd} + \text{db} \cdot \text{mb} \cdot \text{Yi}) / (\text{db} \cdot \text{mb} + 6 \cdot \text{Kb} \cdot \varepsilon^b) \]
\[ \text{Yd} \cdot \text{md} + \text{Yb} \cdot \text{mb} = \text{Ysat} \cdot \text{mt} \]
\[ \text{Td} \cdot \text{md} + \text{Tb} \cdot \text{mb} = \text{To} \cdot \text{mt} \]
\[ \text{Tb} = (\text{rho} \cdot \text{mb} \cdot \text{Ti} \cdot \text{db} \cdot (\alpha g + \text{Yi} \cdot \alpha v) + 6 \cdot \text{hb} \cdot \varepsilon^b \cdot \text{Td}) / (6 \cdot \text{hb} \cdot \varepsilon^b + \text{rho} \cdot \text{mb} \cdot \text{db} \cdot (\alpha g + \text{Yi} \cdot \alpha v)) \]
\[ \text{rho} \cdot \text{ub} \cdot (\text{Yd} - \text{Yi}) - (6 \cdot \text{Kb} \cdot \text{rho} \cdot \varepsilon^b \cdot \text{db}) \cdot (\text{Yb} - \text{Yd}) = (1 - \varepsilon) \cdot (\text{rho} \cdot \text{aif}^* \text{Ky}^* (\text{Ysat} - \text{Yd})) \]

Heat & Mass Transfer
\[ \text{Kb} = (\text{Umvf1}/3) + (4 \cdot \text{Dm}^* \cdot \text{ub} / (\text{db}))^{0.5} \]
Appendix 2 (Continued)

\[ Ky = \left( \frac{Dm}{\psi^*dp^*\phi} \right) \left( 2 + (1.8*Remvf^{0.5}*Sc^{0.33}) \right) \]
\[ hb = \left( \frac{Umf1*rho^*\alpha^*g/3}{3} \right) + \left( 4*\alpha^*g*rho^*Kg^*\epsilon^*ub/(\pi^*db) \right) \]

; *** Solving ODE's by Fourth Order Runge-Kutta method ***
call RK4('TWO,'y,'t,2) ; standard RK

Procedure: RK4
; Classical 4th-Order Runge-Kutta method
; Input Variables: EQ,y,x,ne

; Notation: EQ name of the function with the 1st-order equations
; y master list with names of lists representing
; the unknown functions
; x independent variable (list)
; ne number of the 1st-order equations
; K master list with names of lists of RK coefficients
; 'K#1 through 'K#4
; @yi,@ye auxiliary lists

; Description: This procedure function is an implementation of the classical
; 4th-order Runge-Kutta method for numerical integration of sets of ordinary
; differential equations represented by 1st-order equations

; Usage notes:
; 1. The list x and the initial conditions in the 1st elements of lists
; y1, y2, ... must be set prior to calling RK4.
; 2. The set of 1st-order differential equations must be defined in a
; function the name of which is passed as the value of EQ. The form
; of equations must be as follows:
; \[ y'[i] = f(x, y[1], y[2], ..., y[n]) \]
; The names y', y, x or any other names are local to that function and
; may be freely chosen; they map into the list names Kj and @ye, and
; into the current value of independent variable xi in this function.
; 3. Parameter variables may be used for transmitting the values of equation
; constants (if there are any) directly from the Variable Sheet to the
; Function Subsheet specifying the 1st-order equations.
; 4. Procedure RK4 can handle any number of 1st-order linear and nonlinear
; ordinary differential equations. The master list y (i.e. the list
; with the name passed onto y as symbolic value when calling RK4)
; must contain as its elements appropriate names of subordinate lists.

for j=1 to 4
'K[j][1]:= j ; seeding a matrix of RK coefficients
Appendix 2 (Continued)

next j
for e=1 to ne
call blank(y[e],2,length(y[e])); error indicates missing
next e; element in master list y
xi:= x[1]
for i=2 to length(x)
call statmsg(’Solving,’at,x,x[i])
for e=1 to ne
’@yi[e]:= y[e][i-1]; error at i=2 indicates missing
next e; e-th initial condition
call listcopy(’@yi,’@ye)
h:= (x[i]-xi)/2
for j=1 to 3
Kj:= ’K[j]
call apply(EQ,Kj,’@ye,xi)
if mod(j,2) then xi:= xi + h
if j=3 then h:= 2*h
for e=1 to ne
’@ye[e]:= ’@yi[e] + h*Kj[e]
next e
next j
call apply(EQ,’K#4,’@ye,xi)
for e=1 to ne
y[e][i]:= ’@yi[e] + (’K#1[e] + 2*(’K#2[e] + ’K#3[e]) + ’K#4[e])*h/6
next e
next i
call delete(’@yi)
call delete(’@ye)
for i=1 to 4
call delete(’K[i])
next i
call delete(’K)

Function: TWO
;Comment: 1st order equations
;y[1]=dC/dT
;y[2]=dT/s/dt
Parameter variables:
rho,rhop,ub,Yd,Yi,Yb,Kb,ε,εb,ε0,db,dp,αg,αv,αs,αw,Ti,Tb,Td,hb,λ,Dab,r, Cs
Input Variables: y’,y,t
y’[1]= (rho*ub*(Yd-Yi)-(6*Kb*rho*εb/db)*(Yb-Yd))/(-rhop*(1-ε)*(1-εb))
y’[2]= (rho*ub*(αg+Yi*αv)*(Ti-Td)+(6*hb*εb/db)*(Tb-Td)-λ*(rho*ub*(Yd-Yi)-(6*Kb+rho*εb/db)*(Yb-Yd)))/((αs+αw*y[1])*(1-εb)*(1-ε0)*rhop)
;y[1]=-(π^2*Dab/6r^2)*(y[1]-Cs)
Appendix 2 (Continued)

Variables:

<table>
<thead>
<tr>
<th>Name</th>
<th>Unit</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>SCFM</td>
<td>operational vol. air flow rate</td>
</tr>
<tr>
<td>A</td>
<td>m</td>
<td>operational vibrational amplitude</td>
</tr>
<tr>
<td>ω</td>
<td>rad/s</td>
<td>operational angular frequency</td>
</tr>
<tr>
<td>dp</td>
<td>cm</td>
<td>measured mean particle diameter</td>
</tr>
<tr>
<td>T</td>
<td>K</td>
<td>temperature</td>
</tr>
<tr>
<td>s</td>
<td>n.d.</td>
<td>spericity</td>
</tr>
<tr>
<td>g</td>
<td>cm/s^2</td>
<td>accel due to gravity</td>
</tr>
<tr>
<td>π</td>
<td>n.d.</td>
<td>pi</td>
</tr>
<tr>
<td>rhop</td>
<td>g/cm^3</td>
<td>particle density</td>
</tr>
<tr>
<td>rho</td>
<td>g/cm^3</td>
<td>air density</td>
</tr>
<tr>
<td>c</td>
<td>n.d.</td>
<td>vfb coefficient</td>
</tr>
<tr>
<td>ε0</td>
<td>n.d.</td>
<td>bed voidage (static)</td>
</tr>
<tr>
<td>Hd</td>
<td>cm</td>
<td>bed height (dynamic)</td>
</tr>
<tr>
<td>H</td>
<td>cm</td>
<td>bed height (static)</td>
</tr>
<tr>
<td>Yi</td>
<td>%</td>
<td>humidity</td>
</tr>
<tr>
<td>Ab</td>
<td>cm^2</td>
<td>cross sectional bed area</td>
</tr>
<tr>
<td>k</td>
<td>n.d.</td>
<td>coeffic of collision elasticity</td>
</tr>
<tr>
<td>j</td>
<td>n.d.</td>
<td>lift &amp; fall time / vib. period</td>
</tr>
<tr>
<td>Kg</td>
<td>cal/cm<em>s</em>c</td>
<td>thermal conductivity of air</td>
</tr>
<tr>
<td>Dab</td>
<td>cm/s</td>
<td>binary diffusivity</td>
</tr>
<tr>
<td>φ</td>
<td>n.d.</td>
<td>mole frac. nondiffusing component.</td>
</tr>
<tr>
<td>Ti</td>
<td>K</td>
<td>temp in</td>
</tr>
<tr>
<td>αV</td>
<td>cal/g*c</td>
<td>specific heat</td>
</tr>
<tr>
<td>αg</td>
<td>cal/g*c</td>
<td>specific heat</td>
</tr>
<tr>
<td>αs</td>
<td>cal/g*c</td>
<td>specific heat</td>
</tr>
<tr>
<td>αw</td>
<td>cal/g*c</td>
<td>specific heat</td>
</tr>
<tr>
<td>λ</td>
<td>cal/g</td>
<td>latent heat of vaporization</td>
</tr>
<tr>
<td>BV</td>
<td>ft^3</td>
<td>bed volume</td>
</tr>
<tr>
<td>vib</td>
<td></td>
<td>vibrational factor</td>
</tr>
<tr>
<td>U</td>
<td>cm/s</td>
<td>operational air velocity</td>
</tr>
<tr>
<td>Umf</td>
<td>cm/s</td>
<td>min. fluidiz. air velocity</td>
</tr>
<tr>
<td>Vmf</td>
<td>SCFM</td>
<td>min. fluidiz. vol. air rate</td>
</tr>
<tr>
<td>Umvf1</td>
<td>cm/s</td>
<td>min. vib. fluidiz. air velocity</td>
</tr>
<tr>
<td>Vmf1</td>
<td>SCFM</td>
<td>min. vib. fluidiz. vol. air rate</td>
</tr>
<tr>
<td>db</td>
<td>cm</td>
<td>bubble diameter</td>
</tr>
<tr>
<td>uT</td>
<td>cm/s</td>
<td>terminal air velocity</td>
</tr>
<tr>
<td>ub</td>
<td>cm/s</td>
<td>bubble air velocity</td>
</tr>
<tr>
<td>Vd</td>
<td>SCFM</td>
<td>dense phase vol. air rate</td>
</tr>
</tbody>
</table>
Appendix 2 (Continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vt</td>
<td>SCFM</td>
<td>vol. air rate</td>
</tr>
<tr>
<td>Vb</td>
<td>SCFM</td>
<td>bubble phase vol. air rate</td>
</tr>
<tr>
<td>md</td>
<td>s^-1</td>
<td>dense gas flow rate / unit vol of bed</td>
</tr>
<tr>
<td>mt</td>
<td>s^-1</td>
<td>gas flow rate / unit vol of bed</td>
</tr>
<tr>
<td>mb</td>
<td>s^-1</td>
<td>bubble gas flow rate / unit vol of bed</td>
</tr>
<tr>
<td>g</td>
<td>m^2/s</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>Re</td>
<td>n.d.</td>
<td>bed reynolds number</td>
</tr>
<tr>
<td>Sc</td>
<td>n.d.</td>
<td>bed schmidt number</td>
</tr>
<tr>
<td>ε</td>
<td>n.d.</td>
<td>bed voidage</td>
</tr>
<tr>
<td>εb</td>
<td>n.d.</td>
<td>bed voidage bubble frac.</td>
</tr>
<tr>
<td>aif</td>
<td>cm^-1</td>
<td>interfacial area/bed vol</td>
</tr>
<tr>
<td>Pwsat</td>
<td>Pa</td>
<td>saturation pressure</td>
</tr>
<tr>
<td>Ysat</td>
<td>%</td>
<td>humidity at saturation</td>
</tr>
<tr>
<td>Yb</td>
<td>%</td>
<td>bubble phase humidity</td>
</tr>
<tr>
<td>Yd</td>
<td>%</td>
<td>dense phase humidity</td>
</tr>
<tr>
<td>Kb</td>
<td>Cm/s</td>
<td>mass transfer coeff dense to bubble</td>
</tr>
<tr>
<td>Ky</td>
<td>Cm/s</td>
<td>mass transfer coeff particle to gas</td>
</tr>
<tr>
<td>hb</td>
<td>Cal/cm^2 s C</td>
<td>heat transfer coeff bubble to dense</td>
</tr>
<tr>
<td>To</td>
<td>K</td>
<td>temp out</td>
</tr>
<tr>
<td>Tb</td>
<td>K</td>
<td>temp bubble phase</td>
</tr>
<tr>
<td>Td</td>
<td>K</td>
<td>temp dense phase</td>
</tr>
</tbody>
</table>
Table 12 – Sieving Data Tabulation

<table>
<thead>
<tr>
<th>U.S. Sieve #</th>
<th>Sieve Size (mm)</th>
<th>W_i (g)</th>
<th>P_i (%)</th>
<th>Σ P_i (%&lt;)</th>
<th>log d_i</th>
<th>W_i log d_i</th>
<th>log d_i - log d_{gw}</th>
<th>W_i(log d_i - log d_{gw})</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.330</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
<td>1.056</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.423</td>
<td>0.00</td>
<td>0.00</td>
<td>100.00</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.680</td>
<td>2.55</td>
<td>6.07</td>
<td>93.93</td>
<td>0.906</td>
<td>2.313</td>
<td>0.323</td>
<td>0.824</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.760</td>
<td>11.83</td>
<td>65.82</td>
<td>0.757</td>
<td>8.960</td>
<td>0.174</td>
<td>2.063</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3.360</td>
<td>13.03</td>
<td>30.97</td>
<td>0.609</td>
<td>7.931</td>
<td>0.025</td>
<td>0.332</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.000</td>
<td>11.22</td>
<td>26.66</td>
<td>8.19</td>
<td>4.802</td>
<td>-0.155</td>
<td>-1.737</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1.168</td>
<td>2.91</td>
<td>6.92</td>
<td>1.26</td>
<td>0.200</td>
<td>0.582</td>
<td>-0.383</td>
<td>-1.117</td>
</tr>
<tr>
<td>20</td>
<td>0.840</td>
<td>0.32</td>
<td>0.75</td>
<td>0.51</td>
<td>0.002</td>
<td>0.001</td>
<td>-0.581</td>
<td>-0.184</td>
</tr>
<tr>
<td>30</td>
<td>0.590</td>
<td>0.11</td>
<td>0.27</td>
<td>0.24</td>
<td>-0.146</td>
<td>-0.016</td>
<td>-0.729</td>
<td>-0.082</td>
</tr>
<tr>
<td>40</td>
<td>0.420</td>
<td>0.05</td>
<td>0.13</td>
<td>0.11</td>
<td>-0.297</td>
<td>-0.016</td>
<td>-0.880</td>
<td>-0.048</td>
</tr>
<tr>
<td>50</td>
<td>0.297</td>
<td>0.03</td>
<td>0.07</td>
<td>0.04</td>
<td>-0.446</td>
<td>-0.014</td>
<td>-1.029</td>
<td>-0.032</td>
</tr>
<tr>
<td>70</td>
<td>0.210</td>
<td>0.02</td>
<td>0.04</td>
<td>0</td>
<td>-0.596</td>
<td>-0.010</td>
<td>-1.179</td>
<td>-0.020</td>
</tr>
<tr>
<td>Summation</td>
<td>42.078</td>
<td>100.00</td>
<td>24.532</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ d_{gw} = \log^{-1} \left[ \frac{\sum W_i \log d_i}{\sum W_i} \right] = 3.829 \text{ mm} \]  

(37)

\[ S_{\log} = \sqrt{\frac{\sum W_i (\log d_i - \log d_{gw})^2}{\sum W_i}} = 1.228\text{E-08 non. dim.} \]  

(38)

\[ S_{gw} = \frac{1}{2} \left( d_{gw} \left[ \log^{-1} S_{\ln} - (\log^{-1} S_{\ln})^{-1} \right] \right) = 2.491\text{E-07 mm} \]  

(39)
### Table 13 – Regression Statistics for A/D Calibration

<table>
<thead>
<tr>
<th>Input Channel #</th>
<th>Number of Observations</th>
<th>Correlation Coefficient (R²)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>51</td>
<td>1.000</td>
<td>0.0005</td>
<td>1.0006</td>
<td>0.9996</td>
</tr>
<tr>
<td>2</td>
<td>41</td>
<td>1.000</td>
<td>0.0002</td>
<td>1.0003</td>
<td>0.9998</td>
</tr>
<tr>
<td>3</td>
<td>51</td>
<td>1.000</td>
<td>0.0001</td>
<td>1.0002</td>
<td>0.9999</td>
</tr>
<tr>
<td>4</td>
<td>27</td>
<td>1.000</td>
<td>0.0007</td>
<td>1.0001</td>
<td>0.9998</td>
</tr>
<tr>
<td>5</td>
<td>39</td>
<td>1.000</td>
<td>0.0029</td>
<td>1.0002</td>
<td>0.9996</td>
</tr>
<tr>
<td>6</td>
<td>51</td>
<td>1.000</td>
<td>0.0001</td>
<td>1.0002</td>
<td>0.9999</td>
</tr>
<tr>
<td>7</td>
<td>39</td>
<td>1.000</td>
<td>0.0022</td>
<td>1.0002</td>
<td>0.9997</td>
</tr>
<tr>
<td>8</td>
<td>39</td>
<td>1.000</td>
<td>0.0011</td>
<td>1.0002</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

### Table 14 – Regression Analysis of Variance for the A/D Calibration

<table>
<thead>
<tr>
<th>Input Channel #</th>
<th>Sum of Square Regression</th>
<th>Sum of Square Residual</th>
<th>Mean Square of Regression</th>
<th>Mean Square of Residual</th>
<th>F-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.420</td>
<td>1.352E-05</td>
<td>4.421</td>
<td>2.7604E-07</td>
<td>1.602E+07</td>
</tr>
<tr>
<td>2</td>
<td>2.296</td>
<td>1.200E-05</td>
<td>2.296</td>
<td>3.060E-07</td>
<td>7.491E+07</td>
</tr>
<tr>
<td>3</td>
<td>4.421</td>
<td>8.230E-05</td>
<td>4.421</td>
<td>1.680E-07</td>
<td>2.630E+08</td>
</tr>
<tr>
<td>4</td>
<td>102.369</td>
<td>1.090E-05</td>
<td>102.369</td>
<td>4.360E-07</td>
<td>1.090E-88</td>
</tr>
<tr>
<td>5</td>
<td>308.689</td>
<td>3.130E-04</td>
<td>308.689</td>
<td>8.450E-06</td>
<td>1.60E-122</td>
</tr>
<tr>
<td>6</td>
<td>4.420</td>
<td>6.890E-07</td>
<td>4.420</td>
<td>1.410E-08</td>
<td>3.140E+08</td>
</tr>
<tr>
<td>7</td>
<td>308.703</td>
<td>1.720E-04</td>
<td>308.703</td>
<td>4.640E-06</td>
<td>6.647E+07</td>
</tr>
<tr>
<td>8</td>
<td>308.767</td>
<td>4.580E-05</td>
<td>308.766</td>
<td>1.240E-06</td>
<td>2.500E+08</td>
</tr>
</tbody>
</table>
Appendix 4 (Continued)

Table 15 – Regression Statistics for Mass Flow Meter Calibration

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>39</td>
<td>0.994</td>
<td>0.0753</td>
<td>1.0157</td>
<td>0.9996</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of Square Regression</th>
<th>Sum of Square Residual</th>
<th>Mean Square of Regression</th>
<th>Mean Square of Residual</th>
<th>F-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.013</td>
<td>3.67E2</td>
<td>3.67E2</td>
<td>5.67E-3</td>
<td>6.477E5</td>
</tr>
</tbody>
</table>

Figure 49 – Flow Meter Calibration Plot
Appendix 4 (Continued)

Table 16 – Regression Statistics for the Differential Pressure Sensor Calibration

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>0.9997</td>
<td>0.0329</td>
<td>1.0103</td>
<td>0.9953</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of Square Regression</th>
<th>Sum of Square Residual</th>
<th>Mean Square of Regression</th>
<th>Mean Square of Residual</th>
<th>F-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>84.472</td>
<td>2.06E-2</td>
<td>84.472</td>
<td>1.0857E-3</td>
<td>7.7842E5</td>
</tr>
</tbody>
</table>

Figure 50 – Differential Pressure Sensor Calibration Plot
Appendix 4 (Continued)

Table 17 – Inlet Thermocouple Calibration Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Merged Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>23.081</td>
<td>100.259</td>
<td>20.848</td>
<td>21.357</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.459</td>
<td>0.524</td>
<td>0.650</td>
<td>0.471</td>
</tr>
<tr>
<td>Median</td>
<td>23.010</td>
<td>100.556</td>
<td>21.493</td>
<td>21.697</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.052</td>
<td>2.344</td>
<td>2.906</td>
<td>3.105</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.451</td>
<td>0.086</td>
<td>0.344</td>
<td>0.741</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.195</td>
<td>-0.667</td>
<td>-0.540</td>
<td>-0.168</td>
</tr>
<tr>
<td>Range</td>
<td>7.720</td>
<td>8.626</td>
<td>11.230</td>
<td>15.462</td>
</tr>
<tr>
<td>Minimum</td>
<td>19.511</td>
<td>95.052</td>
<td>14.210</td>
<td>11.769</td>
</tr>
<tr>
<td>Maximum</td>
<td>27.231</td>
<td>103.678</td>
<td>25.441</td>
<td>27.231</td>
</tr>
<tr>
<td>Count</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Conf. Level(95.0%)</td>
<td>0.961</td>
<td>1.097</td>
<td>1.360</td>
<td>1.550</td>
</tr>
</tbody>
</table>

Table 18 – Outlet Thermocouple Calibration Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Merged Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>21.663</td>
<td>101.413</td>
<td>22.071</td>
<td>21.812</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.516</td>
<td>0.550</td>
<td>0.442</td>
<td>0.514</td>
</tr>
<tr>
<td>Median</td>
<td>22.014</td>
<td>101.023</td>
<td>22.582</td>
<td>22.330</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.309</td>
<td>2.462</td>
<td>1.977</td>
<td>2.297</td>
</tr>
<tr>
<td>Sample Variance</td>
<td>5.332</td>
<td>6.060</td>
<td>3.907</td>
<td>5.765</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-0.682</td>
<td>-0.948</td>
<td>-1.144</td>
<td>-0.237</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.053</td>
<td>0.250</td>
<td>-0.475</td>
<td>-0.457</td>
</tr>
<tr>
<td>Range</td>
<td>8.301</td>
<td>8.301</td>
<td>6.035</td>
<td>8.333</td>
</tr>
<tr>
<td>Minimum</td>
<td>16.163</td>
<td>97.819</td>
<td>18.405</td>
<td>16.131</td>
</tr>
<tr>
<td>Maximum</td>
<td>24.464</td>
<td>106.120</td>
<td>24.439</td>
<td>24.464</td>
</tr>
<tr>
<td>Count</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Conf. Level(95.0%)</td>
<td>1.081</td>
<td>1.152</td>
<td>0.925</td>
<td>1.089</td>
</tr>
</tbody>
</table>
Figure 51 – Linear Regression for Drying Parameter Determination and Drying Curves, 144.7°C
Appendix 5 (Continued)

Figure 52 – Linear Regression for Drying Parameter Determination and Drying Curves, 117.7°C
Figure 53 – Linear Regression for Drying Parameter Determination and Drying Curves, 137.7°C
Appendix 5 (Continued)

Figure 54 – Linear Regression for Drying Parameter Determination and Drying Curves, 152.4°C
y = 2.9921x - 7.7075
R² = 0.994

Figure 55 – Linear Regression for Drying Parameter Determination and Drying Curves, 104.1°C
### APPENDIX 6 – DRYING MODEL REGRESSION STATISTICS

**Table 19 – Regression Statistics for the Three-Phase Model at 144.7°C**

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.9971</td>
<td>0.0234</td>
<td>1.0529</td>
<td>0.9303</td>
</tr>
<tr>
<td>Sum of Square Regression</td>
<td>Sum of Square Residual</td>
<td>Mean Square of Regression</td>
<td>Mean Square of Residual</td>
<td>F-Ratio</td>
</tr>
<tr>
<td>0.9496</td>
<td>2.75E-3</td>
<td>0.9496</td>
<td>5.490E-4</td>
<td>1.7289E3</td>
</tr>
</tbody>
</table>

**Table 20 – Regression Statistics for the Three-Phase Model at 117.7°C**

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.9962</td>
<td>0.0268</td>
<td>1.0936</td>
<td>0.9664</td>
</tr>
<tr>
<td>Sum of Square Regression</td>
<td>Sum of Square Residual</td>
<td>Mean Square of Regression</td>
<td>Mean Square of Residual</td>
<td>F-Ratio</td>
</tr>
<tr>
<td>1.1303</td>
<td>4.31E-3</td>
<td>1.1303</td>
<td>7.19E-4</td>
<td>1.72E3</td>
</tr>
</tbody>
</table>

**Table 21 – Regression Statistics for the Three-Phase Model at 137.7°C**

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.9991</td>
<td>0.0121</td>
<td>1.0146</td>
<td>0.9488</td>
</tr>
<tr>
<td>Sum of Square Regression</td>
<td>Sum of Square Residual</td>
<td>Mean Square of Regression</td>
<td>Mean Square of Residual</td>
<td>F-Ratio</td>
</tr>
<tr>
<td>0.8635</td>
<td>7.33E-4</td>
<td>0.8635</td>
<td>1.47E-4</td>
<td>5.891E3</td>
</tr>
</tbody>
</table>
Appendix 6 (Continued)

Table 22 – Regression Statistics for the Three-Phase Model at 152.4°C

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.9961</td>
<td>0.02650</td>
<td>1.0714</td>
<td>0.9293</td>
</tr>
<tr>
<td>Sum of Square Regression</td>
<td>Sum of Square Residual</td>
<td>Mean Square of Regression</td>
<td>Mean Square of Residual</td>
<td>F-Ratio</td>
</tr>
<tr>
<td>0.9203</td>
<td>3.51E-3</td>
<td>0.9203</td>
<td>7.02E-4</td>
<td>1.310E3</td>
</tr>
</tbody>
</table>

Table 23 – Regression Statistics for the Three-Phase Model at 104.1°C

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.9908</td>
<td>0.0425</td>
<td>1.1006</td>
<td>0.9077</td>
</tr>
<tr>
<td>Sum of Square Regression</td>
<td>Sum of Square Residual</td>
<td>Mean Square of Regression</td>
<td>Mean Square of Residual</td>
<td>F-Ratio</td>
</tr>
<tr>
<td>1.1709</td>
<td>1.083E-2</td>
<td>1.1709</td>
<td>1.0804E-3</td>
<td>6.488E2</td>
</tr>
</tbody>
</table>

Table 24 – Regression Statistics for the Thin-Layer Model at 144.7°C

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.9941</td>
<td>0.0340</td>
<td>1.0934</td>
<td>0.9157</td>
</tr>
<tr>
<td>Sum of Square Regression</td>
<td>Sum of Square Residual</td>
<td>Mean Square of Regression</td>
<td>Mean Square of Residual</td>
<td>F-Ratio</td>
</tr>
<tr>
<td>0.9744</td>
<td>5.769E-3</td>
<td>0.9745</td>
<td>1.154E-3</td>
<td>8.446E2</td>
</tr>
</tbody>
</table>

Table 25 – Regression Statistics for the Thin-Layer Model at 117.7°C

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.9914</td>
<td>0.0363</td>
<td>1.0833</td>
<td>0.9112</td>
</tr>
<tr>
<td>Sum of Square Regression</td>
<td>Sum of Square Residual</td>
<td>Mean Square of Regression</td>
<td>Mean Square of Residual</td>
<td>F-Ratio</td>
</tr>
<tr>
<td>1.0595</td>
<td>8.899E-2</td>
<td>1.0595</td>
<td>1.317E-3</td>
<td>8.048E2</td>
</tr>
</tbody>
</table>
Appendix 6 (Continued)

Table 26 – Regression Statistics for the Thin-Layer Model at 137.7°C

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.9987</td>
<td>0.0145</td>
<td>1.0224</td>
<td>0.9439</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of Square Regression</th>
<th>Sum of Square Residual</th>
<th>Mean Square of Regression</th>
<th>Mean Square of Residual</th>
<th>F-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.8659</td>
<td>2.090E-4</td>
<td>4.1413E3</td>
</tr>
</tbody>
</table>

Table 27 – Regression Statistics for the Thin-Layer Model at 152.4°C

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.9962</td>
<td>0.0264</td>
<td>1.0641</td>
<td>0.9224</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of Square Regression</th>
<th>Sum of Square Residual</th>
<th>Mean Square of Regression</th>
<th>Mean Square of Residual</th>
<th>F-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.9074</td>
<td>6.980E-4</td>
<td>1.2994E3</td>
</tr>
</tbody>
</table>

Table 28 – Regression Statistics for the Thin-Layer Model at 104.1°C

<table>
<thead>
<tr>
<th>Number of Observations</th>
<th>Correlation Coefficient ($R^2$)</th>
<th>Standard Error</th>
<th>Upper 95% Confidence Limit</th>
<th>Lower 95% Confidence Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>0.9971</td>
<td>0.0218</td>
<td>1.0518</td>
<td>0.9527</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sum of Square Regression</th>
<th>Sum of Square Residual</th>
<th>Mean Square of Regression</th>
<th>Mean Square of Residual</th>
<th>F-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.1666</td>
<td>4.770E-4</td>
<td>2.4449E3</td>
</tr>
</tbody>
</table>
Included in this dissertation are 3 videos produced to help illustrate the dissertation research. The first (Drying.wmv), is a time-compressed representation of a typical drying trial. It is 3:47 minutes long, and utilizes a split screen format. This allows the viewer to watch the vibrofluidized bed drying process while graphs of the data are generated in the side panel. It is useful for linking the observed bed hydrodynamic behavior to the acquired data, throughout a trial.

The second (Vfbd.wmv), is a low-resolution 31 second clip of the citrus processing residue being dried in the vibrofluidized bed dryer. This video allows the viewer an opportunity to quickly view the quality of fluidization achieved in the bed, at the conditions used for the experimental drying trials. The viewer can observe that stable fluidization is achieved, the bed is well mixed, and there are no agglomeration or elutriation problems.

The third video (Intro.wmv), is a video produced for the dissertation defense. It is 6:20 minutes in length and uses video footage from actual processing plants, as well as from the laboratory. It provides the viewer an introduction to the citrus industry, the challenges associated with the disposal of the processing residue, an overview of the existing feedmill process and an introduction to the proposed vibrofluidized bed drying process.
ERIC ROE is a Ph.D. candidate in Chemical Engineering at the University of South Florida (USF). He received his MS in Chemical Engineering from USF. During his time at USF, and in addition to his research into fluidized bed drying, he has been a consultant to the Citrus Industry, and has worked on the State Test Lab Sampler project - funded by the Florida Department of Citrus, the High School Technology Initiative - funded by the National Science Foundation (NSF), and the Florida Center for Manufacturing Education project - also funded by NSF. Prior to his study at USF, he was employed as a technologist in Research and Development at Tropicana Products, Inc., with process and product development responsibilities. His research interests are food engineering, fluidized bed drying, and the integration of engineering and education.