Characterization of Preliminary Breast Tomosynthesis Data: Noise and Power Spectra Analysis

Madhusmita Behera

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Characterization of Preliminary Breast Tomosynthesis Data:

Noise and Power Spectra Analysis

by

Madhusmita Behera

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Biomedical Engineering
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Keywords: mammography, filtering, signal-dependent noise, wavelet-expansion, Fourier transform

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<tr>
<td>BC</td>
<td>Breast Cancer</td>
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<td>ACS</td>
<td>American Cancer Society</td>
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<td>ART</td>
<td>Algebraic Reconstruction Technique</td>
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<td>DQE</td>
<td>Detective Quantum Efficiency</td>
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<td>GFBP</td>
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<td>MAS</td>
<td>Milli-Ampere Seconds</td>
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<td>MRI</td>
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<td>MTF</td>
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CHARACTERIZATION OF PRELIMINARY BREAST

TOMOSYNTHESIS DATA:

NOISE AND POWER SPECTRA ANALYSIS

Madhusmita Behera

ABSTRACT

Early detection, diagnosis, and suitable treatment are known to significantly improve the chance of survival for breast cancer (BC) patients. To date, the most cost effective method for screening and early detection is screen-film mammography, which is also the only tool that has demonstrated its ability to reduce BC mortality. Full-field digital mammography (FFDM) is an extension of screen-film mammography that eliminates the need for film-processing because the images are detected electronically from their inception. Tomosynthesis is an emerging technology in digital mammography built on the FFDM framework, which offers an alternative to conventional two-dimensional mammography. Tomosynthesis produces three-dimensional (volumetric) images of the breast that may be superior to planar imaging due to improved visualization.
In this work preliminary tomosynthesis data derived from cadaver breasts are analyzed, which includes volume data acquired from various reconstruction techniques as well as the planar projection data. The noise and power spectra characteristics analyses are the focus of this study.

Understanding the noise characteristics is significant in the study of radiological images and in the evaluation of the imaging system, so that its degrading effect on the image can be minimized, if possible and lead to better diagnosis and optimal computer aided diagnosis schemes. Likewise, the power spectra behavior of the data are analyzed, so that statistical methods developed for digitized film images or FFDM images may be applied directly or modified accordingly for tomosynthesis applications.

The work shows that, in general, the power spectra for three of the reconstruction techniques are very similar to the spectra of planar FFDM data as well as digitized film; projection data analysis follows the same trend. To a good approximation the Fourier power spectra obey an inverse power law, which indicates a degree of self-similarity. The noise analysis indicates that the noise and signal are dependent and the dependency is a function of the reconstruction technique. New approaches for the analysis of signal dependent noise were developed specifically for this work based on both the linear wavelet expansion and on nonlinear order statistics. These methods were tested on simulated data that closely follow the statistics of mammograms prior to the real-data applications. The noise analysis methods are general and have applications beyond mammography.
CHAPTER 1
INTRODUCTION

Breast cancer is the most commonly diagnosed cancer and the second leading cause of cancer death among women in United States [ACS 2003], following only lung cancer. In 2004, 40,580 people (40,110 women and 470 men) are projected to die of BC [Jemal et al 2004]. Statistics show that the lifetime risk of BC in the United States has almost tripled in the past 50 years. In the 1940’s, a woman’s lifetime risk of BC was 1 in 22 that increased to 1 in 8 in the year 2002 [MBCC 2002].

Although it is primarily a disease of women, about 1% of BCs occur in men [Jemal et al 2003; Anderson et al 2004]. Breast cancer is caused by the uncontrolled growth of cells in the breast. The female breast as illustrated in Fig.1.1 is primarily composed of lobules (milk-producing glands), ducts (milk channels that link the lobules to the nipple), and the stroma. Approximately 90% of BCs begin in the milk ducts, and 10% begin in the lobules of the breast [Wellings et al 1975]. When the cancer cells remain within the ducts, the cancer is referred as in situ and the probability of cure is high. Once the cells have broken through the wall of a duct or lobule, the cancer is called invasive. The most common types of invasive BCs are Ductal Carcinoma and Lobular Carcinoma.
Before the 1990's, BC mortality rates were constant for nearly four decades. During 1989-1995 the BC mortality declined by 1.6% and by 3.5% from 1995-1999 [MBCC 2002]. Most medical experts agree that this decline in the mortality rate can be attributed to the increasing awareness in the public that leads to the early detection of BC followed by proper treatment and regular follow-up. This is in agreement with previous studies that have shown that early detection, diagnosis and suitable treatment can significantly improve the chance of survival for patients with BC [Chan et al 1995; Lester 1984]. This can be successfully accomplished by effective screening methods of BC [Yaffe 2000].
1.1 BC Imaging and Mammography

It is established that early detection of BC can reduce BC mortality. This is most commonly accomplished with regular screening and mammographic imaging. The only definite method of determining the malignancy of the breast tissue is by a biopsy. The breast biopsy involves removing the tissue sample surgically, or with a less-invasive needle core sampling procedure, to determine whether it is cancerous or benign. Most biopsy methods rely on image guidance to help the radiologist or breast surgeon precisely locate the lesion or abnormality within the breast. Imaging techniques of the breast are therefore vital for early detection of cancer, and localization of the suspicious lesion in the breast for a biopsy procedure.

Some of the imaging modalities available today for breast imaging are:

1.1.1 Magnetic Resonance Imaging (MRI) This is a diagnostic procedure that uses magnetic fields and computers to create images of areas inside the breast. With MRI, the contrast between the soft tissues in the breast is 10 to 100 times greater than that obtained with X-rays [Azar 2000]. This is an expensive procedure.

1.1.2 Breast Ultrasound This technique uses sound waves to create an image of the breast tissue and project it onto a computer screen.

1.1.3 Positron Emission Tomography (PET) This diagnostic procedure involves injecting the patient with a radioactive compound that is taken up by suspicious cells. The
positron radioactivity emitted by the compound is recorded by a PET camera and processed by computer. The areas of greatest metabolic activity light up on a computer generated image which assists the radiologist in identifying suspicious tissue or lesion.

1.1.4 Mammography It is an X-ray screening technique that is used to create detailed images of the breast. Among the imaging systems discussed here, the most widely used modality for BC detection and diagnosis is mammography for cost effectiveness and its ability to reduce BC mortality. A mammogram is an X-ray projection of the breast. These projections are usually taken from two-views: 1) Cranio-caudal, where X-rays are passed through the breast from top to bottom, and 2) Medio-lateral, where the X-rays are passed through the breast from the side. The schematic representation of the mammographic imaging system is illustrated in Fig.1.2. When a mammogram is performed, a beam of X-ray is incident on a compressed breast. The energy spectrum of the beam is characteristic of the X-ray target, filter and tube voltage. The intensity of the beam is correlated with the breast size and composition to some degree due to the experience of the X-ray technician involved with the process and the automated exposure control of modern systems; basically larger and more dense the breast implies longer exposure times. The photon interaction with the breast involves both absorption and scattering. The absorption characteristic of a given tissue is dependent upon the spectral character of the incoming beam. The X-ray photons exiting the breast pass through an anti-scatter grid before reaching a phosphorous intensifying screen. If an X-ray photon is absorbed by the screen, light photons are emitted that expose a film or more generally interact with some form of detector. In areas of the breast where large absorption takes
place the signal is weakest and an area where less absorption occurs, the signal is strongest. Thus, the resulting image represents a crude abstraction of the average attenuation properties of the breast above the detector. As the X-rays pass through the breast, they are attenuated by the different tissue densities within the breast.

**Figure 1.2** Schematic Representation of Screen-film Mammography System

[Image from: http://detserv1.dl.ac.uk/Herald/images/MammoSet.gif]
The appearance of a female breast on a mammogram varies due to the differences in X-ray attenuation in the relative amounts of fat, connective and epithelial tissue. Fat appears radiolucent or dark on a mammogram, epithelial and connective tissues are radiographically dense and appear lighter or white in the developed image. Some relevant findings or abnormalities in a mammogram include [Kaul et al 2002]:

*soft-tissue lesions* These are recognized as a mass or an architectural distortion. A *mass* is often defined as a region of increased density usually with a distinct edge, making it distinguishable from the surrounding breast tissue. *Architectural distortions* are irregular breast patterns caused by abnormal tissue.

*microcalcifications* These are seen as small calcium deposits in the breast tissue. They can typically build up in clusters. Depending on their number in a cluster and the overall shape of the cluster they indicate a possible risk of BC.

### 1.2 Screen-Film Mammography

To date, the most cost effective method for screening and early detection is screen-film mammography, which is also the only tool that has demonstrated its ability to reduce BC mortality [Yaffe 2000]. Conventional screen film mammography uses low energy X-rays that pass through a compressed breast during a mammographic examination. The exiting X-rays are absorbed by film (screen film combination), which is then developed into a two dimensional or planar mammographic image, that is interpreted by the radiologist for diagnostic purposes, with the use of a light box normally. Although there are benefits
associated with film-based screening, there are considerable interpretation errors that are in part caused by the image quality, restricted intensity latitude, and close similarities between normal tissue and suspicious tissue. In order to process these images with computer techniques, they must be digitized. Since this work involves the analysis of film-less acquired mammograms, film detected images will not be discussed further.

1.3 Full Field Digital Mammography

FFDM is an advancement of screen-film mammography that eliminates the need for film-processing because the images are detected electronically for their inception. From a subject’s perspective, the examination is the same as in film mammography, where the breast is positioned between two flat plates and lightly compressed. In FFDM system, the screen-film image receptor is replaced with a flat detector which provides an electronic signal that is proportional to the X-ray exposure [Yaffe 2000]. This system is intended to replace conventional film based imaging in the future. FFDM acquired images are ideal for digital or computer processing without further manipulation and are viewed on soft-copy display when viewed by the mammographer. FFDM offers numerous advantages such as digital image management, digital data transfer, digital image processing. With the capability to process the digital images with a computer, new medical applications will emerge, for example, real-time Computer-Aided Diagnosis (CAD), contrast medium imaging etc.

Fig.1.3 illustrates the working of a mammography system [Moore 2001]. All mammography begins with an X-ray source, which projects photons through a patient. In
conventional mammography, the development of a film-screen cassette generates the mammogram. There are two principal methods of detection in the digital version of mammography: indirect, in which a scintillator converts X-rays into visible light that is collected by a solid-state detector; and direct, which involves employing a coating of a material such as amorphous selenium to convert the X-rays into electron-hole pairs for sensing by a transistor array.

Figure 1.3 The Working of a Digital Mammography System

Standard planar mammography techniques, film or FFDM, suffer from the limitation that three-dimensional anatomical information is projected onto a two-dimensional detector. Thus, the spatial arrangement of tissues cannot be preserved, causing the loss of
morphological information. Cancers may be masked by radiographically dense fibro-glandular breast tissue which may encompass the cancer. Likewise, the true character of breast tissue is somewhat lost or obscured.

1.4 Tomosynthesis

Tomosynthesis is a forthcoming technology in digital mammography built on the FFDM framework, which offers an alternative to conventional two-dimensional mammography. Tomosynthesis is a technique that allows the radiologist to view individual planes of the breast, potentially reducing the problem of superimposed structures that may limit conventional mammography techniques. Since this technology is new and very few systems are available today, most of the research on the subject has been limited to using phantom images. The tomosynthesis data (TD) used for this study are images generated from cadaver breasts.

*How Tomosynthesis works* Tomosynthesis combines a conventional tube with a digital X-ray detector and sophisticated computer algorithms. In this method, multiple projection images from different angles are acquired as the X-ray tube is moved in an arc above the stationary breast and digital detector. A reconstruction technique is then applied to capture a volume of three-dimensional information from the series of projection images. The image obtained at each angle is of low radiation dose, with the total radiation dose required for imaging the entire breast being somewhat less than the dose used for a two-view film-screen mammogram. Since the resulting volumetric information is in digital format, it can be reconstructed in any plane. A key aspect of tomosynthesis is that in the
reconstructed volume, the in-plane resolution is 100\(\mu m\), and the inter-plane resolution is 1000\(\mu m\), which is an artifact of limited number of projections. The reconstructed volume will have slices available at various depths or planes within the breast, as illustrated in Fig.1.4. It is anticipated that by stepping through the slices, one can eliminate the superimposed tissues that might be hiding the tumor or malignancy.

**Figure 1.4** The X-ray Beam Creates 2D Projection View of the 3D breast. A 3D reconstruction of the breast can be viewed as a sequence of 2D planes (shown as colored). Thus, the tomosynthesis image data represents two distinct representations (1) the 12 projections that appear similar to regular planar X-ray image but with more variation due to the reduced X-ray strength, and (2) the reconstructed volumetric data

*Prospective benefits of tomosynthesis*  Tomosynthesis allows three-dimensional (volumetric) imaging of the breast and potentially allows BC to be better visualized
without the superimposed dense breast tissue that may obscure BC in conventional mammography. The potential benefit will be greatest in women with radiographically dense breasts. In addition, improved visibility of a lesion, lesion extent and lesion margins may improve specificity and treatment. Tomographic imaging provides three basic advantages over conventional projection mammography as discussed by Dobbins et al [Dobbins et al 2003], which are significant for BC research. First, it allows depth localization of a lesion. Second, it improved conspicuity of structures by removing the visual clutter associated with overlying anatomy. Third, it improves the contrast of local structures by restricting the overall image dynamic range to that of a single slice. These advantages provide an exciting opportunity for researchers to study BC in depth and the risk factors associated with it. This has been illustrated by an example in Fig.1.5.

![Comparison of The Visibility of a Lesion in 2D and 3D Mammograms.](image)

**Figure 1.5** Comparison of The Visibility of a Lesion in 2D and 3D Mammograms.(a) 2D planar mammogram with 2 lesions barely visible. (b)Shows where lesions should be (c, d)Re-sliced planes of 3D reconstruction clearly show each lesion
The focus of this study is on the tomosynthesis breast images that includes planar projection images and volumetric slices acquired from different reconstruction techniques. The methods developed in this work has also been demonstrated on 2-dimensional FFDM data and simulated images for comparison. This study primarily has two aims: (1) to analyze the noise characteristics of the TD and develop general signal-dependent noise models and (2) analyze the power spectra behavior of the TD. A description of the data that are used in this study is provided in the following section.

1.5 Mammography Data

The planar FFDM data analyzed in this work is of 100 $\mu m$/pixel spatial resolution and of 14-bit pixel dynamic range and was acquired with the General Electrics (GE) Senographe 2000D. The tomosynthesis data basically has two representations (1) the 12 projections that are the same as the planar FFDM images acquired from the planar FFDM system and (2) the reconstructed volume consisting of 44 slices.

**Figure 1.5** -continued (e, f) For comparison, shows how lesions would appear in the re-sliced phantom

[Figures (1.4-1.5) from: http://clio.rad.sunysb.edu/mipl/projects/tomosyn.html]
As described in section 1.4, digital tomosynthesis is a technique for producing slice images (volume) using modified conventional X-ray systems with a limited number of projections. By shifting and adding these projection images, specific planes may be reconstructed. Different types of reconstruction algorithms are used to produce the tomosynthesis volume images.

Images used in this study are from cadaver breasts. This is a case-study of one cadaver breast. There is a set of 2-D projections and also 4 different reconstructed datasets. The projection dataset consists of 12 projection images. Usually the 12th frame has only very low dose, and nominally the tube position doesn't change between the 11th and the 12th shot. So, for all practical purposes, only the first 11 images are considered in this study.

The X-ray tube moves in an arc from -25 to +25 degrees with 5 degree increments and multiple projections are acquired. However, the pivot point is located 22.4 cm above the detector, and the tube arm is 44 cm long. Therefore, the effective angle with respect to the center of the detector is smaller; the angles (in degrees) are approximately,

\[
\begin{array}{cccccc}
\end{array}
\]

The image obtained at each angle is of low radiation dose, with the total radiation dose required for imaging the entire breast being somewhat less than the dose used to acquire the two views of a film-screen mammogram.
Digital tomosynthesis is universally practiced with the shift-and-add or backprojection techniques [Dobbins et al 2003]. However, there are several iterative reconstruction techniques described in the literature for the reconstruction of a three-dimensional object from two-dimensional projection images [Colsher 1977]. Colsher discusses three basic approaches for reconstruction, namely, 1) summation approach, 2) Fourier techniques, and 3) algebraic methods. Some iterative approaches for reconstruction are algebraic reconstruction techniques (ART), simultaneous iterative reconstruction technique and iterative least squares technique. Only one of these techniques has been used by GE for generating the volume data along with several backprojection techniques.

The reconstruction algorithms used to generate the volume data for the cadaver breasts are:

- Algebraic Reconstruction Technique
- Standard Filtered Back projection
- Order Statistics-Based Back projection
- Generalized Filtered Backprojection

A brief description of the first three techniques is provided here.

**Algebraic Reconstruction technique (ART)** This is an iterative reconstruction algorithm in which computed projections or ray sums of an estimated image are compared with the original projection measurements and the resulting errors are applied to correct the image estimate. In ART, the corrections are computed and applied on a ray-by-ray or view by view basis. The manner in which the image converges depends on the order in which the ray-sums are considered. In most ART applications, the reconstructed image is assumed
to consist of an array of square pixels which are of uniform density. The computed projections are obtained by summing the values of the pixels whose centers lie within a path of finite width. The average error is then computed and added to the pixels included in the ray sum.

*Filtered Back projection (FBP)* Filtered backprojection as a concept is relatively easy to understand and is one of the popular reconstruction techniques, which is illustrated by an example in Figs. 1.6. Let's assume that we have a finite number of projections of an object which contains radioactive sources (Fig. 1.6(left)). The projections of these sources at 45 degree intervals are represented on the sides of an octagon. Fig.1.6(right) illustrates the basic idea behind back projection, which is to simply run the projections back through the image (hence the name ‘back projection’) to obtain a rough approximation to the original. The projections will interact constructively in regions that correspond to the emittive sources in the original image. A problem that is immediately apparent is the blurring (star-like artifacts) that occur in other parts of the reconstructed image. The optimal way to eliminate these patterns in the noiseless case is through a ramp filter. The combination of back projection and ramp filtering is known as filtered back projection.
Order-Statistics Based Back Projection

This reconstruction method is based on simple backprojection, which generates high contrast reconstructions with minimized artifacts at a relatively low computational complexity [Claus et al 2002]. The first step in this method is a simple backprojection with an order statistics-based operator (e.g., minimum) used for merging the backprojected images into a reconstructed slice. Accordingly, a given pixel value does not generally contribute to all slices. The percentage of slices where a given pixel value does not contribute, as well as the associated reconstructed values, are collected. Using a form of re-projection consistency constraint, projection images are then updated, and the order statistics backprojection reconstruction step is repeated, but now it uses the "enhanced" projection images calculated in the first step.
In the following pages, examples of the tomosynthesis images from the cadavers are illustrated. First, a 2-D projection image of a breast is demonstrated in Fig. 1.7 followed by a volumetric slice from the four reconstructions techniques in Fig.1.8 and Fig.1.9. In this study, the analysis has been performed on the 11 projection images and 44 volume slices. From each breast image (projection and volume), a large rectangular section was excised, which is termed as the Region of Interest (ROI). The experiments and analysis are restricted to this section which is extracted by user-interaction methods from the breast region. Fig.1.10 (top) illustrates the inscribed region or section on the breast, which is to be excised. The associated ROI is shown in Fig.1.10 (bottom)
Figure 1.7 The 2-D Projection Image from the Cadaver Breast
Figure 1.8 Volume Slice Example of a Cadaver from FBP and ART. The left image is obtained from the FBP and the right from the ART. Note that the appearance is not the same. This difference is brought out in the spectral analysis.
Figure 1.9 Volume Slice from GFBP (left) and OSBP (right). Note that they are somewhat similar and are also similar to Fig. 1.8 (right), these likenesses are indicated by the spectral analysis
Figure 1.10 ROI Inscribed Within the Breast (top)

Excised ROI (bottom)
1.6 Image Noise

All radiological images are corrupted with unwanted fluctuations or uncertainties that arise from several sources in the imaging system. These undesirable variations are referred to as *noise* or *mottle* by physicists and radiologists, which is any component in the signal that interferes with the true signal. From a general point of view, a noise may be described as the set of all obstructive signals superimposed on the useful signal at a given location in an image [Bochud 1999]. In the context of medical imaging, the useful signal contains valuable diagnostic information, whereas the noise represents a hindrance in understanding the information relayed by the useful signal. Noise is detrimental to radiological images because it impairs the reliable detection of subtle or low contrast structures.

In mammograms, the dominant cause of this problem arises from statistics of X-ray quanta, which can be compounded by noise from other sources, like structure of fluorescent screen, granularity of film emulsion etc. Understanding the noise characteristics is therefore significant in the study of radiological images and in the evaluation of the imaging system, so that its degrading effect on the image can be minimized, if possible and leads to better diagnosis.

One aim of this work is to analyze the noise in TD, which is approached from the broad framework of signal dependent noise (SDN) analysis. In some cases it may be appropriate to study this type of noise through direct experimentation of the imaging system based on phantom image analyses and in other cases it may be that the images
are analyzed directly, if that is all that is available (if there is no access to imaging equipment). In this work, we follow that latter avenue since the imager is not local. Thus, in this work the general questions are addressed: given an image, are the signal and noise related and if so, what is the functional connection? The methods developed to address these questions are therefore applicable in the wider sense to all types of data that may have signal dependent noise.

This study has aimed to analyze the characteristics of the noise in the images acquired by digital tomosynthesis. The results have been compared with the findings from 2D FFDM images. Two methods for estimating the noise are developed in this work. Prior to applying on mammographic data, these methods are first experimented and studied on simulated images in the blind, which forms a part of this work. Understanding the noise characteristics is significant in the study of radiological images and in the evaluation of the imaging system, so that its degrading effect on the image can be minimized, if possible and lead to better diagnosis and optimal computer aided diagnosis schemes.

1.7 Spectral Analysis

The other aim on this work is to investigate the spectral character of TD. Since the tomosynthesis is a newer technology, very little is discussed about the image characteristics in literature. In this work, the frequency domain characteristics of tomosynthesis breast data are analyzed. Previous work by Heine et al has shown that the power spectra of FFDM images obey an inverse power law, to a good approximation [Heine and Velthuizen 2002]. This also coincides with the spectral analysis of digitized
film data [Heine et al 1999]. In this work, the power spectra of the tomosynthesis projection and volume images have been studied and compared with that of planar FFDM images. This analysis may be helpful in understanding the image correlation and texture properties. The purpose of this work is to develop an understanding of the power spectra behavior so that statistical methods developed for digitized film images or FFDM images, may be applied to the tomosynthesis data.

This work has been conducted as a two-part study: 1) Noise measurements from the simulated and the mammographic (tomosynthesis and FFDM) images, and 2) The spectral analysis of the tomosynthesis data and comparison with regular FFDM data. From this point, the thesis has been organized as follows: Chapter 2 discusses the X-ray imaging system and the different sources of noise in detail. The different types of noise and the related work by others are discussed in this chapter. The two methods for estimating noise are discussed in chapter 3 followed by the results from tomosynthesis images. The spectral analysis of the TD is discussed in chapter 4. Comparisons of the planar and volume slices are made. Conclusions from the work are discussed in chapter 5 including recommendations for future work.
CHAPTER 2

SIGNAL-DEPENDENT NOISE

The predominant noise component in mammograms is best defined as signal dependent due to the photon statistics responsible for the image creation, when considering two dimensional planar X-ray images. Although the focal point of this work is the analysis of tomosynthesis breast images, it is first necessary to consider the broader general idea of signal dependent noise without regard to mammography, which includes a summary of related work. In this chapter, general signal dependent noise models are discussed as well as the specific phenomena related to planar mammography images. Naturally, more emphasis is placed on the latter, which includes a brief exposition on the imaging system as well as the specific noise mechanism. The more general idea of SDN should be taken in the context that the volume images are constructed from planar mammographic projections with various reconstruction techniques. Thus, the ideas that apply to the projections may not correspond exactly to the resulting noise qualities in the volume data.

2.1 Noise Models

Like any physical measurement, the radiographic imaging system consists of errors and uncertainties which may be broadly distinguished as systemic and random errors
Systemic errors remain unchanged with every repetition of the process, such as geometric distortion, miscalibration of the detector, computation errors when the image is reconstructed etc. Random errors, on the other hand vary with every repetition of the same measurement. Certain examples of random noise are film grain noise, photon noise, electronic noise, interference due to scattered radiation etc. This study concentrates exclusively on random noise, specifically on the noise due to X-ray quanta, which is the most dominant form of noise in radiographic images [Barrett 1981].

In the most general terms, noise associated with the resulting image may have both signal dependent and signal independent components. Throughout the course of this study the term signal refers to the 2-D image signal under consideration. The following noise models are useful for a wide variety of situations:

2.1.1 Additive Noise This noise is independent of the signal. The noise model is of the form,

\[ r = s + n \]  

(2.1)

where \( r \) is the noisy signal, \( s \) is the pure signal and \( n \) is signal independent random noise, that may have a spectral form other than flat (other than white noise). Example of this type of noise is thermal noise from the detector.

2.1.2 Multiplicative Noise This noise is modeled as

\[ r = ns \]  

(2.2)
where \( r, s \) and \( n \) are the same as described above. Example of this type is “film-grain” or “speckle” noise.

### 2.1.3 Signal-dependent Noise

The above expression may be extended to a more general model

\[
\begin{align*}
    r &= s + f(s)n_1 + n_2
\end{align*}
\]

where \( r \) is the noisy image, \( s \) is the pure signal, \( f \) is a general function and \( n_1 \) and \( n_2 \) are signal independent random noise processes [Froelich 1981]. The middle term in Eq. (2.3) gives the SDN component of \( r \) and the additive component is given by the last term.

SDN is commonly encountered in many signal and image processing applications [Cunningham 1975]. Multiplicative noise is a limiting case of SDN, where the amplitude of the noise term is proportional to the value of the pure or noise-free signal [Aiazzi et al 1997]. A key aspect of SDN is that a certain amount of signal information is embedded in the noise [Kasturi 1983]. In the event in which the noise term dominates the signal term, the signal information present in the noise term may be greater than the signal term. Hence recovering the signal from the noise term in order to have it in a usable form becomes a significant task.

Several studies have investigated the various forms of SDN. Various methods have been proposed in these studies to estimate the noise and to recover the signal. A brief review of the work performed by others is presented in section 2.3.
2.2 SDN and Mammography

The mammographic image formation process is initiated with the photon output of the X-ray tube. If we assume that a uniform parallel beam interacts with an object (the beam is orthogonal to its face and we assume the object is a cube) that is characterized by a linear attenuation coefficient \( \mu \) and thickness \( t \), there is simple expression that relates the output beam (emerging through the other side of the object) with the incident beam

\[ I_o = I_i \exp(-\mu t) \]  

(2.4)

This is known as Beer’s law and is really a probabilistic expression in that the probability of the transmission is given by \( \frac{I_o}{I_i} = \exp(-\mu t) \). The linear attenuation coefficient \( \mu \) is a function of the spectral character of the incoming beam. Putting this in context of mammography, if we consider a photon incident on the breast above the detector at spatial location \((x, y)\) the probability of transmission through the breast of constant thickness \( t \) at \((x, y)\) is given by

\[ \exp(-\int \mu(x, y, z) \, dt) = \exp(-\mu_z(x, y)t) = P_T(x, y) \]  

(2.5)

Here, \( \mu_z \) may be considered as the average attenuation along the path above the image plane at \((x, y)\). The number of photons incident upon the breast is a statistical quantity that follows a Poisson distribution

\[ p_n(n) = \frac{(m)^n}{n!} \exp(-m) \]  

(2.6)

where the units may be changed to include photon density or total fluence, whichever is needed. Assume the above equation holds in the vicinity of the spatial location \((x, y)\) and is similar for all \((x, y)\), which is a generalization of a uniform beam from a statistical
point. Note that with the Poisson distribution the expected value $<n> = m$, which is also equal to its variance. In this situation, the expected signal and associated noise are proportional. Hence, the noise is signal dependent. Holding $n$ constant for the moment, the probability of $k$ photons transmitting through the breast at $(x, y)$ with transmission probability $p = P_T(x, y)$ given $n$ incident photons may be expressed as a conditional probability that follows a binomial distribution

$$p(k|n) = \frac{n!}{(n-k)!k!} p^k (1-p)^{n-k} \quad (2.7)$$

Including the statistical nature of $n$ gives the total probability of transmitting $k$ photons through the breast when there is an expected number of $m$ incident photons traveling through $(x, y)$. Substitution of Eq.(2.5) and Eq.(2.6) and letting $m = pm$ gives

$$\sum_{n=k} \frac{p(k|n) p_s(n)}{k!} = \frac{(pm)^k \exp(-pm)}{k!} \quad (2.8)$$

The sum initiates at $k$ because $k$ cannot be less than $n$. Thus, the transmitted photons also follow a Poisson distribution with the mean and variance attenuated by the transmission probability. Likewise, if the detector at $(x, y)$ has a detection efficiency, $\varepsilon$ (which is just the probability of detection), the detected signal statistics also follows a Poisson law by the same arguments as above, which leads to what is termed as cascaded random process [Barrett 1981]. These arguments have ignored scattering, beam hardening and heel effects that are associated with the actual imaging process.

Photon noise is the most dominant source of noise in a radiographic image. This results in the final image consisting of a relatively small number of detected quanta since the
radiation dose delivered to the patient is limited. As the dose increases, the number of photons per unit area of the receptor increases, and the relative noise level decreases. The relative noise level is inversely proportional to the square root of the dose, i.e. reduction of the noise level to half will require that the dose is increased four times [Yaffe 2000]; this follows by considering the relative signal to noise ratio $m / \sqrt{m}$. The visibility of low-contrast objects is thus heavily dependent on the relative noise level.

One of the important considerations for the evaluation of any imaging system is the quality of the image produced [Dobbins 1995]. The quality of the image acquired by any type of imaging system can be measured by certain imaging parameters like signal-to-noise ratio, modulation transfer function, detective quantum efficiency etc, which are discussed in the next section.

2.3 Imaging Parameters

The quantitative evaluation of an imaging system, digital or screen-film, can be performed by taking certain imaging concepts into consideration. Although they are not used for the analyses in this work since this work focuses on SDN due to the photon counting process, a brief review of some of these concepts is provided here.

*Modulation transfer function* (MTF) The sharpness of the imaging system is characterized by the MTF, which is basically the Fourier response due to a delta function spatial input.
**Signal-To-Noise Ratio (SNR)** This is the ratio of the useful information to the random fluctuations or noise that can obscure the useful information in the image. High SNR is thus desirable in an imaging system for superior image quality.

**Noise power spectrum** This is a measure of the noise power per unit frequency, sometimes called the *power spectral density*.

**Contrast Resolution** Represents the number of shades of gray that a detector can capture. Flat-panel digital detectors typically offer resolution of 12-14 bits.

**Detective Quantum Efficiency (DQE)** DQE is the measure of combined effect of noise, efficiency and contrast resolution performance of an imaging system. It is expressed as a function of the object detail or spatial frequency. High DQE is a widely accepted measure of improved digital image quality and object detectability.

### 2.4 Related Work on Noise Analysis

A review of related SDN analyses is provided in this section.

Cunningham et al have studied the problem of detecting a known 2-D signal or object in an image corrupted by SDN by approaching it from the classical statistical technique of hypothesis testing [Cunningham et al 1976]. A general solution is formulated by the derivation of a decision rule using a likelihood ratio test for a signal corrupted by an unknown noise which may include SDN. Using the decision rule, the probability of
detection is evaluated from a prior knowledge of the noise and imaging system. The use of this technique is limited by the necessity of accurate prior knowledge of the signal and the imaging system. But it may be useful in evaluating the relative performance of various imaging systems.

Froehlich et al have proposed some estimators for SDN related to film grain. The noise model used for this study is the general model shown in Eq. (2.3) [Froehlich 1981]. The noise terms \( n_1 \) and \( n_2 \) in the model are assumed to be zero-mean and normal (Gaussian distributed) random variables (rv) and the probability density function of the signal is also assumed. One of the estimators proposed is minimum mean square error estimate (MMSE), which exhibits greater sensitivity to SDN term rather than the additive one. The performance of this estimator suffers if the assumed probability distribution function of the signal is not accurate. Another method is derived by weighted spatial averaging for estimating the sample mean. For example, the estimation procedure in an image will be to replace each pixel with the average of that pixel and its eight neighboring pixels.

Kasturi et al have proposed a simple technique for signal recovery from Poisson noise [Kasturi et al 1983 a]. In this work, the noisy image, \( R \), is modeled as

\[
R = \frac{1}{\lambda} P[\lambda S] \tag{2.9}
\]

where \( P[\lambda S] \) represents a Poisson process with \( \lambda \) as the parameter and \( S \) is the noisy pure signal. An estimate of the signal was obtained by computing the mean, which was then subtracted from the noisy signal. A second estimate of the signal was then obtained
from the difference image which still contained some signal information. This estimate was obtained by using the relationship

\[ \hat{S} = \lambda \hat{\sigma}_d^2 \]  

(2.10)

where \( \hat{\sigma}_d^2 \) is the estimate of the local variance of the difference image.

Another study by Kasturi et al has developed some methods for restoring the image by transformation of SDN to additive signal independent noise [Kasturi et al 1983 b]. One method involves transforming the SD process to an additive process using the general noise model given by Eq. (2.3). Another technique is based on a contrast manipulation transformation. However, both the methods described here are bound by the assumption that the noise processes in the general model are normally distributed zero-mean processes.

In a study by Kuan et al, non-stationary 2-D recursive filters were developed for image restoration based on a non-stationary mean, non-stationary variance (NMNV) image model [Kuan et al 1984]. The recursive filters developed here are for restoration of images corrupted with multiplicative and Poisson noise. In the NMNV model, an image is decomposed into a non-stationary mean component and an uncorrelated residual random process that can be characterized by its non-stationary variance. In this model, the non-stationary mean contains the gross structure of the image while the non-stationary variance contains the edge information.
Another study develops a method for image restoration in SDN using a Markovian covariance matrix [Kasturi et al. 1984]. The image distribution is modeled as a spatially non-stationary process having a Markovian covariance model given by

$$R| = S| + [f(s)]N_1| + N_2|$$

(2.11)

where $R|$ and $S|$ are vectors representing the noisy observation and signal respectively, $N_1|$ and $N_2|$ are noise processes. Note that Eq.(2.11) is similar to Eq.(2.3). The signal dependence is modeled as a memoryless spatially stationary process characterized by the matrix $[f(S)]$. The vector $S|$ is estimated based on the observation $R|$, a knowledge of the matrix $[f(S)]$, and the statistical parameters of the vectors $S|$, $N_1|$ and $N_2|$.

Karssemeijer has developed a method for estimation of high frequency noise level as a function of grey value [Karssemeijer 1993] in mammograms. An adaptive approach was used in this study, in which the high frequency noise for each image was determined as a function of the grey level, and this information was used for rescaling the images to equalize image noise.

Aiazzi et al have presented a form of SDN given by [Aiazzi et al. 1997]

$$r(x, y) = s(x, y) + s(x, y)^\alpha \cdot n(x, y)$$

(2.12)

where $r(x, y)$ represents the noisy image value at pixel position $(x, y)$, $s(x, y)$ represents the noise-free image value and $n(x, y)$ is a stationary random process with zero-mean and variance $\sigma_n^2$. The second term in the equation represents the SDN term, which is a special case of Eq. (2.3). The value of $\alpha$ is estimated to be between 0 and 1. For $\alpha = 0$,
the model reduces to additive noise model. From this model the \( \alpha \) parameter is calculated. This is achieved by measuring the mean and variance from several homogenous patches (regions of interest) of the noisy image, and the \( \alpha \) for a pair of patches is given by

\[
\alpha = \frac{\log \sigma_i - \log \sigma_j}{\log \mu_i - \log \mu_j}
\]  

(2.13)

where \( i \) and \( j \) denote the measurements from two different patches. A consistent estimate of \( \alpha \) is then calculated by averaging over all possible pairs.

An algorithm was developed by Rank et al to estimate the noise variance of an image [Rank et al 1999]. In this work, the algorithm is basically organized in three steps. The image is first preprocessed by a difference operator to minimize the influence of the original image. A histogram of local standard deviations is then computed. In the final step the histogram is evaluated in order to receive the desired estimate of the noise variance. This algorithm was designed for image models consisting of only the additive noise component.

In order to put the noise in context with the image structure, work by Bochud et al is discussed here briefly [Bochud et al 1999]. The effect of system noise may be negligible compared to anatomical fluctuations in some situations. The effect of variations in anatomical background on detection tasks has been quantified. Experiments in this work show that the human observer’s behavior was highly dependent on both system noise and the anatomical background. The anatomy has been identified as partly acting as signal and partly acting as pure noise that disturbs the detection process. This dual nature of the
anatomy has been quantified and it was shown its effect varies according to its amplitude and the profile of the object being detected.

Salmeri et al have presented an algorithm to obtain estimations of a Gaussian additive noise [Salmeri et al 2001]. This method uses a fuzzy system that processed certain parameters which can be easily extracted from the image. This method can be applied to distributions other than Gaussian but is limited to additive noise models.

Veldkamp and Karssemeijer have developed a method for detection of microcalcifications in digital mammograms by noise equalization [Veldkamp and Karssemeijer 2000]. In this work, an adaptive approach is optimized by investigating a number of alternative approaches to estimate image noise. The estimation of high frequency noise as a function of gray scale is improved by a new technique for dividing the gray scale in sample intervals and by using a model for additive high frequency noise.

Several other studies [Aghdasi et al 1994] have been conducted on the subject of radiographic noise which is cited in literature. This further emphasizes the importance of analyzing and characterizing noise in radiographic image which can be beneficial for the detection and diagnosis of disease.
CHAPTER 3
NOISE CHARACTERIZATION AND MODELING

3.1 Introduction

In the previous chapter a review of several studies involving noise in an imaging system was provided with an emphasis on SDN from a general point of view. In most of the studies performed on this subject, the noise field in the image models was assumed to be zero-mean noise. That is the signal dependent noise model is $n \times s^\alpha$, where $n$, the noise factor, is zero mean and $s$ is the pure signal raised to a power. While, several methods have been proposed for estimation and detection of the zero-mean type SDN, very little is found in the literature that addresses the other case, where the expected value of the noise factor is not zero mean. As demonstrated below, this is an important consideration when assuming noise models and developing estimation methods dependent on the noise model assumptions. This idea must be taken in the context that in some instances the imaging physics may lead to the proper model, as in the Poisson case for photon counting, and in other cases the data prior to acquisition and manipulation methods may not be known or are inaccessible. Since the work presented here follows the latter avenue, it may be considered as a signal dependent noise detection algorithm. That is, given a signal, is it possible to determine if the noise is signal dependent and if so, what is the functional relationship between the two?
In the first part of this chapter a description of the simulated data that are used for testing is provided. The simulations are significant because they represent the ideal cases that will show whether the ideas are worth pursuing further. The two methods for estimating the form of SDN are also presented in this chapter and the methods are applied to the simulated data for validation purposes.

3.2 Simulated Data

The noise estimation methods described in this chapter are first tested and validated with 2-D simulations. In this section, discussion on the generation of the simulated fields is provided. Two related mammographic simulations are described below.

3.2.1 Simulation One: 1/\(f\) Field

Simulation-one is a 2-D random field that has statistically similar PS as a mammogram, which is referred to as a 1/\(f\) process. This terminology arises from the fact that the 2-D noise power spectra of the field drops off as approximately \(1/ f^{2\beta}\), where \(f\) represents the 2-D frequency coordinates and \(\beta\) is a positive parameter in the neighborhood of 1.5, which to a good approximation describes the mammographic spectra. This simulation was developed as part of a previous study [Heine et al 1999].

The 1/\(f\) simulation is generated by Fourier domain filtering

\[
S_0(f) = H(f)S(f)
\]  

(3.1)

with \(f = (f_x, f_y)\), where \((f_x, f_y)\) are 2-D Cartesian coordinates in the frequency domain. \(S(f)\) is the Fourier transform (FT) of the simulation input field, \(s(x, y)\), which is a noise
process that is distributed proportional to a modified hyperbolic Bessel function. $H(f)$ is the frequency domain transfer function given by

$$H(f_x, f_y) = (f_x^2 + f_y^2)^{-\beta/2}$$

(3.2)

where $\beta = 1.5$.

$S_0(f)$ is the FT of the output or simulation process, $S_0(x, y)$, which can be obtained by taking the inverse FT. Fig.3.1 illustrates the simulated field generated by this process. The image shown below appears similar to a mammogram, but does not have similar signal dependent noise properties. In the next section, it is shown how this may be achieved.

Figure 3.1 The Simulated 1/f Field
3.2.2 Simulation Two: Mammographic SDN Simulation

In this method, a $1/f$ image is generated as the first step, which is used as the raw image or input image for this simulation. In order to simulate a mammogram, the thickness of a compressed breast is assumed to be 5 cm. In a human breast, the attenuation of glandular tissue is estimated to be 0.9/cm and 0.5/cm for fat tissue [Highnam 1999]. Thus if the X-ray traversed through a 5cm breast, the natural log of the transmission probabilities would range over the interval (2.5, 4.5). Using these values, the $1/f$ image was mapped between the attenuation values of fat and glandular tissue (which are 2.5 and 4.5 for a breast of 5 cm thickness). For a typical X-ray tube, at the tube voltage of 28 kVp, the photon flux rate is given as $0.366 \times 10^6$ photons/mAs/mm$^2$ [Highnam 1999]. If the exposure value measured in mAs (milli-Ampere seconds) is assumed to be 25 and the pixel area as 1 mm$^2$, then the average number of photons over one pixel area of an image at 30 µm is given by

$$\text{(photon flux rate)} \times \text{(mAs)} \times (30 \times 10^{-3})^2 \approx 10,000$$  \hspace{1cm} (3.3)$$

The probability of transmission of a photon through each pixel (shown in Eq.(2.5)) is given by

$$\begin{align*}
p_T &= \exp(-z) \\
\end{align*}$$  \hspace{1cm} (3.4)$$

where, $z$ is the resulting image obtained by mapping the $1/f$ image into the attenuation values of 2.5 and 4.5 of a breast of 5cm thickness.

For each pixel position a rv, $w$, is picked at random from a Poisson distribution with $<w>$ =10000. This gives the maximum number of X-ray photons that may pass through the breast above the detector at the position $(x, y)$. For each photon a uniform rv, $u$, 

40
distributed over the interval (1,100) is picked at random. If the transmission probability is less than \( u/100 \), the photon is transmitted through to the detector, otherwise it is absorbed. This is repeated for each photon for the given location above the detector. The process is then repeated for each pixel. In this case we have the prefect linear detector with an efficacy of 100 \%, implying that every photon interacting with the detector is detected. This resulted in the formation of an image which is considered to be a simulated mammogram of the same size as the input image \( z \), which is illustrated in Fig.3.2.

![Figure 3.2 Simulated Image Generated as a True Mammogram](image-url)
3.3 Noise Estimation

Two methods of noise estimation and SDN modeling are presented in this section. The methods are first tested on simulated data of known noise characteristics and validated with blind simulated data.

A 512 × 512 size 1/f image is generated (shown in Fig.3.1). A 512 × 512 size, zero-mean Gaussian noise field is generated using the random noise generator function in IDL [RSI systems], which is illustrated in Fig.3.3.

The signal-dependent noise is given by

\[ r = n \times s^\alpha \]  \hspace{1cm} (3.5)

where, \( n \) is a zero-mean Gaussian noise field, and \( s \) is the pure signal or the image created as a 1/f field with \( \alpha = 2 \) in this case and \( \beta = 1.5 \) from Eq. (3.2). In this case, note that \( s \) is not a conventional deterministic signal but is a statistical entity itself that follows from the simulation-one method. The variance of \( r \) is given by

\[ \langle r^2 \rangle = \langle n^2 \rangle \times \langle s^{2\alpha} \rangle \]  \hspace{1cm} (3.6)

The importance Eq. (3.6) will become clear when modeling the signal dependent noise behavior. The SDN in Eq.(3.5) is then added to the signal term giving the noisy image

\[ f = s + r \]  \hspace{1cm} (3.7)

Fig.3.4 illustrates the noisy image \( f \).
Figure 3.3 A Zero-mean Gaussian Noise Field
3.3.1 Method I: The Filtering Method

One approach to estimate the noise-signal relation is to apply some method that separates the two components. For the moment, two assumptions are be made (1) the signal dependent noise field is not correlated with the signal term, and (2) the spectral character of the noise term is flat, implying that is white noise. The merits of these assumptions will be discussed later in the work. These two assumptions relate to the signal and noise

Figure 3.4 The Noisy Image $f$ which is Contaminated with SDN with $\alpha = 2$
in following way: the signal varies slowly (long range correlation) due to the $1/f$ behavior and has more power located near the zero frequency region, whereas the noise term has its power distributed evenly. Thus, a filtering operation may separate the signal from the noise term if the assumptions hold true. The wavelet transform has been used here for this application.

*Wavelet Expansion* The noisy image $f$ is expanded into a sum of uncorrelated images by applying wavelet expansion method that was used in a previous study [Heine et al 1997]

$$f = d_1 + d_2 + \cdots + d_j + f_j$$  \hspace{1cm} (3.8)

where, $d_j = f_{j-1} - f_j$. The expansion may be terminated for any value of $j$. The important relation here is that $d_j = f_{j-1} - f_j$. The $d$ images are difference images that contain the detail information as the image $f_{j-1}$ is blurred to the next coarse resolution $f_j$. The $d_j$ images get coarser with increasing value of $j$, implying that the $d_1$ image contains finer detail than the $d_2$ image and the $f_j$ is a half resolution and smoothed version of $f_{j-1}$. This latter network may be considered as the output of a filter bank, where the outputs are linearly independent. Figs.(3.5 – 3.7) illustrate the $d_1$, $d_2$ and $f_1$ versions of the image shown in Fig.3.4. For the purpose of this work, only $d_1$ and $f_1$ images are considered for noise modeling. If a noise field is white, $1/4$ of the power will be contained in the $d_1$ component. Thus under the assumptions described above, the $d_1$ is a good approximation for the noise while the $f_1$ image approximates the true signal.
Figure 3.5 $d_1$ Image

Figure 3.6 $d_2$ Image
Signal and Noise Measurements An $8 \times 8$ size window or box is shifted across the $f_1$ image. For every location of the box, the mean signal, $m$ is calculated and stored. A similar procedure is then applied to the $d_1$ image. But in this case the variance $\sigma^2$ is tabulated. The stored average values are then sorted in ascending order. The spatial location of each average value in the final sorted arrangement is retained. The estimated variance is then aligned such that the resulting combination forms ordered pairs of the ascending averages (the independent variable) and the associated noise variance estimates from the same spatial location. Since the data is integer, there will be repeated values along the independent axis (signal axis). When this case exists, the associated noise terms
are averaged to generate the resultant ordered pair. A box car sliding average of length 10 is applied to the noise signal prior to the modeling.

The result as applied to the Fig.3.4 is shown in Fig.3.8 where the noise variance (jagged) is plotted as a function of the mean, \( m \). A polynomial fit is applied to this function which is shown as the solid curve. Polynomials from degree 0 to 5 are fitted to this curve and the fit with the minimum error is retained as the best fit. The polynomial fitted is of the form

\[
y = a + b x^{2a}
\]  

(3.9)

which follows from Eq 3.6 with an added degree of freedom indicated by the additive constant \( a \). In Fig.3.9, the theoretical curve is plotted against the empirical curve showing the linearity between the two curves. Since \( \alpha \) was given a value of 2 in this particular case, the function is estimated as a fourth degree polynomial (the independent variable is the local sorted mean signal) gave the best fit. As a counter example, the value of \( \alpha \) was set to 0 in Eq.(3.5). In this case plotting the noise as a function of signal as shown in Fig.3.10 produces a constant function; clearly indicating no functional dependency of the noise on the signal. The noise is termed “additive” in this case with approximately zero slope.

Hence, we see that given a known dependency of the noise on the signal, the functional form of the noise can be estimated with this method. This is validated in the following section by using blind data, in the case where the functional dependency of the noise is assumed unknown.
Figure 3.8 The Signal Average Versus Noise Variance. Empirical curve (jagged line) and theoretical curve (solid line) resulting from the curve fitting analysis. The form of the noise is given by $y = 550.84 + 2.5 \times 10^{-9} x^4$.

Figure 3.9 The Theoretical Curve (jagged) Against the Empirical Curve (solid) Shown in Fig. 3.8. This has the effect of making the relation linear when the polynomial relation is in agreement. A linear fit (solid) is applied with a slope, $m = 0.98 \pm 0.001$ and correlation, $r = 0.99$.
Figure 3.10 This Shows the Non-signal Dependent Case. When the noise is not related to the signal, the outcome is a straight line with approximately zero slope. This is encountered with additive random white noise.

Validation of Method 1 It was demonstrated in the previous section that the method is useful for finding the noise-signal relation. Now a blind image is generated using simulation method 2 (Poisson process), which is a different phenomenon than the signal dependent simulation developed in Eqs. 3.5 and 3.7. This image is decomposed into $d_1$ and $f_1$ images by wavelet expansion (discussed above and shown in Fig.3.7) and the signal noise measurements follow the method-I prescription. Plotting the noise as a function of signal is shown in Fig. 3.11. The noise variance in this case is found to be a linear function of the signal which is to be expected due to the Poisson statistics where the expected value is equal to the variance.
Figure 3.11 The Empirical Curve (jagged line) is Linearly Fit (solid line). This shows noise variance is a linear function of the signal with slope $m = 11.54x \pm 0.04$ and $r = 0.98$

3.3.2 Method II: The Order-Statistics Method

In this subsection another method is developed for estimating the signal-noise relation. The necessity for this will become clear in the following sections. The image generated in Eqs.(3.5-3.7) is used as the raw image. As shown in the previous section, a known value of $\alpha$ is set for testing purposes.

*Signal and Noise Measurements* As in the previous method, an $8 \times 8$ size window or box is shifted across the raw image in both directions (vertical and horizontal). At each location of the box, the pixel values were sorted in ascending order and the median value, $q$, approximates the average (analogous to method one). From each location of the box, $q$ of the sorted values and the minimum pixel value (min) and the maximum pixel value
(max) were calculated. The noise standard deviation is estimated at each box site by the following operation

\[
\sigma = k \frac{|q_{\text{max}}| + |q_{\text{min}}|}{2}
\]  

(3.7)

The above expression is an approximation and the constant \(k\) must be determined. This represents a non-linear operation that is sensitive to the image correlation and distribution. This was studied with simulations that indicate \(k = 1/\sqrt{6}\) approximately for mean-symmetric type distributions, which require further investigation. The analysis continues in exactly the same fashion as in method I from this point. The results of the technique are displayed below in Fig.3.12 on the same simulations as displayed in Fig 3.8 above with similar results. The linearity between the theoretical and the empirical curve is shown in Fig.3.13.

![Figure 3.12](image)

Figure 3.12 The Jagged Curve (empirical) Shows the Noise as a Quadratic Function of Signal. It is fitted with a 4th order polynomial (solid curve) given by \(y = 1467.12 + 2.3 \times 10^{-9} x^4\)
Figure 3.13 Theoretical Curve against the Empirical Curve (jagged). This makes the relation linear when the fit is a reasonable approximation. A linear fit (solid) is applied with a slope, $m = 1.02 \pm 0.06$ and correlation $r = 0.97$, which shows the technique produces the expected behavior.

Validation The method is validated with the same Poisson process as in the previous case. The relation is linear as previously and is displayed in Fig. 3.14 with the slope $m = 21.7 \pm 0.05$ and the linear correlation coefficient, $r = 0.91$.

The two methods predict the same form when considering the simulation presented in Fig. 3.1 when considering the scaling constants. However, in the Poisson case (Fig.3.2), the scaling constants are of the same magnitude but differ by roughly a factor of two.
Figure 3.14 The Empirical Curve (jagged line) is Linearly Fitted (solid line). It shows that the noise is a linear function of the signal given by $y = 21.7x - 17102.6$

The two methods discussed above have been tested with a multiplicative model zero-mean case and validated with a Poisson process. The two methods predict the same functional relations for these simulations under the original assumptions, which may not always be the case. In the multiplicative case, changing the DC bias of the multiplicative noise field to something other than zero mean alters the original assumption of the noise character. These ideas are illustrated in Figs. (3.15-3.18). The same simulation displayed above is used for this demonstration with the noise term in Eq. (3.5) having a mean value that is ten times its variance. The results obtained by applying the two methods are illustrated below.
Figure 3.15 The Noise Variance is Plotted as a Function of the Mean Signal. The noise shown here is estimated using method I. It is evident that this method does not predict the true character of the noise in the model.

Figure 3.16 The Noise Variance (jagged) is Plotted as a Function of the Mean Signal. It is fitted with a 4th degree polynomial of the form, $y = 18209.59 + 1.7 \times 10^{-10} x^4$. The noise
shown here is estimated using method II. The result is consistent with the results shown in previous sections for zero-mean noise.

Thus, it is seen that method I fails to estimate the true character of the noise when the noise term contains a large mean value. As the mean in the noise term is small or approaches zero, this method provides results close to Eq.(3.9) but it is not exactly accurate. This is further illustrated in Figs.(3.17-3.18). In the first figure the noise is shown when the mean in the noise is five times the variance and the second figure shows the case when the mean is equal to the variance.

Figure 3.17 The Noise Variance (jagged) is Shown When the Noise Term Contains Mean Value Equal to Five Times the Standard Deviation. It is fitted with a polynomial (solid)of the form $y = -97840.82 + 0.023x^2$
Figure 3.18 The Noise Variance when Mean of Noise is One. The noise variance when the mean in the noise is equal to the standard deviation and is fitted with a polynomial (solid) of the form $y = -2387.98 + 2.7 \times 10^{-3} x^3$. Thus, it is seen that as the mean in the noise gets closer to zero, the estimated noise gets closer to its true form.

3.4 Noise Analysis of Tomosynthesis Data

Two methods for estimating SDN have been discussed in section 3.3. Both methods were applied to the TD and the methods produce consistent results; the results that are presented here follow method-one. The noise characteristics as estimated from the projection data and the volume data are analyzed and compared. Likewise, comparisons are referenced to the standard planar FFDM images.
3.4.1 Projection Data

The results for one projection example are shown in Fig.3.19. In the figure, the noise variance is plotted as a function of the mean signal. A linear fit has been applied to the curve showing that the noise is a linear function of the signal, which is of the form

\[ n(s) = ms + b \]  

where, \( n(s) \) refers to the noise variance, \( s \) is the mean signal, \( m \) is the slope of the line fitted and \( b \) is the constant term. The empirical curve (jagged line) is shown fitted with the theoretical curve (solid line). This result is found to be consistent across the 11 projection images, with a varying slope of the fitted line. The average slope across the 11 projections, \( <m> = 0.24 \pm 0.02 \), and average correlation coefficient, \( <r> = 0.86 \pm 0.03 \). The slope of the line for each projection is plotted in Fig.3.20.

![Figure 3.19](image)

**Figure 3.19** The Noise Variance (jagged) as a Function of the Mean Signal. A linear fit is applied to it (solid) with the slope \( m = 0.20 \pm 0.004 \), and linear correlation coefficient, \( r = 0.87 \)
The average noise power across the 11 projection images was calculated from the first detailed image obtained by the wavelet expansion of the raw image. This image contains three-fourth of the noise power of the raw image if we consider the noise as wide-band white noise. This image captures the effective average interference of the SDN and electronic thermal noise. The plot in Fig.3.21 shows the average or effective noise power for each of the projection image.
Figure 3.21 The Noise Power for Each of the 11 Projection Images (asterisks). The solid line represents the general trend.

3.4.2 Volume Data

The four sets of reconstructed volume data are analyzed and presented in this section.

Figs. (3.22-3.23) illustrates the functional form of the noise as estimated from the volume data. In these figures, the noise variance is plotted as a function of the mean signal. The relationship was modeled by considering polynomial from zero through fifth degree and applying error analysis. A linear fit showed the least error indicating that the noise is a linear function of the signal and is of the form given by Eq.(3.8). The empirical curve (jagged line) is shown fitted with the theoretical curve (solid line). The linear form of the
noise estimated from the two methods is consistent across the 44 volume slices for each volume set. The average slope and the average linear correlation coefficients of 44 slices for each volume set are tabulated in table 3.1.

The slope of the line for each volume set as a function of the slice is plotted in Figs. (3.24-3.25). The correlation between the slopes of the four sets has been tabulated in table 3.2.

As in the previous case the average power was calculated from the first detailed image from the wavelet expansion of each volume slice for the 4 sets. This is plotted as a function of the volume slice, shown in Fig. 3.26, which illustrates the GFBP data. A linear fit is applied to it which has a slope, \( m = 89.69 \), with linear correlation coefficient \( r = 0.88 \). The average power measure from each volume set is found to be following this trend.
Figure 3.22 The Noise Estimated From GFBP (top) and OSBP (bottom). It is shown as function of the mean signal, fitted with the theoretical line (solid). The form of the noise is found to be linear in both cases shown here with the slope, $m = -15.49 \pm 0.051$ (top) with correlation coefficient, $r = -0.93$ and $m = -1.4 \pm 0.012$ (bottom), with the correlation coefficient, $r = -0.91$
Figure 3.23 The Noise Estimated From FBP (top) and ART (bottom). It is shown as function of the mean signal, fitted with the theoretical line (solid). The form of the noise is found to be linear in both cases shown here. For FBP, the slope $m = -3.78 \pm 0.03$ and $r = -0.90$, and for ART, $m = -0.12 \pm 0.01$, and $r = -0.94$
Figure 3.24 The Slope of the Line for GFBP (top) and OSBP (bottom). This is the slope of the line fitted with the noise estimated across the 44 volume slices for each reconstruction set.
Figure 3.25 The Slope of the Line for FBP (top) and ART (bottom). This is the slope of the line fitted with the noise estimated across the 44 volume slices for each reconstruction set.
**Table 3.1** The Average Slope of the Line and the Average Linear Correlation Across the 44 Slices of Each Volume Set

<table>
<thead>
<tr>
<th>RECONSTRUCTION SET</th>
<th>AVERAGE SLOPE, $&lt;m&gt; \pm ERROR$</th>
<th>AVERAGE CORRELATION, $&lt;r&gt; \pm ERROR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFBP</td>
<td>-15.64 $\pm$ 1.35</td>
<td>-0.92 $\pm$ 0.01</td>
</tr>
<tr>
<td>OSBP</td>
<td>-1.37 $\pm$ 0.057</td>
<td>-0.91 $\pm$ 0.03</td>
</tr>
<tr>
<td>FBP</td>
<td>-3.86 $\pm$ 0.75</td>
<td>-0.85 $\pm$ 0.07</td>
</tr>
<tr>
<td>ART</td>
<td>-0.12 $\pm$ 0.009</td>
<td>-0.93 $\pm$ 0.02</td>
</tr>
</tbody>
</table>

**Table 3.2** The Correlation Between the Slopes of the Line (Shown in Fig.3.24 and Fig.3.25) Fitted Over the Volume Slices of the Four Reconstruction Sets

<table>
<thead>
<tr>
<th>RECONSTRUCTION SET I</th>
<th>RECONSTRUCTION SET II</th>
<th>CORRELATION BETWEEN THE SLOPES OF I AND II</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFBP</td>
<td>OSBP</td>
<td>0.25</td>
</tr>
<tr>
<td>GFBP</td>
<td>ART</td>
<td>-0.62</td>
</tr>
<tr>
<td>GFBP</td>
<td>FBP</td>
<td>-0.05</td>
</tr>
<tr>
<td>OSBP</td>
<td>ART</td>
<td>0.14</td>
</tr>
<tr>
<td>OSBP</td>
<td>FBP</td>
<td>0.016</td>
</tr>
<tr>
<td>ART</td>
<td>FBP</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Figure 3.26 The Average Noise Power (diamonds). This is calculated from the first detailed image of the 44 volume slices for one volume set shown with a line (solid) fitted over it with slope $m = 89.69 \pm 3.58$, with linear correlation coefficient $r = 0.88$. The average power for all four sets of volume images follow similar trend as shown here.

3.4.3 FFDM Data

The noise is estimated from regular FFDM data using the methods described in this work. Fig.3.27 illustrates the noise variance (jagged) as a function of the mean signal and fitted with a line (solid). The noise is found to be a linear function of the signal. Previous studies have shown that the noise from the digital mammography system, estimated as a function of the X-ray exposure, exhibits a linear behavior [Cooper 2003]. This is expected, since the FFDM detector has a linear response with the X-ray exposure [Vedantham et al 2000].
Figure 3.27 The Noise Variance from FFDM Data (jagged) Shown as a Function of the Mean Signal. It is fitted with a line (solid). The noise is found to a linear function of the signal with the slope, \( m = 0.075 \pm 0.03 \) and \( r = 0.96 \)

3.5 Discussion

From the noise analysis, the slopes of the linear fit give the scaling between incremental changes in the noise relative to the signal. The work shows that the noise-signal dependency is strongest for the GFBP method and weakest for the ART method. Also note that the relationship is stronger with the volume data sets in comparison to the projection sets. It is also interesting to note that within a reconstruction method, the slopes are similar although not correlated across the reconstruction sets; this holds except for the FBP method. In as much that the wavelet image captures or represents the
effective noise, the work shows, that the projection angle has little influence on the randomness as is evident from Fig. 3.21, which is consistent with the ensemble slope average and variations of the projections.

The detector response in the FFDM system is linear with respect to the X-ray exposure. Thus, the resulting pixel value, PV is related to the exposure, E, by the relation,

\[ PV = mE + c \] [Vedantham et al 2000],

where \( m \) and \( c \) are constant parameters. Since \( c \) is small, the PV and PV variance relation should follow the Poisson relation.
CHAPTER 4

SPECTRAL ANALYSIS OF TOMOSYNTHESIS DATA

To date, very little analysis has been published relating to the spectral characteristics of the TD, since it’s a newer technology and not many systems are in clinical use. In this work, the frequency domain characteristics of the TD are studied and analyzed. Previous work by Heine et al shows that the power spectra of FFDM planar data obey an inverse power law, to a good approximation [Heine et al 2002]. An effort has been made to understand the power spectra behavior of the tomosynthesis (projection and volume) data so that statistical methods developed for digitized film images and FFDM images may be adapted to this data. In addition to modeling the tomosynthesis spectra, the PS of the FFDM data are also discussed for comparison purposes. The PS behavior in relation to SDN has also been discussed with examples in this chapter, which is relevant in understanding and analyzing the noise characteristics in images.

Fourier methods are applied to estimate and analyze the PS of the TD. The analysis technique applied here, referred to as the constant ring model, was developed previously for planar FFDM analysis [Heine and Velthuizen 2002].
4.1 Power Spectrum Estimation

The power spectrum (PS), or more aptly termed the spectral density, describes the characteristics of the data in the frequency domain. Figs. 4.1 illustrates a typical example of an image in the Fourier domain as obtained in Cartesian coordinate systems.

Figure 4.1 Raw Image (left) and the Image in Frequency Domain (right)
The FT of an image \( f(x, y) \) of size \( M \times N \) is given by

\[
F(u, v) = \frac{1}{MN} \sum_{x=0}^{M} \sum_{y=0}^{N} f(x, y) \exp(-j2\pi(\frac{ux}{M} + \frac{vy}{N}))
\]  

(4.1)

where, \( u \) and \( v \) denote the frequency domain coordinates. The PS of \( f(x, y) \) is approximated from this operation by

\[
s(u, v) = |F(u, v)|^2
\]

(4.2)

### 4.2 Constant Ring Model

The constant ring method provides a summary analysis of the spectral behavior of the data. In this method, the power is analyzed by integrating over constant ring widths in a radial coordinate system. For this analysis, the spectrum is divided into 51 sections. This provides a ring width of 5/51 or about 0.098 cycles/mm per ring. The ring that covers the zero frequency range \([0-0.098]\) is not included in the analysis to avoid any dc bias. The other 50 equally spaced rings are analyzed. Since the ROIs are rectangular in size and not square, the rings appear elliptical instead of circular.

**Integration of the rings** Approximating the spectral character of a mammogram as a 1/\( f \) process and integrating the power spectral density over equally spaced rings in radial coordinates gives

\[
P_i(n) = 2\pi c \int_{dn}^{d(n+1)} \frac{f \, df}{f^{2\beta}} = k[(n + 1)^{-2\beta+2} - n^{-2\beta+2}]
\]

(4.3)

where \( k \) denotes all constant factors, \( n \) is an integer that refers to the ring position, and \( d \) is the constant ring width (\( \approx 0.098 \)). Here, \( n \) represents the frequency index; increasing \( n \)
indicates increasing frequency. Here, $f$ represents the two-dimensional frequency domain coordinates and $\beta$ is a positive parameter. The subscript $i$ is the image index. $P_i(n)$ provides the total power in the radial frequency range that covers \([n d, (n+1) d]\). Eq. (4.3) provides a means for parametric spectral modeling. However, the operation described by Eq. (4.3) may also be presented empirically by integrating the power spectra of the image without regard to parametric assumptions.

Normalization A measure of the fractional power per ring is provided by normalizing Eq.(4.3), which is given by

$$p_i(n) = \frac{P_i(n)}{\sum_{n=1}^{50} P_i(n)} \quad (4.4)$$

where, the lower case $p$ denotes the fractional power per ring. The normalization provides the relative power per ring, with respect to the total power in the 50 rings. This ensures that all images are treated with the same weight in the analysis. The index $i$ indicates the data form or representation.

4.3 Spectral Modeling of the Tomosynthesis Data

The above method is applied to the various forms of TD in this section. The intent of the work here is to examine if the TD shows similar behavior as that of other mammography data, particularly the FFDM data. This work involves modeling the projection data and the four sets of volume data as a $1/f$ process. Note in the figures that the projection image and the first three sets of the volume images have been theoretically modeled, but Fig.4.6 showing the PS of the volume slice from FBP has not been modeled.
4.3.1 Projection Data

The normalized power spectra (solid curve) as a function of the frequency index is plotted and theoretically modeled (diamonds) with Eq.(4.3), as illustrated in the following figures Figs.4.2. The top figure shows the PS of the projections image followed by PS displayed on a log scale (bottom) that shows the minute differences. The divergence of the curve from the theoretical model at the tail as seen in the log display is due to the noise in the data in the high frequency region.

*Ensemble Power Spectra* The ensemble PS was obtained by averaging over the 11 projection images. Fig.4.3 shows the ensemble behavior of the PS with the standard deviation curves, displaying the error margins. This is displayed in an un-normalized scale in Fig.4.4. The projections show the $1/f^\beta$ behavior with the average

$<\beta> = 1.36 \pm 0.04$
Figure 4.2 The PS of the Projection Data (top) and Log Display (bottom). The integrated PS (solid curve) of the projection data is displayed over the theoretical $1/f$ model (diamonds) with $\beta = 1.30$. The figure is displayed in a log scale to show minute differences.
Figure 4.3 The Normalized Ensemble PS of Projection Data. The normalized ensemble power (solid) of the 11 projection images is shown with the standard deviation curves given by $<\beta> = 1.36 \pm 0.04$. The (average + deviation) is shown in diamonds and the (average - deviation) is shown as (+). This is displayed in a log scale.
Figure 4.4 The Absolute Ensemble PS (solid) of Projection Data It is shown with the standard deviation curves given by given by $\beta = 1.36 \pm 0.043$. The (average + deviation) is shown in diamonds and the (average - deviation) is shown as (+). This is displayed in a log scale.

4.3.2 Volume Data

The spectral modeling of the volume data from each reconstructions set are displayed in Figs.(4.5-4.8)

*Ensemble Behavior* The ensemble PS was obtained by averaging over the 44 volume slices. Fig.4.9 shows the normalized ensemble behavior of the PS with the standard deviation curves and Fig 4.10 shows the same on an absolute scale. The plots are displayed on a log scale to show minute differences.
**Figure 4.5** The PS of a Volume Slice from the ART Method (diamonds) It is displayed over the theoretical $1/f$ model (solid) with $\beta = 1.56$

**Figure 4.6** The PS of a Volume from the GFBP Method (diamonds) It is displayed over the theoretical $1/f$ model (solid) with $\beta = 1.49$
Figure 4.7 The PS of a Volume Slice from the OSBP Method (diamonds). It is displayed over the theoretical $1/f$ model (solid) with $\beta = 1.46$.

Figure 4.8 The PS of a Volume Slice from the FBP Method. It clearly indicates a different behavior than that of other volume images. This is consistent with the appearance of the reconstructed data (see the figures in chapter 1).
Figure 4.9 The Normalized Ensemble PS from Volume Slices (solid). It is shown with the standard deviation curves given by $\langle \beta \rangle = 1.426 \pm 0.069$. The (average + deviation) is shown in diamonds and the (average- deviation) is shown as (+)

Figure 4.10 The Absolute Ensemble PS from Volume Slices (solid). It is shown with the standard deviation curves given by $\langle \beta \rangle = 1.426 \pm 0.069$. The (average + deviation) is shown in diamonds and the (average- deviation) is shown as (+)
The above analysis indicates that there is little variation across the volume slices. The volume slices show the $1/f^\beta$ behavior with the average $<\beta>=1.426 \pm 0.069$.

### 4.3.3 Spectral Comparison between the three volume sets

The spectral characteristics of the three volume sets showing $1/f$ behavior were compared by performing a paired $t$ test. The 44 PS measurements obtained from each volume set were compared with the same from every other set. This results in 50 pair wise comparisons, which are the $p$-values that cover each frequency division. Fig.4.11 shows the $p$-values from the $t$-statistic comparing two volume sets.

![Figure 4.11](image_url)  

*Figure 4.11* The Comparison of Two Sets of Volume Data. The plot shows the $p$-values for each frequency index obtained from $t$-statistic. The actual points are displayed as diamonds.
This result was consistent between every two volume sets. From the *t-statistic*, it was evident that the spectral characteristics of the volume images obtained from different reconstruction techniques are similar at a significance level of 0.01 in the low frequency region. In the high frequency region (above 12 cycles/mm), they are much different.

The spectral analysis shows that (1) the $1/f$ behavior is not dependent of the projection angle, and (2) three of the reconstruction techniques as well as the projections approximately follow the $1/f$ model and are similar to previous work in modeling mammograms.

### 4.4 SDN and Power Spectra Behavior

In this section, certain aspects of SDN (signal-dependent noise) are discussed in relation to Fourier spectrum, by means of an example. A $1/f$ image, $s$, is generated (discussed in section 3.2) and a Gaussian noise field $n$ of the same size as $s$ is generated. Modulating the image $s$ with $n$ gives

$$z(x, y) = n(x, y) s(x, y),$$  \hspace{1cm} (4.6)

where, $x$ and $y$ represent the pixel position in the image domain. Taking the FT of Eq.(4.6) gives

$$Z(u, v) = N(u, v)*S(u, v),$$  \hspace{1cm} (4.7)

where the capitals denote the Fourier domain. Eq.(4.7) represents a standard convolution integral. From this, example, two cases are inspected that relate to the expected behavior of $n(x, y)$. 

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Case 1  This is termed as the zero-mean case. In this case noise field \( n \) is zero-mean and stationary. The resulting PS of \( z \) is flat, indicating that it is a white noise, as illustrated in Fig.4.12.

![Figure 4.12 The PS of an Image when the Noise Modulating it is Zero-mean](image)

The “flat” argument follows this reasoning: the FT of the noise field will also be a random noise field that meanders about the zero axes in the frequency domain. Thus the convolution gives a meandering process with no structure.

Case 2  This is termed as the non-zero mean case. In this case the \( n(x, y) \) has an appreciable mean value relative to it variance and is stationary. In this case the PS of
$z(x, y)$ is similar to $s(x, y)$. The power spectra for both fields are shown in Figs.4.13 and Fig.4.14, respectively.

**Figure 4.13** The PS of $z(x, y)$

**Figure 4.14** The PS of $s(x, y)$
This result shows that when the noise modulating a signal has a non-zero mean, it is not possible to separate the PS of the noisy image from the pure image. The noise term acts as a reproducing kernel in this case and returns the PS of the pure image [Heine et al. 2004]. The correlation between the two images shown in Figs.(4.13-4.14) was found to be 1.0.

In this case the FT of the noise field follows the previous example everywhere except at zero frequency region. If the mean is appreciable, the FT of \( n \) will appear as a spike at the zero frequency. Hence, Eq. (4.7) is acting like a delta function convolved with the FT of \( s \), which just returns the spectrum of \( s \) (in an average sense).

This result is relevant for the estimation of SDN in images. This characteristic of the power spectra behavior in images in presence of noise with a large mean has shown a significant impact in the results of the two noise estimation methods that are discussed in chapter 3.
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this work, the noise and spectral characteristics of the preliminary tomosynthesis data are analyzed. Two methods are derived for the estimation of signal-dependent noise. Both methods have proven to be successful in estimating the functional form of the noise in the event of the pure noise term modulating the signal being zero-mean. Method I (the filtering method) was discovered to be unsuccessful in estimating the form of the noise in the event of the pure noise term having a large-mean. On the other hand, method II (order-statistics method) was proven to be successful on both events. The failure of the filtering technique arises from the fact that in the event of the noise having a large mean, the spectrum of the SDN and the pure signal cannot be differentiated (discussed in chapter 4).

The noise from the tomosynthesis projection data and four sets of reconstructed volume is estimated as a linear function of the signal, as illustrated in chapter 3. The functional form of noise estimated from the FFDM data is found to be linear as well. Since the tomosynthesis technique is built on the FFDM framework, these results are consistent.
The spectral characteristics of the projections follow a $1/f$ behavior, which shows a similar nature as mammograms. The images from reconstruction sets followed the $1/f$ behavior except the FBP, which deviated from this behavior. Comparing the spectral characteristics of the other three volume sets, the images show similarity in the low frequency region.

### 5.2 Recommendations

Characterization of the images provided a better understanding of the data acquired from the tomosynthesis system. The TD can be investigated in more detail with images from living women, when available. Since the projection and volume images show similar characteristics as regular mammograms, existing statistical methods for mammography data may be modified for tomosynthesis applications for further research. Multiresolution analysis of the data could be an extension of the work presented in this study. Further research could be conducted to compare the volume images from different reconstruction techniques to determine the technique that produces the best results for clinical applications. Texture analysis of the projection and volume images may be an interesting area of research and an extension of this study. Since tomosynthesis is a new technique, there is much scope for further study.
REFERENCES


