Effects of using graphing calculators with a numerical approach on students' learning of limits and derivatives in an applied calculus course at a community college

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Effects of Using Graphing Calculators with a Numerical Approach on Students’ Learning of Limits and Derivatives in an Applied Calculus Course at a Community College

by

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A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy Department of Secondary Education College of Education University of South Florida

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Dedication

I dedicate this dissertation to my loving wife Kalaivani for providing me with the love, patience, understanding, and guidance that was needed to reach this goal. I also dedicate this to my wonderful sons Vishnu and Krishna for their love, patience, and understanding. I further dedicate this to my loving parents Arumugam and Leelavathy.
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<td>American Mathematical Association of Two-Year Colleges</td>
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<td>ANOVA</td>
<td>Analysis of Variance</td>
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<tr>
<td>ANCOVA</td>
<td>Analysis of Covariance</td>
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<td>CAS</td>
<td>Computer Algebra System</td>
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<td>GC</td>
<td>Graphing Calculator</td>
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<td>MAA</td>
<td>Mathematical Association of America</td>
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<td>MWF</td>
<td>Monday-Wednesday-Friday</td>
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<td>NCTM</td>
<td>National Council of Teachers of Mathematics</td>
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Effects of Using Graphing Calculators with a Numerical Approach on Students’ Learning of Limits and Derivatives in an Applied Calculus Course at a Community College

Arumugam Muhundan

ABSTRACT

This study examined the effects of using graphing calculators with a numerical approach designed by the researcher on students’ learning of limits and derivatives in an Applied Calculus course at a community college. The purposes of this study were to investigate the following: (1) students’ achievement in solving limit problems (Skills, Concepts, and Applications) with a numerical approach compared to that of students who solved limit problems with a traditional approach (primarily an algebraic approach); and (2) students’ achievement in solving derivative problems (Skills, Concepts, and Applications) with a numerical approach compared to that of students who solved derivative problems with a traditional approach (primarily an algebraic approach).

Students (n = 93) in all four daytime sections of an Applied Calculus course in a community college participated in the study during the spring 2005 semester. One of two MWF sections and one of two TR sections served as the treatment groups; the other two sections served as the control groups. Two instructors other than the researcher participated in the study. Instructor A taught one treatment group (a TR section) and one
control group (a MWF section); instructor B taught one treatment group (a MWF section) and one control group (a TR section).

Dependent variables were achievement to solve skill, concept, and application limit problems and skill, concept, and application derivative problems, measured by two teacher-made tests. A pretest administered on the first day of class determined that no significant difference existed between the groups on prerequisite algebra skills. Separate ANCOVA tests were conducted on the skill, concept, and application portions of each of the limit and derivative exams.

Data analyses revealed the following: (1) there was no significant difference found on the skill portion of the limit topic (unit 1 exam) due to instruction or to instructor; (2) there was a significant difference found on the concept portion of the limit topic due to instruction and to instructor; (3) there was a significant difference found on the application portion of the limit topic due to instruction but not due to instructor; (4) the interaction effects between instructor and instruction were not significant on the skill, concept, and application portions of the limit topic; (5) there was a significant difference found on the skill portion of the derivative topic (unit 2 exam) due to instruction but not due to instructor; (6) there was a significant difference found on the concept portion of the derivative topic due to instruction and to instructor; (7) there was a significant difference found on the application portion of the derivative topic due to instruction but not due to instructor; and (8) the interaction effects between instructor and instruction were not significant on the skill, concept, and application portions of the derivative topic. All significant differences were in favor of the treatment group.
Chapter 1

Introduction

Concerned mathematics educators search for new and efficient methods to improve students’ understanding and performance in mathematics courses (Douglas, 1986; Heid, 1997; Kaput, 1992; Waits & Demana, 2000). Over the years, there have been many calls for the reform of mathematics education in order to improve students’ performance and understanding. Among them, the most consistent recommendation from the mathematics community is that all mathematics courses take full advantage of the availability of calculators and computers (American Mathematical Association of Two-Year Colleges [AMATYC], 1995, 1999; Dunham, 1999; Heid, 1997; Kaput, 1992; National Council of Teachers of Mathematics [NCTM], 1974, 1980, 1989, 2000; Waits & Demana, 2000).

Educators believe technology is useful in mathematics education and technology has been changing at an increasing rate. The use of computers in education first appeared in the 1960s. The first electronic calculator, a four-function model, was manufactured in 1970 by Canon, Inc. The first computer algebra system (MuMath), a computer program, was first introduced in mathematics in 1979. The first graphing calculator (Casio-fx-7000G) appeared in 1986. The first computer algebra system-added graphing calculators (“supercalculators”, like the TI 92) were manufactured in 1996. In 1998, flash ROM
technology was introduced in calculators which allows software applications to be run on the calculators. Hence, in a relatively short period of time technology has made tremendous strides. These calculators, known as hand-held calculators, and computers have impacted and will continue to impact mathematics education.

The NCTM has been a promoter of the use of technology in mathematics classrooms from the beginning (NCTM, 1974, 1980, 1989, 2000). As early as 1974, the NCTM recommended the use of the earliest four-function calculators in schools. The early research on calculators convinced the NCTM to make further recommendations for using these technologies in mathematics education. In its 1980 publication, *An Agenda for Action: Recommendations for School Mathematics*, the NCTM stated: “Mathematics programs must take full advantage of the power of calculators and computers at all levels” (p. 8). Later, in its *Curriculum and Evaluation Standards*, the NCTM (1989) emphasized that “Computer technology is changing the ways we use mathematics; consequently, the content of mathematics programs and methods by which mathematics is taught are changing” (p. 2). In the same publication, the NCTM stated: “The teaching of mathematics is shifting from primary emphasis on paper-and-pencil calculations to the full use of calculators and computers” (1989, p. 83), and therefore, “scientific calculators with graphing capabilities will be available to all students at all times” (p. 124).

In the NCTM’s latest document, *Principles and Standards for School Mathematics*, the council emphasized the importance of the use of technology in mathematics classrooms by stating, “technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’
learning” (2000, p. 24) and “when technology tools are available, students can focus on decision making, reflection, reasoning, and problem solving” (2000, p. 24).

Other national and international mathematical organizations, such as the AMATYC and the Mathematical Association of America (MAA), have also consistently made recommendations for the use of appropriate calculators and computers in mathematics courses. The AMATYC, in its *Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus* (1995), adopted a standard that “Mathematics faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities it presents as a medium of instruction” (p.15). Later, in its 1999 *Crossroads in Mathematics: Programs Reflecting the Standards*, AMATYC again emphasized, “Students will use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of their results” (p. 9).

Many mathematics educators have also made recommendations to use appropriate technology in mathematics courses. For example, Corbitt (1985) supported the use of technology in mathematics education by saying that “The major influence of technology on mathematics education is its potential to shift the focus of instruction from an emphasis on manipulative skills to an emphasis on developing concepts, relationships, structures, and problem-solving skills” (p. 244). Dossey, Mullis, Lindquist, and Chambers (1988) wrote that the use of technology could improve students’ performance in mathematics:

Improving mathematics performance will require educators’ best efforts to upgrade the curriculum, modify classroom instruction, and use new teaching materials, including technological resources. … The rapid pace of technological
progress necessitates a revised set of priorities for mathematics instruction. To improve their understanding of mathematics and their ability to solve mathematical problems, students need the benefit of instruction that emphasizes practical experience in solving problems and opportunity to use calculators and computers (p. 15).

The constant recommendations for the use of technology in mathematics encouraged mathematics educators to initiate projects and studies in mathematics courses with the use of technology. Evidence shows that many forms of technology have been used and tested in various areas in mathematics (Demana & Waits, 1998; Dunham, 1999; Heid, 1988; Hembree & Dessart, 1986, 1992; Kaput, 1992; Penglase & Arnold, 1996). These studies provided evidence that using technology in mathematics education has a great potential to shift the instructional focus from an emphasis on manipulative skills to an emphasis on developing concepts, reasoning skills, and problem solving skills. These studies concluded that the use of technology in appropriate ways improves students’ attitudes towards mathematics, increases students’ motivation level, and improves students’ mathematics achievement.

There are many areas in mathematics where technology has played and can play an important role to make teaching and learning mathematics easier. Calculus is one area in mathematics that can benefit from technology, as calculus is often difficult for many students (Douglas, 1986; Ferrini-Mundy & Graham, 1991; Smith, 1996; Steen, 1987; Tall, Smith, & Piez, 2004).

**Calculus and Technology**

Calculus is considered a rich subject in the modern world with applications in many areas, such as engineering, the physical and biological sciences, and business.
Therefore, those programs require students to have at least one semester of calculus. Unfortunately, calculus is probably the most unsuccessful course offered in higher education (Gordon & Hughes-Hallett, 1991). Ferrini-Mundy and Graham (1991) reported that approximately 50% of the students enrolled annually in the first semester calculus course either withdraw or fail the course. Students’ unsuccessfulness in calculus filters them out of business, science and engineering fields (White, 1987). For many students, calculus is a major stumbling block on the road to their professional careers.

To understand the calculus concepts and how these concepts can be applied to other fields, one needs a strong algebraic background. A weak mathematical background in terms of lack of function concepts, algebraic manipulations, and geometric visualizations is one reason why students are not successful in calculus classes (Douglas, 1986; Ferrini-Mundy & Graham, 1991; Tall et al., 2004).

Many calculus reforms initiated in the 1980s addressed the high attrition rate in calculus and the lack of student understanding of the concepts of calculus (Barnes, 1997; Douglas, 1986; Smith, 1996; Steen, 1987; Tall et al., 2004). In all the calculus reforms and reports, the primary ideas were to develop an alternate curriculum that is more conceptual and application oriented than the traditional curriculum, to make use of multiple representations of mathematical concepts, to develop a variety of teaching methods for calculus, and to use technology. Many studies in calculus reforms have focused on using technology to increase the emphasis on conceptual understanding while decreasing the emphasis on routine skills (Crocker, 1990; Heid, 1984; Judson, 1990).

Although many of these studies involved the usage of computers and a variety of computer algebra systems (CAS), this technology is not widely available for students
because of its high cost, and the required training to operate these systems. Waits and Demana (1996) agreed: “Dependence on only desktop computers and expensive software housed in computer laboratories is a major barrier to implementing serious technology-based curriculum reform in mathematics” (p. 713). The invention of graphing calculators (GCs) became a solution for this problem. Foley (1987) stated “The advantage of computers over calculators disappeared in early 1986 when Casio introduced the fx-7000G, a programmable scientific calculator” (p. 28).

**Graphing Calculators (GCs)**

The GC was first introduced by Casio in 1986. Since then, many manufacturers, including Texas Instruments, Casio, Sharp, and Hewlett-Packard, have produced more sophisticated GCs. GCs are programmable calculators that have standard computer processors, display screens, and built-in software. Due to increasingly sophisticated technology, GCs have begun to assume more and more computer-type capabilities.

Before the discovery of GC technology, computers and CASs were the only available technology in college level-mathematics courses. Today's easy access to technology in the form of GCs creates new ways for learning through graphical and numerical representations. Because of their low cost, portability, and capability, a GC is an appropriate technology tool to use in upper-level mathematics courses. Particularly, the speed, accuracy, and capabilities of current graphing calculators have led many mathematics educators to believe that more emphasis should be placed on numerical methods in calculus classes and less emphasis on techniques of differentiation and integration (Demana & Waits, 1998; Dunham, 1999; Ferrini-Mundy & Gaudard, 1992;
They believed that this approach can enhance students’ understanding in calculus concepts.

The initial reactions to this technology in mathematics education are generally positive (Dunham & Dick, 1994). Demana and Waits (1998) stated that students who use GCs experience a rich mathematics curriculum that allows them to focus on realistic applications. They also believed that the full use of GC technology could deepen students’ understanding about mathematics concepts. Vonder Embse (1992) mentioned that “The large screen display, graphics capability, exploratory functions of graphing and multiline display calculators afford students and teachers opportunities to investigate, compare, and explore concepts and problem situations in better ways than when using standard hand-held calculators or no technology at all” (p. 65).

GCs have become more popular among students and teachers for several reasons. GCs offer relatively large screens, interactive graphics, and on-screen programming and other built-in features, such as zoom-in, zoom-out, trace, and table. Many of these capabilities were previously available only on a mainframe or a microcomputer.

The invention of hand-held GCs was, in fact, a major breakthrough in mathematics education. The powerful capabilities, together with the decreasing cost, have made the use of GCs a choice for technology use in mathematics classrooms. “The greatest benefits seem to come from this technology that is under student and teacher control, promotes student exploration and enables generalization” (Demana & Waits, 1992, p.94). Ruthven (1995) supported the use of GCs and mentioned “they offer not simply a mechanism for calculating and drawing but a medium for thinking and learning” (p. 232).
Major mathematics organizations such as the NCTM, AMATYC, and MAA and experts in mathematics education support the use of GC technology in mathematics courses, including calculus. Several projects and studies have been initiated in mathematics courses because of the calculator’s power, and a strong recommendation from the mathematics community. The usage of GCs among students and teachers has grown rapidly.

In fact, today a GC is required for students who enroll in any mathematics courses in college algebra and beyond in many universities and community colleges. Also, GCs are required or allowed on many standard mathematics exams, such as PSAT, SAT, and AP calculus by the College Board (Waits, Leinbach, & Demana, 1998). The use of GCs in school mathematics has also increased in many parts of the world, including in Australia, Canada, and many countries in Europe (Waits & Demana, 2000).

Because of the potentially strong influence the technology can have on mathematics education, educators have observed its impact on mathematics. Waits, Leinbach, and Demana (1998) warned that,

At least one-fourth of the material that was typically taught in a US high school “trig/functions” course or college “precalculus/college or algebra/trig” and even some chapters in textbooks dealing with paper-and-pencil computation methods became obsolete and disappeared from the curriculum because hand-held scientific calculators provided better ways to “compute” than paper-and-pencil methods. The same thing (obsolescence) will soon happen with paper-and-pencil symbolic algebraic manipulations common today because of student use of inexpensive hand-held computer algebra systems that now exist and soon will proliferate (p.1).

Although major mathematics organizations and experts in mathematics education support the use of GC technology, some raise questions and concerns about the impact of
using this technology in mathematics courses. Foley (1988) raised some important questions related to the presence and affordability of this technology:

1. How will these calculators affect mathematics education at the community college?
2. How should these calculators affect it?
3. How should calculators influence what is taught and how it is taught?
4. How can calculators improve students’ understanding of mathematics? (p. 29)

Harvey (1992) raised a similar concern. He made the following suggestions:

1. We need to analyze carefully the content that we presently teach and that we would like to teach.
2. Once the content of the mathematics curriculum has been examined, we need to determine the ways that particular tools can help us teach that content.
3. We must not cling to our present ways of teaching (p. 145).

In addition, Dunham (1999) and Heid (1997) emphasized that curriculum development, assessment, the method of instruction, and required training and instructional materials for instructors need to be addressed in the use of GCs in mathematics education.

These questions and concerns are increasingly important with the proliferation of GC technology in mathematics education. The mathematics education community has a responsibility to react positively to the available technology and careful research studies are needed to answer these questions and validate the concerns.

While mathematics educators are working on evaluating the impact of the technology in mathematics education, the use of many forms of technology continues to
grow. In particular, the use of GCs in mathematics courses by teachers and students cannot be stopped. Using a regular scientific calculator is a norm in almost every mathematics course in any school. Now, using a GC has gradually become a norm in many mathematics courses in colleges and universities. With GCs now available and affordable, Foley (1988) warned that students are likely to buy and use them whether or not the mathematics teachers do. The AMATYC (1995) wrote about the availability of this technology in students’ hands: “students will use it [GC] whenever they realize its power, regardless of whether professors allow it or not in their classrooms. Mathematics faculty must adapt to this reality and help students use technology appropriately so that they can be competitive in the workforce and adequately prepared for future study” (p. 55). Iseri (2003) agreed: “The issue, it seems, is not so much on whether to use technology [graphing calculator] in mathematics teaching, but when and how” (p. 1). Therefore, the question to the mathematics community is not whether a GC is allowed in mathematics courses but how it is and should be used in students’ mathematics learning.

**Purpose of the Study**

Among these concerns of the use of GC technology in mathematics, the following question motivated this study: How effectively can and should the GC technology be used so that the usage will have a positive effect on students’ understanding of mathematics, particularly in *limit* and *derivative* topics in an Applied Calculus course?

Research evidence shows that the use of GC technology has a positive impact on students’ calculus learning (Dunham, 1999; Heid, 1997; Penglase & Arnold, 1996; Waits & Demana, 2000). However, it is also evident that using the GC technology alone in
mathematics courses may not make any differences in students’ conceptual understanding (Cassity, 1997; Dunham, 1999; Heid, 1997; Waits & Demana, 2000). Carefully designed instructional materials need to accompany the use of GC technology in the mathematics course.

The main purpose of the proposed study, therefore, was to examine the effects of the use of GCs with a numerical approach and the researcher-developed instructional materials on students’ learning of limits and derivatives in an Applied Calculus course (MAC 2233) at a community college. The course requires students to have a GC (the mathematics department recommends a TI 83 GC) for MAC 2233 course at this college. The instructional materials were developed by the researcher with the use of a TI 83 GC and are divided into four unit lessons. The purpose of these lessons is to guide students to use this powerful calculator in their understanding of selected calculus topics, limits and derivatives. More details of these lessons will be discussed in chapter 3.

The general research question that this study sought to answer is “To what degree can the use of GCs with a numerical approach and instructional materials developed by the researcher affect community college Applied Calculus students’ learning of limits and derivatives?” In particular, the study sought to answer the following research questions:

1. How does the students’ achievement in solving limit problems with a numerical approach compare to that of students who solved limit problems with a traditional approach (primarily an algebraic approach) in an Applied Calculus course?

2. How does the students’ achievement in solving derivative problems with a numerical approach compare to that of students who solved derivative problems
with a traditional approach (primarily an algebraic approach) in an Applied Calculus course?

The following research hypotheses are used to seek the answers for these research questions:

1. Students (GCGL group) who receive instruction with a numerical approach will have higher achievement in routine (skill oriented) limit problems than students (GC group) who receive instruction with a traditional approach (primarily an algebraic approach).

2. The students in the GCGL group will have higher achievement in conceptual oriented limit problems than the students in the GC group.

3. The students in the GCGL group will have higher achievement in related applications of limits than the students in the GC group.

4. The students in the GCGL group will have higher achievement in routine (skill oriented) derivative problems than the students in the GC group.

5. The students in the GCGL group will have higher achievement in conceptual oriented derivative problems than the students in the GC group.

6. The students in the GCGL group will have higher achievement in related applications of derivative problems than the students in the GC group.

**Significance of the Study**

It is often noted by calculus educators that students in calculus show lack of understanding in the concepts of calculus topics (Douglas, 1986; Ferrini-Mundy & Graham 1991; Gordon & Hughes-Hallett, 1991; Tall et al., 2004; Waits & Demana,
One of the criticisms is that students may learn how to answer limit and derivative problems (typically, routine skill oriented problems) but not necessarily understand the concept of the limit and derivative. In fact, getting an answer for a problem without knowing the meaning of it does not help students gain conceptual understanding in mathematics. Ferrini-Mundy and Graham (1991) criticized that “[The] calculus course frequently becomes one of memorization of techniques and procedures with little time focused on the concepts” (p. 29). Similarly, Ferrini-Mundy and Gaudard (1992) noted: “The old calculus becomes a litany of procedures and template problems which too often results only in giving students some rather mindless algebra practice” (p. 58). They further mentioned that “Many students who can find derivatives mechanically and solve problems using them nevertheless often have little idea [about] what a derivative actually means” (p. 62).

This criticism about lack of students’ understanding in calculus still continues. Recently, the AMATYC, in its 2003 report, *A Vision: Mathematics for the Emerging Technologies*, wrote, “Knowing how to differentiate and integrate functions is worthless unless students know how these operations are used and can interpret the results in the context of a real-life situation” (p. 8).

Calculus, which is about 350 years old, is considered a rich subject by the mathematics community. Unfortunately, students do not think of calculus in the same way. Gordon & Hughes-Hallett (1991) mentioned:

Students who come out of calculus do not have any feel for the beauty and grandeur of the subject and an appreciation for its power to solve dynamic problems in almost all areas of human endeavor. Instead, the students have been mired in a series of mindless mechanical manipulations that many believe to be the substance and raison d’etre of calculus (p. 50).
Further they blamed the role of the calculus textbooks as a part of the reason why
students do not value the power of calculus. They noted: “Historically, the development
of calculus was always ‘problem-driven’; it is only the standard course and the associated
textbooks that are ‘technically-driven’ ” (p. 54). They further mentioned that in the
calculus curriculum the evaluation of limits becomes a mechanical process to produce an
answer to a variety of artificial questions; differentiation is a manipulative process to
produce an answer, but not to answer a question of any substance; and integration is a
mechanical process to produce an answer which matches the expression at the end of the
book.

Tall et al. (2004) pointed out that students do not see the power of calculus
because of the way calculus instruction is presented to them. He suggested that the
derivative of a function should be introduced as a rate of change of a function. He wrote,
“Students see differentiation as a sequence of some algebraic manipulations applied for a
specific symbolic expression, rather than a conceptual idea of ‘rate of change’.”

Analyzing these various concerns about students’ conceptual understanding in
calculus topics, this study aimed to promote students’ conceptual understanding of limits
and derivatives. My teaching experience in the Applied Calculus course at a community
college over the last 15 years has given me an opportunity to understand students’
difficulties in this course. The experience shows that one of the reasons for students’
difficulties in this course is their weak mathematical background in terms of lack of
function concepts, algebraic manipulations, and geometric visualizations.

There are two entry level-courses in calculus that are offered in colleges and
universities. One is a regular Calculus-I course (MAC 2311) and the other is an Applied
Calculus course (MAC 2233). Students get to choose one of these calculus courses based on their major and requirements. As mentioned earlier, this study was conducted in an Applied Calculus course at a community college. The Applied Calculus students, the accessible population, at this college, like other colleges in the state, are even weaker in function concepts and algebraic skills than the Calculus-I students. Part of the reason is that, in this college, the prerequisites for MAC 2311 are Precalculus Algebra (MAC 1140) and Trigonometry (MAC 1114); however, the prerequisite for applied calculus is only Basic College Algebra (MAC 1105). Previously, MAC 1140 was the prerequisite for the Applied Calculus course; the state has lowered the algebra requirements for the Applied Calculus course, perhaps because many students were not able to register for the course. Hence, it is unfortunate that less algebra is now required for the Applied Calculus course and students typically experience greater difficulty in the class than do students in Calculus I. It is, therefore, a challenge for teachers to teach calculus concepts to students who are weak in function concepts and algebraic manipulative skills.

The Applied Calculus course, also known as Business Calculus in some colleges, is a calculus course designed to fulfill requirements for business and other non-mathematics majors but not for mathematics and science majors. For many students, this course is probably their last mathematics course. The main purpose of the course is to study calculus applications in students’ related majors. Rigorous calculus is not a goal of the course. Topics include limits, differentiation, and integration of algebraic, exponential, and logarithmic functions, integration techniques, and related applications in the management, business, and social sciences.
Limits of Functions

Students in the Applied Calculus course study limits of functions as the first topic in their calculus learning. They spend a great deal of time studying limit problems without studying the formal definition (the delta-epsilon definition) of a limit. Because, perhaps, the formal definition of the limit of a function is too abstract for students to understand at this level, Applied Calculus students are introduced to the concept of the limit of a function through examples; most Applied Calculus textbooks, if not all, do not even mention the formal definition of the limit of a function.

Generally, therefore, a limit problem can be approached by three methods: Algebraic, Numerical, and Graphical. An algebraic approach to a limit problem depends highly on algebraic techniques and manipulations. Further, it is also possible that any algebraic techniques or manipulations may not work at all for certain limit problems. Students may not do well by this method if they are weak in their algebraic skills. A numerical approach to a limit problem depends on computational skills. This method may be time consuming if no calculators are allowed. The required computations are relatively easy and quick with GCs using special features such as a table feature. The third approach, a graphical approach, requires a GC but not all functions give “nice” graphs. Also, students need to be warned about the misleading behaviors and limitations of graphs. It is possible, of course, that a limit problem can be approached by a combination of these three methods.

For the purpose of this study, an algebraic and a numerical approach are discussed in the following section.
Algebraic Approach to Limit Problems

Consider the following examples of limit problems found in a typical Applied Calculus textbook in the chapter on the limits of functions:

Example 1. Given \( f(x) = \frac{x^3 + 3x^2 + 4x + 12}{2x^2 + 5x - 3} \), find \( \lim_{x \to -3} f(x) \), if it exists.

Answer: Even though finding the limit of \( f(x) \) as \( x \) approaches -3 does not mean finding the function value at \( x = -3 \), if the function is a continuous function then the above two values will be the same. If students "substitute \( x = -3 \)" in the expression, they get "0/0" (indeterminant form). So they simplify the expression by factoring each polynomial in the numerator and the denominator before substituting \( x = -3 \). The steps may look like this:

\[
\lim_{x \to -3} \frac{x^3 + 3x^2 + 4x + 12}{2x^2 + 5x - 3} = \lim_{x \to -3} \frac{(x^2 + 4)(x + 3)}{(2x - 1)(x + 3)} = \lim_{x \to -3} \frac{(x^2 + 4)}{(2x - 1)} = \frac{-13}{7}
\]

Therefore, \( \lim_{x \to -3} f(x) = \frac{-13}{7} \).

Example 2. Given \( f(x) = \frac{x - 4}{\sqrt{x} - 2} \), find \( \lim_{x \to 4} f(x) \), if it exists.

Answer: If students "substitute \( x = 4 \)" in the expression, they again get "0/0" (indeterminant form). This time students simplify the expression by rationalizing the denominator and then are directed to "substitute \( x = 4 \)". The steps may look like this:

\[
\lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2} = \lim_{x \to 4} \frac{(x - 4)(\sqrt{x} + 2)}{\sqrt{x} - 2} = \lim_{x \to 4} \frac{(x - 4)(\sqrt{x} + 2)}{(x - 4)} = \lim_{x \to 4} \sqrt{x} + 2 = 4
\]

Therefore, \( \lim_{x \to 4} f(x) = 4 \).
In these two examples, students obtain the correct answers for the problems, but it is doubtful how much of the limit concept of a function is understood. Students use their algebraic knowledge (factoring knowledge in example 1 and rationalization knowledge in example 2) to complete the problems. This approach relied upon algebraic techniques and manipulations. At the same time, this method of doing these limit problems provided students the opportunity to reinforce their algebraic skills without focusing on the meaning of the concept of the limit of a function. Furthermore, this approach may not work if the expressions in the given function cannot be factored or rationalized. Instead of using these algebraic techniques to find the limits, the limits can be found with the following numerical method.

**Numerical Approach to Limit Problems**

Graphing calculators, such as the TI 83, have a table feature that can be used to find limits numerically as indicated below.

To answer example 1, first enter the expression in the calculator as a function

\[ y_1 = \frac{(x^3 + 3x^2 + 4x + 12)}{(2x^2 + 5x - 3)} \]

and then use the table feature to get the following:
The table is actually providing the function values at various \( x \) values. From the table it is clear that the function approaches \(-1.857\) (correct to 3 decimal places) as \( x \) approaches \(-3\) from both sides. Note that the exact answer for this problem was \(-\frac{13}{7}\), approximated to 3 decimal places as \(-1.857\). Also it is worth noting that the function is undefined at \( x = -3 \).

Likewise for example 2, first enter the expression as the function
\[
y_1 = \frac{(x - 4)}{(\sqrt{x} - 2)}
\]
and then use the table feature to obtain the following:
From the table it is clear that the function approaches 4 (correct to 3 decimal places) as $x$ approaches 4 from both sides. Note that the exact answer was 4 as well. Also, the function is undefined at $x = 4$.

When a limit problem is done numerically using a table, it is easier for students to understand the left hand limit and the right hand limit concepts which are crucial in understanding the limit concept of a function.

**Derivatives of Functions**

The second topic the Applied Calculus students study is *differentiation*. Differentiation is a process of finding the *derivative* of a given function. Because the derivative of a function at a given point is defined through a limit process, understanding the limit of a function is important to understand the derivative of the function.

In the derivative problems, first, one has to apply the correct derivative rule(s) for the given function; second, one has to perform the correct algebraic steps in order to get the derivative in a simple form. Finding the derivative of a function can be a tedious procedure in many real-world applications.

The first and direct application of finding the derivative of a function at a given value is that the derivative value provides the *rate of change* of the function at the given value. The class of *rate of change* applications is one of the important applications that can be solved by the concept of derivative. To solve a rate of change application problem, the derivative of the particular function needs to be obtained from the derivative rule(s) and then the derivative of that particular function needs to be evaluated for a given $x$ value. The second part is relatively easy because students simply “plug – in” the $x$ value
into the derivative they obtain. Of course, if students make mistakes in finding the
derivative, then the “plug – in” part gives an incorrect answer.

Consider the following examples of derivative problems found in a typical
Applied Calculus textbook in the chapter on the derivative of functions. As before, an
algebraic approach is first discussed and then a numerical approach for the same problem
is discussed.

**Algebraic Approach to a Derivative Problem**

Given \( f(x) = \frac{20(10 + 7x)}{1 + 0.02x} \), find \( f'(5) \).

Answer: Students first need to note that the given function is in a quotient form, and use
the *quotient rule* to get \( f'(x) \) and then substitute \( x \) with 5 to get the answer. The steps
may look like:

\[
\begin{align*}
 f'(x) &= \frac{(1 + 0.02x)(20) - 20(10 + 7x)(0.02)}{(1 + 0.02x)^2} \\
 &= \frac{140 + 2.8x - 4 - 2.8x}{(1 + 0.02x)^2} \\
 &= \frac{136}{(1 + 0.02x)^2}.
\end{align*}
\]

Then \( f'(5) = \frac{136}{(1 + 0.02(5))^2} = \frac{136}{1.21} = 112.3966942 \), and is rounded to 112.4.

Students may get the solution by using the necessary formula and some algebraic work.
Again, the concern is: Do students understand the meaning of the number 112.4 in the
answer they found?

At the same time, the derivative of \( f(x) \) at \( x = 5 \), \( f'(5) \), can be found by using
the following numerical method:

**Numerical Approach to the Derivative Problem**

Given \( f(x) = \frac{20(10 + 7x)}{1 + 0.02x} \), find \( f'(5) \).
From the definition of the derivative, \( f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}. \)

Therefore, this derivative problem is a limit problem; thus, students can solve this as a limit problem with the same numerical method they have used earlier. The work is as follows:

To use a calculator to find this limit, use \( x \) for \( h \) (the calculator has a variable key as \( x \)) and let \( y_1 = f(5) = \frac{20(10 + 7(5))}{(1 + 0.02(5))} \), \( y_2 = f(5 + x) = \frac{20(10 + 7(5 + x))}{(1 + 0.02(5 + x))} \), and \( y_3 = \frac{y_2 - y_1}{x} \).

Therefore, \( f'(5) = \lim_{x \to 0} y_3. \)

To find the limit of \( y_3 \) as \( x \) approaches 0, the following \( x \) values are entered in the table feature of a TI-83 graphing calculator. Students are able to get the following:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>112.24</td>
</tr>
<tr>
<td>0.1</td>
<td>112.42</td>
</tr>
<tr>
<td>0.01</td>
<td>112.4</td>
</tr>
<tr>
<td>0</td>
<td>ERROR</td>
</tr>
<tr>
<td>0.001</td>
<td>112.39</td>
</tr>
<tr>
<td>0.01</td>
<td>112.38</td>
</tr>
<tr>
<td>( \cdot 1 )</td>
<td>112.19</td>
</tr>
</tbody>
</table>

From the table, students can conclude that \( \lim_{x \to 0} y_3 = 112.4 \). That is, \( f'(5) = 112.4. \)
Note that because finding the derivative of a function at a given $x$ value is a limit problem, the whole process of finding the limit was done numerically with help from a graphing calculator. Because students have experience in doing limit problems, they can approach this derivative problem as doing a limit problem. A numerical approach using a graphing calculator for finding the limit of a function is a consistent approach that students can use to find the derivative at a given number. While using a graphing calculator for this problem, it is possible for students to focus on concept development with derivatives.

For example, the other benefit of doing the problem this way is that the table gives the average rate of change of that function on various intervals around the given $x$ value. That is, in the previous example, students are able to determine the average rate of change of $f(x)$ on various intervals around $x = 5$. For example, the average rate of change of $f(x)$ between $x = 5$ and $x = 4.9$ is 112.6 (see the first number in the previous table). Also, students notice that the average rate of change of $f(x)$ on an interval approaches the instantaneous rate of change of $f(x)$ as the intervals approach zero.

Both finding the limit of a function and the derivative of a function have traditionally relied upon a high level of algebraic manipulations. As mentioned earlier, students in an Applied Calculus course are not strong in algebraic skills and often get frustrated if they do not get the correct answer or the correct form of the answer that the book provides. This frustration causes students to have difficulty in focusing on calculus concepts. Therefore, teachers in these classes spend time teaching students to use necessary algebraic skills in order to find the correct answer for the limit and derivative
problems. Thus, teachers have less time, if any, to spend using these limits and
derivatives to solve the related real world applications.

The limit and derivative problems are approached by a numerical method with the help of a TI 83-GC and researcher-developed instructional materials in this study. Then the same approach is used to solve the related applications in limits and derivatives so that students can gain conceptual understanding of limits and derivatives.

Definition of Terms

*Regular (Non-Graphing) Calculator (NGC)* is either a basic calculator or a scientific calculator. A basic calculator is a calculator that has the four-functions (addition, subtraction, multiplication, and division) with an eight-digit or more display with floating decimals. A scientific calculator is a calculator that has trigonometric and logarithmic functions in addition to the basic features.

*Graphing Calculator (GC)* is a calculator that has a 2.5 by 2.5 inch display screen with some advanced features such as graphing, table, solving matrices, etc. in addition to the features that a NGC has.

*Computer Algebra System (CAS)* is a computer program that is run in a computer or built-in a graphing calculator. It has the capability to do algebraic operations symbolically.

*Hand-Held Calculator* is a calculator that is either a basic, scientific, graphing calculator, or CAS-added graphing calculator.

*Numerical Method* is an approach that is used to solve limit problems and derivative problems using the table feature of a TI 83 graphing calculator as explained in the previous worked examples.
Traditional Approach is an approach that is used to solve limit problems and derivative problems using primarily algebraic techniques and algebraic manipulations as shown in the previous worked examples.

Delimitations

The study was conducted in the spring of 2005 at a public community college in southwest Florida that serves two counties. According to the college 2003-2004 Factbook, the college service area has a population of approximately 617,000 of which 82.2% is 18 years or older. In spring 2004, the college enrollment was 8393 students, of whom 3357 were full-time and 5036 were part-time; the gender ratio is 64% female to 36% male. In the same term, the average age of full-time students was 23 years and the average age of part-time students was 28 years. The generalization of this study is limited to the students who enroll in an Applied Calculus course at the community college that is described here.

This study was conducted in all of the available daytime sections of an Applied Calculus course. Therefore the generalization of this study is limited to such sections of the course. Also, the generalization of this study is limited to the courses that have the same contents as this course.

Limitations

There are only two instructors who participated and both are males. Any influence of this selection cannot be controlled because these were the two available instructors for this course during the time of the study.
Communication between students enrolled in different sections of the course cannot be controlled. In fact, due to the availability of resources outside the classroom, such as the math lab and personal relationships that existed between students, there is a possibility that students from different sections would have communicated with each other and would have shared instructional materials and methods between the treatment and control groups.

Summary

Evidence shows that technology can have a positive impact on students’ conceptual understanding in mathematics, including calculus. Various organizations and mathematics educators consistently recommend the use of technology in mathematics education, including calculus. Calculus is a subject that can benefit from the use of technology, as calculus is a difficult subject for many students.

At the same time, using technology in any mathematics course without giving careful thought and supporting instructional materials will not ensure students’ understanding in mathematics. Carefully designed guided lessons along with appropriate technology can improve students’ understanding in mathematics. Ellington (2000) stated that “curriculum designed specifically for instruction with [graphing] calculators can enhance student achievement in operational and problem solving skills” (p. 177) and recommended that “teachers should design lessons which integrate calculator-based explorations of word problems and mathematical concepts with regular instruction” (p. 178).
Also, it should be noted that there has been a critical need for these supported instructional materials in mathematics courses. Dunham (1999) also emphasized such a need: “There is a critical need for research in instructional design to create curricula that use calculators to their best advantage, to find effective materials to combat calculator-induced errors, and to evaluate programs that incorporate calculators” (p. 23).

Understanding these concerns in mathematics education and an attempt to respond to such a critical need, particularly in calculus, this study examines the effects of using GCs with a numerical approach and the researcher-developed instructional materials on community college students’ learning of limits and derivatives in an Applied Calculus course (MAC 2233). It is the researcher’s hope that teaching calculus topics with this method along with the use of GCs and the supported instructional materials can increase students’ conceptual understanding in limits and derivatives in the Applied Calculus course.
Chapter 2  

Literature Review

This study was designed to investigate the hypotheses that the effects of using graphing calculators (GCs) with a numerical approach and the researcher-developed instructional materials to limit and derivative problems will increase students’ conceptual understanding in limit and derivative topics in an Applied Calculus course at a community college. An Applied Calculus or Business Calculus course is a part of the calculus curriculum and several closely related areas in the literature were included in this chapter.

The chapter is organized into four sections. The first section examines technology usage in mathematics education, particularly regular (non-graphing) calculator (NGC) usage, computer and computer algebra system (CAS) usage, and graphing calculator (GC) usage in mathematics education. The second section discusses students’ errors and misconceptions in calculator usage. The third section focuses on the contents of calculus, particularly, limits and derivatives and students’ knowledge and learning in those areas. The fourth section discusses the role of technology in calculus, particularly computer and CAS usage, and GC usage in limit and derivative topics.
Research on Technology Usage in Mathematics Education

Technology has taken on an increasingly important role in mathematics education for quite some time as it gains support from the mathematics community (AMATYC, 1995, 2003; Demana & Waits, 1990, 1992, 1998; Dunham, 1999; Heid, 1988, 1997; Kaput, 1992, NCTM, 1989, 2000; Suydam, 1982; Suydam & Brosnan, 1994). There are several forms of technology that are widely used in mathematics education. These include regular (non-graphing) calculators (NGC), computers, computer algebra systems (CASs), microworlds, dynamic geometry tools, calculator-based laboratory devices (CBLs), microcomputer-based laboratory devices (MBLs), graphing calculators (GC), and CAS-added GCs (Heid, 1997). Pea (1987) called these technologies “cognitive technologies” because these technologies help “transcend the limitations of the mind…in thinking, learning, and problem-solving activities” (p. 91). Heid (1997) supported the use of these cognitive technologies noting, “the use of a cognitive technology has the potential for affecting subject matter, curriculum, instruction, learning styles, and problem-solving activities” (p. 7).

Because of the appearance of these cognitive technologies, many areas in mathematics education required changes. Some of the changes are the way students study, the way the instruction is delivered, the way questions are asked on exams, the way assessments are made, and the way curriculum is developed. Bitter (1987) stated this as, “Traditional mathematics curriculum components are being outdated as new technology expands its capabilities” (p. 46). Although there were several forms of technology available, mainly NGCs, GCs, computers, and CASs are the technology forms that are widely accepted in schools, colleges, and in many universities to promote
students’ understanding in mathematics. Over the last twenty years, several dissertation studies and projects were initiated in mathematics courses focusing on increasing the emphasis on conceptual understanding while decreasing the emphasis on routine skills with the use of these technologies (Dunham, 1999; Heid, 1988, 1997; Judson, 1990; Palmiter, 1991; Penglase & Arnold, 1996). The following section focuses on the research in mathematics education with the use of NGCs.

**Research on Usage of Regular (Non-Graphing) Calculators (NGCs) in Mathematics Education**

NGCs were probably the first piece of technology that received attention in mathematics education. NGCs have become more popular among students and teachers because they are simple, inexpensive, and friendly to use. The NCTM (1974) was probably the first organization that recommended the use of these calculators in schools and many mathematics educators welcomed the recommendation. Bell (1978) supported the use of calculators in school and mentioned that, “Calculators can provide a direct alternative to the arithmetic and computational methods that make-up the principal component of the first eight years of mathematics training as schools move away from ‘answer – oriented’ instruction to a concentration on the more important concepts” (p. 405). Kaput (1992) wrote that “…heavy use of calculators in the early grades as part of instruction and assessment does not harm computational ability and frequently enhances problem-solving skill and concept development” (p. 534).

Troutman & Lichtenberg (1995) also supported this little machine: “The minicalculator is truly a revolutionary device…a calculating machine that is accurate, inexpensive, durable, and small. Not only is the use of this machine in the teaching of
mathematics appropriate, but it will have a direct and significant consequence for the school… We will be spending more of our allotted time emphasizing mathematical ideas and less time on long and tedious computational procedures” (p. 38). Many educators believed that calculators could be used to aid algorithmic instruction; facilitate concept development; reduce the demand for memorization; enlarge the scope of problem solving; provide motivation and encourage discovery, exploration, and creativity.

One of the earliest technology-based projects was started in the Ohio State University (OSU) by Waits and Leitzel in 1974 (Waits & Leitzel, 1976). The project was an effort to reform the college remedial mathematics curriculum that required the use of the very earliest four-function calculators (NGCs) by all students. The results of this project motivated the developers to develop two other major projects at the OSU: Approaching Algebra Numerically (AAN) and Calculator and Computer Precalculus (C² PC) project. Among these two, the C² PC project targeted mainly precalculus courses and it was widely conducted in many high schools, colleges, and universities. Computers, CASs, and GCs were heavily used in this project. A discussion on this project is covered later in this chapter.

Much research has occurred on the effects of NGCs on student achievement in basic skills, in problem solving, and on student attitudes, typically showing positive results. Many of these studies were analyzed and reported by several researchers, with three such review reports being examined in this section.

Suydam (1976) was perhaps the first to give a report on NGC-based studies. She reviewed 24 studies and reported that most studies favored the use of calculators but most of them were conducted with poor research designs. In 1982, she again conducted a
research review on the effects of using calculators in mathematics classes (Suydam, 1982). In the research review of 75 studies, Suydam reported that 95 comparisons were drawn between the achievement scores of groups that used or did not use calculators within traditional instruction. In 43 comparisons, the calculator group had a significantly higher mean score than the noncalculator group; in 47 comparisons, the difference was not significant; and in 5 comparisons, the noncalculator group had a significantly higher mean score. She further reported that the use of calculators will result in as high or higher achievement as with paper-and-pencil and recommended that “calculators are good for promoting achievement …” and that “all students can benefit from using calculators” (p. 27).

Hembree and Dessart (1986) conducted another meta-analysis on the effects of NGCs in precollege mathematics courses (Hembree & Dessart, 1986). The purpose of their study was to integrate the findings of the research on students using calculators in learning mathematics in grades K-12. The analysis included 53 dissertations, 12 journal articles, 12 Educational Resources Information Center reports, a project report, and an unpublished report. They found advantages for average students in calculator using groups when assessed with non-calculator problem-solving tests. Students in calculator groups at all ability levels showed positive effects when calculators were allowed on posttests. Hembree and Dessart concluded:

Students who use calculators in concert with traditional instruction maintain their paper-and-pencil skills without apparent harm. Indeed, a use of calculators can improve the average student’s basic skills with paper-and-pencil, both in basic operations and in problem solving (Hembree & Dessart, 1986, p. 96).
They also reported that students using calculators possess a better attitude towards mathematics and an especially better self-concept in mathematics than noncalculator students for all grade and ability levels. They also conducted a follow-up meta-analysis of 88 studies in 1992. With the exception of one study, Hembree and Dessart found that the studies showed positive effects on students’ problem solving abilities and attitudes towards mathematics (Hembree & Dessart, 1992).

In 1996, Smith conducted a meta-analysis on 24 studies with calculator use from 1984 to 1995 and reported that the calculator had positive effects for students in third grade, grades seven through ten, and twelfth grade and had no significant effect on achievement for grades four through six and grade eleven. Further, he stated that problem solving, computation, and conceptual understanding were the areas that provided positive results.

From these review reports there was strong evidence that the use of NGCs in fact had positive effects on students’ understanding in mathematics. Because of NGCs’ limited capabilities, many of those studies reviewed in the meta-analyses were in entry-level mathematics courses. The research on upper-level mathematics courses was mainly conducted with the use of computers, various CASs, and GCs. The review of studies with the use of computers and various CASs follows in the next section.

**Research on Usage of Computers and CASs in Mathematics Education**

Computer technology was first brought to education in various disciplines in the 1960s in the form of mainframe units (Kaput 1992). However, the cost and size of a mainframe computer and needed training to use a mainframe were main factors that prevented their widespread use in schools and colleges. Only after the invention of
microcomputers have many areas in education, including mathematics education, begun to benefit from the computer’s power (Demana & Wait, 1990, 1992, 1998; Dunham, 1999; Heid, 1988, 1997; Kaput, 1992).

The Ohio State University’s C² PC project was one of the first projects that took advantage of the available technology. The main purpose of the C² PC project was to improve the mathematics preparation of college bound high school students through the use of computers and GCs (Waits & Demana, 1998). In particular, the project provided a computer-graphing-intensive precalculus curriculum to improve the preparation of calculus-intending students. Computers were used exclusively in the project for the first two years. Because of the discovery of GCs, the project continued after the first two years with the use of GCs. The project created instructional materials that made effective use of computer-and-calculator-based graphing to improve student understanding of functions and strengthen student problem-solving skills.

As an outgrowth of the C² PC project, several high schools, colleges, and universities adopted the project in their precalculus curriculum. As a result of this project, developers claimed that they observed students learning to value mathematics, the participating students became more flexible problem solvers, and the technology that they used gave students a better feeling about mathematics (Waits & Demana, 1998).

Computer technology in mathematics education was mainly used as tutoring, computer-managed teaching, simulation, and programming to solve problems (Kulik, Kulik, & Cohen, 1980). CASs, such as Mathematica, Derive, Maple, Mathcad, and MuMath, are computer programs that allow a user to solve algebraic and calculus-based problems. The potential of computers and CASs technology on teaching and learning
mathematics was widely initiated in the 1980s. The NCTM (1989) understood the power of this piece of technology and suggested that mathematics programs should use the full power of computer technology at all levels. They made the point that students can solve a problem with a greater degree of accuracy with the computer, and the computer can remove the necessity for long, complicated computations. The use of computers also offers the potential of solving real world applications that were not previously possible in the mathematics curriculum by paper and pencil techniques (Heid, 1988, 1997; Kaput, 1992; Waits & Demana, 1996, 2000). In fact, with the use of computers as an instructional tool, teachers can interact more with students and give students the opportunity to build on their skills, conceptual understanding, and problem solving abilities in mathematics.

Because the purpose of this study is particularly the effects of using GCs in a calculus course, the further discussion of the literature review first focuses on GC-based studies in mathematics education that are related to this study and later it discusses the literature review on computers, CAS, and GC-based studies in calculus.

**Research on Usage of Graphing Calculators in Mathematics Education**

Until the discovery of GCs, only computers and CASs were used as instructional technologies in college level mathematics courses. The cost, teacher training, and other factors prevented the widespread use of these technologies in schools and colleges. It may not be possible for every school to set up computer-based instruction in every mathematics classroom. GCs became a solution for this problem. GCs are programmable calculators that have standard computer processors, display screens, and built-in software. Due to increasingly sophisticated technology, GCs have begun to assume even
more computer type capabilities. Foley (1987) and Dick (1992) called GCs “hand-held computers”, and Waits and Demana (1996) called GCs “pocket computers” for their power. Heid (1997) claimed, “Perhaps the single most important technological influence on the high school and early college mathematics classroom has been the graphing calculator” (p. 21). Because of their low cost, portability, speed, accuracy, and other advanced capabilities, the use of GCs became a better choice for use in upper-level mathematics courses.

Many positive things have been said about GCs by mathematics educators (Foley, 1988; Ruthven, 1990; Demana & Waits, 1998; Dunham & Dick, 1994; Dunham, 1999; Heid, 1997). Demana and Waits (1998) have continuously supported the use of GCs in mathematics, noting, “the use of hand held technology can provide more classroom time for the development of better understanding of mathematical concepts by eliminating the time spent on mindless paper and manipulations” (p. 5).

The first GCs appeared in 1986 and research studies began to appear in 1990. There were a number of studies related to the use of GCs conducted in a wide range of areas in mathematics such as precalculus, calculus, statistics, geometry, trigonometry, and algebra on such topics as function concepts, graphing concepts, modeling, limit concept, and derivative concept. A few studies addressed issues on equity, gender differences, students’ errors and misconceptions; development of spatial visualization skills, and problem-solving skills with the use of GCs. Some other studies examined students’ and teachers’ impact in terms of beliefs, attitudes, and perceptions in GC usage.

Research on GCs, however, is still relatively new and the direction for GC use in mathematics education is still unclear (Penglase & Arnold, 1996). Early research on GC
use in mathematics had mixed results. Many studies and projects reported increases in student achievement, understanding of mathematical concepts, attitudes towards mathematics, and problem-solving ability when comparing GC groups to non-GC groups (Allison, 2000; Carter, 1995; Cassity, 1997; Dunham, 1999; Dunham & Dick, 1994; Ellington, 2000; Heid, 1997). At the same time, there were some studies showing no difference between the GC-usage group and non-GC groups (Army, 1991; Girard, 2002). There were also many studies showing mixed results in mathematics achievement between GC-groups and non-GC groups (Blozy, 2002; Dimiceli, 1999; Ellison, 1993; Estes, 1990; Ganter, 2001; Oster, 1994; Penglase & Arnold, 1996).

Dunham and Dick conducted one of the earliest reviews on studies in mathematics education that used GC technology (Dunham & Dick, 1994). Their finding generally supported the use of GCs in mathematics education. They reported:

The early reports from research indicate that graphing calculators have the potential dramatically to affect teaching and learning mathematics, particularly in the fundamental areas of functions and graphs. Graphing calculators can empower students to be better problem solvers. Graphing calculators can facilitate changes in students’ and teachers’ classroom roles, resulting in more interactive and exploratory learning environments (p. 444).

Penglase and Arnold conducted a critical review on published dissertation studies during 1990 to 1995 that examined the effects of GCs in high school and college mathematics (Penglase & Arnold, 1996). In their review, they sought to answer the following questions:

1) How did the GC benefit student achievement in mathematics?
2) What kind of learning environment allowed for maximum benefits to be attained?
Penglase and Arnold mentioned that the majority of the studies they reviewed focused on two major areas: a) testing the effects of the use of GCs within specific areas of mathematical study; and b) making judgments regarding the effectiveness of such use. From their finding, they concluded that the studies using GCs in mathematics education had mixed results and that the GC research failed to provide clear direction to mathematics education. They stated that most studies that favored the use of GCs did not show significant differences between GC and non-GC groups. Penglase and Arnold’s review found that students’ understanding of the connection between functions and their graphs, capabilities with spatial visualization skills, and attitudes toward mathematics were the areas that provided positive results. They, however, questioned the GC usage and testing procedures in several studies and suggested a need for new methods to evaluate students who have been exposed to GC technology.

Heid (1997) also provided a review on the research studies using GCs. She reported that the review studies provided positive results; in particular, the GC usage in those studies increased conceptual understandings of graphs and functions, understanding of connections among a variety of representations, and students’ problem-solving ability. At the same time, she criticized these studies as, “In light of the almost uniformly positive results, it is important to note that most of these studies reported on projects that involved students in different mathematical activities as they worked with graphics calculators. They did not merely place calculators in students’ hands in the context of an unchanged curriculum” (p. 23).

Again in 1999, Dunham provided a thorough research review on studies that used GCs in various ways. She mentioned that the GC-based studies improved students’
problem solving ability, conceptual understanding, and computational skills. Dunham also stated that the review of some studies supported that the use of GCs in mathematics classes helped some special populations, such as low-ability and at-risk students, students with learning disabilities, and non-traditional students, etc.

Ellington (2000) conducted a meta-analysis on the effects of hand-held calculators (NGCs and GCs) on precollege students in mathematics classes. This analysis contained fifty-three calculus-based studies from 1984 through 2000. Her finding in the GC usage sector included the following: when the GC was a significant element in all aspects of high school mathematics classes, the basic operational skills of students improved; and students who used GCs during mathematics instruction had better attitudes toward mathematics than their non-GC counterparts.

There were numerous studies conducted in mathematics education with the use of GCs in various topics in various courses. The current study is interested in the effect of using GCs in an Applied Calculus course. Further, to learn calculus concepts students need to have a strong foundation in precalculus topics including function concepts and problem-solving ability. Also, to learn calculus concepts with the use of GCs, students need to know how a GC can be used in a correct and effective way. Therefore, the following review sections concentrate on studies that focused on students’ understanding in precalculus topics, including function concepts and calculus concepts with the use of GCs, and the studies that focused on students’ difficulties, errors, reluctance, and misconceptions in using GCs in their mathematics learning.
Graphing Calculator Usage in Precalculus Topics

There were a number of studies with the use of GCs conducted at the precalculus level in high school, colleges, and universities to examine the effect on students’ understanding. For example, Oster (1994) examined the effects of instruction using a GC on conceptual and procedural understanding in precalculus. Three teachers and 98 students participated in this quasi-experimental study to examine aspects of constructivism as applied to the instructional use of a GC in college-level precalculus. The experimental group was taught precalculus graphics strategies with the use of a GC and the control group was taught using traditional teaching methods. Pretest-posttest and end-of-treatment surveys were used as instruments in this study. The results of the study showed a significant increase in students’ conceptual knowledge but no evidence was found to support significance for students’ procedural or overall achievement in the precalculus topics. Oster, however, claimed that the result of her study supported the constructivist view that recommends teachers involve students in an interactive problem-solving situation. Also, Oster reported that the student and teacher survey responses showed generally positive views on the use of the GCs for learning and teaching in precalculus topics.

Doenges (1996) also conducted a study to examine the effect of using a GC in high school precalculus classrooms. She examined several factors in her study, such as gender, spatial skills, achievement, confidence, and attitudes in using GCs. Doenges selected six sections of a precalculus course in four high schools for her study, with a total of 134 students. She concluded that confidence-with-calculator scores were significantly higher than confidence-without-calculator scores. She also reported that the
majority of the students regarded GCs positively, with most negative comments coming from females and low spatial ability students. The significant differences in the confidence-with-calculator scores favored the males.

A significant amount of research has been conducted into the effectiveness of the GC as a tool for instruction and into students’ learning about functions and graphing concepts. For example, Carter (1995) investigated the effects of GCs on student achievement and understanding of the function concept. The treatment group was introduced to function concepts graphically with the use of a GC and the control group was taught in a traditional manner. Pretest-posttest, two questionnaires, and an interview were used as instruments in this study. From the data collection and analysis, Carter concluded that there was a significant gain between pretest and posttest scores for both groups and the treatment group showed more improvement than the control group. He also reported that the GC instruction produced a favorable influence on student achievement; however, the difference in the outcomes for the two groups was not statistically significant.

Carter’s study also examined the effects of GCs on students’ difficulties and misconceptions with the function concept. From the questionnaires and interviews, the researcher concluded that the students who used GCs understood function transformations, understood the connections between the graphical and algebraic representations, were able to make connections between a point on the graph and the two distinct values that the point encodes, were able to solve nonroutine problems, and were more active than students not using a GC.
Rich (1990) conducted a study to examine the ways in which the use of GC as a teaching tool affects precalculus students’ learning of functions and related concepts. Two classes used a Casio fx-7000G calculator and three control groups studied the same materials without the use of the GCs. She reported that there was indication that students who used a GC had a better understanding of the relationship between an algebraic function and its graph; comprehended that problems in algebra can be solved graphically as well as through algebraic manipulation; and tended to do more conjecturing and generalizing. Rich also claimed that the use of the GC in precalculus provided a new problem-solving technique, and changed classroom dynamics.

Problem solving is another area where the use of GCs provided some positive results. For example, Allison (2000) sought to determine the impact of the GC on four high school students’ mathematical thinking while solving problems. The students were given both contextual nonroutine problems and noncontextual exploratory problems. From the interviews with the students, Allison was able to observe the following: GCs amplified the speed and accuracy of students’ problem-solving strategies; GCs encouraged the students to use graphical approaches to solve problems; and GCs enhanced the students’ ability to focus on reasoning for their answers to the problems. The students, however, commented that the GC added time to the problem-solving when syntax errors occurred.

Cassity (1997) reported that when GCs were used in a college algebra course, spatial visualization and mathematical confidence were improved but no gender difference was observed. She, however, pointed out that many studies with a GC focused on procedural or algorithmic understanding rather than conceptual understanding.
A large portion of the body of GC research has focused on students’ achievement in various mathematics courses; only a few studies examined students’ errors and misconceptions that are related to GC technology (Dunham, 1999; Gaston 1990; Mitchelmore and Cavanagh 2000; Tuska 1992). Dunham (1999) referred to these errors as “calculator-induced errors”. In order to use GCs in their mathematics learning effectively, students need to be observed and informed about their GC based errors and misconceptions. The next section discusses some of these new classes of students’ errors and misconceptions.

Students’ Errors and Misconceptions in Calculator Usage

Evidence shows that a new class of errors and misconceptions that students experience are introduced by GCs (Dunham, 1999; Gaston 1990; Mitchelmore and Cavanagh 2000; Tuska 1992). This is an area that needs careful attention but it often receives less attention. Mitchelmore and Cavanagh (2000) pointed out that,

Researchers have rarely investigated how individual students actually use a GC. In particular, although there is anecdotal evidence that students occasionally misinterpret the graphic image, we [they] have found no systematic research on the types of misconceptions which arise or their causes (p. 254).

Tuska (1992) attempted such a study. She studied students’ general errors in GC-based precalculus classes and identified eight GC associated misconceptions. These misconceptions fell into four categories: misunderstanding of the domain of a function, misunderstanding of the end behavior and asymptotic behavior of functions, misconception of the solution of inequalities, apparent misbelief that every number is rational.
Mitchelmore and Cavanagh (2000) investigated students’ difficulties in operating a GC. This particular study investigated how grades 10 and 11 students interpret linear and quadratic graphs on a Casio fx-7400G through clinical interviews. Their finding showed that students’ errors in using a GC were attributable to four main causes: a tendency to accept the graphic image uncritically without attempting to relate it to other symbolic or numerical information; a poor understanding of the concept of scale; an inadequate grasp of accuracy and approximation; and a limited grasp of the processes used by the calculator to display graphs.

Another study by Gaston (1990) reported that certain students in remedial and non-remedial precalculus classes exhibited a reluctance to use calculators and/or had difficulty with calculator use in classes where such use was permitted. The study was conducted with community college students who took Basic Arithmetic, Introductory Algebra, Intermediate Algebra, and Trigonometry courses. The data was collected through students’ responses to the researcher-made questionnaire and a series of interviews with certain students. Her finding showed that students’ reluctance to use calculators was observed in students who had poor attitudes toward the use of calculators in the mathematics classroom; had little perception of the usefulness of calculators; had little experience with calculator use; and/or were unable to achieve the successful integration of appropriate levels of mathematical competency and calculator competency.

The next section discusses the contents of calculus that are related to this study and students’ knowledge in calculus, in particular with limits and derivatives. The final part of this chapter examines the role of technology in calculus instruction.
Calculus and Students’ Knowledge in Limits and Derivatives

The first part of this section reviews the contents of calculus, particularly in limits and derivatives. Understanding the nature of the contents of calculus will help to understand how students learn these calculus topics and also the knowledge they have in these topics.

*Differentiation* and *integration* are two main components in calculus. The history of calculus shows that the development of calculus by Newton (1642-1727) and Leibniz (1646-1716) resulted from the investigation of the following problems:

1. Finding the slope of a tangent line to the graph of a given function at a given point.
2. Finding the area of a region bounded by the graph of a function.

The first problem led to the creation of *differential* calculus in which students learn the derivative of functions and related applications. The second problem led to the creation of *integral* calculus in which students learn the antiderivative of functions, definite integrals, and related applications.

Finding the derivative of a function is developed from the idea of a *limit* of the function. Likewise, the limit concept is a foundation for the development of integration as well. So, understanding the limit concept is crucial in understanding the derivative and integration.

Students who enter into a calculus course, therefore, first study the limit topics. However, it is acknowledged by mathematics educators that students have difficulties in
understanding the limit concept (Barnes, 1997; Bergthold, 1999; Davis & Vinner, 1986; Tall et al., 2004; Smith, 1996). Bergthold (1999) wrote that the limit concept is harder for students to understand because the mathematical definition of limit differs substantially from intuitive limit ideas.

The formal definition of the limit of a function, the delta – epsilon definition, was first given by Cauchy (1789 – 1857). The definition was given as follows:

\[
\lim_{x \to c} f(x) = L \text{ is that for each } \varepsilon > 0 \text{ there exists a } \gamma > 0 \\
\text{such that if } 0 < |x - c| < \gamma, \text{ then } |f(x) - L| < \varepsilon. 
\]

This formal definition, however, for the limit concept of a function is rather complex to understand for students who just enter into a calculus class. Barnes (1997) stated, “the limit concept is inherently difficult and causes problems no matter how it is taught, partly because many students’ intuitive ideas are in conflict with the formal definition” (p.1). Tall, Smith, and Piez (2004) agreed: “the formal limit is a grievously difficult concept to use as a foundation for teaching the calculus” (p. 6). Teaching the limit of a function through the formal definition of a limit is, in fact, a difficult task for mathematics teachers.

It is also noted by calculus educators that even students who have passed a calculus class could not define the limit of a function correctly. “Students who have studied calculus are often unable to define limit correctly, or to explain why the concept of limit is fundamental to calculus” (Davis & Vinner, 1986).

After the limit section is covered, students in a calculus course study derivatives of functions. The derivative of a function at a given point is defined as the slope of the
function at that point. Further, the slope of a function at a particular point is a measure of how quickly the output \((y)\) changes as the input \((x)\) changes at that point. This is called the *instantaneous rate of change* or simply the *rate of change* of the function at that point. Thus, the problem of finding the derivative of a function at a point is mathematically equivalent to finding the rate of change of the function at that given point. The process of finding the derivative of a function at a point is known as differentiation. Therefore, all applications that are related to finding the rate of change of functions require finding the derivative of those functions. A description of finding the derivative of a function at a point is given as follows:

Let \(A = (x, f(x))\) be the given point on the graph of \(f(x)\). Choose another point that is close to the given point on the graph of \(f(x)\). Say, the second point \(B = (x+h, f(x+h))\). Then the slope of the line (called secant line) that goes through these two points is \(\frac{f(x+h) - f(x)}{(x+h) - x}\). Now the argument is that when \(h\) approaches 0, the point B approaches the point A. Therefore, when \(h\) approaches 0, the secant line \(AB\) approaches the tangent line at point A. So, when \(h\) approaches 0, the slope of the secant line \(AB\) approaches the slope of the tangent line at A. That is, the slope of the tangent line to the graph of \(f(x)\) at \(x\) is \(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\). Now, because the tangent line goes through the point A on the graph of the given function, the slope of \(f(x)\) at \(x\) equals to \(\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\) and is known as the derivative of \(f(x)\) at \(x\).
Students, therefore, need to understand that finding the derivative of a function at a point is, in fact, a limit problem.

Like students’ difficulties with the limit concept, students experience difficulties with the concept of derivatives (Smith, 1996; Tall, 1990; Tall et al., 2004). Some argue that students have difficulties in understanding the derivative because of the way the derivative is presented. For example, Tall (1990) thought that students experience difficulties in understanding the derivative concept because the derivative is presented as a limit problem, not because of the students’ inability to grasp such an abstract concept. His argument was that the limiting process may be ‘intuitive’ in a mathematical sense but not in a cognitive sense, therefore students have difficulties in understanding the derivative concept.

To understand the calculus concepts and how these concepts can be applied to other fields, one needs a strong algebraic background. A weak mathematical background in terms of lack of function concepts, algebraic manipulations, and geometric visualizations is one reason why students are not successful in calculus classes (Douglas, 1986; Ferrini-Mundy & Graham, 1991). As a result of these students’ difficulties, students study calculus courses by memorizing the techniques and procedures with little time focused on the concepts (Ferrini-Mundy & Graham, 1991; Tall et al., 2004).

Educators and researchers are concerned about students’ difficulties in calculus topics. They look for new ways to deliver calculus instruction so that students will have a better understanding of what they study in calculus. Technology has been thought by many to aid calculus instruction in order to improve students’ conceptual understanding.
in calculus concepts. The next section examines such studies in which technology, particularly GCs, computers and CASs were used in calculus.

The Role of Technology in Calculus

Like other areas in mathematics, calculus is one of the areas in mathematics that has been and can benefit from technology, as calculus is often difficult for many students (Douglas, 1986; Ferrini-Mundy & Graham, 1991; Tall et al., 2004). Mostly, computers, various CASs, and GCs are widely used in calculus courses to improve students’ learning in the subject. Waits and Demana (1996) state that “Computer generated numerical, visual, and symbolic mathematics is revolutionizing the teaching and learning of calculus” (p. 712).

A concerned mathematics faculty group initiated calculus reform in the 1980s to address the high attrition rate in calculus and the lack of student understanding of the concepts of calculus. In their official report, Toward a Lean and Lively Calculus, they wrote about their ideas (Douglas, 1986). Immediately after that report, over six hundred mathematicians, scientists, and educators gathered for the Calculus for a New Century Colloquium sponsored by the National Academy of Sciences and the National Academy of Engineering. A well known report, Calculus for a New Century: A Pump, Not a Filter, came out of the conference and discussed issues in calculus instruction (Steen, 1987). Similar calculus reform movements initiated in Britain (Tall, 1992) and in Australia (Barnes, 1997) generated the same concerns in calculus instruction.

In all calculus reforms, the primary ideas were to develop an alternate curriculum that is more conceptual and application oriented than the traditional curriculum, to make
use of multiple representations of mathematical concepts, to develop a variety of teaching methods for calculus, and to use technology.

The calculus reformers have recommended the widespread use of technology in the form of computers, CASs, and appropriate graphing and symbolic calculators. The increasing availability of computers and highly sophisticated new generations of calculators helped to push the development of calculus reforms forward. Hughes-Hallet (1991) wrote “I believe that in calculus most of the ideas should be presented in three ways: graphically and numerically, as well as in the traditional algebraic way. Technology is invaluable here” (p. 33). These calculus reformers’ strong belief is that using technology will help students in the following ways:

1. To free students from algebraic manipulations.
2. To reduce the drudgery of calculations.
3. To use visualization to understand abstract ideas.
4. To explore “what if” situations.

Most of these studies were conducted with the use of computers and various computer software and CASs and a few studies were conducted with the use of GCs.

**Research on Usage of Computers and CAS in Calculus**

Since the beginning of the calculus reform movement, there have been many calculus studies initiated and conducted with the use of different forms of technology in high schools, colleges, and universities (Crocker, 1990; Freese, Lounesto, & Stegenga, 1986; Heid, 1988; Hughes-Hallet, 1991; Judson, 1990). Most of the studies emphasized less time on paper and pencil methods and more time on applications, problem solving, and concept development in calculus. For example, the *Harvard Core Calculus*
Consortium Project emphasized the importance of multiple representations with technology that should be used in calculus classes in order to understand calculus concepts (Hughes-Hallet, 1991). The project emphasized that most of the ideas in calculus should be presented in three ways: graphically, numerically, and algebraically with the appropriate use of technology, and therefore, the project was known as the “Rule of Three”. The project had five major thrusts:

1. De-emphasizing the current stress on manipulative skills by achieving a balance among visual interpretations of the concepts, numerical interpretations of the ideas, and the traditional manipulative approaches.
2. Presenting a more intuitive approach to the concepts and methods of calculus to improve student understanding.
3. Introducing more modern mathematical ideas.
4. Including a wider variety of more realistic applications to reflect better the modern uses of calculus in the client disciplines.
5. Incorporating the use of appropriate technology (computers, CASs, and sophisticated graphing calculators) to improve student understanding of the ideas of calculus.

This project later expanded to be referred to as the “Rule of Four”, including a “writing” or “verbal descriptions” component (Ferrini-Mundy & Gaudard, 1992). This component emphasizes the need for students to explain their mathematical work.

The other well-known calculus project, called “Project CALC”, was developed and conducted at Duke University between 1989 and 1993. The project, developed by Moore and Smith, created a new calculus course at Duke University that differed from
the traditional calculus course in several fundamental ways. The traditional course emphasized acquisition and memorization of pencil-and-paper computational skills whereas the content of the project was based on solving real world problems with the use of the computer lab. The project developers claimed that these projects provided evidence that students’ conceptual understanding can be improved by using technology and suggested placing more emphasis on conceptual development and less emphasis on computational skills.

Ganter (2001) evaluated calculus reform projects that were conducted with the use of computers, or CASs, or GCs during 1988 to 1998 that were mainly funded by the National Science Foundation (NSF). Ganter observed that the common belief from the project developers was that the students in the reform classes did at least as well as those in the traditional classes. He found positive student achievement in some projects but not in all.

Many individual mathematics researchers conducted studies in various topics in calculus with the use of computers and of CAS technology. Heid (1984, 1988, & 1997) was one of the first to investigate the effects of technology in calculus classes. For example, for her doctoral dissertation study, Heid (1984) examined the effects of an applied calculus course during which students focused on concepts and applications and used the CAS to execute routine symbolic manipulations. She participated in her own study and compared a traditional calculus and a treatment group who used a CAS called muMATH. The study lasted for twelve weeks with data that was collected from interview transcripts, conceptual comparison questions, and final examination results. Heid claimed that “Computers have decreased the time and attention usually directed towards mastery
of computational skills and have lent flexibility to the analysis of problem situations” (p. 24). She concluded that the treatment group had a better understanding in the calculus concepts than did the control group by saying, “students from the experimental classes spoke about the concepts of calculus in more detail, with greater clarity, and with more flexibility than did students of the comparison group” (p. 21).

A similar study was conducted by Judson in 1990 to examine the effects of using a CAS (MAPLE) in an introductory calculus class. The results of the study indicated that there were no significant differences between the experimental group who used the CAS and the control group who was taught with a traditional approach in skills and concepts.

Palmiter (1991) conducted a study on integration techniques in a calculus course with a CAS (MACSYMA). The experimental group used the CAS as an aid in their homework problems and the control group learned those integration techniques with a paper and pencil method. The results showed that the experimental group gained significantly in achievement over the control group. She also claimed that the use of technology increased student attitude and confidence in mathematics. The author admitted some internal threats to the study because it lasted 5 weeks for the experimental group but 10 weeks for the control group.

A study by Cunningham (1991) examined the effects on achievement of using computer software to reduce hand-generated symbolic manipulation in freshmen calculus. The study used a pretest-posttest design to compare treatment and control group performance on calculus computation and conceptual skill measures. He concluded that the use of software improved achievement and did not do harm when access was denied.
He further stated that success required instructor use in the classroom in tandem with extensive student use both outside of the classroom and on tests.

Most CAS programs offer multiple representations that allow students to use graphical, numerical, and analytical representations to solve calculus problems. For example, Porizo (1994) examined the effects of using three instructional approaches in three first-year calculus classes. One group was treated with a CAS program (Mathematica), the second group was treated with the use of a GC (TI 81), and the third class received a traditional approach. He found that students in the CAS group made stronger connections between graphical and symbolic representations than did students in the other two groups.

A study by Connors (1995) investigated relationships between gender and achievement as well as gender and attitudes in computer and non-computer groups in a first year college calculus. The study contained both quantitative and qualitative components. She collected final exam scores from four semesters and attitude survey from two semesters. Connors concluded that the students in the experimental group performed significantly better on the common final exam in fall of 1993 and female students in the experimental group benefited more than the other group. She also concluded from the attitude survey that the students in the experimental group showed a positive result but there was no gender difference in the attitude results.

The benefits of using CAS and computers in calculus instruction are being discussed and various projects and studies are being conducted. Yet, the usage of these forms of technology in calculus instruction is still limited because of cost and required teacher preparation and training. The GC technology that was introduced in 1986 offers
similar capability to the capability possible with CASs. But because the GCs are portable and inexpensive, this technology has become a better choice for educators to use in mathematics classrooms. The next section examines calculus studies with the use of GCs.

**Research on Usage of Graphing Calculators in Calculus**

Researchers took advantage of the GC technology in various mathematics courses, including calculus, to improve students’ understanding because of its affordability, capability, and portability. Research studies in calculus with GC technology, like other areas in mathematics, were initiated beginning in the early 1990s (Barton, 1995; Bergthold, 1999; Blozy, 2002; Dunham, 1999; Ellison, 1993; Estes, 1990; Heid, 1997; Stiles, 1994).

There are two entry level-courses in calculus that are offered in colleges and universities. One is a regular calculus course (Calculus-I, MAC 2311) and the other is an Applied Calculus course (MAC 2233). Students get to choose one or the other based on their major and requirements. This study was conducted in Applied Calculus at a community college and examined the effects of a numerical method along with the use of GCs and the researcher-developed guided lessons in limits, derivatives, and related applications in these two topics. Limits and derivatives are common topics for both calculus courses. Because an Applied Calculus course has a relatively small place in the calculus literature, this section includes a review of research studies in calculus from both of those calculus courses.

Stiles (1994) conducted a study in using GC in first semester calculus. The study explored the use of GC in the relationship between a function and its tangent line, Newton’s method for finding real zeros of polynomials, and curve sketching. The Casio
fx-7000 G graphing calculator was used in the study and Stiles concluded that the usage of the GC in the calculus class improved students’ geometric intuition and improved students’ understanding of calculus.

Bergthold’s (1999) study provided some useful information about students’ early understanding of limit concepts with the use of GCs. The purpose of the study was to identify and describe students’ patterns of analytical thinking and knowledge use in solving limit problems. The research was a qualitative design involving four interviews with each of 10 first-semester calculus students in a university. Written and oral responses were analyzed relative to the researcher-made four-element framework:

1. Analyzing functions locally in graphical and numerical settings;
2. Conjecturing limits from representative graphs and tables;
3. Understanding advantages and limitations of tables and graphs to conjecture limits; and
4. Producing multiple sources of evidence to justify a limit conjecture.

The researcher observed two factors that influenced students’ early understanding of the limit concept: a) students’ knowledge and understanding of functions and b) students’ knowledge and use of GCs. Bergthold pointed out that “Graphing calculators allow easy access to numerous intuitive limit ideas” (p. 5). She further listed benefits of using a GC: graphs and tables can be produced quickly and easily; the trace feature permitted a dynamic sense of the limit process; and the zoom feature sometimes helped to show how smaller viewing windows lead to more accurate limit conjectures. She also reminded students that these features sometimes have drawbacks, such as a) calculator-produced tables and graphs can be misleading due to computational limitations, b) poor
input choices can mislead the limit result, c) the interesting behavior at a particular point is “hiding” between two pixels, and d) misleading graphs are possible in limit problems that involve vertical asymptotes.

Some calculus studies were conducted with the combined use of computers and GCs. For example, Estes (1990) conducted a study to examine the effect of implementing GC and computer technologies as instructional tools and the impact of the technology on the instructor and student in an Applied Calculus class. The treatment group experienced graphics techniques via a microcomputer and a GC and the control group was taught by traditional methods and allowed to use a GC. The pretest-posttest design was used to measure the achievement in both conceptual and procedural knowledge. Estes reported that the experimental group scored significantly higher on conceptual achievement but there was no significant difference on procedural achievement. The examination of the impact of technology on the instructor showed four major categories of problems for instructors to implement the technology: instructional-design, syllabus-schedules, computer-peripherals, and environmental difficulties. The examination of the impact of the technology on the students showed that computer demonstrations along with the use of GCs enhanced conceptual understanding. Further, the student survey data indicated that students believed that the GC and computer were helpful in their learning, if the student understood how to use the technology. Also, the students in the study indicated a preference for the GC over the computer.

Ellison (1993) examined the effect of using computer software and a GC on students’ ability to construct calculus concepts. She used a qualitative-case-study research methodology in her study. Ten students were drawn from two technologically
enhanced sections of *Calculus and Analytical Geometry* offered at a university. A series of interviews were conducted for the purpose of the study. The tasks of the interviews were to see whether students were able

A. to distinguish the graphs of functions and their derivatives;
B. to sketch the graph of the derivative from its parent function;
C. to draw conclusions about characteristics of the derivative from the graph of the derivative; and
D. to link the formal definition of the derivative with a visual image of the limiting slope of secant lines.

Ellison reported that all ten students were able to do task A; six of them were able to do task B; five students were able to do task D; and only four students were able to do task C. Overall she concluded that the technological-based instruction had a positive effect on students’ ability to construct a mental concept image of the derivative but a number of students had only a partially-formed understanding of the connections between derivatives, functions, and graphs. The study, however, reported some serious limitations for the study: One of the two classes from which the ten students were chosen was taught by the researcher; the researcher had a more thorough knowledge of and proficiency with the computer software and GC than the other instructor, and the researcher used GC to a greater extent during class sessions than did the other instructor.

There have been very few studies conducted in mathematics education with the next generation of GCs, known as “supercalculators” or CAS graphing calculators (Blozy, 2002; Keller & Russell, 1997). These calculators (eg., TI-92 and TI-89) not only
have both numeric and graphic features but also have built-in computer algebra systems computer programs.

Blozy (2002) conducted a study to analyze students’ performance in calculus between students who used a non-CAS-GC (TI 83) and students who used a CAS-GC (TI 89). The study also analyzed students’ performance in calculus when those students were not allowed to use any type of graphing calculator at all. In both cases, the study examined students’ performance in concepts, algorithmic skills, graphing and interpretation, and related applications in calculus. A group of 56 students from two different Advanced Placement Calculus classes from a high school participated in Blozy’s study. He collected data from two tests (one allowed the use of a GC and the other did not allow the use of any GC) and a clinical interview with 10 students (five from each group). The study showed mixed results. On the calculator-allowed tests, the CAS-GC group performed significantly better on the algorithmic skills but the non-CAS-GC group performed significantly better on concepts and graphing and interpretation areas. On the calculator-not-allowed tests, overall, students performed equally. He also reported that his clinical interviews showed similar results on the two tests but he was able to notice overwhelmingly that the CAS-GC group approached and solved problems using algebraic representations and the non-CAS-GC group approached and solved problems using graphical and numerical representations.

A study by Barton (1995) examined classroom instructional practices and teacher’s professed conceptions about teaching and learning college calculus in relationship to the implementation of GCs. The study provided information on how the college teacher responded to the call for reform in teaching calculus through a graphing
approach supported by use of GCs. The researcher selected five teachers as subjects out of ten teachers who were assigned to teach the calculus course at a university at that time. She collected data through interviews with the teachers and classroom observations. She concluded that the college teachers’ conceptions of their teaching approach were largely consistent with their instructional practice. Teachers showed skepticism about the usefulness of the use of GCs because of

a) inexperience in operating the GCs.

b) teachers’ limited time both within the classroom and in preparation for class

c) teachers’ rigid conceptions of an appropriate teaching approach to calculus

d) teachers’ strong conceptions toward a theoretical approach emphasizing precise wording of definitions and proofs of theorems

e) lack of interest from students, and

f) the calculator display unit and physical arrangement of the teaching environment.

Summary

This chapter reviewed the literature on technology usage in mathematics education, including NGC, GC, computer, and CAS usage in mathematics education. Then it reviewed students’ knowledge and learning in calculus, particularly with limits and derivatives. Studies indicated that students experience difficulties in limits and derivative concepts. Then the last part discussed how technology played and can play a role in calculus instruction to improve students’ conceptual understanding in limits and derivative concepts.
The literature indicates that several national organizations in mathematics, as well as mathematics educators, have urged the use of technology in mathematics teaching and learning. Several studies have indicated positive results in using technology in mathematics education, including calculus. Many researchers believe that GC technology can play an important role in improving students’ conceptual understanding in mathematics courses, including calculus.

Students in many fields need to take at least one semester calculus for their major. Yet, calculus is an unsuccessful course and a difficult course for many students. These students can benefit from the available sophisticated technology we have today. Several studies with various technology indicated positive results in students’ mathematics learning. A critical need for instructional materials with the use of GCs that would enhance students’ conceptual understanding in calculus topics was observed.

It has been almost 20 years since the first GC appeared in mathematics education. Its usage by students and teachers has been growing at a rapid rate. A national survey indicates that as of 2000, 80% of high school teachers used GC technology in their classrooms. This figure will likely continue to grow.

GCs are powerful and capable of doing things that were not possible before or only possible with computers. The question is whether or not we really take advantage of this powerful calculator. At the same time, there are many questions that remain open when it comes to the usage of technology in mathematics education. For example, a recent report, *Handheld Graphing Technology in Secondary Mathematics: Research Findings and Implications for Classroom Practice* (Burrill, 2000), raises the following questions to the GC technology research community:
1. What is the nature of the tasks for which the technology is used?
2. How do students and teachers choose to use the technology?
3. What is the impact of its use on student understanding?
4. Which students benefit from using technology?

These unanswered questions, perhaps, lead the future GC technology research in the right direction.

This study, however, searched for evidence that students in an Applied Calculus course can enhance their conceptual understanding in limits and derivatives. To examine students’ improvement in understanding these topics, this study provided a numerical method along with the use of GCs and the researcher-developed instructional materials. GCs have great potential that can be used to enhance students’ understanding in limit and derivative topics. In fact, a GC is required for this Applied Calculus course. However, it has been noted from the literature and personal experience that students have problems using this GC effectively. The researcher-developed instructional materials in this study have a component that helps students to use their GCs correctly and effectively by providing instructional supporting materials. Also, it has been noted from the literature and personal experience that students experience a greater difficulty in understanding the limit and derivative concepts. Another component of the researcher-developed instructional materials in this study demonstrated a way of approaching limit and derivative problems numerically with the help of a TI 83 GC so that students can improve their conceptual understanding in limits and derivatives.
Chapter 3

Method

This chapter describes how the study was conducted. It begins by discussing the purpose of the study, the research questions, and hypotheses that are needed to answer the stated research questions. Next, the overall main study design and procedures are described along with information about facilitators, participants, instruments, and data analysis. Also, changes to the study design based on results from a pilot study conducted in the fall of 2004 are discussed.

The purpose of this study was to examine effects of the use of GCs with a numerical approach and the researcher-developed instructional materials on the limit and derivative topics in an Applied Calculus course at a community college. The general research question that this study sought to answer is “To what degree can the use of GCs with a numerical approach and instructional materials developed by the researcher affect community college Applied Calculus students’ learning of limits and derivatives?” In particular, the study sought to answer the following research questions:

1. How does the students’ achievement in solving limit problems with a numerical approach compare to that of students who solved limit problems with a traditional approach (primarily an algebraic approach) in an Applied Calculus course?
2. How does the students’ achievement in solving derivative problems with a numerical approach compare to that of students who solved derivative problems with a traditional approach (primarily an algebraic approach) in an Applied Calculus course?

The following research hypotheses were used to answer the research questions. The first three hypotheses are stated to answer the first research question and the last three hypotheses are stated to answer the second research question.

1. Students (GCGL group) who receive instruction with a numerical approach will have higher achievement in routine (skill oriented) limit problems than students (GC group) who receive instruction with a traditional approach (primarily an algebraic approach).

2. The students in the GCGL group will have higher achievement in conceptual oriented limit problems than the students in the GC group.

3. The students in the GCGL group will have higher achievement in related applications of limits than the students in the GC group.

4. The students in the GCGL group will have higher achievement in routine (skill oriented) derivative problems than the students in the GC group.

5. The students in the GCGL group will have higher achievement in conceptual oriented derivative problems than the students in the GC group.

6. The students in the GCGL group will have higher achievement in related applications of derivative problems than the students in the GC group.
Design of the Study

The main design of the study was a 2 X 2 (Type of Instruction X Instructor) factorial analysis of covariance (ANCOVA). The pretest was used as a covariate. The two types of instructions were: Instruction with a numerical method with the use of TI 83 GC; and Instruction with a traditional approach (primarily algebraic approach). Two instructors (Instructor A and Instructor B) participated in this study. The design of the study was a quasi-experimental treatment-control group.

Variables

Independent Variables

The main independent variable was method of instruction. The two levels of instruction on the selected calculus topics were instruction with a numerical approach with the use of GCs and instruction with the traditional approach (primarily algebraic approach). The other independent variable for this study was instructor. There were two instructors other than the researcher used in the study. The study was designed with two instructors to show that different instructors can replicate the effectiveness of the instructional treatment. It is, however, possible to argue that any differences in performance between the two groups are due to the instructor and not necessarily the method of instruction. It was expected that the effects due to possible differences in individual teaching style were minimal because the instructors had similar characteristics. The instructors are both male, have almost the same number of years of experience in college teaching (one has 16 years of teaching and the other has 17 years of teaching), and have the same number of years of teaching experience in an Applied Calculus course.
and both have positive reputations among students. Further, to help minimize effects due to possible differences in individual teaching style, each instructor taught one Monday-Wednesday-Friday (MWF) section and one Tuesday-Thursday (TR) section and each instructor taught one treatment group and one control group. Also, to investigate potential interaction effects between the instructors and the method of instruction, instructor was included as a second independent variable.

**Dependent Variables**

The dependent variables were the achievement in solving skill oriented limit problems, conceptual oriented limit problems, and related application problems of limits, skill oriented derivative problems, conceptual oriented derivative problems, and related application problems of derivatives and were measured by researcher-made tests. These dependent variables were measured at two different times during the semester.

**Instruments**

A student initial survey, a pretest, two unit exams, and a classroom observation protocol were used as instruments for this study.

**Student Initial Survey**

All students were given a student initial survey to complete on the first day of class. The purpose of the survey was to obtain information about students, such as their major, previous math courses they have taken, their GC ownership and usage, etc. See Appendix A for the student initial survey.
Pretest

The researcher-made pretest was administered on the first day of the semester. The Basic College Algebra (MAC 1105) course is a prerequisite for the Applied Calculus (MAC 2233) course. Therefore, a set of 13 questions were chosen for the pretest from the objectives of the Basic College Algebra course. The purpose of the pretest was to determine whether students in the four sections were similar in their mathematical ability before they received any new instruction in applied calculus. The pretest was used as a covariate. The question items in the pretest were constructed response rather than multiple choice. See Appendix B for the pretest.

Unit Exams

The unit 1 and 2 exams were used to measure students’ achievement on limit and derivative topics, respectively. In particular, to measure students’ achievement in skill oriented problems, conceptual oriented problems, and application problems, each unit test contained problems for those three areas. The researcher developed these two unit exams. See Appendix C for the two unit exams.

The two unit exams were scored in two different ways. Because these two unit exams were part of the students’ final grade for the course, the instructors who participated in the study graded the tests for their own sections for the purpose of giving students a final grade for the course. For the purpose of the statistical analyses of the study, the researcher graded the two unit exams separately. The two unit exams as well as the pretest were graded and scored by the researcher using the following scoring rubric:
Table 1

<table>
<thead>
<tr>
<th>Point(s)</th>
<th>Guideline</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Correct work and Answer</td>
</tr>
<tr>
<td>1</td>
<td>Partially Correct work and Answer</td>
</tr>
<tr>
<td>0</td>
<td>Incorrect Work or No Response</td>
</tr>
</tbody>
</table>

Class Observation Protocol

A classroom observation protocol was developed by the researcher to use every time he visited the treatment and control groups during the period of the study. The researcher visited all experimental groups and control groups once a week to observe that the progress in the classes was in agreement with the course syllabus and the instructional methods were being utilized as planned in the main study. See Appendix D for the description of the classroom observation protocol.

In addition to the classroom protocol, each of the instructors completed a course-implementation log sheet after every class (see Appendix I). The categories listed on the log sheet were created to learn how each class was run. A free response section was provided to list details of any problems that occurred. Observations made on this were included with the class observation protocol.
Validity and Reliability

Content Validity

To test for content validity of the instruments, the pretest and the two unit exams were given to two faculty members who have extensive experience in these two courses. These two faculty members were chosen from the same department but were not the two participating instructors or the researcher. The faculty members were asked to evaluate the content, number of questions, and the timing of the pretest and two unit exams. Also, they were asked to evaluate any ambiguousness, wording, grammar, consistency, and reasonability of each item in all tests. Appropriate changes and corrections to these exam items were made by the researcher based on the suggestions and corrections that were received from the faculty members who reviewed the tests.

Reliability

The pretest, unit 1 exam, and unit 2 exam internal consistency were measured using Cronbach’s alpha and found to be .86, .78, and .72, respectively. These coefficients support the reliability of the tests based on the number of questions on the instruments and the size and variability of the groups (Gall, Borg, & Gall, 1996).

To ensure the consistency of the grading process of all tests, the researcher and another faculty member who was not participating in the study but who has extensive experience in this course scored 10 anchor papers chosen randomly from the pretest and the two unit exams in the study. Both instructors graded 10 anchor papers for all three of these tests using the scoring rubric in Table 1. To assess the reliability of the grading, a total score for each student for each test from each grader was obtained and correlation coefficients were computed to obtain inter-rater reliability coefficients between the
graders. The inter-rater reliability coefficients for the pretest, unit exam 1, and unit exam 2 were .93, .98, and .95, respectively. According to Gall et al. (1996), the obtained inter-rater reliability coefficients validated the reliability of the grading.

Procedure

Participants

The study was conducted in the spring of 2005 at a public community college in southwest Florida that serves a two county area with a population of approximately 617,000. In spring 2004, the college enrollment was 8393 students; 3357 students were full-time and 5036 students were part-time. The gender ratio is 64% female to 36% male. In the same term, the average age of full-time students was 23 years and the average age of part-time students was 28 years. Table 2 shows the demographic information of the community college as of the 2003-2004 school year.

Table 2

<table>
<thead>
<tr>
<th>College Demographic Information</th>
<th>Percentages (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Race</td>
<td></td>
</tr>
<tr>
<td>Caucasian</td>
<td>78.1</td>
</tr>
<tr>
<td>African-American</td>
<td>10.0</td>
</tr>
<tr>
<td>Hispanic</td>
<td>6.2</td>
</tr>
<tr>
<td>American Indian &amp; Asian</td>
<td>2.3</td>
</tr>
<tr>
<td>Non-U.S. Residents</td>
<td>1.6</td>
</tr>
<tr>
<td>Unknown Race</td>
<td>1.8</td>
</tr>
</tbody>
</table>
The target population was community college students enrolled in an Applied calculus course in the United States. The sample for this study consisted of students who enrolled in four daytime sections of an Applied Calculus course (MAC 2233) scheduled during the 2005 spring semester at the community college where the study was conducted. The class size is limited to 30 students per section for this course at this college. However, due to students’ work schedules and their schedules of other classes, earlier sections (MWF 8 A.M. and TR 8 A.M.) have smaller enrollments than later sections (MWF 11 A.M. and TR 12:30 P.M.).

The attrition was monitored during the study period. Because the study period was the first four weeks of the semester, attrition was not expected to be a significant problem for this study. However, the students’ attendance was recorded during the study period. If a student missed more than half of the class meetings during the study, his or her score on the tests was not included in the statistical analyses. Although the students were not randomly selected from the target population, it is reasonable to assume that the students were fairly representative of the population because community college students are very similar around the country, according to Watkins (1992). There was a total of 93 students enrolled for the four sections of MAC 2233. The number of students who registered for these four sections by instructors and groups are reported in Table 3. These students were not randomly assigned to these four sections. That is, students who registered for these four sections knew who their instructor would be.
Table 3

Number of Students Registered for the Four Sections by Instructors and Groups

<table>
<thead>
<tr>
<th></th>
<th>Control</th>
<th>Treatment</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor A</td>
<td>17</td>
<td>30</td>
<td>47</td>
</tr>
<tr>
<td>Instructor B</td>
<td>18</td>
<td>28</td>
<td>46</td>
</tr>
<tr>
<td>Total</td>
<td>35</td>
<td>58</td>
<td>93</td>
</tr>
</tbody>
</table>

Selection of Treatment and Control Groups

Four daytime sections (between 8:00 AM and 2:00 PM) of the Applied Calculus course were offered in the spring of 2005 at the college, including two (8 A.M and 11 A.M.) Monday-Wednesday-Friday (MWF) sections and two (8 A.M. and 12:30 P.M.) Tuesday-Thursday (TR) sections. Two other sections of this course were also offered at the college in the evening between 5:30 PM and 9:00 PM. Typically, the students who take day classes are full-time students and are younger and the students who take evening classes are part-time students and are older. To keep the study among a similar population, only the four daytime sections were chosen to participate in the study.

Two groups (11 A.M. MWF section and 12:30 P.M. TR section) served as treatment groups and the other two groups (8 A.M. MWF section and 8 A.M. TR section) served as control groups. Two instructors other than the researcher participated in the study and each instructor taught a MWF schedule class and a TR schedule class and each instructor taught one treatment and one control group.

Due to scheduling concerns between the participating instructors, A and B, and the researcher, both participating instructors knew which two sections they were going to
teach but they did not know which section was treatment and which section was control until the first day of the classes. The MWF schedule classes meet 53 minutes for three days a week and the TR schedule classes meet 80 minutes for two days a week. To avoid the treatment variable being confounded by the number of days the classes meet each week, one of the two MWF sections served as a treatment group and the other MWF section served as a control group; the same arrangement existed for the two TR sections. The two treatment groups received instruction with a numerical method with the use of GCs and the researcher-developed instructional materials; the two control groups received instruction in a traditional manner (primarily algebraic approach) with the use of GCs in limit and derivative topics.

A flip of a coin between the two TR sections determined that TR at 12:30 P.M. (by Instructor A) was assigned as a treatment group. Therefore, instructor A’s MWF at 8 A.M. section was assigned as a control group. This selection determined instructor B’s MWF at 11 A.M. section as a treatment group and TR at 8 A.M. as a control group. Table 4 below summarizes the selection of treatment and control groups.

Table 4

<table>
<thead>
<tr>
<th>Section</th>
<th>Time Slot</th>
<th>Class Length</th>
<th>Instructor</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>MWF Class</td>
<td>8:00 – 8:53 A.M.</td>
<td>53 minutes</td>
<td>A</td>
<td>Control</td>
</tr>
<tr>
<td>MWF Class</td>
<td>11:00 – 11:53 A.M.</td>
<td>53 minutes</td>
<td>B</td>
<td>Treatment</td>
</tr>
<tr>
<td>TR Class</td>
<td>8:00 – 9:20 A.M.</td>
<td>80 minutes</td>
<td>B</td>
<td>Control</td>
</tr>
<tr>
<td>TR Class</td>
<td>12:30 – 1:50 P.M.</td>
<td>80 minutes</td>
<td>A</td>
<td>Treatment</td>
</tr>
</tbody>
</table>
Facilitators

Two full-time instructors (Instructor A and Instructor B) in the mathematics department who were scheduled to teach these four sections participated in the main study. The researcher is also a full-time faculty member in the same department and has been teaching at least one section of this course for the last 15 years but did not teach any section of this course in the spring of 2005.

The two facilitating instructors have positive reputations among students and instructors and extensive experience teaching the Applied Calculus course at this college.

Table 5 below summarizes the demographic data of the instructors.

Table 5

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Gender</th>
<th>Race</th>
<th>Age</th>
<th>Highest Degree</th>
<th>Years Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Male</td>
<td>Caucasian</td>
<td>45</td>
<td>Ph.D Mathematics Education</td>
<td>16</td>
</tr>
<tr>
<td>B</td>
<td>Male</td>
<td>Caucasian</td>
<td>56</td>
<td>M.S Mathematics Education</td>
<td>17</td>
</tr>
</tbody>
</table>

The Studied Course

The study was conducted in the Applied Calculus (MAC 2233) course at a community college. This course, also known as Business Calculus, is a three-credit course and is designed to fulfill requirements for business and non-mathematics majors. For many students, this is the last mathematics course they need to graduate. Topics in this course include limits, differentiation, and integration of algebraic, rational,
exponential, and logarithmic functions, and related applications of limits, differentiation, and integration in the management, business, and social sciences. The main purpose of this course is to study calculus applications in the related majors. Rigorous calculus is not a goal of this course.

**Textbook**

A required text, *Calculus for Managerial, Life, and Social Sciences, 6th Edition*, by Tan (2003) was used for both groups. Chapters 2 through 6 were covered during the course. See Appendix E for the course syllabus and a tentative academic calendar.

**Graphing Calculator**

A graphing calculator was required for this course. Students were allowed to use any type of graphing calculator but any CAS-added graphing calculator (like TI 92 or TI 89) was not allowed during any exams because of their symbolic manipulative capabilities. The mathematics department strongly recommends a TI 83 model because the TI83 is a powerful graphing calculator in terms of its affordability, portability, and capability. It is also user friendly and a popular graphing calculator among college students. This study focused on instruction in limit and derivative topics in the Applied Calculus course with a numerical approach with the use of the TI 83 graphing calculator. Both groups were allowed to use their GCs in the classroom, for homework, and for all exams.
Researcher-Developed Instructional Materials

In addition to the text and a graphing calculator, the researcher developed a set of instructional materials in four unit lessons along with the use of the TI 83 graphing calculator to use in the study. See Appendix F for the entire unit lessons. The purpose of developing the instructional materials is given below.

Generally, there are three types of representations possible to teach mathematical concepts such as limits and derivatives in calculus. These multiple representations are namely graphical, analytical, and numerical representations. A graphical representation is a way of approaching a particular concept via visual images. For example, the graph (picture) of a given function can be used to explain the behavior and properties of the given function. A problem with this approach is that, in general, the functions that are described in real world applications (in the limit and derivative topics) may not produce “nice” graphs.

An analytical representation uses mainly algebraic techniques, properties, and manipulations to explain a mathematical concept. Because this approach depends highly on algebraic techniques and manipulations, this approach may not work well for students if they are weak in their algebraic skills. Further, it is quite possible that any algebraic techniques or manipulations may not work at all for certain mathematical problems (in the limit and derivative topics).

A numerical representation refers to the use of numerical computations to explain or solve mathematical problems. Because this approach depends highly on computational work, this method may be a time consuming process if no calculators are allowed. The
required computations are relatively easy and quick with GCs using special features such as the table feature.

As stated earlier, the purpose of this study was to examine students’ performance in learning limit and derivative topics with a numerical approach using GCs. Therefore, the researcher developed two unit lessons for the topic of limits (Lesson 3) and derivatives (Lesson 4). That is, these two lessons focused on a numerical approach to find limits and derivatives of functions and related applications in limits and derivatives. These two lessons were used only in the treatment groups. Each unit lesson included an instructor note section that provided an instructional guide along with a variety of examples for instructors to use and student activity section that consisted of questions for students as practice.

Because the GC technology is used in the previous unit lessons, one needs to make sure that students are using this technology correctly and effectively. The researcher constantly observed students’ lack of confidence and familiarity, errors, and misconceptions when using a GC in this course. Similar observations were made by some in the literature (Dunham, 1999; Gaston 1990; Mitchelmore and Cavanagh 2000; Tuska 1992). Understanding this problem fully and in order to help students use a GC correctly and confidently, the researcher developed two unit lessons (Lesson 1 and 2) just to help students learn to use the GC effectively. These two lessons, therefore, were used in both treatment and control groups. These unit lessons were previously used by the researcher in his Applied Calculus classes. The researcher received positive comments from students for the supplied guided lessons. These four unit lessons were given to three faculty members (two from the same college where the researcher teaches and one from a nearby
university) to evaluate the contents of the unit lessons. The faculty members were encouraged to make any comments, corrections, and/or suggestions on the unit lesson pages as needed. Those unit lessons were collected from the faculty panel and appropriate changes were made by the researcher based on the suggestions received from the faculty members. The brief contents of each lesson are given in Table 6.

Table 6

<table>
<thead>
<tr>
<th>Unit Lesson</th>
<th>Contents of Each Unit Lesson</th>
</tr>
</thead>
</table>
| 1           | A. Entering arithmetic and algebraic expressions in a graphing calculator.  
               B. Simplifying arithmetic expressions by using a graphing calculator.  
               C. Graphing functions (equations) with a graphing calculator.  
               D. Student activities. |
| 2           | A. Finding function values using a graphing calculator.  
               B. Student activities. |
| 3           | A. Finding the limit of a function as $x$ approaches a number.  
               B. Finding the limit of a function as $x$ approaches a number from the right and left.  
               C. Finding the limit of a function as $x$ approaches $\infty$ and $-\infty$.  
               D. Student activities. |
| 4           | A. Finding the derivative of a function at a given value.  
               B. Finding the slope of a function at a given point.  
               C. Finding the rate of change of a function at a given value.  
               D. Student activities. |

**Instructor Preparation**

One month prior to the main study, the two participating instructors were given a copy of the researcher-developed unit lessons, including student handouts. Then the researcher met the participating two instructors as a team four times before the end of the
fall of 2004. The purpose of the meeting was to explain the purpose, procedure, and method of the study and to review the details of the treatments that both the treatment and control groups received during the study. The researcher provided instruction on using the researcher-developed unit lessons along with the use of the TI 83 graphing calculator during the meeting. Each meeting lasted about 30 to 45 minutes.

When the main study began in the spring of 2005, the researcher met the instructors together once a week outside the classrooms throughout the study period. The purpose of these meetings was to provide opportunities for the instructors to share any problems implementing the plans, discuss the progress, share successful strategies, and ask questions. In addition to the meetings, the researcher visited all treatment and control groups unannounced once a week during the study to assure that the methods and the procedures were being employed as planned.

**Treatment Phase**

Table 7 provides brief information about the time schedules of the treatment phase including the first day activity for the study.
Table 7

<table>
<thead>
<tr>
<th>Weekly Time Table for the Study</th>
<th>Descriptions of Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Week Number</strong></td>
<td><strong>Sections were randomly assigned to both facilitating instructors as one treatment group and one control group for each instructor. The instructors met students and explained the students’ role in the study.</strong></td>
</tr>
<tr>
<td>1- 2</td>
<td><strong>Student Demographic Information and Student Initial Survey were given to all groups.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Pretest was given to all groups.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Sections 2.1, 2.2, and 2.3 were covered for all groups. These 3 sections were review of functions and both treatment and control groups were taught these sections along with the use of TI 83 GCs and the researcher-developed guided unit lessons 1 and 2.</strong></td>
</tr>
<tr>
<td>2- 3</td>
<td><strong>Sections 2.4 and 2.5 were covered for all groups. These two sections were the topic of the limit of functions. The two treatment groups were taught this topic with the use of TI 83 GCs and the researcher-developed guided unit lesson 3. The two control groups were taught without the researcher-developed guided unit lessons.</strong></td>
</tr>
<tr>
<td>3-4</td>
<td><strong>Section 2.6 was covered for all groups. This section was the topic of the derivative of functions. The two treatment groups were taught this topic with the use of GCs and the researcher-developed guided unit lesson 4. The two control groups were taught without the researcher-developed guided unit lessons.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Unit 2 Exam was given to all groups.</strong></td>
</tr>
</tbody>
</table>

On the first day of class, instructors briefly explained the purpose of the research study to students. Students were informed that participation in the study was optional, but that they would receive extra credit for scores obtained on all student-activity homework assignments. All students enrolled for these sections agreed to participate in the study and
students were asked to sign a consent form (see Appendix K). The students, however, were not told whether their group would be a treatment group or a control group. Next, students were asked to complete a demographic information sheet (see Appendix J) and a student initial survey (see Appendix A). The survey focused on the student’s major, previous math courses, and graphing calculator ownership and usage. Also, the pretest (see Appendix B) was administered the first day of class.

The treatment phase began in the second day of class with the review of functions for all groups. Both treatment and control groups followed the same curriculum with the same textbook, guidelines, and time schedules. All subjects took the same pretest and unit exams at the same scheduled times and were graded based on the same score rubric previously discussed.

As mentioned earlier, for the Applied Calculus course, a graphing calculator (GC) is required. TI 83 model GCs were used by all students and the instructors. Although a GC is required for this course, prior experience showed that many students come to this course with less experience and lack of familiarity with a GC. As a result, students constantly use this machine incorrectly and interpret its outcome incorrectly. Therefore, the first two unit lessons out of the four unit lessons developed by the researcher were designed to meet students’ needs in order to use the TI 83 GC correctly and efficiently. Therefore, these lessons were utilized in both treatment and control groups. All students in the treatment and control groups were given a copy of these two lessons as handouts. In these handouts, the instruction of operating the machines, possible mistakes and misconceptions by students, a number of worked out examples, and a homework activity section were discussed. Also, according to the syllabus, all students need to spend one
week reviewing some algebra topics that they previously learned, including functions. Therefore, all students in both groups reviewed these sections along with the researcher-developed unit lessons 1 and 2.

After the review sections, both groups of students studied the limit of functions. The two treatment groups of students learned the limit topic with the researcher-developed unit lesson 3 that focused on solving limit problems with a numerical method with the use of TI 83 GCs. The students in the treatment groups were given a copy of this lesson as a handout. In this handout, the instruction of solving a variety of limit problems with worked out examples and a home-work activity section were discussed. That is, the two treatment groups of students used the table feature of a TI 83 GC along with the researcher-developed unit lesson 3 to solve limit problems, including applications. The two control groups of students, however, learned the same topic in a traditional manner that includes primarily algebraic techniques to solve limit problems. The students in the control groups were not given the researcher-developed unit lesson 3. At the end of the first treatment phase, all students took the same unit 1 exam which they were given 53 minutes to complete. This test consisted of limit problems from three areas: skill oriented, concept oriented, and related applications. Each portion was graded and scored separately.

Immediately after the unit 1 exam, all students learned derivatives of functions with the limit definition. The two treatment groups were taught to find the derivative of a function at the given $x$ value as a limit problem with the help of the researcher-developed unit lesson 4. That is, the students in the treatment groups were introduced to derivative problems as “rate of change” problems; thus, they approached the derivative
problems the same way they previously solved the limit problems. All students in the
treatment groups were given a copy of this lesson as a handout. In this handout, the
instruction of solving a variety of derivative problems with worked out examples, and a
homework activity section were discussed. The control groups were introduced to the
derivative with the same limit definition but they were taught to find the derivative in a
traditional manner that includes primarily algebraic techniques and then substituted the
given $x$ value at the end. Both groups learned to solve the related applications of
derivatives by whatever method they learned to find the derivative for the given
functions. At the end of the second and the last treatment phase, all students took the
same unit 2 exam which they were given 53 minutes to complete. This test consisted of
derivative problems from three areas: skill oriented, concept oriented, and related
applications. Each portion was graded and scored separately. The treatment phase ended
with the unit 2 exam and the entire treatment phase lasted for four weeks.

Even though it was expected that students would solve the exam problems by the
method they were taught in the classroom, they could have solved a problem by any
method but they were required to show the work of their method. It is possible that
students from both the treatment and control groups could have interacted outside the
classrooms. Also, the college has a math lab where students can visit and receive help on
their math courses. To find out the interaction between students in the treatment and
control groups and how students prepared (studying with the same class mates, studying
with other class mates, or receiving any tutoring help) for both unit exams, a student
questionnaire was given to students to complete after every unit test was completed.
A discussion on the results of the student questionnaires is given in chapter 5. See Appendix H for the student questionnaire.

**Data Analysis**

The data analysis was divided into three parts. The first part examined the initial group comparisons with the pretest test scores. Descriptive statistics, including means and standard deviations for the pretest for all four groups were reported. A mean comparison graph and a boxplot graph for the pretest data by instructors were also shown. Then the pretest test scores were tested with a simple one-way ANOVA. An alpha level of .05 was used for the test. According to Stevens (1986), the following are the ANOVA assumptions: (1) the observations are normally distributed on the dependent variable in each group; (2) the population variances for the groups are equal; and (3) the observations are independent. Assumption (1) was examined by plotting pretest scores and using histograms. Inspection showed that the scores were approximately normally distributed. Further, as reported in Table 8 (chapter 4), the values of skewness and kurtosis for the treatment and control groups were within -1 to +2, indicating that there were no major departures from normality. Assumption (2) was examined using Levene statistic. The pretest data was found to have homogeneous variances. It was assumed that the students were independent (Assumption (3)) of one another since the pretest was administered in the classrooms on the first day of class.

The second part examined the test scores of the unit 1 exam on limits for all four groups. Mean comparison graphs were shown for each portions of skill, concept, and application of the unit 1 exam for the treatment and control groups. Then three separate ANCOVA tests were conducted on the skill, concept, and application portions of the
unit 1 exam. The third part is similar to the second part but examined the test scores of the unit 2 exam on derivatives. An alpha level of .05 was used in all ANCOVA tests. According to Stevens (1986), the following are the ANCOVA assumptions: (1) the observations are normally distributed on the dependent variable in each group; (2) the population variances for the groups are equal; (3) the observations are independent; (4) a linear relationship exists between the dependent variable and the covariate; and (5) the slope of the regression line is the same in each group.

Assumption (1) was examined by plotting the scores of the unit 1 and 2 exams and using histograms. Inspection showed that the scores were approximately normally distributed. Further, as reported in Tables 10 and 18 (chapter 4), the values of skewness and kurtosis for the treatment and control groups were within -1.61 to +1, indicating that there were no major departures from normality. Assumption (2) was examined using Levene statistic. The unit 1 and 2 exam data were found to have homogeneous variances. Even though it was assumed that the students were independent (Assumption (3)) of one another, there is some concern. The lecture settings for instruction for all groups provide independent observations in the classrooms but it was almost impossible to control the students’ independence outside the classroom. The examinations for assumptions 4 and 5 showed that the data used in the ANCOVA failed to meet these assumptions. Part of the reason is, as discussed in chapter 4, that the correlations between the pretest and unit 1 and 2 exams were found to be very low. Therefore, the pretest score might not be a good measurement for testing the groups’ equivalence. Students’ previous math scores or grades could have been used as a covariate in all ANCOVA tests instead.
Finally, the next section provides a summary of the pilot study that was conducted prior to the main study. A full report of the pilot study, including the data analysis, is given in Appendix G.

**The Pilot Study**

A pilot study was conducted in the fall of 2004 by the researcher prior to the main study. The purpose of the main study was to examine the effects of using GCs with a numerical approach and the researcher-developed instructional materials in limits and derivatives in an Applied Calculus course at a community college. It is the researcher’s belief that a numerical method with the use of a TI 83 GC and supported instructional materials would improve students’ understanding of limit and derivative topics. The purpose of the pilot study was to evaluate the characteristics of the used numerical method with the use of a TI 83 GC and researcher-developed instructional materials in the limit and derivative topics of the Applied Calculus course and to determine critical situations that might be encountered during the main study.

It was expected that the pilot study would help to explore whether the instruction with the numerical method along with the use of TI 83 GC and instructional materials could lead to better understanding on limits and derivatives among community college students. Furthermore, the pilot study was used to access the validity and reliability of all exams that were used in the main study. The researcher used parallel forms of the pretest, and two unit exams in his pilot study during the fall of 2004. The following changes were made to the main study as a result of the pilot study:
1. In the pilot study, there were two parts on each unit exam. The first part of each unit exam dealt with a number of skill-oriented problems for measuring students’ ability to solve these routine (skill-oriented) problems. The second part dealt with a number of applications for measuring students’ conceptual understanding for a particular topic. Some of these problems were based on graphs. It was noted in the pilot study that some students were able to solve the application problems but did not solve the graphical-based questions and vice versa. Therefore, to learn how students understand a particular component of the topic, each unit exam was changed to have three parts: a number of skill oriented problems; a number of conceptual oriented problems based on graphs; and a number of application problems.

2. In the pilot study, the researcher-developed materials (four unit lessons) were used only in the treatment group. Because the first two unit lessons were only about how to use a TI 83 GC effectively, it was decided that these two unit lessons would be given to both treatment and control students.

3. A number of items that would be helpful to students were added to the student activity sections in all instructional guided lessons.

4. A number of items that would be helpful to students were added to the instructor notes sections in all instructional guided lessons.

5. Some question items were edited for purposes of clarity.

6. Both unit exams were shortened slightly because of timing issues.
Summary

This study was conducted to examine effects of the use of GCs with a numerical approach and the researcher-developed instructional materials on the limit and derivative topics in an Applied Calculus course at a community college. The pilot study supported the validity and the reliability of all instruments.

The main study involved four day-time sections with 87 participants and 2 instructors as facilitators. Two sections served as treatment groups and received instruction with the numerical approach along with the researcher-developed unit lessons and TI 83 graphing calculators on limit and derivative topics; the other two sections served as control groups and received instruction on those two topics with the traditional manner (primarily algebraic approach). Hypotheses were stated to answer stated research questions. Instruments used to gather data included the pretest and two unit exams to test hypotheses. To analyze the data gathered with these instruments, a simple one-way ANOVA (pretest) and a set of two-factorial ANCOVA statistical tests (unit 1 and 2 exams) were used. The next chapter discusses the results of the data analysis.
Chapter 4

Results

This chapter begins by restating the research questions and hypotheses that were tested in order to answer the research questions for the study. Next, the data analysis is discussed in three parts. The first part discusses the descriptive statistics and an ANOVA test for the pretest. The purpose of this part is to examine the initial equality of the study because full random assignment of students to the study groups was not possible. The second part discusses the descriptive statistics of the dependent measures of the entire unit 1 exam on limits and each subset (Skills, Concepts, and Applications) of the unit 1 exam and then necessary ANCOVA tests that are needed to answer the study’s first three research hypotheses. The third discussion is similar to the second part but on the entire unit 2 exam on derivatives and each subset (Skills, Concepts, and Applications) of the unit 2 exam and then necessary ANCOVA tests that are needed to answer the study’s last three research hypotheses.

Research Questions and Hypotheses

The general research question that this study sought to answer is “To what degree can the use of GCs with a numerical approach and instructional materials developed by the researcher affect community college Applied Calculus students’ learning of limits and derivatives?” In particular, the study sought to answer the following research questions:
1. How does the students’ achievement in solving limit problems with a numerical approach compare to that of students who solved limit problems with a traditional approach (primarily an algebraic approach) in an Applied Calculus course?

2. How does the students’ achievement in solving derivative problems with a numerical approach compare to that of students who solved derivative problems with a traditional approach (primarily an algebraic approach) in an Applied Calculus course?

The following research hypotheses were used to answer the research questions. The first three hypotheses are stated to answer the first research question and the last three hypotheses are stated to answer the second research question.

1. Students (GCGL group) who receive instruction with a numerical approach will have higher achievement in routine (skill oriented) limit problems than students (GC group) who receive instruction with a traditional approach (primarily an algebraic approach).

2. The students in the GCGL group will have higher achievement in conceptual oriented limit problems than the students in the GC group.

3. The students in the GCGL group will have higher achievement in related applications of limits than the students in the GC group.

4. The students in the GCGL group will have higher achievement in routine (skill oriented) derivative problems than the students in the GC group.

5. The students in the GCGL group will have higher achievement in conceptual oriented derivative problems than the students in the GC group.
6. The students in the GCGL group will have higher achievement in related applications of derivative problems than the students in the GC group.

**Data Analysis of the Pretest**

The pretest data was used to determine whether students in the treatment and control groups were similar in their prerequisite mathematical ability before they received any instruction for the calculus topics. The pretest items were selected from College Algebra (MAC 1105), the prerequisite course to Applied Calculus (MAC 2233).

A total of 93 students showed up in the selected four sections of the Applied Calculus course on the first day of class and all of them took the pretest. The first week is also the drop-add period at the college. Six students dropped the course after the first day for various reasons that were not related to the study. Also, the other 87 students stayed in the course and took the unit 1 exam but only 82 students’ scores were considered for the statistical analysis because 5 students were absent 50% or more during the first experiment phase. Descriptive statistics for the pretest (n = 82) for all four groups are reported in Table 8. Small effect sizes of 0.24 and 0.08 were obtained for the classes of instructors A and B, respectively, and favored the treatment group for both instructors. A mean comparison graph for the pretest data by instructors is shown in Figure 1 and a boxplot graph to illustrate the spread of the data is shown in Figure 2. A visual comparison from Figure 1 also indicates that the mean score of the control group was slightly lower than the mean score of the treatment group for both instructors A and B.
Table 8

Means, Standard Deviations, Skewness, and Kurtosis for the Pretest by Instructors

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>M</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Min</th>
<th>Max</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control A</td>
<td>16</td>
<td>23.00</td>
<td>6.98</td>
<td>0.03</td>
<td>-0.60</td>
<td>11</td>
<td>35</td>
<td>0.24</td>
</tr>
<tr>
<td>Treatment A</td>
<td>25</td>
<td>24.48</td>
<td>5.60</td>
<td>-0.24</td>
<td>1.84</td>
<td>9</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>Control B</td>
<td>15</td>
<td>23.40</td>
<td>8.54</td>
<td>-0.50</td>
<td>0.15</td>
<td>6</td>
<td>37</td>
<td>0.08</td>
</tr>
<tr>
<td>Treatment B</td>
<td>26</td>
<td>23.96</td>
<td>7.32</td>
<td>0.13</td>
<td>-0.44</td>
<td>8</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

*Note:* Maximum possible score on pretest was 40 points.
Control A = Control group for Instructor A, Treatment A = Treatment group for Instructor A, Control B = Control group for Instructor B, Treatment B = Treatment group for Instructor B.

![Figure 1: Mean Comparison for the Pretest for the Treatment and Control Groups by Instructors](image)

Figure 1: Mean Comparison for the Pretest for the Treatment and Control Groups by Instructors
To determine whether the control groups and treatments groups were statistically-significantly different on the pretest, the means of these groups were compared using a one-way ANOVA and the results are reported in Table 9.

Table 9

<table>
<thead>
<tr>
<th>One-Way ANOVA for the Pretest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
</tr>
<tr>
<td>Sum of Squares</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Between Groups</td>
</tr>
<tr>
<td>24.81</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>8.27</td>
</tr>
<tr>
<td>0.17</td>
</tr>
<tr>
<td>0.918</td>
</tr>
<tr>
<td>Within Groups</td>
</tr>
<tr>
<td>3844.80</td>
</tr>
<tr>
<td>78</td>
</tr>
<tr>
<td>49.29</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>3869.61</td>
</tr>
<tr>
<td>81</td>
</tr>
</tbody>
</table>

Figure 2: Boxplot Graph for the Pretest for the Treatment and Control Groups by Instructor
The ANOVA table confirmed that the groups were not statistically-significantly different, \( F(3, 78) = 0.17, p = .92 \). That is, there was no statistically significant difference between the groups in their mathematical ability prior to their study of Applied Calculus.

**Data Analysis of Unit 1 Exam on Limits**

The unit 1 exam was used to measure students’ achievement on limit topics of Applied Calculus. The exam contained problems from three areas: Skills, Concepts, and Applications. The means, standard deviations and other descriptive statistics for the entire unit 1 exam for all groups by instructors are reported in Table 10. Also, Figure 3 shows a visual comparison of the mean scores of the control groups and treatment groups for the entire unit 1 exam for each instructor. The graph shows that the mean score of the treatment group is higher than the mean score of the control group for both instructors A and B.

Table 10

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>M</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control A</td>
<td>16</td>
<td>33.69</td>
<td>7.53</td>
<td>-0.01</td>
<td>-0.83</td>
<td>18</td>
<td>48</td>
</tr>
<tr>
<td>Treatment A</td>
<td>25</td>
<td>41.88</td>
<td>10.22</td>
<td>-0.27</td>
<td>-0.56</td>
<td>20</td>
<td>57</td>
</tr>
<tr>
<td>Control B</td>
<td>15</td>
<td>41.73</td>
<td>11.25</td>
<td>-0.34</td>
<td>-0.89</td>
<td>21</td>
<td>60</td>
</tr>
<tr>
<td>Treatment B</td>
<td>26</td>
<td>45.38</td>
<td>10.62</td>
<td>-1.05</td>
<td>0.83</td>
<td>20</td>
<td>59</td>
</tr>
</tbody>
</table>

*Note*: Maximum possible score on unit 1 exam was 60 points.
Control A = Control group for Instructor A, Treatment A = Treatment group for Instructor A, Control B = Control group for Instructor B, Treatment B = Treatment group for Instructor B.
As mentioned earlier, the unit 1 exam has three portions: Skills, Concepts, and Applications. Each portion is graded and scored separately for testing any statistically significant differences between the groups on each of these portions. First, the means and standard deviations of the skill, concept, and application portions of the unit 1 exam were obtained (see Table 11).
Table 11

Means and Standard Deviations for Unit 1 Exam on Limits by Skills, Concepts, and Applications

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Skill M</th>
<th>Skill SD</th>
<th>Concept M</th>
<th>Concept SD</th>
<th>Application M</th>
<th>Application SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control A</td>
<td>16</td>
<td>14.37</td>
<td>2.83</td>
<td>10.81</td>
<td>7.09</td>
<td>8.50</td>
<td>1.27</td>
</tr>
<tr>
<td>Treatment A</td>
<td>25</td>
<td>15.48</td>
<td>3.28</td>
<td>15.88</td>
<td>6.87</td>
<td>10.52</td>
<td>2.42</td>
</tr>
<tr>
<td>Control B</td>
<td>15</td>
<td>15.00</td>
<td>4.15</td>
<td>15.73</td>
<td>6.77</td>
<td>9.93</td>
<td>1.98</td>
</tr>
<tr>
<td>Treatment B</td>
<td>26</td>
<td>16.07</td>
<td>4.55</td>
<td>19.77</td>
<td>5.95</td>
<td>10.62</td>
<td>2.35</td>
</tr>
</tbody>
</table>

*Note: Maximum possible score on unit 1 exam was 60 points. Maximum possible scores for skill, concept, and application portions were 20, 26, and 14, respectively. Control A = Control group for Instructor A, Treatment A = Treatment group for Instructor A, Control B = Control group for Instructor B, Treatment B = Treatment group for Instructor B.*

The next section discusses the statistical analysis of the skill, concept, and application portions of the unit 1 exam for any statistically significant differences between the control and treatment groups with three separate ANCOVA tests with the pretest as covariate. First, Table 12 reports the results of the ANCOVA test for the skill portion of the unit 1 exam.
Table 12

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>5.92</td>
<td>1</td>
<td>5.92</td>
<td>0.40</td>
<td>.527</td>
</tr>
<tr>
<td>Instructor (Ir)</td>
<td>7.13</td>
<td>1</td>
<td>7.13</td>
<td>0.49</td>
<td>.488</td>
</tr>
<tr>
<td>Instruction (In)</td>
<td>0.01</td>
<td>1</td>
<td>0.01</td>
<td>0.01</td>
<td>.981</td>
</tr>
<tr>
<td>Ir X In</td>
<td>21.94</td>
<td>1</td>
<td>21.94</td>
<td>1.49</td>
<td>.225</td>
</tr>
<tr>
<td>Error</td>
<td>1131.01</td>
<td>77</td>
<td>14.69</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As reported in Table 12, there were no statistically significant differences on instruction and instructor effects between the groups found for the skill portion of the unit 1 exam, $F(1, 77) = 0.01$, $p = .98$ and $F(1, 77) = 0.49$, $p = .49$, respectively. Also, there was no interaction effect between instruction and instructor found in the data, $F(1, 77) = 1.49$, $p = .23$.

Table 13 reports the result of the ANCOVA test for the concept portion of the unit 1 exam.
Table 13

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>35.76</td>
<td>1</td>
<td>35.76</td>
<td>0.82</td>
<td>.369</td>
</tr>
<tr>
<td>Instructor (Ir)</td>
<td>374.79</td>
<td>1</td>
<td>374.79</td>
<td>8.55</td>
<td>.005</td>
</tr>
<tr>
<td>Instruction (In)</td>
<td>380.07</td>
<td>1</td>
<td>380.07</td>
<td>8.67</td>
<td>.004</td>
</tr>
<tr>
<td>Ir X In</td>
<td>4.28</td>
<td>1</td>
<td>4.28</td>
<td>0.10</td>
<td>.756</td>
</tr>
<tr>
<td>Error</td>
<td>3376.86</td>
<td>77</td>
<td>43.86</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table confirmed that the effect of instruction was significant, F(1, 77) = 8.67, p = .004, and the effect of instructor was also significant, F(1, 77) = 8.55, p = .005 on the concept portion of the unit 1 exam. But the interaction effect between instruction and instructor was not significant, F(1, 77) = 0.10, p = .756.

Table 14 reports the result of the ANCOVA test for the application portion of the unit 1 exam.

Table 14

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>1.57</td>
<td>1</td>
<td>1.57</td>
<td>0.34</td>
<td>.561</td>
</tr>
<tr>
<td>Instructor (Ir)</td>
<td>11.29</td>
<td>1</td>
<td>11.29</td>
<td>2.44</td>
<td>.122</td>
</tr>
<tr>
<td>Instruction (In)</td>
<td>33.92</td>
<td>1</td>
<td>33.92</td>
<td>7.34</td>
<td>.008</td>
</tr>
<tr>
<td>Ir X In</td>
<td>8.38</td>
<td>1</td>
<td>8.38</td>
<td>1.81</td>
<td>.182</td>
</tr>
<tr>
<td>Error</td>
<td>355.75</td>
<td>77</td>
<td>4.62</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The table confirmed that the effect of instruction was significant, 
F(1, 77) = 7.34, p = .008, and the effect of instructor was not significant, 
F(1, 77) = 2.44, p = .122 on the application portion of the unit 1 exam. Also, the 
interaction effect between instruction and instructor was not significant, F(1, 77) = 1.81, 
p = .182.

In addition to the ANCOVA tests on the skill, concept, and application portions of 
the unit 1 exam on limits for any statistically significant differences, some further 
analysis was conducted to help understand how students in each group scored on each of 
those sections. First, it should be noted that the maximum possible points for each portion 
of the unit 1 exam vary. The maximum possible score for the entire unit 1 exam was 60 
points and the maximum possible scores for the skill portion, concept portion, and 
application portion are 20, 26, and 14, respectively. Therefore, the mean score percent of 
the skill, concept, and application portions of the unit 1 exam was calculated and is 
reported in Table 15.
Table 15

Mean Percents for Unit 1 Exam on Limits by Skills, Concepts, and Applications

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Skill %</th>
<th>Concept %</th>
<th>Application %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M</td>
<td>Earned</td>
<td>M</td>
</tr>
<tr>
<td>Control A</td>
<td>16</td>
<td>14.37</td>
<td>71.9</td>
<td>10.81</td>
</tr>
<tr>
<td>Treatment A</td>
<td>25</td>
<td>15.48</td>
<td>77.4</td>
<td>15.88</td>
</tr>
<tr>
<td>Control B</td>
<td>15</td>
<td>15.00</td>
<td>75.0</td>
<td>15.73</td>
</tr>
<tr>
<td>Treatment B</td>
<td>26</td>
<td>16.07</td>
<td>80.1</td>
<td>19.77</td>
</tr>
</tbody>
</table>

*Note:* Maximum possible score on unit 1 exam was 60 points. Maximum possible scores for skill, concept, and application portions were 20, 26, and 14, respectively. Control A = Control group for Instructor A, Treatment A = Treatment group for Instructor A, Control B = Control group for Instructor B, Treatment B = Treatment group for Instructor B.

Also, visual comparisons of the percent mean scores of the control and treatment groups by instructors on the Skill portion (Figure 4), Concept portion (Figure 5), and Application portion (Figure 6) are presented.

![Figure 4: Mean Comparison for the Skill Portion of the Unit 1 Exam on Limits for all Groups by Instructors](image-url)
Several interesting results can be noted in the percent mean scores of the unit 1 exam. The most notable observation is that each treatment group outperformed their respective control group in the skill, concept, and application portions of the limit topic (unit 1 exam). In general, groups scored highest in the skill portion, with scores on the application portion next highest, and the scores on the concepts the lowest. That is, all four groups scored higher (between 71.9% and 80.1%) on the skill portion than the application or concept portion. Comparing the percent mean scores, Treatment B group
scored the highest and Control A group scored the lowest in every portion of the unit 1 exam.

These observations made on the skill, concept, and application portions of the unit 1 exam were confirmed by the calculated effect sizes and are reported in Table 16 below. That is, both instructors had small to large (according to Cohen, 1969) effect sizes but positive, favoring the treatment groups.

Table 16
Effect Sizes for the Skill, Concept, and Application Portions of the Unit 1 Exam on Limits

<table>
<thead>
<tr>
<th>Instructor</th>
<th>Skill</th>
<th>Concept</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor A</td>
<td>0.36</td>
<td>0.73</td>
<td>1.09</td>
</tr>
<tr>
<td>Instructor B</td>
<td>0.25</td>
<td>0.64</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Furthermore, one needs to note that the question items in each portion of the unit 1 exam on limits differ in terms of the number of questions, content, difficulty level, etc. Therefore, to help understand how students scored on each item of the skill, concept, and application portions of the unit 1 exam on limits, the percent score of each group by item type was calculated and is reported in Table 17.
Table 17
Percent Scores for Each Item Type within Skill, Concept, and Application Portions for Unit 1 Exam on Limits by Groups

<table>
<thead>
<tr>
<th></th>
<th>Control A</th>
<th>Treatment A</th>
<th>Control B</th>
<th>Treatment B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Skill Portion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #s 1 and 2:</td>
<td>75</td>
<td>77</td>
<td>78</td>
<td>82</td>
</tr>
<tr>
<td>Finding the limit of a function as $x$ approaches a number.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #s 3, 4, and 5:</td>
<td>72</td>
<td>78</td>
<td>77</td>
<td>80</td>
</tr>
<tr>
<td>Finding the limit of a function as $x$ approaches a number from the left and/or from the right.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #6:</td>
<td>68</td>
<td>75</td>
<td>70</td>
<td>78</td>
</tr>
<tr>
<td>Finding the limit of a function as $x$ approaches $\infty$ and/or $-\infty$.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Concept Portion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #7:</td>
<td>36</td>
<td>57</td>
<td>48</td>
<td>66</td>
</tr>
<tr>
<td>Explaining the meaning of the limit of a function as $x$ approaches a number.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #s 8, 9, and 10:</td>
<td>46</td>
<td>64</td>
<td>65</td>
<td>80</td>
</tr>
<tr>
<td>Finding the limit of a function as $x$ approaches a number from the left and the right from the graph of the function.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #11:</td>
<td>43</td>
<td>62</td>
<td>65</td>
<td>83</td>
</tr>
<tr>
<td>Finding the limit of a function as $x$ approaches a number from the given table.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Application Portion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #12(a) and 12(b):</td>
<td>82</td>
<td>87</td>
<td>84</td>
<td>90</td>
</tr>
<tr>
<td>Finding the function value and interpreting the answer.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #12(c):</td>
<td>48</td>
<td>68</td>
<td>60</td>
<td>67</td>
</tr>
<tr>
<td>Finding the limit of a function as $x$ approaches a number from the left and interpreting the answer.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #13:</td>
<td>52</td>
<td>70</td>
<td>68</td>
<td>70</td>
</tr>
<tr>
<td>Finding the limit of a function as $x$ approaches $\infty$ and interpreting the answer.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Some interesting results can also be noted in the percent mean scores of each question type for the skill, concept, and application portions of the unit 1 exam for all four groups. Among the skill oriented limit questions, all groups scored lower on the question of “finding the limit of a function as $x$ approaches $\infty$ and/or $-\infty$” than any other questions. This difficulty might be associated with students’ understanding of the meaning of $\infty$ and $-\infty$. On the concept portion of the limit topic, the question of “explaining the meaning of the limit of a function as $x$ approaches a number” was the hardest question for all groups. On the application portion of the limit topic, the question of “finding the limit of a function as $x$ approaches a number from the left and interpreting the answer” was the hardest question for all groups. A possible explanation for students’ poor performance on this type of question is that students, in general, dislike an “interpret your answer” part of a question. Experience teaching Applied Calculus course has shown that even good students tend to avoid answering such questions.

**Data Analysis of Unit 2 Exam on Derivatives**

The unit 2 exam was used to measure students’ achievement on derivative topics of Applied Calculus. This exam also contained problems from three areas: Skills, Concepts, and Applications. Table 18 reports the mean scores and other descriptive statistics for the entire unit 2 exam for all groups by instructors. Figure 7 shows a visual comparison of the mean scores of the control groups and treatment groups for the entire unit 2 exam for each instructor. The graph shows that the mean score of the treatment group is higher than the mean score of the control group for both instructors A and B.
Table 18

Means, Standard Deviations, Skewness, and Kurtosis for the Entire Unit 2 Exam on Derivatives by Instructors

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>M</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control A</td>
<td>14</td>
<td>27.57</td>
<td>4.14</td>
<td>0.45</td>
<td>0.24</td>
<td>21</td>
<td>36</td>
</tr>
<tr>
<td>Treatment A</td>
<td>23</td>
<td>34.09</td>
<td>6.08</td>
<td>0.08</td>
<td>0.31</td>
<td>20</td>
<td>46</td>
</tr>
<tr>
<td>Control B</td>
<td>14</td>
<td>29.00</td>
<td>8.89</td>
<td>0.36</td>
<td>0.27</td>
<td>13</td>
<td>45</td>
</tr>
<tr>
<td>Treatment B</td>
<td>25</td>
<td>35.96</td>
<td>7.39</td>
<td>0.04</td>
<td>-1.61</td>
<td>25</td>
<td>46</td>
</tr>
</tbody>
</table>

*Note:* Maximum possible score on unit 2 exam was 48 points.
Control A = Control group for Instructor A, Treatment A = Treatment group for Instructor A, Control B = Control group for Instructor B, Treatment B = Treatment group for Instructor B.

Figure 7: Mean Comparison for the Entire Unit 2 Exam on Derivatives for all Groups by Instructors

As mentioned earlier, the unit 2 exam was also divided into three portions as skills, concepts, and applications. These three portions of the unit 2 exam were graded and scored separately for all groups and the means and standard deviations of these portions are reported in Table 19.
Table 19

Means and Standard Deviations for Unit 2 Exam on Derivatives by Skills, Concepts, and Applications

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Skill M</th>
<th>Skill SD</th>
<th>Concept M</th>
<th>Concept SD</th>
<th>Application M</th>
<th>Application SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control A</td>
<td>14</td>
<td>12.86</td>
<td>2.69</td>
<td>5.00</td>
<td>3.55</td>
<td>9.71</td>
<td>1.59</td>
</tr>
<tr>
<td>Treatment A</td>
<td>23</td>
<td>14.78</td>
<td>2.35</td>
<td>8.22</td>
<td>4.27</td>
<td>11.13</td>
<td>1.87</td>
</tr>
<tr>
<td>Control B</td>
<td>14</td>
<td>12.14</td>
<td>3.66</td>
<td>8.07</td>
<td>4.45</td>
<td>8.79</td>
<td>2.72</td>
</tr>
<tr>
<td>Treatment B</td>
<td>25</td>
<td>14.76</td>
<td>2.83</td>
<td>10.36</td>
<td>5.23</td>
<td>10.84</td>
<td>3.04</td>
</tr>
</tbody>
</table>

*Note:* Maximum possible score on unit 2 exam was 48 points. Maximum possible scores for skill, concept, and application portions were 18, 16, and 14, respectively. Control A = Control group for Instructor A, Treatment A = Treatment group for Instructor A, Control B = Control group for Instructor B, Treatment B = Treatment group for Instructor B.

The next section discusses the statistical analysis of the skill, concept, and application portions of the unit 2 exam for any statistically significant differences between the control and treatment groups. Three separate ANCOVAs were conducted with the pretest as covariate on these three portions. An alpha level of .05 was used in each case and Table 20 reports the results of the ANCOVA test for the skill portion of the unit 2 exam.
Table 20

2 X 2 ANCOVA for the Skill Portion of Unit 2 Exam on Derivatives

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>2.67</td>
<td>1</td>
<td>2.67</td>
<td>0.33</td>
<td>.569</td>
</tr>
<tr>
<td>Instructor (Ir)</td>
<td>2.42</td>
<td>1</td>
<td>2.42</td>
<td>0.30</td>
<td>.588</td>
</tr>
<tr>
<td>Instruction (In)</td>
<td>93.81</td>
<td>1</td>
<td>93.81</td>
<td>11.50</td>
<td>.001</td>
</tr>
<tr>
<td>Ir X In</td>
<td>2.38</td>
<td>1</td>
<td>2.38</td>
<td>0.29</td>
<td>.591</td>
</tr>
<tr>
<td>Error</td>
<td>579.23</td>
<td>71</td>
<td>8.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table confirmed that the effect of instruction was significant, F(1, 71) = 11.50, p = .001, and the effect of instructor was not significant, F(1, 71) = 0.30, p = .588 on the skill portion of the unit 2 exam. Also, the table found that the interaction effect between instruction and instructor was not significant, F(1, 71) = 0.29, p = .591.

Table 21 reports the results of the ANCOVA test for the concept portion of the unit 2 exam.
The table confirmed that the effect of *instruction* was significant, $F(1, 71) = 5.21, p = .025$, and the effect of *instructor* was also significant, $F(1, 71) = 5.92, p = .018$ on the concept portion of the unit 2 exam. But, the table found that the interaction effect between *instruction* and *instructor* was not significant, $F(1, 71) = 0.25, p = .617$.

Table 22 reports the result of the ANCOVA test for the application portion of the unit 2 exam.
Table 22

2 X 2 ANCOVA for the Application Portion of Unit 2 Exam on Derivatives

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>27.46</td>
<td>1</td>
<td>27.46</td>
<td>4.88</td>
<td>.030</td>
</tr>
<tr>
<td>Instructor (Ir)</td>
<td>6.48</td>
<td>1</td>
<td>6.48</td>
<td>1.15</td>
<td>.287</td>
</tr>
<tr>
<td>Instruction (In)</td>
<td>38.21</td>
<td>1</td>
<td>38.21</td>
<td>6.79</td>
<td>.011</td>
</tr>
<tr>
<td>Ir X In</td>
<td>1.09</td>
<td>1</td>
<td>1.09</td>
<td>0.19</td>
<td>.661</td>
</tr>
<tr>
<td>Error</td>
<td>399.72</td>
<td>71</td>
<td>5.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table confirmed that the effect of instruction was significant, $F(1, 71) = 6.79, p = .011$, and the effect of instructor was not significant, $F(1, 71) = 1.15, p = .287$ on the application portion of the unit 2 exam. Also, the table found that the interaction effect between instruction and instructor was not significant, $F(1, 71) = 0.19, p = .661$.

As before, some further analysis was conducted to help understand how students in each group scored on each of those portions. It should be again noted that the maximum possible points for each portion of the unit 2 exam also vary. The maximum possible score for the entire unit 2 exam was 48 points and the maximum possible scores for the skill portion, concept portion, and application portion are 18, 16, and 14, respectively. Therefore, the mean score percent of skill, concept, and application portions of the unit 2 exam was calculated and is reported in Table 23.
Table 23

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Skill M</th>
<th>% Earned</th>
<th>Concept M</th>
<th>% Earned</th>
<th>Application M</th>
<th>% Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control A</td>
<td>14</td>
<td>12.86</td>
<td>71.4</td>
<td>5.00</td>
<td>31.3</td>
<td>9.71</td>
<td>69.4</td>
</tr>
<tr>
<td>Treatment A</td>
<td>23</td>
<td>14.78</td>
<td>82.1</td>
<td>8.22</td>
<td>51.4</td>
<td>11.13</td>
<td>79.5</td>
</tr>
<tr>
<td>Control B</td>
<td>14</td>
<td>12.14</td>
<td>67.4</td>
<td>8.07</td>
<td>50.4</td>
<td>8.79</td>
<td>62.8</td>
</tr>
<tr>
<td>Treatment B</td>
<td>25</td>
<td>14.76</td>
<td>82.0</td>
<td>10.36</td>
<td>64.8</td>
<td>10.84</td>
<td>77.4</td>
</tr>
</tbody>
</table>

*Note:* Maximum possible score on unit 2 exam was 48 points. Maximum possible scores for skill, concept, and application portions were 18, 16, and 14, respectively.

Control A = Control group for Instructor A, Treatment A = Treatment group for Instructor A, Control B = Control group for Instructor B, Treatment B = Treatment group for Instructor B.

Further, visual comparisons of the percent mean scores of the control and treatment groups by instructors on the Skill portion (Figure 8), Concept portion (Figure 9), and Application portion (Figure 10) are presented.

![Figure 8: Mean Comparison for the Skill Portion of the Unit 2 Exam on Derivatives for all Groups by Instructors](image)
Several interesting results can be noted in the percent mean scores of the unit 2 exam. As in the unit 1 exam, the most notable observation was that each treatment group of both instructors outperformed their control groups on the skill, concept, and application portions of the derivative topic (unit 2 exam). Every group in both treatment and control groups scored highest (between 67.4% and 82.1%) on the skill portion, second highest on the application portion (between 62.8% and 79.5%), and lowest (between 31.3% and 64.8%) on the concept portion. That is, again it was found that the
concept portion was the hardest for all groups. Comparing the percent mean scores, the Treatment B group scored the highest on the skill and concept portions of the topic of derivative, but Treatment A scored the highest on the application portion; Control B group scored the lowest in the skill and application portions and Control A group scored the lowest on the concept portion.

These observations made on skill, concept, and application portions of the unit 2 exam were confirmed by the calculated effect sizes and are reported in Table 24 below. That is, both instructors had medium to large (according to Cohen, 1969) effect sizes but positive, favoring the treatment groups.

<table>
<thead>
<tr>
<th>Table 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect Sizes for the Skill, Concept, and Application Portions of the Unit 2 Exam on Derivatives</td>
</tr>
<tr>
<td>Skill</td>
</tr>
<tr>
<td>Instructor A</td>
</tr>
<tr>
<td>Instructor B</td>
</tr>
</tbody>
</table>

As noticed earlier on the unit 1 exam, the question items in each portion of the unit 2 exam also differ in terms of the number of questions, content, difficulty level, etc. Therefore, to help understand how students scored on each item of the skill, concept, and application portions of the unit 2 exam on derivatives the percent score of each group by item type was calculated and is reported in Table 25.
Table 25

Percent Scores for Each Item Type within Skill, Concept, and Application Portions for Unit 2 Exam on Derivatives by Groups

<table>
<thead>
<tr>
<th></th>
<th>Control A</th>
<th>Treatment A</th>
<th>Control B</th>
<th>Treatment B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Skill Portion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finding the derivative of a function at a given ( x ) value.</td>
<td>75</td>
<td>85</td>
<td>71</td>
<td>84</td>
</tr>
<tr>
<td>Question #2:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finding the slope of a function at a given point.</td>
<td>68</td>
<td>79</td>
<td>64</td>
<td>80</td>
</tr>
<tr>
<td><strong>Concept Portion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #4:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explaining the meaning of the derivative of a function at a given ( x ) value.</td>
<td>25</td>
<td>46</td>
<td>42</td>
<td>59</td>
</tr>
<tr>
<td>Question #s 5, 6, and 7:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determining the derivative of the given graph of a function at given points as positive, negative, and zero.</td>
<td>36</td>
<td>55</td>
<td>56</td>
<td>68</td>
</tr>
<tr>
<td>Question #8:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Determining the point of the given graph of a function at which the derivative is the greatest.</td>
<td>33</td>
<td>53</td>
<td>53</td>
<td>66</td>
</tr>
<tr>
<td><strong>Application Portion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Question #s 3(a) and 9(a):</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finding the average rate of change of a function on a given interval.</td>
<td>84</td>
<td>89</td>
<td>80</td>
<td>87</td>
</tr>
<tr>
<td>Question #s 3(b) and 9(b):</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finding the instantaneous rate of change of a function at a given value.</td>
<td>64</td>
<td>77</td>
<td>56</td>
<td>75</td>
</tr>
<tr>
<td>Question #10:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finding the derivative of a function at a given value and interpreting the answer.</td>
<td>60</td>
<td>72</td>
<td>52</td>
<td>70</td>
</tr>
</tbody>
</table>
Some interesting results can also be noted in the percent mean scores of each question type for the skill, concept, and application portions for the unit 2 exam for all four groups. Among the skill oriented derivative questions, all groups scored lower on the question of “finding the slope of a function at a given point” than the other type question, “finding the derivative of a function at a given \( x \) value. Even though these two questions are exactly the same, students often do not connect the fact that the derivative of a function at a given \( x \) value gives the slope of the function at that \( x \) value. On the concept portion of the derivative topic, the question of “explaining the meaning of the derivative of a function at a given \( x \) value” was the hardest question for all groups. This finding was similar to a finding on the limit topic where explaining the meaning of the limit of a function as \( x \) approaches a number was the hardest question for all groups. On the application portion of the derivative, the question of “finding the derivative of a function at a given value and interpreting the answer” was the hardest question for all groups. Here also, students failed to connect the fact that the derivative of a function at a given value in an application gives the rate of change of the function at that given value. Further, students again dislike the “interpret your answer” part of the question.

As this study compared the achievements in solving skill, concept, and application portions on the limit (unit 1 exam) and derivative (unit 2 exam) topics, the researcher also looked at Pearson product-moment correlations of the following. The correlations between the pretest and each portion of the unit 1 exam and between the portions of the unit 1 exam are reported in Table 26. The correlations between the pretest and each portion of the unit 2 exam and between the portions of unit 2 exam are reported in Table 27.
Table 26

Correlations Between the Pretest and Each Portion of the Unit 1 Exam on Limits and Between the Portions

<table>
<thead>
<tr>
<th></th>
<th>Skill</th>
<th>Concept</th>
<th>Application</th>
<th>Pretest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concept</td>
<td>.38*</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Application</td>
<td>.44*</td>
<td>.48*</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>.08</td>
<td>.11</td>
<td>.09</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: Total Number of Students Used in Correlation Computations was 82.
*Correlation is significant at the .01 level.

Table 27

Correlations Between the Pretest and Each Portion of the Unit 2 Exam on Derivatives and Between the Portions

<table>
<thead>
<tr>
<th></th>
<th>Skill</th>
<th>Concept</th>
<th>Application</th>
<th>Pretest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concept</td>
<td>.28*</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Application</td>
<td>.48*</td>
<td>.10</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Pretest</td>
<td>.01</td>
<td>.19</td>
<td>.30*</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: Total Number of Students Used in Correlation Computations was 76.
*Correlation is significant at the .01 level.

Tables 26 and 27 indicated that the correlations between the pretest and each portion of both unit 1 and 2 exams were very low except the correlation between the application portion of the unit 2 exam and pretest (the correlation was .30, see Table 27) and that was significant. The pretest was administrated on the first day of class and
students typically did not expect to take an exam on the first day of class. Even good
students can do poorly on the pretest if they had a longer gap between mathematics
courses. It was noted in the initial student survey that 14% of students took the
prerequisite course 2 years ago, another 14% took it more than 2 years ago, 6 students
(about 5%) took it more than 5 years ago, and 2 students (about 2%) took the course 10
years ago. It was quite possible that these students did not remember anything from their
last math course. However, it was noted by the researcher that about 5% who scored low
on the pretest made perfect scores on the unit 1 and 2 exams. One other possible
explanation for the low correlations was that, because the pretest score was not a part of
students’ final grade, some students did not do their best on such a test.

Summary

The analysis of the data does show some positive effects of using the GCs with
the proposed numerical approach in the study to understand better concepts and
application problems in both limit and derivative topics. Statistically significant
differences between the control and treatment groups were found on the concept and
application portions of the limit topic (unit 1 exam) and on the skill, concept and
application portions of the derivative topic (unit 2 exam), but not on the skill portion of
the limit topic. Also, there were no statistically-significant differences found on the
interaction effect between the instruction and instructor on any portions of the limit and
derivative topics.

It is, however, important to note from the data that the treatment groups of both
instructors outperformed their control groups in each portion of both limit and derivative
topics. Thus, apparently, one can at least conclude that the used numerical approach along with the use of GCs and instructional materials in this study helped the students learn the limit and derivative topics better. The next chapter (Chapter 5) discusses the results of each of the stated hypotheses along with the implications and limitations of this study.
Chapter 5

Discussion

This study examined the effects of using graphing calculators (GC) with a numerical approach and the researcher-developed instructional materials on limits and derivatives in an Applied calculus course at a community college. The general research question that this study sought to answer is “To what degree can the use of GCs with a numerical approach and instructional materials developed by the researcher affect community college Applied Calculus students’ learning of limits and derivatives?” In particular, the study sought to answer the following research questions:

1. How does the students’ achievement in solving limit problems with a numerical approach compare to that of students who solved limit problems with a traditional approach (primarily an algebraic approach) in an Applied Calculus course?

2. How does the students’ achievement in solving derivative problems with a numerical approach compare to that of students who solved derivative problems with a traditional approach (primarily an algebraic approach) in an Applied Calculus course?

When limit and derivative topics were taught with two instructional approaches (numerical versus algebraic), students’ achievement was measured on those topics in three different areas: Skills, Concepts, and Applications. This study was interested in determining the treatment and control groups’ performance on these areas due to the
treatment effect. In the preceding chapter the details of the procedure of the study were described and the results of the statistical tests for the collected data were presented in order to answer the stated research questions.

On the first day of class, all students took the pretest and an ANOVA test on the pretest confirmed that there was no statistically significant difference on prior mathematical ability (pretest) between the control and treatment groups. The first treatment phase began with the study of the limit topic. The students in the treatment group solved the limit problems (skill, concept, and application types) with the use of GCs and a numerical approach and the students in the control group solved the limit problems in a traditional manner (primarily an algebraic approach). Both groups used GCs but the treatment groups were provided with the researcher-developed instructional materials (the groups used unit lesson 3 on limits). Immediately after the end of the first treatment phase, all students took a test on the limit topic (unit 1 exam) and the problems were graded separately as skill, concept, and application portions.

**Hypotheses Results of the First Research Question**

The following research hypotheses (Numbers 1, 2, and 3) were used to answer the first research question on the limit topic.

1. Students (GCGL group) who receive instruction with a numerical approach will have higher achievement in routine (skill oriented) limit problems than students (GC group) who receive instruction with a traditional approach (primarily an algebraic approach).
2. The students in the GCGL group will have higher achievement in conceptual oriented limit problems than the students in the GC group.

3. The students in the GCGL group will have higher achievement in related applications of limits than the students in the GC group.

Three separate ANCOVA tests on the skill, concept, and application portions of the limit topic with the pretest as covariate were conducted and the results are briefly given below:

- There was no statistically significant difference found on the skill portion of the limit topic (unit 1 exam) due to instruction or to instructor.
- There was a statistically significant difference found on the concept portion of the limit topic (unit 1 exam) due to instruction and to instructor.
- There was a statistically significant difference found on the application portion of the limit topic (unit 1 exam) due to instruction but not due to instructor.
- The interaction effects between instructor and instruction were not statistically significant on the skill, concept, and application portions of the limit topic (unit 1 exam).

Even though there was no statistically significant difference found between the treatment and control groups on the skill portion of the limit topic, the study supported that students (treatment group) who solved the skill oriented limit problems with the numerical method were better able to solve the concept and application problems in the
limit topic than the students (control group) who solved the skill oriented limit problems algebraically. That is, the numerical approach in the limit topic helps students to do better in the concepts and applications of limit problems. Therefore, this finding supports the second and third research hypotheses but did not support the first research hypothesis.

Hypotheses Results of the Second Research Question

As mentioned earlier, immediately after the first unit 1 exam on limits, the second treatment phase began with the study of the derivative topic. Again, the students in the treatment group solved the derivative problems (skill, concept, and application types) with the use of GCs and a numerical approach and the students in the control group solved the derivative problems in a traditional manner (primarily algebraic approach). Again, both groups used GCs but the treatment groups were provided with the researcher-developed instructional materials (the groups used the unit lesson 4 on derivatives). At the end of the second treatment phase, all students took a test on the derivative topic (unit 2 exam) and the test problems were graded again separately as skill, concept, and application portions.

The following three research hypotheses (Numbers 4, 5, and 6) were used to answer the second research question on the derivative topic.

4. The students in the GCGL group will have higher achievement in routine (skill oriented) derivative problems than the students in the GC group.

5. The students in the GCGL group will have higher achievement in conceptual oriented derivative problems than the students in the GC group.
6. The students in the GCGL group will have higher achievement in related applications of derivative problems than the students in the GC group.

Three separate ANCOVA tests on the skill, concept, and application portions of the derivative topic with the pretest as covariate were conducted and the results are briefly given below:

- There was a statistically significant difference found on the skill portion of the derivative topic (unit 2 exam) due to *instruction* but not due to *instructor*.
- There was a statistically significant difference found on the concept portion of the derivative topic (unit 2 exam) due to *instruction* and to *instructor*.
- There was a statistically significant difference found on the application portion of the derivative topic (unit 2 exam) due to *instruction* but not due to *instructor*.
- The interaction effects between instructor and instruction were not statistically significant on the skill, concept, and application portions of the derivative topic (unit 2 exam).

The ANCOVA tests supported the fact that the students (treatment group) who used the numerical method were better able to solve not only the skill oriented derivative problems but also the concept and application problems on derivatives than the students (control group) who solved the derivative problems algebraically. Therefore, this finding supports the fourth, fifth, and sixth research hypotheses.
One might wonder why the treatment group students did better than the control group students on the skill portion of the derivative topic but not on the skill portion of the limit topic. One explanation was that treatment group students first solved the skill oriented limit problems with the numerical method and then solved the skill oriented derivative problems; these derivative problems were presented as limit problems. So the experience the treatment students gained in the early part of the study (limit topic) may have helped them to see a direct relation to the second part (derivative topic); therefore, they may have done better than their control group counterparts. Also, at the beginning of the treatment period, students were slow in entering the expressions and setting-up the correct table in their GCs in order to find the required limits. Students in the control group solved the skill portion of the limit and derivative problems algebraically but the types of algebraic work they had to do in both sections were different. That is, the algebraic experience the control students gained from the first topic (limit) may not have helped in the second topic (derivative).

As noted earlier, students in all groups were able to score higher on the skill portion than the concept and application portions in the limit and derivative topics. It has been mentioned by many in the literature that students are able to solve routine (skill) limit and derivative problems whether they understand the concept of limit and derivative or not (Ferrini-Mundy & Graham, 1991; Tall et al., 2004). The finding of this study also confirmed those claims.
Limitations of the Study

Several limitations of the research method of this study must be noted when interpreting conclusions drawn from the results of this study.

Group Selection: Full random assignment of students to the study groups was not possible. All available four day-time sections of Applied Calculus during spring 2005 at the college were chosen to participate in this study. Two other sections of this course offered in the evening during spring 2005 at the same college were not chosen because of the student differences in the population. Two of the four day-time sections were offered on TR at 8 a.m. and 12:30 p.m.; the other two were offered on MWF at 8 a.m. and 11:00 a.m. Due to students’ work schedules and their schedules of other classes, morning sections (8 a.m.) have smaller enrollments than later time sections (11 a.m. and 12:30 p.m.). As a result of a flip of a coin, TR at 12:30 (taught by Instructor A) was assigned as a treatment group; therefore, MWF at 11:00 a.m. (taught by Instructor B) was assigned as the other treatment group. That is, the two larger (a total of 58 students) and later time sections (11 a.m. and 12:30 p.m.) ended up serving the treatment groups and the two smaller (a total of 32 students) and earlier time sections (8:00 a.m.) ended up serving the control groups.

Sample Size: A total of 93 students (17 students were in Control A; 18 students were in Control B; 30 students were in Treatment A; and 28 students were in Treatment B) enrolled for the course and took the pretest administrated on the first day of class. The first week is also the drop-add week for the college. Students who were misplaced or did not meet the prerequisite needed to drop the class. When the treatment phase began on the second day of class, 6 students had dropped out (Control A class lost one student,
Control B lost 2 students, Treatment A class lost 2 students and Treatment B class lost 1 student). Further, students’ attendance was monitored; if a student missed 50% or more of the classes during the treatment period, his or her test score was not included for the statistical analysis. A total of 87 students took the unit 1 exam; because of absences, only 82 students had test scores considered for the unit 1 exam; 16 students in Control A, 15 students in Control B, 25 students in Treatment A, and 26 students in Treatment B. Therefore the ANOVA test on pretest and three ANCOVA tests on unit 1 exam were conducted with n = 82. After the second treatment phase, a total of 79 students took the unit 2 exam and 3 students’ scores were not included because they missed 50% or more of the classes; therefore, only 76 students had test scores considered for the unit 2 exam: 14 students in Control A, 14 students in Control B, 23 students in Treatment A, and 25 students in Treatment B. Therefore, the three ANCOVA tests on unit 2 exam were conducted with n = 76.

Facilitators: Two instructors A and B participated and both were white and male. As mentioned earlier, both have some similarities but they certainly have differences such as teaching style, the belief about using technology (GC), and the amount of experience they have in using GC technology. The participating two instructors in this study were volunteers. Applying an instructional method by others under volunteer conditions might be subject to bias in terms of motivation, belief, and responsibility. However, the participating instructors showed positive attitudes and worked with the researcher to promote the goals of this study. Also, it is a policy in the department that the names of the instructors are published in the semester schedules so students know who is teaching what section. One other factor was that one of the instructors teaches the
prerequisite course (MAC 1105) for the Applied Calculus course and the other one never teaches MAC 1105. If students took and passed MAC 1105 with a particular instructor then they tend to stay with the same instructor for the next mathematics course if they have a choice. It was not clear whether students who had the instructor previously would do better than the students who did not.

Participants: As mentioned earlier, the students knew who their instructor would be. But they were not told whether they were in the treatment or the control group. There was no statistical difference between the groups on the pretest scores. From the students’ initial survey, it was noted that students were different in terms of their mathematical experiences and background and the familiarity with their calculator ownership. About 92% of the students took MAC 1105 as the prerequisite course with about 27% of them completing the course in the previous semester, 37% in the last year, 14% 2 years ago, and the other 14% more than 2 years ago. About 3% are permitted into this class by their placement scores and about 5% had Precalculus (MAC 1140) or higher. About 86% of students had used a GC in their previous math course and about 99% used the TI 83 model (the model that was used in the study). Further, it was not clear how many students had repeated this course.

Measurement Error: The dependent variables were the achievement scores of the skill, concept, and application portions of the unit 1 and 2 exams that were measured using paper and pencil instruments. Although the scores from these measures were moderately reliable, they were limited by the testing conditions (e.g., the time and the day of the testing).
Instrument Items: Although the content validity of the instruments’ items was measured, the items selected for skill, concept, and application portions for the limit and derivative topics were subjective. Further, there are different types of problems that may be chosen for the same category of skill, concept, and application. Therefore, just because a student answered a type of problem correctly in a category does not necessarily mean the student can answer another type of problem in the same category. It is also possible that solving an application problem in a topic is often associated with the concept of that topic. That is, a question or a part of a question in the application portion could be a question for the concept portion. Also, the number of questions for each portion was limited because of the available testing time.

Selection of the Course: The study was carried out in an Applied Calculus course (MAC 2233) and the topics under investigation were limits and derivatives at a community college. The population of a community college reflects the surrounding community, and therefore, the results of this study might not be generalized to all students taking Applied Calculus. Further, the limit and derivative topics are taught in other calculus courses (Calculus I, MAC 2311) too. The difference is that if students take MAC 2233, then that is the only calculus course they have to take; if students take MAC 2311 then most probably students have to take additional calculus courses (Calculus II and Calculus III). So the populations of MAC 2233 and MAC 2311 are different; therefore, the results of this study may not be generalized to any calculus course.

Students’ Preparation before the Exams: Even though students in the treatment groups and control groups received different treatments, the interaction between students and the type of help students received from the available math lab and private tutoring
cannot be controlled. Students were required to fill out two student questionnaires (Appendix H) right after each unit exam to learn about how they prepared for these exams. The majority of them (about 80%) studied by themselves and the other students (about 15%) studied either with students from the same class or the other classes. There are a few students (about 5%) who received help from private tutors before they took the exam. These situations might have influenced some students’ performance.

**ANCOVA Assumptions:** The assumptions of ANCOVA tests were discussed earlier but any violations of these assumptions might be a threat to the finding of this study.

**Implications for Practice**

It was repeatedly mentioned in the literature that students are able to do better on skill oriented problems than the concept and application problems on the limit and derivative topics whether they understand the topic or not (Ferrini-Mundy & Graham, 1991; Tall et al., 2004). In this study also, all groups except one group (Treatment B) on the limit topic and one group (Control A) on the derivative topic scored better on the skill portion than the concept and application portions of the limit and derivative topics. However, the results are encouraging in that both treatment groups did better than their control groups on the concept and application portions of the limit and derivative topics and the skill portion of the derivative topic when the proposed numerical approach with the use of GCs was presented on these topics.

As reported earlier, about 86% of the students had used a GC in their previous mathematics course but it was not clear at what level the calculator was used. Because of
its capabilities, the GC can be used as a regular four-function calculator to do just basic operations. This was suspected because when students were asked on the initial survey about their comfort level of using a GC, about 25% felt very comfortable, another 25% felt moderately comfortable, and 50% felt not comfortable about using their calculator even though they had previously used it for at least one semester. It seemed like some students used a GC just as a regular four-function calculator, not as a GC. Therefore, when a GC is required for a course instead of a regular calculator, special features of that GC must be utilized by both instructors and students. Otherwise, a part of the available time in the next course has to be spent on learning how to use a GC.

Also, when students are taught certain topics or certain types of problems by a specific method (like the numerical approach), the approach must be consistent throughout their mathematics learning whenever possible. For example, finding the rate of change of functions is a type of application Applied Calculus students need to solve throughout the semester at different times as they learn different functions. If students have to solve a rate of change problem algebraically, they need to apply different types of algebraic techniques, manipulations, and/or formulas to solve the problem based on what type of function is given in that problem. If students learned how to solve a rate of change problem on a function by a numerical approach with the help of a GC, students will be ready to solve a rate of change problem on any type of function because the procedures and the amount of work needed to do the problem are the same.

Instructors’ beliefs, knowledge, and willingness are important factors to implement new teaching methods successfully in classrooms. Cross (1990) noted that classroom research should be planned and used with teacher involvement to try better
teaching methods. This would be successful if instructors are provided the needed training for the technology that is intended to be used for a course. They also need to be provided with detailed instructional materials that focus different ways of teaching various mathematics concepts. Also, instructors should be aware not only of what the GC is capable of doing but also the things a GC can’t do, its limitations, and misleading behaviors along with other pitfalls.

Students’ beliefs, knowledge, and willingness are equally important factors to implement new teaching methods successfully in their learning. Schoenfeld (1985) recognized that a student’s belief system is an important factor in his/her ability to learn concepts. Also, there is a perception in students’ minds that if their class is under a “treatment”, they “are being used”; they do not necessarily consider the treatment as a way of trying to improve their understanding. Students’ attitudes about the new method are not known nor any changes in attitudes towards mathematics learning. For many students (about 87%) the Applied Calculus course is the last mathematics course they must pass to graduate; therefore, they do not necessarily think that this new approach will help them much in the long run.

**Conclusions and Recommendations for Future Research**

As stated earlier, the purpose of this study was to identify a method with the available graphing technology that helps improve students’ conceptual understanding in the limit and derivative topics. Otherwise, these topics are hard for many students (Ferrini-Mundy & Graham, 1991; Tall et al., 2004). Several studies and projects reported positive results about using appropriate pieces of technology along with the instruction to

The experience with the use of GC technology in mathematics courses and other studies showing positive results in this area motivated the researcher to try a numerical approach with the GC technology in limit and derivative topics in Applied Calculus at a community college. The researcher proposed a numerical method with the use of GC that can be used to solve limit and derivative topics. This method with various examples was given as a set of supplemental handouts to all treatment students (unit lesson 3 and 4).

But the researcher realized that an important initial and basic step needed is knowing how to operate the piece of technology comfortably. The researcher constantly observed students’ lack of confidence and familiarity, errors, and misconceptions when using a GC in this course. For example, students had trouble noticing the difference in syntax, even in simple cases like $3/2^8$ and $3/(2^8)$ or the difference between $x/x+3$ and $x/(x+3)$.

Understanding these concerns and in order to help students to use a GC correctly and confidently, the researcher prepared two unit lessons just to help students learn to use the GC effectively. These two unit lessons (unit lesson 1 and 2) were given as a set of supplemental handouts to all students. Then the other two unit lessons (unit lesson 3 on limits and unit lesson 4 on derivatives) were used in the treatment groups. The use of these unit lessons and numerical methods in the treatment groups in this study yielded some positive results in learning limit and derivative topics in calculus.

With today’s technology sophistication, a friendly usable and affordable graphing technology like the TI 83 has a lot to offer students and instructors to promote students’
conceptual understanding in difficult topics such as the limit and derivative. Only a fraction of its capability is being used by instructors and students, if it is used at all. It is the mathematics community’s responsibility to promote new ways with available technology that would help students’ understanding in mathematical topics and to conduct research studies on those new methods to validate its usefulness.

This study mainly focused on one type of application (rate of change) with certain types of functions (Algebraic Functions) in the derivative topic with the proposed numerical method along with the use of a GC. Further research studies are needed to inquire about this numerical approach on the same type of application (rate of change) but with other types of functions (e.g., Transcendental Functions). Also, it would be worthwhile for studies to investigate the effects of this method on other types of applications in derivative topics (finding the maximum and minimum, etc).

Also, research studies are needed to inquire into the effect of this approach on the integration topic (this topic also develops from the idea of limit concept) that calculus students study immediately after they study the derivative topic.

This study was a quantitative study. The conclusions of this study were made based on the data collected from the test scores. A portion of a qualitative study (for example, student interviews) would have given a better picture of students’ understanding in the topics studied. Therefore, future researchers in this area need to consider building a qualitative study portion when they design the intended studies.
References


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Appendices
Appendix A: Student Initial Survey

1. What is your major? If undecided, please state so.

2. Did you take the course MAC 1105 College Algebra? Yes: _____ No: _____
   If not, what course did you take as a prerequisite for the applied calculus course?

3. When did you take this course?
   Last semester: ________ Last year: __________ Other (specify): __________

4. Do you plan to take any other mathematics course after this course? If yes, please indicate which course.

5. Do you plan to transfer to a university? Yes: _____ No: _____ Maybe: ______

6. Before this course, have you ever used a graphing calculator in a mathematics course? Yes: _____ No: _____
   If the answer is “Yes”, go to question #7; if the answer is “No”, skip question #7 and go to question #8.

7. a) For which mathematics courses have you used a graphing calculator?

   __________________________________________________________________________
   
   b) Specify the type of the graphing calculator:

   __________________________________________________________________________

   c) Was it required for the course? Yes: _____ No: _____
Appendix A: (Continued)

d) How often did you use it in this class?
   A. Daily         B. 2 to 3 times a week       C. 2 to 3 times a month
   D. once a month  E. Never

e) How useful was it in your mathematics course?
   A. Very useful   B. Useful                C. Moderately useful  D. Not useful at all

8. What type of graphing calculator will you use in this Applied Calculus course?

   ____________________________________________

9. What are the reason(s) for choosing that type of graphing calculator?
   Circle all that apply.
   A. Cost                     B. Recommended to you            C. Given to you by somebody
   D. Capabilities of the calculator     E. Other reason (specify):

10. How comfortable, in terms of proficiency, are you using your graphing calculator?
    A. Very comfortable         B. Comfortable
        C. Moderately Comfortable D. Not comfortable

11. How helpful do you expect the graphing calculator to be to your understanding and
    learning of calculus in this course?
    A. Very helpful         B. Somewhat helpful         C. Not helpful       D. Not sure

12. What features (if any) do you like the best about your graphing calculator?
    ____________________________________________

13. Is there anything you do not like about your graphing calculator?
    ____________________________________________

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Appendix B: Pretest

MAC 2233 Pretest Name:
Spring 2005

Show all work. Section: MWF or TR

1. Given \( g(x) = -x^2 + 6x \), find the following:
   a) \( g(4) \)  
   b) \( g(-3) \)

2. Given \( f(x) = 3x - 10 \), find the following:
   a) \( f(a) \)  
   b) \( f(a + h) \)

3. Given \( f(x) = \begin{cases} x^2 - 3x + 8 & x < 4 \\ -4x + 1, & x \geq 4 \end{cases} \), find a) \( f(2) \)  
   b) \( f(4) \)

4. Find the slope of the line given by the following equation: \( 3x - 4y = 8 \).
Appendix B: (Continued)

5. Solve the following equation: \( 2x^2 - 3x = 20 \)

6. Find the domain of the following function: \( f(x) = \sqrt{2x - 3} \)

7. Sketch a line that has a positive slope.

8. Find the slope and the \( y \) intercept of \( 5x - 3y = 8 \).

9. Factor the following polynomial: \( 2t^3 + t^2 - 15t \)
10. Find the domain of \( f(x) = \frac{x - 2}{x^2 + x} \)

11. Find the slope of the line that passes through the points (4,-3) and (6,8).

12. Consider the following graph for a function \( f(x) \) and the following:

   a) \( f(2) \)

   b) the \( x \) value such that \( f(x) = 0 \)

13. A population of town is given by \( P(x) = 52340 + 28x^{3/2} + 15x \), where \( P(x) \) denotes the population \( x \) months from now.

   a) Find \( P(0) \) and interpret your answer

   b) Find \( P(9) \) and interpret your answer
Appendix C: Unit Exams

MAC 2233   Unit 1 Exam: Limits   Name:
Spring 2005   Section: MWF or TR

Show all work.

Find the limit of each of the following functions, if it exists, by any method. If the limit doesn’t exist, then state so and explain why the limit doesn’t exist. If the answer seems to be $\infty$ or $-\infty$, state so. Show your work!

1. $f(x) = \frac{2x^2 - 7x - 4}{x - 4}; \quad \lim_{x \to 4} f(x) =$

2. $f(x) = \frac{\sqrt{x} - 5}{x - 25}; \quad \lim_{x \to 25} f(x) =$

3. $f(x) = \frac{|x - 2|}{x - 2};$
   a) $\lim_{x \to 2^-} f(x) =$
   b) $\lim_{x \to 2^+} f(x) =$
   c) $\lim_{x \to 2} f(x) =$

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Appendix C: (Continued)

4. \( f(x) = \frac{2x + 7}{x - 3}; \quad \lim_{x \to 3^-} f(x) = \)

5. \( f(x) = \begin{cases} 
-2x + 1, & x < 0 \\
0, & x \geq 0 
\end{cases} \)

a) \( \lim_{x \to 0^-} f(x) = \)

b) \( \lim_{x \to 0^+} f(x) = \)

c) \( \lim_{x \to 0} f(x) = \)

6. \( f(x) = \frac{10 - 3x - 4x^2}{7 + 3x^2}; \quad \lim_{x \to -\infty} f(x) = \)

7. Suppose that for a function \( f(x) \), it is given that \( \lim_{x \to 5} f(x) = 3 \). Explain what this means.
Appendix C: (Continued)

8. Consider the graph of $f(x)$ and find the following. If the answer seems to be $\infty$ to $-\infty$, state so.
   a) $f(-2)$
   b) $\lim_{x \to -2} f(x)$
   c) $\lim_{x \to 2} f(x)$
   d) $\lim_{x \to -2} f(x)$

9. Consider the graph of $g(x)$ and find the following. If the answer seems to be $\infty$ to $-\infty$, state so.
   a) $g(2)$
   b) $\lim_{x \to 2} g(x)$
   c) $\lim_{x \to 2} g(x)$
   d) $\lim_{x \to 2} g(x)$

10. Consider the graph of $f(x)$ and find the following. If the answer seems to be $\infty$ to $-\infty$, state so.
    a) $f(2)$
    b) $\lim_{x \to 2} f(x)$
    c) $\lim_{x \to 2} f(x)$
    d) $\lim_{x \to 2} f(x)$
Appendix C: (Continued)

11. Consider the following table. The function values of \( f(x) \) are computed for different \( x \) values. Is it reasonable to estimate the limit, \( \lim_{x \to 4} f(x) \)? If yes, give the answer for the limit and if not possible, tell why that is not possible.

\[
\lim_{x \to 4} f(x) =
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>3.9</th>
<th>3.99</th>
<th>3.999</th>
<th>4</th>
<th>4.001</th>
<th>4.01</th>
<th>4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>46.81</td>
<td>47.88</td>
<td>47.988</td>
<td>Error</td>
<td>48.012</td>
<td>48.12</td>
<td>49.21</td>
</tr>
</tbody>
</table>

12. The cost \( C(x) \), in thousands of dollars, of removing \( x\% \) of a city’s pollutants discharged into a river is given by \( C(x) = \frac{93x}{100 - x} \).

   a. Find \( C(25) \) and interpret your answer.

   b. Can you find \( C(100) \)? Why or why not?

   c. Find \( \lim_{x \to 100} C(x) \) and interpret your answer.

13. The local game commission decided to stock a lake with trout. To do this, 200 trout were introduced into the lake. The population of the trout can be approximated by

\[
P(t) = \frac{20(10 + 7t)}{1 + 0.02t}, \quad t \geq 0, \text{ where } t \text{ is time in months since the lake was stocked.}
\]

Find \( \lim_{x \to \infty} P(x) \) and interpret your answer.
1. \( f(x) = 2x^2 - 10x + 25 \), find \( f'(3) \).

2. Given \( f(x) = \frac{3}{x} \), find the slope of \( f(x) \) at the point \( (2, \frac{3}{2}) \).
Appendix C: (Continued)

3. Let \( f(x) = x^2 - 4x \).

   a) Find the average rate of change of \( f(x) \) with respect to \( x \) in the interval from \( x = 5 \) to \( x = 5.5 \) and \( x = 5 \) to \( x = 5.1 \).

b) Find the (instantaneous) rate of change of \( f(x) \) at \( x = 5 \).

4. Consider the graph of \( f(x) \). The tangent line to the graph of \( f(x) \) at (-1, 2) is drawn and the equation of the tangent line is given as \( y = -2x \). What is the value of \( f'(-1) \)? Explain your answer.
Appendix C: (Continued)

Consider the points A, B, C, D, E, and F on the graph of the function $f(x)$ and answer the questions (5) – (8).

5. At what point(s) the derivative of $f(x)$ is positive?

6. At what point(s) the derivative of $f(x)$ is negative?

7. At what point(s) the derivative of $f(x)$ is zero?

8. At what point the value of the derivative of $f(x)$ is the greatest?
Appendix C: (Continued)

9. Under a set of controlled laboratory conditions, the size of the population, \( P(t) \), of a certain bacteria culture at time \( t \) (in minutes) is described by the function \( P(t) = 3t^2 + 2t + 1 \).

   a. Find the average rate of the population of the bacteria between \( t = 5 \) minutes and \( t = 10 \) minutes; \( t = 5 \) minutes and \( t = 7 \) minutes

   b. Find the (instantaneous) rate of change of the population of the bacteria at \( t = 5 \) minutes.

10. The quarterly profit (in thousands of dollars) of Cunningham Realty is given by
    \( P(x) = -x^2 + 7x + 30, \quad (0 \leq x \leq 50) \)
    where \( x \) (in thousands of dollars) is the amount of money Cunningham spends on advertising per quarter.

    a. Find \( P(10) \) and interpret the result.

    b. Find \( P'(10) \) and interpret the result.
Appendix D: Class Observation Protocol

The researcher visited all experimental groups and control groups once a week. The purpose of the visit was to make sure the groups are following the lessons as planned in the study. All groups used GCs throughout the semester. The experimental groups used TI 83 GCs and the researcher-developed instructional guided lessons to learn limit and derivative problems with numerical approach. The control groups solved the problems in those topics in a traditional approach (algebraic approach). Instructors A and B and all students in the treatment groups were given a copy of the entire four units as handouts.

The researcher visited and observed all sections to insure that the progress in the classes are in agreement with the course syllabus and the instructional methods are being utilized as planned in the main study. The researcher made class observations in the following manner:

For the Treatment Groups:

1. Observed that students in this group studied the review sections with the use of TI 83 GCs and the researcher-developed unit lessons #1 and #2. The unit lessons have examples that helped students to use their GCs effectively and that reinforced the function concepts.

2. Observed that the students in this group learned the limit problems with the table feature of a TI 83 GC and the researcher-developed unit lesson #3. The unit lesson has numerous examples to make students understand the nature of the limit problems, how to solve those problems numerically with the table feature of a TI 83 GC, and interpret the answers to those problems. After that students learned to solve related applications of
Appendix D: (Continued)

limits. The researcher observed that the instruction was carried out as planned.

3. Observed that students in this group learned the derivative problems as limit problems and solved them with the table feature of a TI 83 GC (the same way they have solved the limit problems previously) and the researcher-developed unit lesson #4. The unit lesson has numerous examples to make students understand the meaning of the derivative problems, how to solve those problems numerically with the table feature of the GC, and interpret the answers to those problems. After that students learned to solve related applications of derivatives with the help of the unit lesson and a TI 83 GC. The researcher observed that the instruction was carried out as planned.

For the Control Groups:

1. Observed that students in this group studied the review sections (Lessons 1 and 2) with the use of TI 83 GCs only. It was noted that the instruction was done as planned without any treatment effect influences.

2. Observed that students in this group studied the limit problems the way that was done in the textbook with the use of TI 83 GCs only. It was noted that the instruction was done as planned without any treatment effect influences. No unexpected events noted by the researcher.

3. Observed that students in this group studied the derivative problems the way that was done in the textbook with the use of TI 83 only. It was noted
Appendix D: (Continued)

that the instruction was done as planned without any treatment effect influences.

The instructor observed in all sections that the instruction for the particular session was done as planned in terms of time management, amount of material covered for the session, and the type of instruction delivered to the students as planned in the main study. The researcher took notes of each session visited.
Appendix E: Course Syllabus

Tentative Academic Calendar Spring 2005 - MAC 2233 APPLIED CALCULUS

<table>
<thead>
<tr>
<th>Week</th>
<th>Sections</th>
<th>Topics Covered</th>
<th>Suggested Homework Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1</td>
<td>Functions and Their Graphs</td>
<td>2.1-p.58:3-33(eoo),37-47(o),51,53,55,62</td>
</tr>
<tr>
<td></td>
<td>2.2</td>
<td>The Algebra of Functions</td>
<td>2.2-p.75: 1-41(eoo),47,51,55,57,67-70</td>
</tr>
<tr>
<td></td>
<td>2.3</td>
<td>Functions &amp; Mathematical Models</td>
<td>2.3-p.87:1-17(eoo),19,36,39,45,55,65</td>
</tr>
<tr>
<td>2</td>
<td>2.4</td>
<td>Limits</td>
<td>2.4-p.111:1-77(eoo),59,79</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>One-Sided Limits</td>
<td>2.5-p.127:1-19(o),21-107(eoo)</td>
</tr>
<tr>
<td></td>
<td>2.6</td>
<td>The Derivative</td>
<td>2.6-p.148:1-57(eoo)</td>
</tr>
<tr>
<td>3</td>
<td>Review</td>
<td>Exam 1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.1</td>
<td>Basic Rules of Differentiation</td>
<td>3.1-p.168: 1-69(eoo)</td>
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<tr>
<td></td>
<td>3.2</td>
<td>The Product and Quotient Rules</td>
<td>3.2-p.182: 1-61(eoo)</td>
</tr>
<tr>
<td>5</td>
<td>3.3</td>
<td>The Chain Rule</td>
<td>3.3-p.195:1-85(eoo)</td>
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<tr>
<td></td>
<td>3.4</td>
<td>Marginal Functions in Economics</td>
<td>3.4-p.212:1,5,9,13,23,27,31,33</td>
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<tr>
<td></td>
<td>3.5</td>
<td>Higher-Order Derivatives</td>
<td>3.5-p.219:1-45(eoo)</td>
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<tr>
<td>6</td>
<td>3.6</td>
<td>Implicit Differentiation &amp; Related Rates</td>
<td>3.6-p.231:3-51(eoo),53,55,57,59</td>
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<tr>
<td></td>
<td>3.7</td>
<td>Differentials</td>
<td>3.7-p.241:1-45(eoo)</td>
</tr>
<tr>
<td>7</td>
<td>Review</td>
<td>Exam 2</td>
<td></td>
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<tr>
<td>8</td>
<td>4.1</td>
<td>Applications of the 1st Derivative</td>
<td>4.1-p.262:9,19-57(eoo),63,69,73,77,81</td>
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<tr>
<td></td>
<td>4.2</td>
<td>Applications of the 2nd Derivative</td>
<td>4.2-p.280:1-41(eoo),49,61,63,65,69,75,81</td>
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<tr>
<td></td>
<td>4.3</td>
<td>Curve Sketching</td>
<td>4.3-p.296:1-65(eoo)</td>
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<tr>
<td>9</td>
<td>4.4, 4.5</td>
<td>Optimization Problems</td>
<td>4.4-p.311:1-69(eoo)</td>
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<td></td>
<td></td>
<td>4.5-p.325: 1-25(eoo)</td>
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<tr>
<td>10</td>
<td>Exam 3</td>
<td>5.1, 5.2</td>
<td>Exponential and logarithmic Functions</td>
</tr>
<tr>
<td>11</td>
<td>5.3</td>
<td>Compound Interest</td>
<td>5.3-p.359:1-37(eoo)</td>
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<td></td>
<td>5.4</td>
<td>Differentiation of Exponential Fns</td>
<td>5.4-p.368:1-73(eoo)</td>
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<td></td>
<td>5.5</td>
<td>Differentiation of Logarithmic Fns</td>
<td>5.5-p.379:5-65(eoo),64</td>
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<tr>
<td>12</td>
<td>5.6</td>
<td>Review Exam 4</td>
<td>Exponential Fns as Math Models</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>Spring Break</td>
<td></td>
</tr>
<tr>
<td></td>
<td>College Closed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>6.1</td>
<td>Antiderivatives and Integration</td>
<td>6.1-p.407:1-85(eoo)</td>
</tr>
<tr>
<td></td>
<td>6.2</td>
<td>Integration by Substitution</td>
<td>6.2-p.409:1-65(eoo)</td>
</tr>
<tr>
<td>15</td>
<td>6.3</td>
<td>Area and the Definite Integral</td>
<td>6.3-p.430:3,7,15</td>
</tr>
<tr>
<td></td>
<td>6.4</td>
<td>Fundamental Theorem of Calculus</td>
<td>6.4-p.439:1-49(eoo)</td>
</tr>
<tr>
<td></td>
<td>6.5</td>
<td>Evaluating Definite Integrals</td>
<td>6.5-p.449:1-49(eoo)</td>
</tr>
<tr>
<td>16</td>
<td>6.6</td>
<td>Area between Two Curves</td>
<td>6.6-p.461:1-51(eoo)</td>
</tr>
<tr>
<td></td>
<td>6.7</td>
<td>Applications of Definite Integral</td>
<td>6.7-p.478:1-17(eoo)</td>
</tr>
<tr>
<td>17</td>
<td>Review</td>
<td>Exam 5</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td>FINAL EXAMS</td>
<td></td>
</tr>
</tbody>
</table>

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MAC 2233                             APPLIED CALCULUS                                   Spring 2005

Instructor:        Math Office:  
Instructor’s Office:    Math Office  phone: 
Instructor’s Phone:    Math Lab: 
Instructor’s e-mail:    Math Lab Hrs 
Office hours:      Adjuncts available prior to or after class or specific times available by appointment

PREREQUISITES:  MAC 1105 with a grade of “C” or better or equivalent. Students already with credit for MAC 2311 cannot subsequently get credit for this course. Student Enrollment in any mathematics course is contingent upon approval of the mathematics department. This means that students who have been misplaced may have their schedule changed.

COURSE DESCRIPTION:  Topics in this course include limits, differentiation and integration of algebraic, exponential and logarithmic functions, integration techniques and related applications in the management, business, and social sciences. This course is not designed for science majors. Course performance standards are available at www.mccfl.edu/academ/math/math.htm and in the Math Labs.

TEXT:  Calculus for Managerial, Life and Social Sciences, by Tan, 6th edition

MATERIALS: A graphing calculator is required. It is allowed during exams. The TI-83 model is strongly recommended. Calculators with symbolic manipulation capabilities (e.g. TI-89, TI-92) will not be allowed for use during exams.

ADDITIONAL MATERIALS: Student Solutions manual is available in the bookstore.

EXAMINATIONS:  There will be 5 exams and a required comprehensive final examination. NO MAKE-UP EXAMS WILL BE GIVEN.

GRADING:  Your grade in the course is determined by the percentage of points earned during the semester. A grade of 60% or better must be earned on the final exam in order to pass the course.

<table>
<thead>
<tr>
<th>POINTS</th>
<th>SCALE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Exams 500</td>
<td>90 – 100% = A</td>
</tr>
<tr>
<td>Quizzes/Participation/Projects/Homework* 100-200</td>
<td>80 – 89% = B</td>
</tr>
<tr>
<td>Final Exam (cumulative) 200</td>
<td>70 – 79% = C</td>
</tr>
<tr>
<td></td>
<td>60 – 69% = D</td>
</tr>
<tr>
<td></td>
<td>0 -- 59% = F</td>
</tr>
</tbody>
</table>

*Instructor will choose composition of these points.

GORDON RULE:  This course meets the Florida State Board of Education Rule Number 6A-10.30. For the purpose of this rule, a grade of “C” or better shall be considered successful completion.

ATTENDANCE:  All late arrivals, early departures and absences must be discussed and cleared with the instructor. More than 3 hours of unexcused absences or excessive tardiness may result in your withdrawal from the course.

WITHDRAWAL:  March 16, 2005 is the last day to withdraw with a “W”. If you remain in the course after that date, you must receive a grade. Withdrawal after the midpoint may be granted only by the Associate Dean of Instruction and shall be based on major extenuating circumstances. As of Fall 1997, a student is allowed a maximum of three attempts per course. On the third attempt the student cannot be withdrawn and must receive a grade. More information regarding this policy is available in the current Manatee Community College Catalog or the class schedule.
Appendix F: Researcher-Developed Instructional Lessons:  
Unit Lessons of TI 83 Graphing Calculator Usage in an  
Applied Calculus Course

Unit Lesson 1: A Guide that Helps to Use a TI 83 Graphing Calculator

A. Entering Arithmetic and Algebraic Expressions in a Graphing Calculator  
B. Simplifying Arithmetic Expressions by Using a Graphing Calculator  
C. Graphing Functions (Equations) with a Graphing Calculator  
D. Student Activities for Lesson 1

Unit Lesson 2: Function Values and the Table Feature of a  
TI 83 Graphing Calculator

A. Finding Function Values by Using a Graphing Calculator  
B. Student Activities for Lesson 2

Unit Lesson 3: Limits of Functions

A. Finding the Limit of a Function as \( x \) Approaches a Number  
B. Finding the Limit of a Function as \( x \) Approaches a Number from the Right and Left  
C. Finding the Limit of a Function as \( x \) Approaches ∞ and -∞  
D. Student Activities for Lesson 3

Unit Lesson 4: Derivatives of Functions

A. Finding the Derivative of a Function at a Given Value  
B. Application: Finding the Rate of Change of a Function  
C. Student Activities for Lesson 4

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Lesson 1: A Guide that Helps to Use a TI 83 Graphing Calculator

Instructor Notes:

Introduction

A graphing calculator can be a good learning tool in studying mathematics concepts, if it is used correctly and appropriately. Otherwise, its use could seriously harm students’ mathematics learning. If students are going to use a graphing calculator as a tool, they need to learn to “communicate” with their calculators. For example, if students work with \( y = \frac{1}{x+3} \) and enter it in a calculator as \( y = 1/x + 3 \), the calculator understands this as \( y = 1 + 3 \), and therefore, gives a different result than expected.

Suppose students have an exponential function, such as \( y = e^{2x} \). If it is entered in a calculator as \( y = e^{(2x)} \), the calculator understands the syntax as the correct exponential function; however, if it is entered as \( y = e^{(2)x} \), the calculator understands the syntax as a linear function, \( y = e^{2x} \). Likewise, when students perform an operation such as 0.00014 \( \cdot 5 \) in a graphing calculator, the answer appears as 7E-4. The calculator gives the answer in scientific notation and students need to know that means 0.0007, not just 7 or something else. Therefore, it is important that students understand what they are typing in their calculator and also what they are reading from their calculator.
Appendix F: (Continued)

A. Entering Arithmetic and Algebraic Expressions in a Graphing Calculator

When using graphing calculators in a mathematics course, the first thing students need to know is how an arithmetic or algebraic expression needs to be entered in the calculator. If students fail to do this properly, the calculator gives a false result or no result at all. This section addresses how mathematical expressions need to be entered in a graphing calculator.

a) The minus key and the negative key in a graphing calculator

A minus sign, indicating the operation, and a negative sign, indicating the opposite of a number or an expression, are represented by distinct symbols on a graphing calculator. The symbol may not be a problem when it is used in a paper-pencil mode, but it is often a problem when it is entered in a graphing calculator. Students need to know the locations of these two keys. The minus key, " - " is located in the right column, third key from the bottom; the negative key, " (-)" is located in the bottom of the second right column. For example, 10 minus 7 gives 3 as an answer and 10 negative 7 gives an error message.

b) The role of a parenthesis key

The parenthesis key is an important key for students to understand. In some cases a parenthesis key needs to be placed, in some cases a parenthesis key should not be placed, and in other cases placing a parenthesis key may not make any difference. Consider the following examples:

1. Negative of five squared means -5² so the answer is -25. In this case, if it is entered as (-5)² the answer is 25. Therefore a parenthesis key should not be placed.
Appendix F: (Continued)

2. a) Negative five squared means \((-5)^2\) so the answer is 25. In this case, if it is entered as \(-5^2\) the answer is -25. Therefore a parenthesis key should be placed.

b) The expression \(\frac{1}{5+3}\) equals \(\frac{1}{8}\) or 0.125. But, if it is entered as \(1/5+3\), the answer is 3.2 because the calculator understands the entered expression as \(\frac{1}{5}+3\). However, if it is entered as \(1/(5+3)\), the calculator gives the correct answer. Therefore a parenthesis key should be placed.

3. The equation \(y = \frac{2x}{x+1}\) is entered as either \(y = 2x/(x+1)\) or \(y = (2x)/(x+1)\) in a graphing calculator and produces the same graph. Therefore a parenthesis key for the numerator does not make any difference in this case.

The following table gives additional examples to emphasize the importance of using the minus key, negative key, and parenthesis key. Furthermore, the table identifies a list of common mistakes that are experienced by many students when they use graphing calculators. These mistakes are not necessarily common mistakes for students when they work with paper-pencil. The purpose of this lesson is to address the mistakes students make and provide students with a guide that helps them to use their calculators properly.
### Appendix F: (Continued)

<table>
<thead>
<tr>
<th>Desired Expression</th>
<th>Correct way to input</th>
<th>Incorrect way to input</th>
</tr>
</thead>
<tbody>
<tr>
<td>- x + 5</td>
<td>- x + 5 (negative of x) + 5</td>
<td>- x + 5 (minus x) + 5</td>
</tr>
<tr>
<td>- 5^2</td>
<td>- 5^2</td>
<td>(-5)^2</td>
</tr>
<tr>
<td>(−7)^2</td>
<td>(−7)^2</td>
<td>- 7^2</td>
</tr>
<tr>
<td>(\frac{3 \cdot 4^2 - 5 \cdot 2 + 7}{3 - 2 \cdot 5 + 2})</td>
<td>((3 \cdot 4^2 - 5 \cdot 2 + 7)/(3 - 2 \cdot 5 + 2))</td>
<td>(3 \cdot 4^2 - 5 \cdot 2 + 7 / 3 - 2 \cdot 5 + 2)</td>
</tr>
<tr>
<td>√(x + 2)</td>
<td>√(x + 2)</td>
<td>√(x) + 2</td>
</tr>
<tr>
<td>(\frac{1}{x + 3})</td>
<td>1/(x + 3)</td>
<td>1/x + 3</td>
</tr>
<tr>
<td>(x^{2/3})</td>
<td>(x^{(2/3)})</td>
<td>(x^{2/3})</td>
</tr>
<tr>
<td>((-x)^4)</td>
<td>((-x)^4)</td>
<td>- (x^4)</td>
</tr>
<tr>
<td>(\ln x^3)</td>
<td>(\ln(x^3))</td>
<td>((\ln x)^3)</td>
</tr>
<tr>
<td>(\frac{3x^2 - 4x + 5}{x - 5})</td>
<td>((3x^2 - 4x + 5)/(x - 5))</td>
<td>(3x^2 - 4x + 5 / x - 5)</td>
</tr>
<tr>
<td>(e^{x^2})</td>
<td>(e^{(2x)})</td>
<td>(e^{(2)x})</td>
</tr>
<tr>
<td>(\ln x + 3)</td>
<td>(\ln(x) + 3)</td>
<td>(\ln(x + 3))</td>
</tr>
</tbody>
</table>
Appendix F: (Continued)

B. Simplifying Arithmetic Expressions by Using a Graphing Calculator

In this section, students learn how to use a graphing calculator to simplify a given arithmetic expression. Students can simplify any arithmetic expression by correctly entering the expression in a graphing calculator.

Example 1:

Simplify the expression by using a graphing calculator.

\[
\frac{-3 \cdot 4^2 + 6 \cdot 4 - 7 \cdot -3}{5 \cdot 3 + 4 - 2 \cdot 3^2}
\]

Enter the expression as \((-3 \cdot 4^2 + 6 \cdot 4 - 7 \cdot -3)/(5 \cdot 3 + 4 - 2 \cdot 3^2)\) and press <ENTER>. The answer -3 appears on the screen.

Example 2:

Simplify the expression by using a graphing calculator.

\[
\frac{5 \cdot 2^3 + 5 \cdot 4 - 2}{4 \cdot 3 + 5 - 2^2 \cdot 3^2}
\]

Enter the expression as \((5 \cdot 2^3 + 5 \cdot 4 - 2)/(4 \cdot 3 + 5 - 2^2 \cdot 3^2)\) and press <ENTER>. The answer -3.789473684 appears on the screen.
Appendix F: (Continued)

Because the result is a decimal representation of the exact answer, students can change the result to a fraction which is the exact answer to the expression.

Press <MATH> and select option 1: → Frac.

Ans → Frac appears on the screen. Press <ENTER> to get the exact answer, $\frac{72}{19}$, on the screen.
Example 3:

Simplify the expression by using a graphing calculator.

\[
\frac{0.0025 \cdot 0.07 \cdot 5}{1.6}
\]

Enter the expression as \(0.0025 \cdot 0.07 \cdot 5 / 1.6\) and press \(<\text{ENTER}>\). The answer 5.46875E - 4 appears on the screen.

The answer is in scientific notation, meaning the answer is 0.000546875.

Example 4:

Simplify the following expressions by using a graphing calculator.

\[a) \ \frac{\sqrt{10 \cdot 3 - 5} + 7 \cdot 2}{\sqrt{2 \cdot 4 + 1} + 6} \quad b) \ \frac{\sqrt{23 \cdot 3 + 15 + 11 - 2 \cdot 3}}{16 - 3^2}\]

\[a) \ \text{Enter the expression as } (\sqrt{10 \cdot 3 - 5} + 7 \cdot 2)/(\sqrt{2 \cdot 4 + 1} + 6) \text{ and press } <\text{ENTER}>.\]

The answer 2.111111111 appears on the screen.
Appendix F: (Continued)

Because the answer is a decimal form, students may change it to a fraction form.

Press <MATH> and select option 1: → Frac.

![](image1)

**Ans → Frac** appears on the screen. Press <ENTER> then the exact answer, \(\frac{19}{9}\), appears on the screen.

![](image2)

b) Enter the expression as \((\sqrt{(23 \cdot 3 + 15) + 11 - 2 \cdot 3})/(16 - 3^3)\) and press <ENTER>.

The answer 2.023593056 appears on the screen.
Because the answer is a decimal form, students may change it to a fraction form.

Press <MATH> and select option 1: → Frac.

Ans → Frac appears on the screen. Press <ENTER>. This time the same decimal number appears on the screen.
Appendix F: (Continued)

Students could be asked why this happens, providing an opportunity for the instructor to explain that the answer to the expression is an irrational number and therefore a fraction form cannot be obtained for the answer.
C. Graphing Functions (Equations) with a Graphing Calculator

In this section, students learn to use a graphing calculator to graph a given function or equation.

Example 1:

Graph \( y = |x| - 4 \) with a graphing calculator.

Press \(<\text{Y} = >\) to enter the equation as \( y_1 = \text{abs}(x) - 4 \). The absolute value, \( \text{abs} \), key is listed under \(<\text{Math}>\) key and then under \( \text{NUM} \) option.

Press \(<\text{WINDOW}>\) to make sure that the window is in standard mode. In the standard mode, the window has the following properties:

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
```

Otherwise, press \(<\text{Zoom}>\) and select option 6:Zstandard to get the standard window as previously mentioned.
Appendix F: (Continued)

Press <GRAPH> to obtain the following graph on the screen.

Example 2:

Graph \( f(x) = \sqrt{x-3} + 2 \) with a graphing calculator.

Press <Y = > to enter the equation as \( y_1 = \sqrt{(x-3)} + 2 \).

Press <GRAPH> to obtain the following graph on the screen.
Appendix F: (Continued)

Example 3:

Graph \( g(x) = (x - 2)^2 + 13 \) with a graphing calculator.

Press \(<Y = >\) to enter the equation as \( y_1 = (x - 2)^2 + 13 \).

When students press \(<\text{GRAPH}>\) to view the graph, they do not see any graph on the screen.

Students need to understand that the standard window is not always the correct window on which to view the graph. The graph of the given function is a parabola with vertex at
Appendix F: (Continued)

(2, 13). Students need to change the viewing window. Press <WINDOW> and change $Y_{\text{max}} = 25$, leaving all other values as they are.

Then press <GRAPH> to view the following graph on the screen.

Example 4:

Graph $2x + 3y = 10$ with a graphing calculator.

Because the equation needs to be entered as “$y =$” in the calculator, first students need to solve for $y$ and then enter the resulting equation in the calculator.

$2x + 3y = 10$, then $3y = -2x + 10$, and then $y = \frac{-2}{3}x + \frac{10}{3}$.

Now press <Y = > to enter the equation as $y_1 = (-2/3)x + 10/3$. 

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Appendix F: (Continued)

Press <GRAPH> to obtain the following graph on the screen.

Example 5:

Graph \( f(x) = \begin{cases}  x^2, & x < 1 \\ -2x + 5, & x \geq 1 \end{cases} \) with a graphing calculator.

Because the calculator does not have a piecewise function key, students need to enter the function as a sum of those two equations with the restrictions. All inequality keys are listed under TEST key. Press <2nd> MATH to access the TEST key. The function is entered as follows:

Press <Y = > to enter the equation as \( y_1 = (x^2)(x < 1) + (-2x + 5)(x \geq 1) \).
Appendix F: (Continued)

Press <GRAPH> to obtain the following graph on the screen.
Appendix F: (Continued)

D. Student Activities For Lesson 1

Worksheet #1A: Entering Expressions in a Graphing Calculator

For each of the following expressions, write the way you would enter it in your graphing calculator. Show the written expressions in the given table.

1. \( \frac{5}{4x} - 7 \)
2. \( \frac{3x}{5x - 6} \)
3. \( \frac{x^2 - 3}{2x^3 + 10} \)
4. \( \sqrt{2x - 13} \)
5. \( \sqrt{5x - 7} + 3x \)
6. \( 3|x - 5| - 4 \)

Recording Table

<table>
<thead>
<tr>
<th>Expression</th>
<th>Write the expression the way you would enter it in the calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{5}{4x} - 7 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{3x}{5x - 6} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{x^2 - 3}{2x^3 + 10} )</td>
<td></td>
</tr>
<tr>
<td>( \sqrt{2x - 13} )</td>
<td></td>
</tr>
<tr>
<td>( \sqrt{5x - 7} + 3x )</td>
<td></td>
</tr>
<tr>
<td>( 3</td>
<td>x - 5</td>
</tr>
</tbody>
</table>
Worksheet #1B: Simplifying Arithmetic Expressions with a Graphing Calculator

1. Enter the following arithmetic expressions in your calculator and then simplify. If the answer is a decimal, convert the decimal to a fraction if possible; if the answer is in scientific notation, convert it to a standard number. Record the work on the given sheet.

   a) \(-4^2\)  
   b) \(\frac{2 \cdot 3 - 4^2 \div 2}{-7 + 4 \cdot 2}\)  
   c) \(\frac{2}{8^3}\)  
   d) \(\sqrt{3 - 4 \cdot 5 + 6} - 3 \cdot 4^2\)  
   e) \(\frac{\sqrt{2}}{3}\)  
   f) \(\sqrt{2 \cdot 4 - 6 + 3 - 5 \cdot 2 \cdot (-3)^2}\)  
   g) \(\frac{3}{2 \cdot 5^2}\)  
   h) \(0.00045\cdot 2\)  
   i) \(\frac{0.002 \cdot 0.0034}{2.5}\)

Recording Table

<table>
<thead>
<tr>
<th>Expression</th>
<th>Copy the expression the way that is entered in the calculator</th>
<th>Write the answer from the calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-4^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{2 \cdot 3 - 4^2 \div 2}{-7 + 4 \cdot 2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{2}{8^3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sqrt{3 - 4 \cdot 5 + 6} - 3 \cdot 4^2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{\sqrt{2}}{3})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sqrt{2 \cdot 4 - 6 + 3 - 5 \cdot 2 \cdot (-3)^2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{3}{2 \cdot 5^2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.00045\cdot 2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\frac{0.002 \cdot 0.0034}{2.5})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Worksheet #1C: Graphing Functions (Equations) with a Graphing Calculator

Graph the following equations with your graphing calculator. Record the work on the given sheet. Adjust the window when necessary. Copy the graphs from the calculator on the other side.

1) \( y = \sqrt{x} - 3 \)  
2) \( f(x) = |x + 4| - 2 \)  
3) \( g(x) = \frac{x - 3}{x + 2} \)  
4) \( f(x) = \sqrt{x + 3} \)  
5) \( g(x) = \frac{3}{4}x - \frac{7}{3} \)  
6) \( y = \frac{3}{4}x - \frac{7}{3} \)  
7) \( 4x - 3y = 14 \)  
8) \( f(x) = 50 \)  
9) \( \begin{array}{ll}
    f(x) = \begin{cases} 
        3x - 4, & x < -2 \\
        x^2 + 3, & x \geq -2 
    \end{cases}
\end{array} \)

Recording Table

<table>
<thead>
<tr>
<th>Problem</th>
<th>Write the way the function that it is entered in the calculator</th>
<th>Copy the graph from the calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \sqrt{x} - 3 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) =</td>
<td>x + 4</td>
<td>- 2 )</td>
</tr>
<tr>
<td>( g(x) = \frac{x - 3}{x + 2} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Appendix F: (Continued)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Write the way the function that it is entered in the calculator</th>
<th>Copy the graph from the calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \sqrt{x + 3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( g(x) = \frac{3}{4x} - \frac{7}{3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = \frac{3}{4} x - \frac{7}{3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 4x - 3y = 14 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = 50 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x) = \begin{cases} 3x - 4, &amp; x &lt; -2 \ x^2 + 3, &amp; x \geq -2 \end{cases} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Lesson 2: Function Values and the Table Feature of a TI 83 Graphing Calculator

Instructor Notes:

The table feature in a graphing calculator is a useful feature to determine the values of a function for various $x$ values.

Consider the following examples.

Example 1:

Given $f(x) = -x^2 + 3x - 4$, find the following:

a) $f(-4)$  

b) $f(-1)$  

c) $f(0)$  

d) $f(3)$  

e) $f(10)$  

f) $f(100)$

Instead of substituting the given $x$ value in each case, students can enter the function as $y_1$ in a graphing calculator and then use the table feature to find the required function values. Before entering the function in the calculator, students need to set up the table as follows. This is required for the first time only.

Setting up the Table:

Press the $<2nd>$ and $<WINDOW>$ keys to obtain the $<TBL SET>$ screen. The default $TABLE SET$ screen looks like this.
Appendix F: (Continued)

Set up the table by using the arrow keys to bring the cursor down to highlight the option

**Ask** at the **Indpnt** prompt; press **<ENTER>**. Keep the option **Auto** highlighted at the

**Depend** prompt. That is, the screen should look like this:

With this setup, the calculator asks for an input (the independent variable, \( x \) value); when given a value for \( x \), the calculator will automatically evaluate the corresponding output value (the dependent variable, \( y \) value).

Now enter the problem. Press **<Y>** to enter the function as \( y_1 = -x^2 + 3x - 4 \).
Appendix F: (Continued)

Press the <2nd> and <Graph> keys to access the <TABLE> screen. The table screen looks like this:

<table>
<thead>
<tr>
<th>X</th>
<th>Y₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-32</td>
</tr>
<tr>
<td>-1</td>
<td>-8</td>
</tr>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>3</td>
<td>-74</td>
</tr>
<tr>
<td>10</td>
<td>-9704</td>
</tr>
</tbody>
</table>

Then at the $X =$ prompt enter the first $x$ value, -4, and press <ENTER>, then again at the $X =$ prompt enter the second $x$ value, -1, and press <ENTER>. Continue to enter given all $x$ values in this manner. The table looks like this after entering all given $x$ values.

The $Y₁$ column gives the function values for the given $x$ values. That is,

a) $f(-4) = -32$   b) $f(-1) = -8$   c) $f(0) = -4$

d) $f(3) = -4$   e) $f(10) = -74$   f) $f(100) = -9704$
Example 2:

Given \( f(x) = \begin{cases} 
5x^2 + x - 2, & x < -1 \\
3x - 14, & x \geq -1 
\end{cases} \), find the following:

\begin{align*}
\text{a) } f(-3) & \quad \text{b) } f(-1) & \quad \text{c) } f(0) \\
\text{d) } f(4) & \quad \text{e) } f(8.6) & \quad \text{f) } f(53)
\end{align*}

Press \( <\text{Y}> \) to enter the function as \( y_1 = (5x^2 + x - 2)(x < -1) + (3x - 14)(x \geq -1) \).

Press the \(<\text{2nd}>\) and \(<\text{GRAPH}>\) keys to access the \(<\text{TABLE}>\) screen. Then at the \( X = \) prompt enter the first \( x \) value and press \(<\text{ENTER}>\), and then next \( x \) value and so on.

The table looks like this after entering all given \( x \) values.

\[
\begin{array}{c|c|c}
X & Y_1 & \\
-3 & 40 & \\
-1 & -17 & \\
0 & -14 & \\
4 & -2 & \\
8.6 & 11.8 & \\
53 & 145 & \\
\end{array}
\]

The \( Y_1 \) column gives the function values for the given \( x \) values. That is,

\begin{align*}
\text{a) } f(-3) = 40 & \quad \text{b) } f(-1) = -17 & \quad \text{c) } f(0) = -14
\end{align*}
Appendix F: (Continued)

d) \( f(4) = -2 \)  
\[ e) \quad f(8.6) = 11.8 \]
\[ f) \quad f(53) = 145 \]

Example 3:
The number of commercial FM radio stations in the United States can be modeled by
\[ f(x) = 2149.6(1.036)^x \]
for \( 1 \leq x \leq 26 \), where \( x \) represents the number of years since 1969 and \( f(x) \) represents the number of commercial FM radio stations (Source: U.S. Census Bureau, www.census.gov).

A. Find the following and interpret your answers.
\[ a) \ f(1) \quad b) \ f(3) \quad c) \ f(10) \quad d) \ f(25) \]

B. (i) Is it possible to compute \( f(-4) \) and \( f(28) \) ?
(ii) Is it meaningful to compute \( f(-4) \) and \( f(28) \)? Why or why not?

Answer:

A. Compute the function values as before.

Press \( \text{<Y>} \) to enter the function as \( y_1 = 2149.6(1.036)^x \).
Appendix F: (Continued)

Then press the <2nd> and <GRAPH> keys to access <TABLE> and enter the given $x$ values to obtain the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$Y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2227</td>
</tr>
<tr>
<td>3</td>
<td>2390.2</td>
</tr>
<tr>
<td>10</td>
<td>3061.6</td>
</tr>
<tr>
<td>25</td>
<td>5204.2</td>
</tr>
</tbody>
</table>

That is, the following answers are obtained for part A.

a) $f(1) = 2227$  \hspace{1cm} b) $f(3) = 2390.2$

c) $f(10) = 3061.6$  \hspace{1cm} d) $f(25) = 5204.2$

Thus, the number of FM radio stations in the US was 2227 in 1970, about 2390 in 1972, about 3061 in 1979 and about 5204 in 1994.

B. (i) It is possible to compute $f(-4)$ and $f(28)$ by using the table feature of the calculator as before.

(ii) The computations are not meaningful because the given $x$ values -4 and 28 are out of the given domain $1 \leq x \leq 26$. In other words, the given mathematical model may not work for any $x$ values other than $1 \leq x \leq 26$. 

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Student Activities For Lesson 2

Work Sheet #2: Evaluating Functions with a Graphing Calculator

Determine the function values for the given $x$ values in each case. Enter the given function in your calculator as $y_1$ and then use <TABLE> feature to find the function values. Write the way that the function is entered in your calculator. Then use the table to write the answer.

1. Given $f(x) = \frac{2x + 5}{x - 4}$, find the following:

   a) $f(0)$  
   b) $f\left(-\frac{1}{2}\right)$  
   c) $f(4)$

Answer:

Write the way the function is entered in the calculator:

Write the answers from the table:

   a) $f(0) =$  
   b) $f\left(-\frac{1}{2}\right) =$  
   c) $f(4) =$

2. Given $g(x) = |x - 7.2| - 10$, find the following:

   a) $g(-29)$  
   b) $g\left(-\frac{2}{3}\right)$  
   c) $g(4.3)$

Answer:

Write the way the function is entered in the calculator:

Write the answers from the table:

   a) $g(-29) =$  
   b) $g\left(-\frac{2}{3}\right) =$  
   c) $g(4.3) =$
Appendix F: (Continued)

3. Given $f(x) = 12$, find the following:

   a) $f(-5)$  
   b) $f(0)$  
   c) $f\left(\frac{13}{4}\right)$

Answer:

Write the way the function is entered in the calculator:

Write the answers from the table:

   a) $f(-5) =$  
   b) $f(0) =$  
   c) $f\left(\frac{13}{4}\right) =$

4. Given $f(x) = \begin{cases} -x^2 + 3x - 5, & x < -3 \\ \frac{1}{2}x - 10, & x \geq -3 \end{cases}$, find the following:

   a) $f(-4)$  
   b) $f(-3)$  
   c) $f(2.4)$

Answer

Write the way the function is entered in the calculator:

Write the answers from the table:

   a) $f(-4) =$  
   b) $f(-3) =$  
   c) $f(2.4) =$

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5. The number of milligrams of cholesterol consumed each day per person in the United States can be modeled by $C(x) = 0.11x^2 - 4.04x + 445.02$, $1 \leq x \leq 23$
where $x$ represents the number of years since 1974 and $C(x)$ represents the number of milligrams of cholesterol consumed each day per person. (Source: U.S. Department of Agriculture).

A. Compute the following function values with your calculator (using the \texttt{TABLE} feature) and interpret the answers.

a) $C(1)$

b) $C(4)$

c) $C(23)$

B. (i) Is it possible to compute $C(0)$ and $C(35)$?

(ii) Is it meaningful to compute $C(0)$ and $C(35)$? Why or why not?

Answer:

Write the way that the function is entered in your calculator.

Then use the table to write the answer.

A. a) $C(1) =$

b) $C(4) =$

c) $C(23) =$

B. (i)

(ii)
Appendix F: (Continued)

Lesson 3: Limits of Functions

Instructor Notes:

Introduction

The notion of a limit of a function is fundamental to the study of calculus. The investigation of the following two problems led to the creation of calculus:

1. Finding the slope of a tangent line to the graph of a given function at a given point.
2. Finding the area of a region bounded by the graph of a function.

The first problem led to the creation of differential calculus and the second led to the creation of integral calculus. The idea of a limit is used in the process of solving these two problems. Therefore understanding the limit concept is important in calculus.

Generally, limit problems can be approached graphically, analytically, or numerically. The graphical method depends on the graph of the function; if the graph of the function is not given or not easy to get, this method may not be a good choice. The analytical method depends on how well the expression in the function can be simplified; if the expression in the function is hard to simplify or not possible to simplify, this method does not help. The numerical method can be used for any limit problem as long as the function rule is given. These numerical computations are easy to perform with the table feature of a graphing calculator. This method also helps to understand the meaning of the limit of a function.
Appendix F: (Continued)

In this lesson students learn to solve limit problems numerically by using the table feature of a graphing calculator and develop understanding of the limit concept. This lesson is divided into three sections. The first section deals with the problems of finding the limit of a function as \( x \) approaches a number; the second section discusses the problems of finding the limit of a function as \( x \) approaches a number from the right or left; and the last section examines problems of finding the limit of a function as \( x \) approaches \( \infty \) and \( -\infty \).

A. Finding the limit of a function as \( x \) approaches a number

The question is to find the limit of a function \( f(x) \) as \( x \) approaches a number \( a \), or in notation, find \( \lim_{x \to a} f(x) \). The meaning of this question is: Does \( f(x) \) approach a number as \( x \) approaches \( a \) from the right and the left? Students enter the function in the calculator as \( y_1 \) and use the table to find the corresponding function values as \( x \) approaches \( a \) from the left and the right. If the function approaches a particular value, say \( l \), as \( x \) approaches \( a \) from the left and the right, then the function has a limit of \( l \) as \( x \) approaches \( a \), this is written as \( f(x) \to l \) as \( x \to a \), or in notation, \( \lim_{x \to a} f(x) = l \). Otherwise the limit of \( f(x) \) does not exist as \( x \) approaches \( a \).

Consider the following examples.

Example 1: Given \( f(x) = \frac{4x-12}{x^2-9} \), find \( \lim_{x \to 3} f(x) \), if it exists.

Press \( <Y> \) to enter the function as \( y_1 = (4x-12)/(x^2-9) \).
Appendix F: (Continued)

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen and enter $x$ values of 2.9, 2.99, and 2.999 to find the corresponding function values as $x \to 3$ from the left ($x \to 3^-$). Enter $x$ values of 3.1, 3.01, and 3.001 to find the corresponding function values as $x \to 3$ from the right ($x \to 3^+$). Now observe what happens to the function value when $x = 3$. The calculator gives this table.

The above table is the same as the following table. Note the function values as $x \to 3$ from both the left and right.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2.9</th>
<th>2.99</th>
<th>2.999</th>
<th>3</th>
<th>3.001</th>
<th>3.01</th>
<th>3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0.678</td>
<td>0.668</td>
<td>0.667</td>
<td>Error</td>
<td>0.667</td>
<td>0.666</td>
<td>0.656</td>
</tr>
</tbody>
</table>
Appendix F: (Continued)

The values in the table suggest that the limit of the function as \( x \to 3 \) from both sides is 0.667, that is, \( \lim_{x \to 3} \frac{4x-12}{x^2-9} = 0.667 \). Also, note that the function is undefined (error message in the calculator) when \( x = 3 \).

**Example 2:** Find \( \lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} \), if it exists.

Press <Y> to enter the function as \( y_1 = (\sqrt{x} - 5)/(x - 25) \).

![Graph of function](image)

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter \( x \) values of 24.9, 24.99, and 24.999 to find the corresponding function values as \( x \to 25^- \). Enter \( x \) values of 25.1, 25.01, and 25.001 to find the corresponding function values as \( x \to 25^+ \). Also, find the function value when \( x = 25 \). The calculator gives this table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.9</td>
<td>0.1001</td>
</tr>
<tr>
<td>24.99</td>
<td>0.10001</td>
</tr>
<tr>
<td>24.999</td>
<td>0.10000</td>
</tr>
<tr>
<td>25.001</td>
<td>ERROR</td>
</tr>
<tr>
<td>25.01</td>
<td>0.09999</td>
</tr>
<tr>
<td>25.1</td>
<td>0.0999</td>
</tr>
</tbody>
</table>

| \( x = 25.1 \) | \( y_1 = \text{ERROR} \) |
Appendix F: (Continued)

The above table can be rewritten as follows. Note the function values as \( x \to 25 \) from both the left and right.

<table>
<thead>
<tr>
<th>( x )</th>
<th>24.9</th>
<th>24.99</th>
<th>24.999</th>
<th>25</th>
<th>25.001</th>
<th>25.01</th>
<th>25.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0.1001</td>
<td>0.10001</td>
<td>0.1</td>
<td>Error</td>
<td>0.1</td>
<td>0.09999</td>
<td>0.0999</td>
</tr>
</tbody>
</table>

The values in the table suggest that \( \lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} = 0.1 \). Also, note that the function is undefined (error message in the calculator) when \( x = 25 \).

Example 3: Find \( \lim_{x \to -2} \frac{|x + 2|}{x + 2} \), if it exists.

Press \(<Y>\) to enter the function as \( y_1 = abs(x + 2)/(x + 2) \).

Press the \(<2nd>\) and \(<GRAPH>\) keys to access the \(<TABLE>\) screen. Enter \( x \) values of -2.1, -2.01, and -2.001 to find the corresponding function values as \( x \to -2^- \). Enter \( x \) values of -1.9, -1.99, and -1.999 to find the corresponding function values as \( x \to -2^+ \). Observe the function value when \( x = -2 \). The calculator gives this table.
The above table can be rewritten as follows. Note the function values as \( x \to -2 \) from both the left and right.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2.1</th>
<th>-2.01</th>
<th>-2.001</th>
<th>-2</th>
<th>-1.999</th>
<th>-1.99</th>
<th>-1.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>Error</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

From the values in the table, it appears that \( f(x) \to -1 \) as \( x \to -2^- \) and \( f(x) \to 1 \) as \( x \to -2^+ \). Because \( f(x) \) approaches two different values as \( x \) approaches -2 from the left and right, \( f(x) \) does not have a limit as \( x \to -2 \) and therefore, \( \lim_{{x \to -2}} \frac{|x+2|}{x+2} \) does not exist. Also, note that the function is undefined (error message in the calculator) when \( x = -2 \).

**Example 4:** Find \( \lim_{{x \to 3}} \frac{2x-5}{x^2-9} \), if it exists.

Press \(<Y>\) to enter the function as \( y_1 = (2x-5)/(x^2-9) \).
Appendix F: (Continued)

Press the \textlt<2nd> and \textlt<GRAPH> keys to access the \textlt<TABLE> screen. Enter $x$ values of 2.9, 2.99, and 2.999 to find the corresponding function values as $x \to 3^-$. Enter $x$ values of 3.1, 3.01, and 3.001 to find the corresponding function values as $x \to 3^+$. Observe the function value when $x = 3$. The calculator gives this table.

The above table can be rewritten as follows. Note the function values as $x \to 3$ from both the left and right.

\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
$x$ & 2.9 & 2.99 & 2.999 & 3 & 3.001 & 3.01 & 3.1 \\
\hline
$f(x)$ & -1.356 & -16.36 & -166.4 & Error & 166.97 & 16.972 & 1.9672 \\
\hline
\end{tabular}
Appendix F: (Continued)

The values in the table suggest that \( f(x) \to -\infty \) as \( x \to 3^- \) and \( f(x) \to \infty \) as \( x \to 3^+ \) so

\[
\lim_{x \to 3^-} \frac{2x - 5}{x^2 - 9} \text{ does not exist. Also, note that the function is undefined (error message in the calculator) when } x = 3.
\]

Example 5: Given \( f(x) = \begin{cases} -2x + 8, & x < 3 \\ x - 1, & x \geq 3 \end{cases} \), find \( \lim_{x \to 3^-} f(x) \), if it exists.

Press <Y> to enter the function as \( y_1 = (-2x + 8)(x < 3) + (x - 1)(x \geq 3) \).

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter \( x \) values of 2.9, 2.99, and 2.999 to find the corresponding function values as \( x \to 3^- \). Enter \( x \) values of 3.1, 3.01, and 3.001 to find the corresponding function values as \( x \to 3^+ \). The calculator gives this table.
The above table can be rewritten as follows.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2.9</th>
<th>2.99</th>
<th>2.999</th>
<th>3</th>
<th>3.001</th>
<th>3.01</th>
<th>3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2.2</td>
<td>2.02</td>
<td>2.002</td>
<td>2.001</td>
<td>2.01</td>
<td>2.1</td>
<td></td>
</tr>
</tbody>
</table>

Thus, $f(x) \to 2$ as $x \to 3$ from both sides and therefore, $\lim_{x \to 3} f(x) = 2$. 
Appendix F: (Continued)

B. Finding the limit of a function as $x$ approaches a number from the right and left

Suppose the question is to find the limit of a function $f(x)$ as $x$ approaches a number $a$ from the right, denoted $\lim_{x \to a^+} f(x)$, or to find the limit of a function $f(x)$ as $x$ approaches a number $a$ from the left, denoted $\lim_{x \to a^-} f(x)$. Consider the following examples.

Example 1: Find $\lim_{x \to 4^+} \frac{5x}{x - 4}$, if it exists.

Press $<Y>$ to enter the function as $y_1 = 5x / (x - 4)$.

Press the $<2nd>$ and $<GRAPH>$ keys to access the $<TABLE>$ screen. Enter $x$ values of 4.5, 4.1, 4.01, 4.001, and 4.0001 to find the corresponding function values as $x \to 4^+$. Also, note the function value when $x = 4$. The table looks like this:
Appendix F: (Continued)

The above table is the same as the following. Note the function values as $x \rightarrow 4^+$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>4</th>
<th>4.0001</th>
<th>4.001</th>
<th>4.01</th>
<th>4.1</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>Error</td>
<td>200005</td>
<td>20005</td>
<td>2005</td>
<td>205</td>
<td>45</td>
</tr>
</tbody>
</table>

From the table, it appears that $f(x) \rightarrow \infty$ as $x \rightarrow 4^+$.

**Example 2:** Find $\lim_{{x \to 4^+}} \frac{{-5}}{{x^2 - x}}$, if it exists.

Press <Y> to enter the function as $y_1 = -5/(x^2 - x)$.

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter $x$ values of 0.5, 0.9, 0.99, 0.999, and 0.9999 to find the corresponding function values as $x \rightarrow 4^+$.

Also, note the function value when $x = 1$. The table looks like this:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ERROR</td>
</tr>
<tr>
<td>1.1</td>
<td>20</td>
</tr>
<tr>
<td>1.9999</td>
<td>5005.05</td>
</tr>
<tr>
<td>0.9999</td>
<td>5005</td>
</tr>
<tr>
<td>0.99</td>
<td>55.556</td>
</tr>
<tr>
<td>0.9</td>
<td>555.56</td>
</tr>
<tr>
<td>0.5</td>
<td>200005</td>
</tr>
</tbody>
</table>

200
Appendix F: (Continued)

The above table can be rewritten as follows:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.5</th>
<th>0.9</th>
<th>0.99</th>
<th>0.999</th>
<th>0.9999</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>20</td>
<td>55.56</td>
<td>505.05</td>
<td>5005</td>
<td>50005</td>
<td>Error</td>
</tr>
</tbody>
</table>

Thus, $f(x) \rightarrow \infty$ as $x \rightarrow 1^{-}$ and therefore, \( \lim_{x \rightarrow 1^{-}} \frac{-5}{x^2 - x} = \infty \).

Example 3: Find \( \lim_{x \rightarrow 0^{+}} \frac{x}{|x|} \), if it exists.

Press <Y> to enter the function as \( y_{1} = x / \text{abs}(x) \).

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter \( x \) values of 1, 0.5, 0.1, 0.01, and 0.001 to find the corresponding function values as \( x \rightarrow 0^{+} \). The calculator gives this table. Also, note that the function is undefined (error message in the calculator) when \( x = 0 \).
The above table can be rewritten as follows.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>Error</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

From the values in the table, it appears that $f(x) \to 1$ as $x \to 0^+$ so $\lim_{x \to 0^+} \frac{x}{|x|} = 1.$
Appendix F: (Continued)

C. Finding the limit of a function as $x$ approaches $\infty$ and $-\infty$.

Examples in this section are about examining the limit of a function as $x$ approaches $\infty$ and $-\infty$. Consider the following examples.

Example 1: Find $\lim_{x \to \infty} \frac{4 + 3x^2}{2x^2 - 9}$, if it exists.

Press <Y> to enter the function as $y_1 = (4 + 3x^2)/(2x^2 - 9)$.

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter $x$ values of 10, 100, 200, 500, 1000 and 5000 to find the corresponding function values as $x \to \infty$.

The tables look like these:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.5916</td>
</tr>
<tr>
<td>100</td>
<td>1.5009</td>
</tr>
<tr>
<td>200</td>
<td>1.5002</td>
</tr>
<tr>
<td>500</td>
<td>1.5</td>
</tr>
<tr>
<td>1000</td>
<td>1.5</td>
</tr>
<tr>
<td>5000</td>
<td>1.5</td>
</tr>
</tbody>
</table>

203
Thus, it appears that \( \lim_{x \to \infty} \frac{4 + 3x^2}{2x^2 - 9} = 1.5 \).

**Example 2:** Find \( \lim_{x \to -\infty} \frac{12}{5x^2 + 4} \), if it exists.

Press \(<Y>\) to enter the function as \( y_1 = \frac{12}{5x^2 + 4} \).

Press the \(<2nd>\) and \(<GRAPH>\) keys to access the \(<TABLE>\) screen. Enter \( x \) values of -10, -100, -200, -500, -1000 and -5000 to find the corresponding function values as \( x \to -\infty \). The table looks like this:
Thus, it appears that \( \lim_{x \to -\infty} \frac{12}{5x^2 + 4} = 0. \)

**Example 3:** Find \( \lim_{x \to \infty} \frac{x^2 - 4x + 5}{2x - 3} \), if it exists.

Press <Y> to enter the function as \( y_1 = \frac{(x^2 - 4x + 5)}{(2x - 3)} \).

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter \( x \) values of 10, 100, 200, 500, 1000 and 5000 to find the corresponding function values as \( x \to \infty \).

The table looks like this:
Appendix F: (Continued)

<table>
<thead>
<tr>
<th>$x$</th>
<th>10</th>
<th>100</th>
<th>200</th>
<th>500</th>
<th>1000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>3.8235</td>
<td>48.756</td>
<td>98.753</td>
<td>248.75</td>
<td>498.75</td>
<td>2498.8</td>
</tr>
</tbody>
</table>

Thus, it appears that $\lim_{x \to \infty} \frac{x^2 - 4x + 5}{2x - 3} = \infty.$

Applications for limits

Example 1:

The concentration (in mg/cubic cm) of a certain drug in a patient’s bloodstream $t$ hours after injection is given by $C(t) = \frac{0.2t}{t^2 + 1}.$

a) Compute $C(0.5), C(1), C(2),$ and $C(3)$ and interpret the answers.

b) Evaluate $\lim_{t \to \infty} C(t)$ and interpret the result.

Press <Y> to enter the function as $y_1 = 0.2x/(x^2 + 1)$.

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter $x$ values of 0.5, 1, 2, and 3 to find the corresponding function values.
a) That is, \( C(0.5) = 0.08, \ C(1) = 0.1, \ C(2) = 0.08, \) and \( C(3) = 0.06. \) This means that the concentrations (in mg/cubic cm) of the drug in the patient’s blood-stream are 0.08, 0.1, 0.08 and 0.06 after 0.5 hour, 1 hour, 2 hours, and 3 hours of injection, respectively.

b) To answer \( \lim_{t \to \infty} C(t), \) enter \( x \) values of 10, 20, 50, 100 and 150 and determine the limit of the function as \( x \to \infty. \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C(x) )</td>
<td>0.0198</td>
<td>0.00998</td>
<td>0.004</td>
<td>0.002</td>
<td>.00133</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Thus, it appears that \( \lim_{t \to \infty} C(t) = 0. \) This means that the concentration of the drug in the patient’s blood-stream is vanishing as time increases.
Appendix F: (Continued)

Example 2:

The average cost/disc in dollars incurred by Herald Records in pressing $x$ videodiscs is given by the average cost function

$$C(x) = 2.2 + \frac{2500}{x}.$$ 

a) Compute $C(50)$, $C(100)$, $C(500)$, and $C(1000)$ and interpret the answers.

b) Evaluate $\lim_{x \to \infty} C(x)$ and interpret the result.

Press $\langle Y \rangle$ to enter the function as $y_1 = 2.2 + \frac{2500}{x}$.

Press the $\langle 2nd \rangle$ and $\langle$GRAPH$\rangle$ keys to access the $\langle$TABLE$\rangle$ screen. Enter $x$ values of 50, 100, 500, and 1000 to find the corresponding function values.
Appendix F: (Continued)

a) That is, $C(50) = 52.20$, $C(100) = 27.20$, $C(500) = 7.20$, and $C(1000) = 4.70$. This means that the average costs are $52.20$/disc, $27.20$/disc, $7.20$/disc, and $4.70$/disc when 50, 100, 500, and 1000 videodiscs are produced, respectively.

b) To answer $\lim_{x \to \infty} C(x)$, enter $x$ values of 10000, 40000, 80000, 100000, 200000, 400000, 1000000 and determine the limit of the function as $x \to \infty$.

Thus, it appears that $\lim_{x \to \infty} C(x) = 2.20$. This means that the average cost of producing $x$ videodiscs will approach $2.20$/disc in the long run. That is, the price does not go below $2.20$ per disc.
Appendix F: (Continued)

D. Student Activities For Lesson 3

Work Sheet #3: Finding the limits of Functions with a Graphing Calculator

Find the limit of each of the following functions, if it exists. In each case, complete the table by using the table feature of your graphing calculator. Then determine the limit, if it exists. If the limit doesn’t exist, then state so and explain why the limit doesn’t exist. If the answer seems to be $\infty$ or $-\infty$, state so.

1. Given $f(x) = \frac{2x^2 - 5x}{x^2 + 3x}$, find $\lim_{x \to 0} f(x)$, if it exists.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-0.1</th>
<th>-0.01</th>
<th>-0.001</th>
<th>0</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: $\lim_{x \to 0} f(x) = \ldots$

2. Given $f(x) = \frac{x + 2}{x^2 + 5x + 6}$, find $\lim_{x \to -2} f(x)$, if it exists.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2.1</th>
<th>-2.01</th>
<th>-2.001</th>
<th>-2</th>
<th>-1.999</th>
<th>-1.99</th>
<th>-1.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: $\lim_{x \to -2} f(x) = \ldots$
Appendix F: (Continued)

3. Given \( f(x) = \frac{\sqrt{x} - 3}{x - 9} \), find \( \lim_{x \to 9} f(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>8.9</th>
<th>8.99</th>
<th>8.999</th>
<th>9</th>
<th>9.001</th>
<th>9.01</th>
<th>9.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to 9} f(x) = \)

4. Given \( f(x) = \frac{x^2 - 4}{-\sqrt{x} + 2} \), find \( \lim_{x \to 4} f(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>3.9</th>
<th>3.99</th>
<th>3.999</th>
<th>4</th>
<th>4.001</th>
<th>4.01</th>
<th>4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to 4} f(x) = \)
Appendix F: (Continued)

5. Given \( f(x) = \frac{x}{|x|} \), find \( \lim_{x \to 0} f(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-0.1</th>
<th>-0.01</th>
<th>-0.001</th>
<th>0</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to 0} f(x) = \)

6. Given \( g(x) = \frac{|x - 5|}{x - 5} \), find \( \lim_{x \to 5} g(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>4.9</th>
<th>4.99</th>
<th>4.999</th>
<th>5</th>
<th>5.001</th>
<th>5.01</th>
<th>5.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to 5} g(x) = \)
Appendix F: (Continued)

7. Given \( g(x) = \frac{|x + 3|}{x + 3} \), find \( \lim_{x \to -3} g(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>-2.999</th>
<th>-2.99</th>
<th>-2.9</th>
<th>-2.8</th>
<th>-2.7</th>
<th>-2.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to -3} g(x) = \)

8. Given \( f(x) = \frac{x - 10}{|x - 10|} \), find \( \lim_{x \to 10^-} f(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>9.6</th>
<th>9.7</th>
<th>9.8</th>
<th>9.9</th>
<th>9.99</th>
<th>9.999</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to 10^-} f(x) = \)
Appendix F: (Continued)

9. Given $f(x) = \frac{3x - 5}{x^2 - x}$, find $\lim_{x \to 0^+} f(x)$, if it exists.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-0.4</th>
<th>-0.3</th>
<th>-0.2</th>
<th>-0.1</th>
<th>-0.01</th>
<th>-0.001</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: $\lim_{x \to 0^+} f(x) = \boxed{}$

10. Given $f(x) = \frac{6 + 5x}{2x - 1}$, find $\lim_{x \to \frac{1}{2}^+} f(x)$, if it exists.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0.5</th>
<th>0.501</th>
<th>0.51</th>
<th>0.52</th>
<th>0.53</th>
<th>0.54</th>
<th>0.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: $\lim_{x \to \frac{1}{2}^+} f(x) = \boxed{}$
Appendix F: (Continued)

11. Given \( h(x) = \frac{8}{2x^2 - 5} \), find \( \lim_{{x \to \infty}} h(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>10</th>
<th>100</th>
<th>1000</th>
<th>2000</th>
<th>5000</th>
<th>8000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{{x \to \infty}} h(x) = \)

12. Given \( g(x) = \frac{3 - 5x^2}{3 + 2x^2} \), find \( \lim_{{x \to \infty}} g(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-10000</th>
<th>-8000</th>
<th>-50000</th>
<th>-2000</th>
<th>-1000</th>
<th>-100</th>
<th>-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{{x \to \infty}} g(x) = \)
Appendix F: (Continued)

13. Given \( f(x) = \frac{4x - 3x^2}{2x - 5} \), find \( \lim_{x \to -\infty} f(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-10000</th>
<th>-8000</th>
<th>-50000</th>
<th>-2000</th>
<th>-1000</th>
<th>-100</th>
<th>-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to -\infty} f(x) = \)

14. Given \( g(x) = \frac{8}{2x^2 - 5} \), find \( \lim_{x \to \infty} g(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>10</th>
<th>100</th>
<th>1000</th>
<th>2000</th>
<th>5000</th>
<th>8000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to \infty} g(x) = \)
Appendix F: (Continued)

15. Given \( f(x) = \begin{cases} -2x^2 + 10, & x < 2 \\ 4x - 1, & x \geq 2 \end{cases} \), find \( \lim_{x \to 2} f(x) \), if it exists.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.9</th>
<th>1.99</th>
<th>1.999</th>
<th>2</th>
<th>2.001</th>
<th>2.01</th>
<th>2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Write the answer from the table: \( \lim_{x \to 2} f(x) = \)

16. The total worldwide box-office receipts for a long-running blockbuster movie are approximated by the function

\[
T(x) = \frac{120x^2}{x^2 + 4},
\]

where \( T(x) \) is measured in millions of dollars and \( x \) is the number of months since the movie’s release.

a) What are the total box-office receipts after the first month? The second month? The fifth month?

b) What will the movie gross in the long run? (Hint: Find \( \lim_{x \to \infty} T(x) \).)
17. A major corporation is building a 4325-acre complex of homes, offices, stores, schools, and churches in the rural community of Glen Cove. As a result of this development, the planners have estimated that Glen Cove’s population (in thousands) \( t \) years from now will be given by

\[
P(t) = \frac{25t^2 + 125t + 200}{t^2 + 5t + 40}.
\]

a) What is the current population of Glen Cove? After 2 years? After 10 years?

b) What will be the population in the long run?
Lesson 4: Derivatives of Functions

Instructor Notes:

Calculus is a study of change. It is a study of how quickly one quantity changes as the other quantity changes. The slope of the quantity (function) is used to measure this change. The slope of a function at a particular point is a measure of how quickly the output \((y)\) changes as the input \((x)\) changes at that point. This is called the instantaneous rate of change or simply the rate of change of the function at that point. The mathematical term for the rate of change of a function at a point is the derivative of the function at that point. The process of finding a derivative is called differentiation.

A. Finding the Derivative of a Function at a Given Value

Example: Finding the derivative of \(f(x)\) at \(x = c\). Or simply finding \(f'(c)\).

Procedure:

\((c, f(c))\) is the given point on the graph of \(f(x)\) where the derivative needs to be found. Choose another point that is close to the given point on the graph of \(f(x)\). Say, the second point is \((c + h, f(c + h))\), where \(h\) is the difference of the \(x\) values of these two points. The slope of the secant line that goes through these two points is

\[
\frac{f(c + h) - f(c)}{(c + h) - c} = \frac{f(c + h) - f(c)}{h}
\]

and this quantity gives the average rate of change of \(f(x)\) on the interval \([c, c + h]\). Then the slope of the tangent line to the graph of \(f(x)\) at \(x = c\) will be the same as the slope of the secant line when \(h\) approaches 0.
That is, the slope of the tangent line to the graph of \( f(x) \) at \( x = c \) is

\[
\lim_{h \to 0} \frac{f(c + h) - f(c)}{h}.
\]

This is also the slope of the function \( f(x) \) at \( x = c \). This quantity is known as the derivative of \( f(x) \) at \( x = c \), which gives the instantaneous rate of change at \( x = c \). Therefore, finding the derivative of a function is a limit problem so students can find this limit in the same way they have found limits in their earlier study of this topic.

Consider the following examples.

Example 1:

Given \( f(x) = 3x^2 - x + 3 \), find the rate of change of \( f(x) \) at \( x = 4 \).

That is, the question is to find \( f'(4) \). Therefore, from the definition of the derivative, the question is to find \( f'(4) = \lim_{h \to 0} \frac{f(4 + h) - f(4)}{h} \).

To use a calculator to find this limit, use \( x \) for \( h \) and let \( y_1 = f(4) = 3(4)^2 - 4 + 3 \),

\[ y_2 = f(4 + x) = 3(4 + x)^2 - (4 + x) + 3, \] and \[ y_3 = \frac{y_2 - y_1}{x}. \] Therefore, \( f'(4) = \lim_{x \to 0} y_3 \).

Press \(<\textbf{Y}>\) to enter the following:

\[
\begin{align*}
y_1 &= 3(4)^2 - 4 + 3 \\
y_2 &= 3(4 + x)^2 - (4 + x) + 3 \\
y_3 &= (y_2 - y_1) / x
\end{align*}
\]
Appendix F: (Continued)

Because the table is only needed for \( y_1 \), deactivate the equations for \( y_1 \) and \( y_2 \) by moving the cursor to the “=” sign of both equations and pressing <Enter>. The screen should look like this:

![Calculator Screen]

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter \( x \) values of 0.1, 0.01, and 0.001 to find the corresponding function values as \( x \to 0^+ \). Enter \( x \) values of -0.1, -0.01, -0.001 to find the corresponding function values as \( x \to 0^- \). The calculator gives this table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>22.3</td>
</tr>
<tr>
<td>0.01</td>
<td>22.03</td>
</tr>
<tr>
<td>0.001</td>
<td>22.003</td>
</tr>
<tr>
<td>0</td>
<td>ERROR</td>
</tr>
<tr>
<td>-0.1</td>
<td>22.7</td>
</tr>
<tr>
<td>-0.01</td>
<td>22.97</td>
</tr>
<tr>
<td>-0.001</td>
<td>ERROR</td>
</tr>
</tbody>
</table>

The above table can be rewritten as follows. Note the function values as \( x \to 0 \) from both the left and right.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-0.1</th>
<th>-0.01</th>
<th>-0.001</th>
<th>0</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_3 )</td>
<td>22.7</td>
<td>22.97</td>
<td>22.997</td>
<td>Error</td>
<td>23.003</td>
<td>23.03</td>
<td>23.3</td>
</tr>
</tbody>
</table>
Appendix F: (Continued)

The values in the table suggest that \( \lim_{x \to 0} y_3 = 23 \). That is, \( f'(4) = 23 \).

The benefit of doing the problem this way is that the table gives the average rate of change of that function on various intervals around \( x = 4 \). Students are able to determine the average rate of change of \( f(x) \) on various intervals around \( x = 4 \). Also, students notice that the average rate of change of \( f(x) \) on an interval approaches the instantaneous rate of change of \( f(x) \) as the intervals approach zero. The meaning of the values of \( y_3 \) in the table is as follows:

<table>
<thead>
<tr>
<th>The Average Rate of Change of ( f(x) ) Between Two Points</th>
<th>The Values of ( y_3 ) from Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Average rate of change of ( f(x) ) between ( x = 4 ) and ( x = 4.1 )</td>
<td>( \frac{f(4.1) - f(4)}{4.1 - 4} = 23.3 )</td>
</tr>
<tr>
<td>The Average rate of change of ( f(x) ) between ( x = 4 ) and ( x = 4.01 )</td>
<td>( \frac{f(4.01) - f(4)}{4.01 - 4} = 23.03 )</td>
</tr>
<tr>
<td>The Average rate of change of ( f(x) ) between ( x = 4 ) and ( x = 4.001 )</td>
<td>( \frac{f(4.001) - f(4)}{4.001 - 4} = 23.003 )</td>
</tr>
<tr>
<td>The Average rate of change of ( f(x) ) between ( x = 3.9 ) and ( x = 4 )</td>
<td>( \frac{f(3.9) - f(4)}{3.9 - 4} = 22.7 )</td>
</tr>
<tr>
<td>The Average rate of change of ( f(x) ) between ( x = 3.99 ) and ( x = 4 )</td>
<td>( \frac{f(3.99) - f(4)}{3.99 - 4} = 22.97 )</td>
</tr>
<tr>
<td>The Average rate of change of ( f(x) ) between ( x = 3.999 ) and ( x = 4 )</td>
<td>( \frac{f(3.999) - f(4)}{3.999 - 4} = 22.997 )</td>
</tr>
</tbody>
</table>
Example 2:

Given \( f(x) = \frac{2x}{x^2 + 1} \), find the rate of change of \( f(x) \) at \( x = 3 \).

From the definition of the derivative, the question is to find \( f'(3) = \lim_{h \to 0} \frac{f(3 + h) - f(3)}{h} \).

To use a calculator to find this limit, use \( h \) for \( x \) and let \( y_1 = f(3) = 2(3)/(3^2 + 1) \),
\( y_2 = f(3 + x) = 2(3 + x)/((3 + x)^2 + 1) \), and \( y_3 = (y_2 - y_1)/x \). Therefore, \( f'(3) = \lim_{x \to 0} y_3 \).

Press <Y> to enter the following:

\[
y_1 = 2(3)/(3^2 + 1)
\]
\[
y_2 = 2(3 + x)/((3 + x)^2 + 1)
\]
\[
y_3 = (y_2 - y_1)/x
\]

Activate only equation \( y_3 \).

Press the <2nd> and <GRAPH> keys to access the <TABLE> screen. Enter \( x \) values of 0.1, 0.01, and 0.001 to find the corresponding function values as \( x \to 0^+ \). Enter \( x \) values of -0.1, -0.01, -0.001 to find the corresponding function values as \( x \to 0^- \). The calculator gives this table.
The above table can be rewritten as follows. Note the function values as \( x \to 0 \) from both the left and right.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-0.1</th>
<th>-0.01</th>
<th>-0.001</th>
<th>0</th>
<th>0.001</th>
<th>0.01</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_3 )</td>
<td>-0.1637</td>
<td>-0.1604</td>
<td>-0.16</td>
<td>Error</td>
<td>-0.16</td>
<td>-0.1696</td>
<td>-0.1665</td>
</tr>
</tbody>
</table>

The values in the table suggest that \( \lim_{x \to 0} y_3 = -0.16 \). That is, \( f'(3) = -0.16 \).

Using the table, students can determine the average rate of change of \( f(x) \) on various intervals around \( x = 3 \). Also, students note that the average rate of change of \( f(x) \) on an interval approaches the instantaneous rate of change of \( f(x) \) as the intervals approach zero. The corresponding average rate of change of \( f(x) \) can be summarized as follows:
The Average Rate of Change of \( f(x) \) Between Two Points | The Values of \( y_3 \) from Table
---|---
The Average rate of change of \( f(x) \) between \( x = 3 \) and \( x = 3.1 \) & \( \frac{f(3.1) - f(3)}{3.1 - 3} = -0.1565 \)
The Average rate of change of \( f(x) \) between \( x = 3 \) and \( x = 3.01 \) & \( \frac{f(3.01) - f(3)}{3.01 - 3} = -0.1596 \)
The Average rate of change of \( f(x) \) between \( x = 3 \) and \( x = 3.001 \) & \( \frac{f(3.001) - f(3)}{3.001 - 3} = -0.16 \)
The Average rate of change of \( f(x) \) between \( x = 2.9 \) and \( x = 3 \) & \( \frac{f(2.9) - f(3)}{2.9 - 3} = -0.1637 \)
The Average rate of change of \( f(x) \) between \( x = 2.99 \) and \( x = 3 \) & \( \frac{f(2.99) - f(3)}{2.99 - 3} = -0.1604 \)
The Average rate of change of \( f(x) \) between \( x = 2.999 \) and \( x = 3 \) & \( \frac{f(2.999) - f(3)}{2.999 - 3} = -0.16 \)
Appendix F: (Continued)

B. Application: Finding the Rate of Change of a Function

Example:

According to the U.S. Labor Department, the number of temporary workers (in millions) is estimated to be

\[ N(t) = 0.025t^2 + 0.255t + 1.505 \quad (0 \leq t \leq 5) \]

where \( t \) is measured in years, with \( t = 0 \) corresponding to 1991.

(Source: Labor Department)

a) Find \( N(0) \), \( N(1) \), and \( N(3) \) and interpret the answers.

b) Find the average rate of change of \( N(t) \)

(i) between \( t = 1 \) and \( t = 5 \)

(ii) between \( t = 1 \) and \( t = 4 \)

(iii) between \( t = 1 \) and \( t = 3 \)

(iv) between \( t = 1 \) and \( t = 2 \)

and interpret the answers.

c) Find the instantaneous rate of change of \( N(t) \) at \( t = 1 \) and interpret the answer.

Answers:

a) To find the function values at different \( t \) values,

Press <Y> to enter the function as \( y = 0.025t^2 + 0.255t + 1.505 \).

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Appendix F: (Continued)

Then press the \texttt{<2nd>} and \texttt{<GRAPH>} keys to access \texttt{<TABLE>} and enter the given $x$ values of 0, 1, and 3 to obtain the following table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$Y_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.505</td>
</tr>
<tr>
<td>1</td>
<td>1.785</td>
</tr>
<tr>
<td>3</td>
<td>2.495</td>
</tr>
</tbody>
</table>

That is, $N(0) = 1.505$, $N(1) = 1.785$, and $N(3) = 2.495$. This means that the number of temporary workers are 1.505 million, 1.785 million, and 2.495 million in 1991, 1992, and 1994, respectively.

b) Recall the definition of the average rate of change of $N(t)$ between the two given $t$ values:

Average rate of change of $N(t)$ between $t = 1$ and $t = 1 + h$ equals

$$\frac{N(1 + h) - N(1)}{h}.$$

Let $y_1 = N(1)$, $y_2 = N(1 + x)$, and $y_3 = \frac{y_2 - y_1}{x}$.

Press \texttt{<Y>} to enter the following:
Appendix F: (Continued)

\[ y_1 = 0.025(1)^2 + 0.255(1) + 1.505 \]

\[ y_2 = 0.025(1 + x)^2 + 0.255(1 + x) + 1.505 \]

\[ y_3 = (y_2 - y_1)/x \]

![Graph showing equations](image)

Make sure that only equation \( y_3 \) is activated.

Press the \(<2nd>\) and \(<GRAPH>\) keys to access the \(<TABLE>\) screen. Enter \( x \) values of 4, 3, 2, and 1.

![Table showing \( y_3 \) values](image)

From the table, conclude that

the average rate of change of \( N(t) \) between \( t = 1 \) and \( t = 5 \) is 0.405,

the average rate of change of \( N(t) \) between \( t = 1 \) and \( t = 4 \) is 0.38,

the average rate of change of \( N(t) \) between \( t = 1 \) and \( t = 3 \) is 0.355, and

the average rate of change of \( N(t) \) between \( t = 1 \) and \( t = 2 \) is 0.33.
Appendix F: (Continued)

Interpretation of the results:

The average rate of change of the number of temporary workers during the years 1992 and 1996 is 0.405 million per year. The average rate of change of the number of temporary workers during the years 1992 and 1995 is 0.38 million per year. The average rate of change of the number of temporary workers during the years 1992 and 1994 is 0.355 million per year. The average rate of change of the number of temporary workers during the years 1992 and 1993 is 0.33 million per year.

c) Finding the instantaneous rate of change of \( N(t) \) at \( t = 1 \) is the same as finding \( N'(1) \).

From the definition of the derivative, \( N'(1) = \lim_{h \to 0} \frac{N(1 + h) - N(1)}{h} \).

Thus, \( N'(1) = \lim_{x \to 0} y_3 \). Using the same table, note the function values of \( y_3 \) by letting \( x \) approach 0.

Thus, the table suggests that \( \lim_{x \to 0} y_3 = 0.305 \) so that \( N'(1) = 0.305 \).

The meaning of \( N'(1) = 0.305 \) is that the rate of change of the number of temporary workers in 1992 is 0.305 million/year.
Appendix F: (Continued)

C. Student Activities For Lesson 4

Work Sheet #4: Finding the Derivatives of Functions with a Graphing Calculator

Find the derivative of each of the following functions at the given value. Use the
definition of the derivative of a function at \( x \), \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \).

1. \( f(x) = 2x^3 - x^2 - 10x + 25 \), find a) \( f'(-2) \) b) \( f'(3) \).

2. \( f(x) = 8x - 11 \), find a) \( f'(-3) \) b) \( f'(4) \).

3. \( f(x) = \frac{3x + 1}{x - 2} \), find a) \( f'(0) \) b) \( f'(2.4) \).

4. \( f(x) = \sqrt{x + 4} \), find a) \( f'(-2) \) b) \( f'(10) \).

5. \( f(x) = (3x - 4)^2 \), find a) \( f'(-2.5) \) b) \( f'\left(\frac{2}{3}\right) \).

6. Find the slope of the tangent line to the graph of the given function at the given point.
   a) \( f(x) = 2x^2 - 5x + 7 \), \( (2, 5) \)
   b) \( f(x) = \frac{x}{x + 2} \), \( (-1, 1) \)
   c) \( f(x) = \sqrt{x - 3} \), \( (4, 1) \)

7. Let \( f(x) = 3x^2 - 2x + 5 \).
   a) Find the average rate of change of \( f(x) \) with respect to \( x \) in the interval from \( x = 2 \) to \( x = 3 \), from \( x = 2 \) to \( x = 2.5 \), and from \( x = 2 \) to \( x = 2.1 \).
   b) Find the (instantaneous) rate of change of \( f(x) \) at \( x = 2 \).
Appendix F: (Continued)

8. A hot-air balloon rises vertically from the ground so that its height after \( t \) sec is
\[
h(t) = \frac{1}{2} t^2 + \frac{1}{2} t \text{ feet (} 0 \leq t \leq 60)\.
\]
   a) What is the height of the balloon at the end of 40 sec?
   b) What is the average velocity of the balloon between \( t = 0 \) and \( t = 40 \)?
   c) What is the velocity of the balloon at the end of 40 sec?

9. Lynbrook West, an apartment company, has 100 two-bedroom units. The monthly profit (in dollars) realized from renting \( x \) apartments is given by
\[
P(x) = -10x^2 + 1760x - 50,000.
\]
   a) Find \( P(50) \) and interpret the result.
   b) Find \( P'(50) \) and interpret the result.
Appendix G: Pilot Study Report

A pilot study was conducted for four and a half weeks in the fall of 2004 by the researcher to investigate the effects of using TI 83 graphing calculators (GCs) along with the researcher-developed instructional guided unit lessons on *limits*, *derivatives* and related applications of *limits* and *derivatives* in an Applied Calculus course at a community college.

The purpose of this pilot study was to evaluate the characteristics of the use of GCs along with the guided unit lessons and determine critical situations that might be encountered during the study. It was expected that the pilot study would help to explore whether the instruction with GCs along with the guided unit lessons could lead to better understanding on *limits*, *derivatives*, and related applications of *limits* and *derivatives* among community college applied calculus students. Furthermore, the pilot study was used to access the validity and reliability of all test instruments, the pretest and the two unit exams.

Procedure

To conduct this study, two intact sections of an Applied Calculus course (MAC 2233) at a community college were selected. The students were not randomly assigned. Thus, this was a quasi-experimental study to compare two instructional methods for solving problems in the following four areas: routine (skill-oriented) *limit* problems, related applications of *limits*, routine (skill-oriented) *derivative* problems, and related applications of *derivatives*. 
Appendix G: (Continued)

Design

A non-equivalent treatment-control group design was used in this pilot study. Dependent variables were ability to solve routine (skill oriented) limit problems, related application problems of limits, routine (skill oriented) derivative problems, and related application problems of derivatives. Variables were measured by researcher-made tests. The dependent variables were measured at two different times with a unit 1 exam and a unit 2 exam. It was expected that the treatment group would do better than the control group on all measures. The main independent variable was method of instruction.

Participants

Two researcher-taught intact applied calculus sections from a community college were used in the pilot study during the fall of 2004. Because the main study is planned for the spring of 2005, the researcher decided to use these two sections for his pilot study. One section, the Tuesday-Thursday (TR) section, met twice a week with each class meeting for 80 minutes from 9:30 – 10:50 a.m.; the other section, the Monday-Wednesday-Friday (MWF) section, met three times a week with each class meeting for 53 minutes from 9:00 – 9:53 a.m. Students were not randomly assigned to these two sections. That is, students who registered for these two sections knew who their instructor would be. Due to students’ work schedules and their schedules of other classes, certain sections at certain time periods have larger enrollments than other sections. At the beginning of the semester, 20 students registered for the TR section and 30 students registered for the MWF section. The TR section was chosen as the treatment group by the outcome of a flip of a coin; thus, the MWF section served as the control group.
Appendix G: (Continued)

Instruments

A pretest and two unit exams were used as instruments for this study. The researcher developed all three tests.

Pretest

A pretest was administered during the first week of the semester. Because the Basic College Algebra course is a prerequisite for the applied calculus course, a set of 20 questions on the content of this course was written. The purpose of the pretest was to determine whether students in the two classes were similar in their mathematical ability before they received any new instruction for the applied calculus course. The pretest is given in Appendix B.

Unit Exams

Two unit exams were used to measure students’ achievement on limits and derivatives. The first part of the unit 1 exam was on routine (skill oriented) limit problems and the second part was on related application problems of limits. The first part of the unit 2 exam was on routine (skill oriented) derivative problems and the second part was on related application problems of derivatives. Both unit exams are given in Appendix C.

To address the validity of the pretest and two unit exams, two experienced mathematics instructors from the same college inspected the tests and found the content of the pretest and two unit exams to be valid measures.
Appendix G: (Continued)

Treatment Phase

For the studied course, Applied Calculus, a textbook and a graphing calculator (GC) are required. The treatment group (GC-GL group) received instruction with the use of TI 83 GCs and the researcher-developed instructional guided lessons; the control group (GC group) received instruction with GCs but without the researcher-developed instructional guided lessons on limits and derivative sections. There were four unit lessons developed by the researcher to use in the pilot study (the entire lessons are given in Appendix F). Although a GC is required for this course, prior experience showed that many students come to this course with less experience and lack of familiarity with a GC. As a result, students constantly use this machine incorrectly and also interpret the calculator’s outcome incorrectly. The first two unit lessons were developed to meet students’ needs in order to use GCs correctly and efficiently.

The students in the Applied Calculus course spent the first week reviewing some algebra topics that they previously learned, including functions. The GC-GL group reviewed these sections along with instructional guided lessons 1 and 2 and the GC group reviewed the sections in a traditional manner. After the first week, both groups of students studied the limit topics for about one and a half weeks. The GC-GL students used the table feature of a TI 83 GC along with the instructional guided lesson 3 to solve limit problems, including applications; the GC students learned the same topic in a traditional way with the textbook. The control group did use GCs for this topic but were not supported by the guided lessons. At the end of the first treatment phase, both groups took the same unit 1 exam. This test had two parts and students were given 53 minutes to
Appendix G: (Continued)

complete the test. The first part of the test consisted of a set of routine (skill oriented) limit problems and the second part of the test consisted of a set of related applications of limits. Each part of the test had a maximum of 50 points. Each part was graded and scored separately.

Immediately after the unit 1 exam, both students learned derivatives of functions with the limit definition. The GC-GL group was taught to find the derivative of a function at the given $x$ value as a limit problem with the help of the instructional guided lesson 4. That is, the students in the GC-GL group were introduced to the derivative problems as “rate of change” problems; thus, they approached the derivative problems the same way they previously solved the limit problems. The GC group was introduced to the derivative with the same limit definition but they found the derivative algebraically in a traditional way and then substituted the given $x$ value at the end. Both groups learned the related applications of derivatives by whatever method they learned to find the derivative for the given functions. At the end of this treatment phase, both groups took the same unit 2 exam. This test also had two parts and students were given 53 minutes to complete the test. The first part of the test consisted of a set of routine derivative problems and the second part of the test consisted of a set of related applications of derivatives. Each part of the test had a maximum of 50 points. Each part was graded and scored separately. The treatment phase ended with the unit 2 exam and it took about four and a half weeks.

Exam Scoring

Multiple-choice questions were not used on the pretest or either unit exam. Therefore, students had to work the problems to answer the questions and students were
Appendix G: (Continued)

required to show their work. Because most questions had multiple steps in order to obtain the final answer, the researcher graded the pretest and the two unit exams using the following scoring rubric:

Table 28

<table>
<thead>
<tr>
<th>Point(s)</th>
<th>Guideline</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Correct work and Answer</td>
</tr>
<tr>
<td>1</td>
<td>Partially Correct work and Answer</td>
</tr>
<tr>
<td>0</td>
<td>Incorrect Work or No Response</td>
</tr>
</tbody>
</table>

To address the validity of the scoring rubric the same instructors who inspected the content validity of the tests reviewed the rubric and found that the content of the rubric was a valid measure. Also, a sample of 10 anchor papers chosen randomly from each of the three tests was used to test for inter-rater reliability. The selected papers were graded by the researcher and other faculty from the mathematics department. The faculty member was different from the participating two instructors in the main study but has extensive experience in this course. A total score for each student for each test from each grader was obtained and correlation coefficients were computed to obtain inter-rater reliability coefficients between the graders. The inter-rater reliability coefficients for the pretest, unit 1 exam, and unit 2 exam were .93, .98, and .95, respectively. According to Gall et al. (1996), the obtained inter-rater reliability coefficients supported the reliability of the scoring.
Appendix G: (Continued)

Data analyses and Results

Pretest

Descriptive statistics for the pretest are reported in Table 29. The mean comparison graph for the pretest is given in Figure 11. Also, a boxplot graph was obtained (see Figure 12) to see how the data of each group are spread out. Although the means of both groups were closer to each other, the control group had more variability than the treatment group. Also, a small effect size of 0.16 was obtained for the pretest.

Table 29

Descriptive Statistics for the Pretest of the Pilot Study

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>M</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>20</td>
<td>31.65</td>
<td>5.82</td>
<td>-1.06</td>
<td>2.32</td>
<td>0.16</td>
</tr>
<tr>
<td>Control</td>
<td>30</td>
<td>30.67</td>
<td>6.10</td>
<td>-.27</td>
<td>-.41</td>
<td></td>
</tr>
<tr>
<td>All Students</td>
<td>50</td>
<td>31.06</td>
<td>5.95</td>
<td>.22</td>
<td>-.55</td>
<td></td>
</tr>
</tbody>
</table>

Note: Maximum possible score on pretest was 60 points.

Figure 11: Mean Comparison for the Pretest for the Treatment and Control Groups
Then the means of these two groups were compared using a one-way ANOVA. An alpha level of .05 was used and the results reported in Table 30.

Table 30

One-way ANOVA for the Pretest of the Pilot Study

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between Groups</td>
<td>11.603</td>
<td>1</td>
<td>11.603</td>
<td>.323</td>
<td>.572</td>
</tr>
<tr>
<td>Within Groups</td>
<td>1723.217</td>
<td>48</td>
<td>35.900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1734.820</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ANOVA table confirmed that the groups were not statistically different, $F (1, 48) = 0.323, p = .572$. That is, there was no statistically significant difference between the groups in their mathematical ability prior to this course.
Appendix G: (Continued)

Unit 1 and Unit 2 Exams

Unit 1 and unit 2 exams were used to measure students’ achievements in limits and derivatives, respectively. The first part of each exam (maximum of 50 points) focused on students’ ability to solve skill-oriented problems and the second part (maximum of 50 points) focused on students’ ability to solve related application problems. Table 31 reports the descriptive statistics for each portion of both unit exams.

Table 31

Descriptive Statistics for Unit 1 and Unit 2 Exams of the Pilot Study

<table>
<thead>
<tr>
<th></th>
<th>Unit 1 Exam</th>
<th>Unit 2 Exam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control</td>
<td>Treatment</td>
</tr>
<tr>
<td>Skill Portion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>29</td>
<td>20</td>
</tr>
<tr>
<td>Mean</td>
<td>35.93</td>
<td>38.70</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.17</td>
<td>6.90</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>.11</td>
<td>-.31</td>
</tr>
<tr>
<td>Skewness</td>
<td>-.69</td>
<td>-.39</td>
</tr>
<tr>
<td>Application Portion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>29</td>
<td>20</td>
</tr>
<tr>
<td>Mean</td>
<td>31.17</td>
<td>36.50</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.66</td>
<td>6.91</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>-.15</td>
<td>.59</td>
</tr>
<tr>
<td>Skewness</td>
<td>.19</td>
<td>-.72</td>
</tr>
</tbody>
</table>

*Note:* Maximum possible score on each portion of both unit exams was 50 points.
Appendix G: (Continued)

The mean comparisons of each part of both unit exams for both groups were noted. The graphs of mean comparisons are given in Figure 13 and Figure 14.

![Figure 13: Mean Comparison for Unit 1 Exam for the Treatment and Control Groups](image1)

![Figure 14: Mean Comparison for Unit 2 Exam for the Treatment and Control Groups](image2)

As this pilot study compared the achievements in solving skill-oriented problems and application problems in limit and derivative topics, the researcher looked at Pearson
Appendix G: (Continued)

product-moment correlations between the pretest and each part of the unit 1 and unit 2 exams for the treatment and control groups. The results are reported in Table 32.

Table 32

<table>
<thead>
<tr>
<th>Exams</th>
<th>Pretest of the Treatment Group</th>
<th>Pretest of the Control Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Skill Portion of Unit 1 Exam</td>
<td>.86*</td>
<td>-.01</td>
</tr>
<tr>
<td>Application Portion of Unit 1 Exam</td>
<td>.91*</td>
<td>-.08</td>
</tr>
<tr>
<td>Skill Portion of Unit 2 Exam</td>
<td>-.24</td>
<td>.14</td>
</tr>
<tr>
<td>Application Portion of Unit 2 Exam</td>
<td>-.18</td>
<td>.24</td>
</tr>
</tbody>
</table>

Note: * p < .01

Then a number of ANCOVAs were used to compare the means of each part of both unit exams with the pretest as a covariate. An alpha level of .05 was used for all tests. First, the mean difference of the skill portion of the *limit* topic (unit 1 exam) between the control and treatment groups was examined. The results are reported in Table 33.
Table 33

**ANCOVA for the Skill Portion of Unit 1 Exam of the Pilot Study**

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>33.540</td>
<td>1</td>
<td>33.540</td>
<td>0.953</td>
<td>.334</td>
</tr>
<tr>
<td>Group</td>
<td>99.286</td>
<td>1</td>
<td>99.286</td>
<td>2.822</td>
<td>.100</td>
</tr>
<tr>
<td>Error</td>
<td>1618.522</td>
<td>46</td>
<td>35.185</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>69046.000</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Although the mean score of the treatment group ($M_T = 38.70$) is slightly greater than the mean score of the control group ($M_C = 35.93$), the ANCOVA table found no significant difference between the groups on the skill portion of the *limit* topic, $F = 2.822, p = .100$. Next, the mean difference of the application portion of the *limit* topic (unit 1 exam) between the control and treatment groups was examined. The results are reported in Table 34.

Table 34

**ANCOVA for the Application Portion of Unit 1 Exam of the Pilot Study**

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>29.205</td>
<td>1</td>
<td>29.205</td>
<td>0.953</td>
<td>.334</td>
</tr>
<tr>
<td>Group</td>
<td>349.943</td>
<td>1</td>
<td>349.943</td>
<td>9.064</td>
<td>.004</td>
</tr>
<tr>
<td>Error</td>
<td>1775.933</td>
<td>46</td>
<td>38.607</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>56630.000</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix G: (Continued)

The ANCOVA table found a statistically-significant difference between the achievement of the groups on the application portion of the *limit* topic, favoring the treatment group, $F = 9.064, p = .004$.

Next, the mean difference of the skill portion of the *derivative* topic (unit 2 exam) between the control and treatment groups was examined with an ANCOVA. The results are reported in Table 35.

Table 35

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>9.093</td>
<td>1</td>
<td>9.093</td>
<td>0.514</td>
<td>.478</td>
</tr>
<tr>
<td>Group</td>
<td>165.369</td>
<td>1</td>
<td>165.369</td>
<td>9.348</td>
<td>.004</td>
</tr>
<tr>
<td>Error</td>
<td>707.633</td>
<td>40</td>
<td>17.691</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>55405.000</td>
<td>43</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ANCOVA table found a statistically-significant difference between the achievement of the groups on the skill portion of the *derivative* topic, favoring the treatment group, $F = 9.348, p = .004$.

Finally, the mean difference of the application portion of the *derivative* topic (unit 2 exam) between the control and treatment groups was examined with an ANCOVA. The results are reported in Table 36.
## Table 36

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Df</th>
<th>Mean Square</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pretest</td>
<td>9.965</td>
<td>1</td>
<td>9.965</td>
<td>0.408</td>
<td>.526</td>
</tr>
<tr>
<td>Group</td>
<td>231.817</td>
<td>1</td>
<td>231.817</td>
<td>9.497</td>
<td>.004</td>
</tr>
<tr>
<td>Error</td>
<td>976.361</td>
<td>40</td>
<td>24.409</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>45522.000</td>
<td>43</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ANCOVA table found a statistically-significant difference between the achievement of the groups on the application portion of the derivative topic, favoring the treatment group, $F = 9.497$, $p = .004$.

### Further Research

To study the effect of using GCs along with the instructional guided lessons on students’ learning of limits and derivatives, a follow-up study is planned with a similar structure to that conducted in the pilot study. Two instructors other than the researcher will participate in the main study to reduce the chance of producing preferential instruction for the treatment group.
Appendix H: Student Questionnaire

Name (Optional): ____________________

Section (Circle One): MWF 8 – 8:53   MWF 11 – 11:53   TR 8 – 9:20   TR 12:30 – 1:50

Please answer the following questions to the best of your knowledge:

1. How did you study for the test?
   A. I studied by myself.  B. I studied with other student(s) who is (are) in this class.
   C. I studied with other student(s) who is (are) in a different class.
   D. I studied with a private tutor  E. Other (Please specify.): ______

2. Have you visited the math lab to get any help for this class?
   Yes: ___  No: ___

3. If you visited the math lab for help, how many times and how long did you spend in the lab?
   Total number of times visited: _____
   Total number of hours spent (approximately): ______

4. If you visited the math lab for help, was it helpful?
   Yes: ____  No: ____  Somewhat helpful: _____

5. If you received any help from the math lab or a private tutor, choose one of the following that fits the nature of the help you received:
   a. Math lab person(s)/Tutor helped me with the same methods that were discussed in my class.
   b. Math lab person(s)/Tutor helped me with the methods that were different from the class methods.
Appendix I: Course Implementation Log

Instructor: __________________

Textbook Section: ____________   Date: ________________

**Control Group** (MWF/TR)

1. In general, were proposed procedures for this section properly implemented? _______

2. If a problem occurred, circle the following that best categorizes the problem(s):
   
   A. Graphing calculator          B. Not enough time          C. Student apathy
   
   D. Student comprehension       E. Other (Please specify): __________________________

3. What was done in attempt to remedy the situation?

________________________________________________________________________

________________________________________________________________________

**Treatment Group** (MWF/TR)

1. In general, were proposed procedures for this section properly implemented? _______

2. If a problem occurred, circle the following that best categorizes the problem(s):

   A. Graphing calculator          B. Not enough time          C. Student apathy
   
   D. Student comprehension       E. Other (Please specify): __________________________

3. What was done in attempt to remedy the situation?

________________________________________________________________________

________________________________________________________________________
Appendix J: Student Demographic Information

Please fill out the information on this sheet to the best of your ability. All information obtained here and elsewhere in this study will remain strictly confidential.

1. Your Full Name: __________________________________________
   Last                       First                        Middle

2. Your Instructor’s Last Name: _________________________________

3. Time of Class Meeting: _____________________________________

4. Your Gender:         Male: _________            Female: ________

5. Your Student ID Number (“GOO” Number): ____________________

6. Your Home Address: __________________________________________
   Street
   __________________________________________
   City, State                                             Zip Code

7. Your Telephone Number: _______________________________________

8. Your Date of Birth: _____________________________________
   Month           Day         Year

9. Your Race (Optional): (Circle one that best describes you.)
   A. African American       B. Asian American       C. Caucasian
   D. Hispanic American      E. Native American      F. Non US Resident
   G. Other Group (Please indicate): _________________________
Appendix K: Consent to Participate in a Research Study

Date ________________

Dear Student Enrolled in MAC 2233: Applied Calculus,

I am very interested in students’ conceptual understanding in limit and derivative topics in Applied Calculus course, and I am conducting some research on this topic. The purpose of the research will be to examine the effect of using graphing calculators with numerical approach in limits and derivatives. To do this, I am asking you to do a pretest and two unit exams. The two unit exams are part of your normal course requirements. I will collect the scores from these three tests to analyze the performance. **Only your instructor and I will have access to the test scores with names attached, which will be kept in my office in the department of mathematics.** Additionally, authorized research personnel, employees of the Department of Health and Human Services and the USF Institutional Review Board may inspect the records from this research project.

There are no anticipated risks involved with your participation. Your participation is voluntary and will not have any effect (positive or negative) on your course grade. You will not be paid for your participation in this study. Results from the study may be shared with other teachers at professional meetings or in published resources for teachers, such as journal articles or books. **However, no actual names or any other information that would in any way personally identify you will ever be used. If needed, pseudonyms will be used.**

There are no direct benefits to you for participation in this research project, but the study may increase research knowledge related to students’ conceptual understanding in calculus.

If you have any questions regarding the research study, please contact me at *** or the Office of Research Compliance (813-974-5638).

If you are willing to participate in this study, please sign below. You may keep a copy of this letter for your records.

Sincerely,

Arumugam Muhundan
Appendix K: (Continued)

University of South Florida
Consent to Participate in a Research Study

I understand that I am being asked to participate in a research study entitled, “Effects of Using Graphing Calculators: A Numerical Approach to Limits, Derivatives, and Related Applications in an Applied Calculus Course at a Community College” By signing this form, I give permission for Arumugam Muhundan to use results from the test scores. I understand that my name will not be given. I understand that my participation is voluntary and will not have any effect (positive or negative) on my course grade.

By signing this form I agree that:
(a) I have fully read this informed consent form.
(b) I have had the opportunity to ask questions about this research project.
(c) I understand the risks and benefits, and I freely give my consent to participate in this research project.
(d) I have been given a signed copy of this informed consent form, which is mine to keep.

Print Name: ________________________________
Social Security Number: ______________________

Home Address: ______________________________

Signature: ________________________ Date: __________

Investigator Statement
I have carefully explained to the subject the nature of the above protocol. I hereby certify that to the best of my knowledge the subject signing this consent form understands the nature, demands, risks and benefits involved in participating in this study.

_____________________________ Arumugam Muhundan

Signature of Investigator Printed Name of Investigator Date

Institutional Approval of Study and Informed Consent
This research study and informed consent form were reviewed and approved by the University of South Florida Institutional Review Board for the protection of human subjects. This approval is valid until the date indicated below. If you have any questions about your rights as a person taking part in a research study, you may contact a member of the Division of Research Compliance at (813) 974-5638.
About the Author

Arumugam Muhundan received his Bachelor of Science in Mathematics from Eastern University in Batticaloa, Sri Lanka and his Master of Science in Mathematics from Florida Atlantic University in Boca Raton, Florida. He entered the Ph.D program at the University of South Florida in 1993.

Since 1990, Mr. Muhundan has been a member of the faculty at Manatee Community College in Bradenton, Florida and has made numerous contributions to his department and college. Mr. Muhundan currently is a member of American Mathematical Association of Two-Year Colleges and the Mathematical Association of America.