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Variance, Violence, and Democracy: A Basic Microeconomic Model of Terrorism

By John A. Sautter

Introduction

Much of the debate surrounding contemporary studies of terrorism focuses upon transnational terrorism. However, historical and contemporary evidence suggests that domestic terrorism is a more prevalent and pressing concern. A formal microeconomic model of terrorism is utilized here to understand acts of political violence in a domestic context within the domain of democratic governance.

Terrorism is the use of violence and threats to intimidate or coerce for political purposes.[1] Unlike purely conventional military conflict where two opposing armies meet on a battlefield, terrorism focuses violence directly on civilians to affect political change through altering public opinion.[2] Within the modern day context, terrorism as a calculated political tool must be thought of within democratic institutions, and for good reason. History shows not only that an insurgent or terrorist organization can influence electoral outcomes, but that as a society becomes more democratic rates of terrorism tend to increase.[3]

This article builds a very basic microeconomic model of terrorist decision making to hypothesize how a democratic government might influence the sorts of strategies that terrorists use. Mathematical models have been used to explain terrorist behavior in the past.[4] However, the bulk of inquires in this area have only focused on the relationship between terrorists and the government, or amongst terrorists themselves.[5] Central to the interpretation of the terrorist conflict presented here is the idea that voters (or citizens) are also one of the important determinants of how a government will respond to acts of terrorism.

Domestic Terrorism and Democracy

A major focus of terrorism studies, especially since September 11, 2001, has been on ideologically motivated, transnational terrorism. However,

whether one views this topic historically or in the contemporary context, transnational and religiously motivated incidents are much less frequent than might at first be obvious. An examination of the ITERATE collection of acts of terrorism during the past thirty years shows that a majority of incidents was neither international in scope nor initiated by religiously motivated groups.[6] Furthermore, most terrorist acts over this period were caused by individuals indigenous to the state in which they occurred, with the victims being largely from those states as well.[7] Without a doubt, domestic terrorism is as important as the transnational case.

There are distinct features that set domestic models of terrorism apart from transnational ones. First, the idea that the victim and the terrorist are from different populations, or countries, is either the assumption made in an international focus, or the issue is completely ignored. In domestic terrorism it is often the case that clandestine groups recruit from all parts of society and share the same nationality with those that they are attacking.[8] Second, transnational models of terrorism rarely, if ever, model the main tool of violent political groups, terror itself. It is often treated as an intangible variable that cannot be accounted for.[9] Indeed, in the transnational context it may very well be impossible to place such a concept into the workings of an economic model, however important it may be to policy outcomes. Finally, asymmetric information and signaling is very different in each.[10] Domestic terrorism, as conceived here, is a method of signaling used by a political dissident to attempt to directly sway a citizenry, whereas transnational terrorism is usually conceived as indirect influence.

Research has demonstrated the importance of democratic institutions in explaining the frequency of terrorism. The prevalence of democratic institutions has a positive relationship with incidences of terrorism.[11] This is generally ascribed to liberal rules and freedoms under a democratic regime, which allow terrorist organizations to organize freely. Indeed, evidence suggests that as government institutional obstruction to democratic expression decreases, so do incidences of terrorism.[12] One way to think of this is by understanding that individuals will be more likely to take advantage of the political process as opposed to violent opposition to it when the costs of political participation are much lower.[13] When there are fewer obstructions to participation, its costs are inherent lower.

The model developed below assumes three different agents. There are terrorists (t), voters (v) and politicians (g).[14] Terrorists aim to commit illegal acts to further their political beliefs. Voters want to maximize safety and stability. Politicians desire re-election and will optimize government anti-terrorism policy in order to maximize re-election. A democratic system of

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government with direct election for representative government is assumed in the model. In short, terrorists attack voter-citizens, voters decide their government, and the government adopts a policy toward terrorists.

Terrorist Utility

$$T = T(D, \pi, \overline{G}) - C(D^*) \quad (1)$$

A terrorist's utility is a function of three different variables. Where D is some amount of damage that a terrorist would like to commit and D^* is the chosen amount of damage that they will attempt. The cost constraint $C(.)$, is a function of the chosen damage that the terrorists will attempt to commit. π is the probability of successfully completing the intended act or acts of terrorism. $\overline{G}$ is a proxy for the mean government policy in the past. Government policy includes counterterrorism efforts, implementing security measures or any other government activity that was implemented to retaliate against or protect from terrorist activities. We assume that $\frac{\partial T}{\partial D}$ and $\frac{\partial T}{\partial \pi}$ are both > 0, or that a terrorist's utility is increasing in both the amount of damage that they are able to inflict, as well as, the probability of success. However, $\frac{\partial T}{\partial G} < 0$, or that as the government policy toward terrorists is increased, a terrorist will get disutility.

Finding the first order condition:

$$\frac{\partial T(D, \pi, \overline{G})}{\partial D} = C'(D^*) \quad (2)$$

The first order condition shows that a terrorist is constrained by the cost of their chosen amount of damage. D^* is the theoretical combination of terrorist acts, D_i, (i = 1,2,...), that amount to different ways of committing violence, or,

$$D^* = \sum_{i}^{m} D_i(G(\overline{D}), \pi) \quad (3)$$

A terrorist will maximize his utility by choosing the optimal D^* that solves equation (2) given a certain probability of success and the current government policy toward terrorism. The government policy from the terrorist perspective is a function of the mean of all past violent activities, $\overline{D}$.

Voter Utility

$$V_i = V(D^*, \pi, G, \rho, \theta(u, D^*)) \quad (4)$$

Voter, or citizen, utility is some function of the same variable inputs: D^*, π, G. Rho is a theoretical parameter that measures the rate at which voters discount past acts of terrorism over time, where $0 \leq \rho \leq 1$.[15] This models the behavior that a voter weights the future much more than the past. Prior to the terrorist attacks on September 11, 2001, most Americans did not perceive a terrorist threat, though the risk of a successful attack was obviously high. Conversely, the overhaul of airline security regulations, a new emphasis on the threat posed by terrorists in governmental intelligence organizations and the establishment of a Homeland Security Administration have significantly reduced the danger of future terrorist attacks that abounded before 9/11. However, the public perceives that the risk of future attacks is in fact higher. Voters are assumed to place more emphasis on future risks than they do on the whole for past events.

As a stochastic variable, u represents all other things in society that could become more important to a voting public than a threat posed by terrorism, including economic conditions, erosion of civil liberties or corruption in government. The parameter $\theta(.)$ is some function of u between zero and one, so that $\frac{\partial \theta}{\partial u} < 0$ and $\frac{\partial^2 \theta}{\partial D^* \partial u} > 0$. The first term illustrates that when damage is held constant other issues in the political atmosphere begin to outweigh the importance of inputs D^*, π, G in the voter utility function, sending $\theta \rightarrow 0$. However, the second term denotes that there is a positive second derivative with respect to D^* and u, showing that theta increases when there is a simultaneous increase in both D^* and u. This makes θ increasing in damage even if other political issues are concurrently in the political debate. It is assumed that in the absence of an increase in D, u, will rise over time as other issues enter into the political landscape. For the purposes of explaining the model, θ is set to one in equation (4) so that a focus can be placed on the variables D^*, π, G.[16]

Voter maximization is also affected by a chance constraint that takes into account the probability of damage.

$$\Pr\{D^* \geq \overline{G}(\overline{D})\} \geq \pi \quad (5)$$

Equation (5) is a probability density function that ensures that the probability of receiving positive damage: $D^* - \overline{G}(\overline{D}) > 0$, is greater than or equal

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to π. In other words, the terrorist act is greater than the historic mean of government policy, thereby incurring damage. D^* is a random variable that, while in the voter utility function, represents the amount of economic damage or violence that terrorists have committed during the current round of attacks. Voters have information regarding the historic mean and standard deviation of D^*since they are aware of the terrorist's density function and the variance of their past acts of violence. While past violent acts of terror provide information to voters on the scope of D^*, it should only be regarded as a theoretical general guide to them. For example, if one knows that in the past suicide bombers have ridden buses, this informs one on the inherent risks of riding the bus. However, this general knowledge does not inform one of the specific time, day and place of an act of terror that is random for the typical citizen.

In the chance constraint, $\frac{\partial D^*}{\partial \pi} < 0$, indicating that as the amount of damage increases, the probability of success decreases. This follows from the fact that generally attacks that kill a larger number of people or inflict more economic damage are normally more complex, costly and involve a higher probability of failure. Equation (5) can be re-written as:[17]

$$[\Pr\{D^* \geq \overline{G}\} \geq \pi] \approx \sigma^{D^*}\gamma(1 - \pi) + \mu^{D^*} - \overline{G}(\overline{D}) \qquad (6)$$

Which combined with equation (4) gives the following voter utility function:

$$V_i = V(D^*, \pi, G; \rho, \theta(u)) + \overline{G}(\overline{D}) - \sigma^{D^*}\gamma(1 - \pi) - \mu^{D^*} \qquad (7)$$

Where $\sigma^{D^*}$and $\mu^{D^*}$are the standard deviation and mean of D^*; and $\gamma(1 - \pi)$ is a function that delineates the number of standard deviations that D^* must be from $\overline{D}$, the mean of all past damage, in order to satisfy the constraint.

Voters are interested in safety and national security. Though D^*, π, G, are exogenous to the voter's utility function, they are seen as maximizing utility when the government pursues a policy, G, that provides for the most stability to society. Following a terrorist attack, voters will evaluate the government based on their perceptions of how the administration responded to the attack. Therefore, voters feel the effect of the following maximization problem:

$$\max_{G} \quad V_i = V(D^*, \pi, G, \rho, \theta(u)) + \overline{G}(\overline{D}) - \sigma^{D^*}\gamma(1 - \pi) - \mu^{D^*} \qquad (8)$$

Solving for the first order condition in equation (8):

$$\frac{\partial V_i}{\partial G} = \frac{\partial V}{\partial D^*} \cdot \frac{\partial D^*}{\partial G} + \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial G} + \frac{\partial V}{\partial G} - \frac{\partial \sigma^{D^*}}{\partial G} \gamma'(1 - \pi) - \frac{\partial \mu^{D^*}}{\partial G} + \frac{\partial \overline{G}}{\partial \overline{D}} \cdot \frac{\partial \overline{D}}{\partial G} = 0 \quad (9)$$

Equation (9) describes the effect on voters' utility with a change in government policy in response to a terrorist attack. For this example, theta is presumed to be equal to one.

Each term describes how a voter reacts to the damage perpetrated by terrorists in light of the government response to the damage and can be interpreted as either an expression of sensitivity to the terrorist violence (alpha) or as a sensitivity to the government response (beta). Accordingly,

$$\alpha_i = \frac{\partial \sigma^{D^*}}{\partial G} \gamma'(1 - \pi) + \frac{\partial \mu^{D^*}}{\partial G} \quad (10)$$

and,

$$\beta_i = \frac{\partial V}{\partial D^*} \cdot \frac{\partial D^*}{\partial G} + \frac{\partial V}{\partial G} + \frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial G} + \frac{\partial \overline{G}}{\partial \overline{D}} \cdot \frac{\partial \overline{D}}{\partial G} \quad (11)$$

Equation (10) is the summation of partial derivatives that equal alpha or the sensitivity to terrorist violence, and equation (11), beta, represents the terms that are equal to the sensitivity to the government response. The difference in alpha and beta, $\beta_i - \alpha_i = 0$, will always be equal to zero. If it did not equal zero, then the government can be understood as not responding to the political violence in a proportionate and effective manner. Therefore in this model, it is assumed that the government will continue to play the game round to round, and will always have a response. The significance of this measure is where these two sides of voter utility meet in policy space. As shown in Figure 1 and Figure 2 below, the important feature of this model is its ability to show why terrorists use the methods that they choose. By expanding the variance and decreasing their mean level of violence, terrorists can decrease government response and increase their signaling potency on the average voter or citizen.

Terrorists are attempting to cause fear and panic in order to sway citizens into pressuring the government into making political concessions. Each one of the expressions in alpha causes negative utility for the voter. The first term, $\frac{\partial \sigma^{D^*}}{\partial G} \gamma' < 0$ is a kind of "terror" term that reflects how an increase in the standard deviation of acts of damage changes with respect to government policy. It illustrates the disutility that a voter receives from an increase in the randomness of violence. The second term, $\frac{\partial \mu^{D^*}}{\partial G} < 0$ represents a "strength" of terror term and depicts the change in the mean level

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of violence associated with terrorism. All of the expressions in the α equation are decreasing in government policy.

The β equation models the voter sensitivity to the government policy. The first term, $\frac{\partial V}{\partial D^*} \cdot \frac{\partial D^*}{\partial G} < 0$, can be called the "violence" term and represents the negative utility that voters receive from the act of damage and the government's initial response to the violence. The next term, $\frac{\partial V}{\partial \pi} \cdot \frac{\partial \pi}{\partial G} > 0$, demonstrates that voters' gain utility from seeing the terrorists' probability of success decrease as the amount of government policy toward terrorism increases. $\frac{\partial V}{\partial G} > 0$, is a "safety" term. Voters garner positive utility from the reassuring feeling and stability that an increase in government policy brings to society. The final expression in the β equation, $\frac{\partial \overline{G}}{\partial \overline{D}} \cdot \frac{\partial \overline{D}}{\partial G} > 0$, is an "updating" term that represents the way voters garner utility from seeing the government change its policy to match the new threat posed by an increase in the $E(D^*)$, or the new expected future value of damage.

Government Utility

$P = P(\varepsilon; \delta)$

$$\text{Where: } \varepsilon = \varepsilon\left(\sum_i^n \alpha_i(D^*, \pi, G), \sum_i^n \beta_i(D^*, \pi, G, \rho, \theta)\right) \qquad (12)$$

The government utility function is only concerned with getting re-elected. The incumbent government will optimize G, given D^* and π, which are both exogenous to $P(.)$, so as to maximize ε, the chance of re-election. Delta is a parameter that represents the rate that office holders discount the future, where $0 \leq \delta \leq 1$.[18] Unlike voters, who place more emphasis on the future, politicians are assumed to place more importance on the present. Indeed, if not elected in the forthcoming round of elections, the government will have no say in the policy toward terrorists anyway. In effect, the sum of voters' utility is nested within the government's utility function and expressed by the way that voters feel toward the government. The parameter "n" is any hypothetical number of voters in a state and "i" represents each individual voter.

Government utility maximization is also constrained by a chance constraint that takes into account an incumbent's chances of getting re-elected.

$$\Pr\{\sum_{i}^{n}\beta > \sum_{i}^{n}\alpha\} \geq \varepsilon \quad (13)$$

Equation (13) is a condition that ensures the government will maximize its counterterrorism policy in an attempt to make the following equality hold: $\beta_i - \alpha_i = 0$. The government desires to make ε, their chance at re-election, equal to one. In other words, the current administration desires to maximize the probability of staying in office. This is dependent on their ability to defeat the terrorists. If the equality stated above does not hold, the government is seen to have lost its ability to fight the terrorists and would thus not be re-elected to office.

Interpreting the Model

An increase in damage only changes the government crackdowns by a marginal amount as compared to a change in the level of α,[19] which would bring a similar change in G and D. An increase in the probability of success for the terrorists arises from an increase in variance of violence, shifting Beta away from the $G = D^*$ line. The flatter the slope of the Beta expansion path, or the higher proportion of damage D compared to the government policy of $G(D^*)$, the higher the probability that violence will succeed with a smaller crackdown from the government. Alpha represents a constrained probability of success, which is inherently determined by the choices of the terrorists in the strategy they will use. The decision to pursue easier targets versus more complex and dangerous ones alters the level of the alpha expansion path in the policy space.

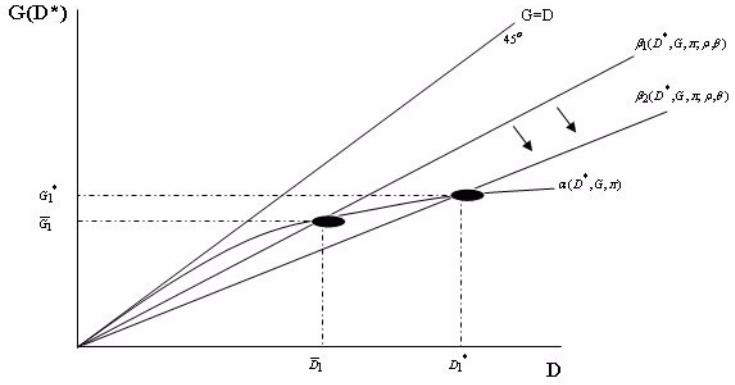

Figure 1: Graph showing the effect of an increase in variance of violent acts in policy space.

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A change in the mean level of violence will raise alpha, increasing *D*. However, notice that under this form of damage policy the terrorists can expect a substantially larger crackdown from the government than if they had increased their variance. Also, notice that a higher mean lowers the probability of success, which would eventually begin to push the beta line toward the $G = D^*$ line, making the government reaction even closer to being the same as the increase in damage. Or, in terms of Figure 1 and Figure 2, we would say that:

$D_1^* - \overline{D}_1 > D_2^* - \overline{D}_2$ and, $G_2^* - \overline{G}_2 > G_1^* - \overline{G}_1$

Terrorists will desire to keep their mean level of violence low, so as to avoid a costly government attack. There will be a better chance of committing more violence and terror on the general public by adopting a strategy that increases the variance of attacks while decreasing its mean level.

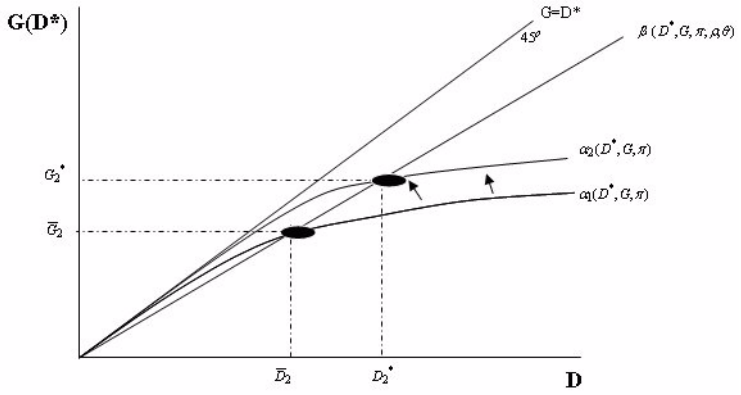

Figure 2: Graph showing the effect of an increase in the mean level of violence in policy space.

Conclusion

Democratic governments should deal with terrorists early and with impunity before they are allowed to gather enough resources for a larger attack. This may seem obvious. However, there is not always the necessary public support to provide resources for a police action against insurgents or to enact the necessary security laws to avert future attacks. What this model suggests is that the government should undertake such actions even when

there is little public support. This conclusion is somewhat at odds with the notion of democracy being a form of government that protects civil liberties. In many respects, this is the paradox of domestic terrorism and democracy: how to provide security for a citizenry when some of those you wish to protect are hostile to their own government.

The connection between electoral government and terrorism is important. The study of how each party involved in the three-way relationship depicted here reacts to the actions of the other two parties can provide insight into how best to manufacture policies aimed at curbing terrorist incidents. One of the most striking results of this investigation is the basic realization that there are trade-offs in a democratic system that a terrorist or insurgent group must make between large attacks which draw the scorn of the public, and thereby the government retribution; as compared to small, more varied, attacks that harass the populace but do not create a large government backlash.

About the Author

John Sautter, Ph.D., is an adjunct professor at Green Mountain College, Poultney, Vermont, and a Captain in the United States Marine Corps. His qualifications include: B.A., New York University, 2001; M.A., University of Nebraska, 2002; Ph.D., University of Nebraska, 2005; J.D. Vermont Law School, 2008; L.L.M. Vermont Law School 2009. The views herein should not be attributed to any of the author's institutional affiliates. The author can be contacted at: *jas276@nyu.edu*.

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14 Though a democratic regime is assumed in the explicit analysis this assumption could easily be relaxed for an interpretation of the authoritarian case, where citizens' approval of the regime would parallel the voters' electoral decision.

15 ρ would appear in an individual voter's utility function in the following form: $\rho = (\frac{1}{1-r})$. It is assumed that $\frac{\partial \rho}{\partial D} < 0$, or that the larger amount of damage terrorists inflict, the less voters discount the past act of violence.

16 It is also assumed that the second derivative with respect to damage is positive, or $\frac{\partial^2 \theta}{\partial D^{*2}} > 0$. This condition allows theta to increase at an increasing rate when damage occurs, making larger amounts of damage disproportionately more harmful than smaller ones.

17 Pr $\{D^* \geq \overline{G}\} \geq \pi$, therefore if we treat both G and D as normalized random variables we get the following, Pr $\{\frac{D^* - \mu}{\sigma} \geq \frac{\overline{G} - \mu}{\sigma}\} \geq \pi$, where $\frac{D^* - \mu}{\sigma}$ is a normalized random variable with a distribution of some function F= $F[\frac{D^* - \mu}{\sigma}]$. Therefore, it follows that we can also say that $F^{-1} = k$. Therefore, our equation can be expressed as $1 - k[\frac{\overline{G} - \mu}{\sigma}] \geq \pi$, which rearranging once again gives us the following, $[\frac{\overline{G} - \mu}{\sigma}] \geq k(1 - \pi)$. Finally, through a little algebra we can express the constraint in the following manner: $\sigma.k(1 - \pi) + \mu - \overline{G} \geq 0$, which allows for the voter utility function to be constrained by the terrorists' manner and probability of success.

18 The discount parameter delta would appear in the office holder's utility function in the following form: $\delta = (\frac{1}{1 + r})$.

19 The α expansion path is increasing at a decreasing rate because of the nature of the relationship between the standard deviation and mean of damage. Since, $\frac{\partial \mu}{\partial \sigma} > 0$, or as the mean increases so does the absolute value of the standard deviation of violence. However, the second derivative is negative, $\frac{\partial^2 \mu}{\partial \sigma^2} < 0$, because as the mean level of damage increases, its probability of success decreases. Conversely, the β expansion path (the government) is linear and additive.

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