CAN GLACIER IN ICE CAVE CUT U-SHAPED VALLEY - A NUMERICAL ANALYSIS

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Abstract
U-shaped valleys are widely found in ordinary glaciers. There are a few huge ice caves in which ice body extends nearly 1km and becomes a glacier in the cave. Can glacier in an ice cave cut the cave and form a U-shaped valley? We use numerical simulation method to study the possibility if a U-shaped valley could exist or not in an ice cave before carrying out costly field work. Our calculation does simulate the formation of U-shaped valley in ordinary glaciers, but the ice cave glacier is too small in size, so that it is not likely to form U-shaped valleys in the lifetime of the cave. Water erosion is the decisive factor in producing the cave.

Introduction
A U-shaped valley, with steep sides and a flat bottom, is formed by the erosion of alpine glaciation (Johnson, 1970; Boulton, 1974). There are a few huge ice caves in which ice body extends nearly 1km and becomes a glacier in the cave, such as Eisriesenwelt Ice Cave in Austria, the largest ice cave of the world. Flow structures both basal and internal can be observed (Hausmann and Behm, 2011). Can glacier in an ice cave cut the cave and form a U-shaped valley? It is an interesting question. The bottom of an ice cave is covered by ice. Geophysical methods could be used to explore the base of glacier, but it usually is not an easy job. Therefore, we use numerical simulation method to study the possibility if a U-shaped valley could exist or not in an ice cave before carrying out costly field work.

Methods
We use a Cartesian coordinate with x-axis across the glacier, z along the glacier, and y perpendicular to the x-y plane pointing up (Figure 1a). Figure 1b is longitudinal glacier profile. α is downglacier slope. We developed a 3D ice flow model coupled with an erosion process. The flow model calculates the velocity field by solving the continuity equation (1) and the Stokes equation (2) for viscous flow.

\[
\text{div}(\vec{v}) = 0 \quad (1)
\]

\[
\frac{\partial \sigma_{ij}}{\partial x_j} - \frac{\partial P}{\partial x_i} + \rho g_i = 0
\]

\[
\sigma_{ij} = 2\eta \varepsilon_{ij} \quad (2)
\]

Where \(\vec{v}\) is the velocity vector; \(i\) and \(j\) are coordinate indices; \(x_i\) and \(x_j\) are spatial coordinates; \(\sigma_{ij}\) is deviatoric stresses tensors; \(P\) is pressure; \(\rho\) is density; \(g_i\) is the i-th component of the gravity vector. For the first order calculation, Newtonian law of viscous friction showed as equation (3) is used.

\[
\sigma_{ij} = 2\eta \varepsilon_{ij} \quad (3)
\]

Where \(\eta\) is viscosity; \(\varepsilon_{ij}\) is the deviatoric strain rate tensors. Glacial flow is treated as a non-Newtonian fluid.
for the erosion law represents the general form of the abrasion law proposed by Hallet (1979). In this paper, erosion rate normal to the bedrock surface was calculated as equation (6). Where \( C \) is an erosion constant equal to \( 10^{-4} \) a m\(^{-1}\) (Harbor, 1992).

\[
E = C u_b^2
\]  

Result - The formation process of an ordinary U-shaped valley

We prescribe a V-shaped cross section with maximum ice thickness of 480m, surface width of 1200m, and downglacier slope of 4° (Figure 2) as the initial glacier and valley geometries. We employ a 3D FEM (Finite Element Method) to solve the Stokes equation with Newtonian law of viscous friction.

![Figure 2. Glacial valley evolution from an initial pre-glacial profile computed with the 3D model for different times.](image)

The complex nature of the glacial erosion has not allowed the development of physically complete models for processes such as glacial abrasion, plucking. But we can couple the ice flow model with an erosion model by introducing a quadratic function of the sliding speed for the calculation of the erosion rate. This assumption for the more exacter calculation, and Glen’s flow law is used as a constitutive relation. Glen’s flow law is showed as below:

\[
\dot{\varepsilon} = A \sigma^n
\]  

Where \( \sigma \) is dominant shear stress; \( \dot{\varepsilon} \) is corresponding shear strain rate; \( A \) is flow parameter; \( n \) is creep exponent.

At the glacier base \((y = GB(x))\), we introduce the basal sliding by linearly relating the sliding speed \( u_b \) to the shear stress acting on the bed \( \tau_b \) (Weertman, 1964),

\[
u_b = -c \tau_b = -c \left( \frac{\partial GB}{\partial x} \tau_{x} + \tau_{y} \right) / \sqrt{1 + \left( \frac{\partial GB}{\partial x} \right)^2}
\]  

The complex nature of the glacial erosion has not allowed the development of physically complete models for processes such as glacial abrasion, plucking. But we can couple the ice flow model with an erosion model by introducing a quadratic function of the sliding speed for the calculation of the erosion rate. This assumption for the formation process is shown in Figure 3. Figure 3a, the three graphs in the left column, shows \( V_y, V_z \) and erosion rate in the valley respectively at time t = 10ka. The velocity at both
edge of the V-shaped valley is most small, and velocity reach maximum at the center upper part of the glacier. The velocity gradient, however, is largest at the central part of the two edges, and smallest at the bottom of the V-shaped valley and the edge at the top of glacier. The velocity gradient is proportional to the shear strain rate and shear stress. Therefore, largest erosion rate occurs at the central part of two edges of the valley as shown in the bottom figure of Figure 3a. Difference between erosion rates at the center of edge and that at the bottom at the valley prompt the transition from the initial V-shaped valley gradually to a U-shaped valley after dozens of kiloyears.

Figure 3b, three graphs in the right column shows similar figures at the time $t=30$ka when the valley approaches a U-shaped valley. The overall pattern of velocity distribution of $V_y$ and $V_z$ are similar to those at time 10ka. However, the velocity gradients are different. The difference in velocity gradient between the bottom and edges becomes more and more small, so are the erosion rates as shown in bottom of Figure 3b. At $t=10$ka, the maximum erosion rate is nearly 60 mm/a at the central part of valley edges, but only 10 mm/a at the valley bottom. The difference is 50mm/a. However, at $t=30$ka, this difference is reduced to less than 20mm/a. If the erosion continues, at $t=60$ka the difference is only

Figure 3. (a) (b) Cross-glacier variation of $V_y$, $V_z$ (Unit: m/s) and erosion rates for times $t=10$ and 30 ka, respectively.
about 3 mm/a. That means, the U-shape is stabilized, and the valley cut deeper and deeper, but the U-shape keeps almost unchanged at this stage. These results are in agreement with 2-D analysis of Seddik, Greve et al., 2009.

**Results - The erosion process in an ice cave**

Glaciers in ice caves are much smaller in size in comparison with ordinary glaciers. We prescribe a V-shaped cross section with maximum ice thickness of 8 m, surface width of 20 m, as the initial glacier and valley geometries. Models of downglacier slope of 4° and 40° are calculated for comparison.

Figure 4 shows the cross-glacier variation of $V_y$, $V_z$ and glacial shape at time $t=50,000$ years. The left column is for $\alpha=4^\circ$ and the right column is for $\alpha=40^\circ$ respectively. Although the patterns of velocity distribution are similar to the large size glacier of Figure 3, the magnitude of velocity is much smaller. For $\alpha=4^\circ$ case, the largest velocity is as well as velocity gradient are only about $3 \text{ mm/a}$.

**Figure 4.** Cross-glacier variation of $V_y$, $V_z$ (Unit: m/s) when $t=50,000$ year; Glacial valley evolution from an initial pre-glacial profile. a. $\alpha = 4^\circ$; b. $\alpha = 40^\circ$
1/1000 of those of the large scale glacier of Figure 3. The erosion rate in ice cave is smaller about 50,000 times than erosion rate of U-shaped valley. In another word, if it is need 1 ka, an enough small time, to form a U-shaped valley of ordinary glacier, it has to take about 50 Ma to form a U-shaped valley in ice cave. It is a time too long for the life of cave. For the case of $\alpha=40^\circ$, both the velocity and the velocity gradient are 10 times greater than that of $\alpha=4^\circ$. However, the erosion rate is still too small for U-shape valley formation in the life time of a cave.

**Discussion and Conclusion**

The size of glacier is very important for obtain enough erosion rate to form U-shaped valley. In an ice cave glacier, the size is too small to reach sufficient erosion rate to form U-shaped valley. Before the ice cut the U-shaped valley, the cave may have been reformed by actions of leakage water. Steep slopes may increase erosion rate, but still too small for ice cave glacier.

From the numerical analysis, it is suggested that U-shaped valley is not likely to be able to form in ice caves.

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**References**
