Probabilistic Assessment of the Friendship Trail Bridge

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PROBABILISTIC ASSESSMENT OF THE FRIENDSHIP TRAIL BRIDGE

FINAL REPORT

Sponsored by:

Hillsborough County

Research Team

Rajan Sen, Ph.D., P.E., Gray Mullins, Ph.D., P.E.,
and Niranjan Pai, Ph.D. P.E.

August 2011
Executive Summary

Description
The Friendship Trail Bridge is the current name of the old Gandy Bridge which carried westbound traffic across Old Tampa Bay until 1995. It was designed to H20 truck loading and was constructed in 1956. The 2.6-mile long bridge has 274 spans, of which 252 are low-level 48ft spans. The typical 48ft span consists of a 30ft 7in wide low-level bridge deck supported by four 3ft 4in deep post-tensioned concrete girders that act compositely with the 7in thick deck slab and are spaced 8ft 6in apart. The four girders are each post-tensioned with four 1 1/8in diameter, Grade 160 post-tensioning (PT) bars. Two of these bars are straight and located in the bottom flange, while the other two have a parabolic profile. Partial-depth post-tensioned diaphragms tie the girders together at third points (16ft on center). The girders resist shear load through the parabolic tendons and do not have additional shear reinforcement.

The bridge was to be demolished when the new Gandy Bridge was opened, however actions from citizens of Hillsborough and Pinellas counties resulted in the two counties assuming joint ownership of the bridge in 1997 and making the bridge available for recreational use.

Condition of Bridge
A bridge inspection by KCA documented severe corrosion induced deterioration to the bridge. These include significant longitudinal cracking on the girder web and soffit along the path of the post-tensioning bars. In addition, post-tensioning bar breakages and multiple concrete spalls were also observed.
The main cause of the observed deterioration in the bridge is corrosion of the post-tensioning bars. Corrosion occurs in such concrete structures once the chloride from salt water diffuses through the concrete cover and reaches the steel.

Objective of the Study
The objective of the study was to determine the probability of collapse of the superstructure of a typical 48 ft span over the next 20 years under its own self-weight and pedestrian loading during which period no repairs are undertaken. The study did not involve any actual inspection of the bridge. Therefore, parameters used in making the probabilistic assessment were taken from the published literature.

Corrosion Assumptions
Corrosion is characterized by two types of deterioration, uniform corrosion and pitting corrosion. Uniform corrosion refers to situations where there is uniform loss of steel section. In this study, the rate of loss of steel diameter is assumed to be 6 mils/year based on data found in the literature. Pitting corrosion refers to localized corrosion where a part of the bar has significant section loss that can lead to bar breakage. This type of corrosion was considered in the study by modeling breakage in post-tensioning bars. The study does not account for any potential benefits of repairs on the behavior of the structure.

Technical Challenges
Accurate prediction of the failure load of the bridge requires the analysis to account for redundancy of the structure arising from the inter-connection of the four girders through the diaphragms and the deck. Furthermore, the analytical approach must accurately account for staged construction, long term creep/shrinkage, non-linear behavior due to concrete cracking and yielding of post-tensioning steel, post-tensioning losses and load redistribution due to post-tensioning bar breakage and creep/shrinkage. Unfortunately, most available prestressed concrete analysis software are intended to be
used for design. They use design code-based simplifications and are therefore not capable of accurate analysis that accounts for the factors required for this analysis.

Structural Analysis Approach
Due to lack of design software capable of meeting the technical requirements noted above, a general purpose finite element code, ANSYS, was used for the study. A three-dimensional model of a typical span was generated using beam elements to model the girder/diaphragm and shell elements to model the deck. Special routines had to be written to accurately model girder post-tensioning using non-linear beam elements. Since the available beam elements could not simultaneously model creep and concrete material non-linearity due to cracking, special multi-step techniques were developed to generate accurate analytical models accounting for long term creep and concrete cracking.

Model Validation
As part of model validation, analytical results from the structural model were calibrated against available test results. The analytical results were shown to agree well with test data from full scale testing of post-tensioned girders from the old Sunshine Skyway Bridge conducted in 1973. The structural model results were further validated by ensuring the model predictions match the design-code based ultimate load predictions for the Friendship Trail Bridge.

Finite Element Simulation
Finite element analysis was performed for the different construction stages considered during design, including post-tensioning of girders, application of non-composite dead load and formation of the composite section. In addition, nine cases were analyzed to assess the impact of partial or complete failure in the girder post-tensioning bars on the structure’s ultimate capacity. In these analyses, the projected steel section loss values after 5, 10, 15 and 20 years were used. These cases were:
Case 1 - All post-tensioning bar areas reduced by 45% (to simulate 2009 level of average post-tensioning bar area loss due to corrosion)
Case 2 – Case 1 and two straight bars broken in all four girders
Case 3 – Case 1 and bottom three bars broken in an interior girder
Case 4 – Case 1 and all bars broken in an interior girder
Case 5 – All post-tensioning bar areas reduced by 59% (to simulate 2029 level of average post-tensioning bar area loss due to corrosion)
Case 6 – Case 5, with PT area reduction applied locally only to 1ft zone at the mid-span (to simulate impact of local area loss and simulate any stress concentration due to sudden section change)
Case 7 – All post-tensioning bar areas reduced by 48% (to simulate 2014 level of average post-tensioning bar area loss due to corrosion)
Case 8 – All post-tensioning bar areas reduced by 52% (to simulate 2019 level of average post-tensioning bar area loss due to corrosion)
Case 9 – All post-tensioning bar areas reduced by 55% (to simulate 2024 level of average post-tensioning bar area loss due to corrosion)

**Finite Element Simulation Results**

Analysis results from all the above cases indicated that due to the redundancy in the structure because of the interaction of multiple girders connected through the diaphragms and the deck, there is sufficient capacity in all the above cases to resist self-weight + pedestrian loading (85 psf). The analysis also indicated that the structural failure mode may be sudden brittle collapse due to girder cracking at the mid-span through the entire girder section. The predicted deflection at the failure load was minimal (0.4in). The lack of ductility occurs because the analysis predicts that the PT bar does not yield at failure. It was found that this was because the change in load on the post-tensioning bar is governed by the axial strain in the composite section and is therefore limited since the change in the composite section axial strain is relatively small when it is due to the lost PT force.
The nine three-dimensional, non-linear finite element analyses did not account for variation in corrosion rate, material properties, geometry and loading. Thus these results are indicative of the response of an average span, not of a span that may be more severely distressed. Probabilistic analysis methods were used to predict the response of such severely distressed spans.

**Probabilistic Analysis Method**

The Monte Carlo method was used to compute the probability of failure of a bridge span for the period from 2009-2029. This method requires an understanding of the variation of all the factors that cause failures, such as loads, material properties and section geometry. These are typically expressed using statistical distributions, such as normal and log-normal distributions.

Statistical parameters defining these distributions for live load, dead load and flexural resistance for prestressed concrete bridge were obtained from the literature. The Monte Carlo method involves generating a large number of samples consistent with the statistical distribution of the variable, such as loads and resistance, and using these to perform the analysis. The results from the large number of analysis provide a good indication of the expected behavior of the system due to variation of the various factors considered. For this study, the likelihood of the load exceeding the flexural resistance was determined using 100,000 statistical samples.

**Probabilistic Analysis Studies**

Monte Carlo analysis of the as-designed case was performed to validate the method by comparison of the results with those found in the literature for prestressed concrete girder bridges. The probability of failure and reliability index obtained from the analysis was found to agree well with published literature.
Monte Carlo method was also used to determine the distribution of post-tensioning bar area loss using equations for corrosion initiation and rate of corrosion found in the literature. A statistical distribution of the likelihood of post-tensioning bar breakage was developed using the information that only 1 in 252 typical spans had a bar breakage. These two statistical distributions were combined with the distribution found in the literature for flexural resistance to obtain a new distribution for flexural resistance of distressed spans for periods from 2009 to 2029 in 5 year increments. These distributions were used to compute the likelihood of failure of the bridge under self-weight plus pedestrian loading and under just self-weight alone.

**Probabilistic Analysis Findings**

Results from the above analysis indicate that the probability of failure of the bridge under pedestrian load increases from 128 in 100,000 to 1569 in 100,000 during the period from 2009 to 2029. Given that analysis with the original design code would have resulted in a probability of failure of 43 in 100,000, the state of the bridge in 2029 represents a significantly higher risk of failure than is currently found acceptable by design codes. The analysis suggests that of the 252 spans, 1 may fail under full pedestrian loading around 2014, 3 spans around 2024 and 4 spans around 2029. The analysis also shows that the bridge has a very low probability of failure (19 in 100,000) under self-weight alone between 2009 and 2029.

**Recommendations**

The analysis performed in this study is theoretical and uses data found in the open literature rather than actual data for the bridge. The results presented indicate that the bridge is unlikely to meet a service life of an additional 20 years at reliability levels required by prevailing design codes while foregoing routine maintenance. In the light of the lower than typical reliability predicted by the analysis, more frequent bridge inspections will be needed to maintain safety in the event the bridge is repaired.
The predictions are critically dependent on assumptions relating to the corrosion rate and statistical distributions of the load and the resistance. The validity of these assumptions needs to be verified from appropriate field inspection of the bridge. Without such verification, it will be unwise to base decisions exclusively on the reported theoretical analysis.
Acknowledgements

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## Contents

Executive Summary

Acknowledgements

List of Figures

List of Tables

1. INTRODUCTION
   1.1 Introduction
   1.2 Bridge Overview
   1.3 Consultant’s repair recommendations
   1.4 Scope of USF Study
   1.5 Outline

2. FINITE ELEMENT MODEL
   2.1 Introduction
   2.2 Typical Span Configuration
   2.3 Typical Span Finite Element Model
   2.4 Materials
   2.5 Boundary Conditions
   2.6 Validation Against Sunshine Skyway Girder Test Results

3. RESULTS: DESIGN CONDITION
   3.1 Introduction
   3.2 Design Equation Based Comparison
   3.3 Finite Element Model Loading Sequence
   3.4 Results

4. RESULTS: DISTRESSED CONDITION
   4.1 Introduction
   4.2 Corrosion Behavior
4.3 Modeling Area Loss 23
4.4 Results 25
5. PROBABILISTIC ASSESSMENT 35
  5.1 Introduction 35
  5.2 Monte Carlo Analysis 35
  5.3 Variable Distributions 36
  5.4 Results 37
  5.5 Probability of Failure 39
6. CONCLUSIONS AND RECOMMENDATIONS 43
  6.1 Conclusions 43
  6.2 Study Limitations 44
  6.3 Recommendations 46
REFERENCES 47
APPENDIX A: DESIGN/CODE EQUATION BASED CALCULATIONS 49
APPENDIX B: FINITE ELEMENT ANALYSIS RESULTS—DESIGN CONDITIONS 61
List of Figures

Figure 2-1  Cross section and side elevation of typical 48ft span. ________________________ 8
Figure 2-2  Girder Post Tensioning details. ________________________________ 9
Figure 2-3  Three dimensional finite element mesh. ___________________________ 10
Figure 2-4  Side elevation of typical girder mesh showing post-tensioning elements
(Note: Not to scale). ________________________________________________________ 11
Figure 2-5  Dimension of test girders from the old Sunshine Skyway bridge [3]. _____ 14
Figure 3-1  Interior Girder Concrete Stress at neutral axis and bottom straight PT bar
stress at mid-span over load steps. _______________________________ 18
Figure 3-2  Effect of creep on girder axial post-tensioning forces (kip). ________ 19
Figure 3-3  Effect of creep on girder shear forces (kip). ___________________ 20
Figure 3-4  Load Case #12 - Interior Beam Bending Moment Diagram, Shear Force
Diagram and Axial Force (units – kip-ft, kip). ________________________________ 21
Figure 4-1  Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial
Force (units kip-ft, kip) – Case 1, Failure Load. ____________________________ 28
Figure 4-2  Deflections (ft) – Case 1, Failure Load. ____________________________ 29
Figure 4-3  Interior Beam Concrete Stress (ksf)- Case 1, Failure Load. _________ 29
Figure 4-4  Interior Girder Concrete Stress at neutral axis and bottom straight PT bar
stress at mid-span under uniform pedestrian loading. _________________________ 30
Figure 4-5  Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial
Force (units kip-ft, kip) – Case 2, Failure Load. ______________________________ 31
Figure 4-6  All Beams Bending Moment Diagram, Shear Force Diagram and Axial Force
(units kip-ft, kip) – Case 3, Failure Load. ________________________________ 32
Figure 4-7  All Beams Bending Moment Diagram, Shear Force Diagram and Axial Force
(units kip-ft, kip) – Case 4, Failure Load. ________________________________ 33
Figure 5-1  Monte Carlo Results of distribution of Friendship Trail Bridge PT Bar diameter
in year 2029. __________________________________________________________ 40
Figure B-1  Load Case #1 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 62
Figure B-2  Load Case #2 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 63
Figure B-3  Load Case #3 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 64
Figure B-4  Load Case #4 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 65
Figure B-5  Load Case #5 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 66
Figure B-6  Load Case #6 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 67
Figure B-7  Load Case #7 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 68
Figure B-8  Load Case #8 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 69
Figure B-9  Load Case #9 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 70
Figure B-10 Load Case #10 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 71
Figure B-11 Load Case #11 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 72
Figure B-12 Load Case #12 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip). 73
List of Tables

Table 4-1 Summary of analysis results. ________________________________ 34
Table 5-1 Variable distributions used for Monte Carlo Analysis __________ 37
Table 5-2 PT Area Variation and Flexural Resistance Coefficient of Variation - 2009 to 2029. ________________________________ 38
Table 5-3 Distribution of load and resistance [9]. ______________________ 41
Table 5-4 Probability of Failure from 2009 to 2029.____________________ 41
1. INTRODUCTION

1.1 INTRODUCTION

A $4.76 million contract, April 15, 2008 pending award to repair the Friendship Trail Bridge was terminated on November 7, 2008. This bridge is one of the longest pedestrian bridges in the country and is a major recreational facility for the Tampa Bay community with over 600,000 citizens using it on an annual basis.

The recommendations for the repair are included in a report [1] prepared by consultants commissioned by the owners (Hillsborough County and Pinellas County). They are based on a thorough inspection of the bridge and its substructure, and are intended to ensure the bridge can be in service for the anticipated remaining service life of 15 to 20 years. This report describes a theoretical probabilistic structural analysis to provide independent data to the owners on the condition of the girders supporting the deck slab in spans located closest to the water-line. The objective of this study is to determine the probability of collapse of the Friendship Trail Bridge under self-weight and pedestrian load in the next 20 years.

1.2 BRIDGE OVERVIEW

The Friendship Trail Bridge is the old Gandy Bridge, which was constructed in 1956 and carried westbound traffic across Old Tampa Bay until 1995. The bridge was to be demolished when the new westbound bridge was opened, however actions from citizens of Hillsborough and Pinellas counties resulted in the two counties assuming joint ownership of the bridge in 1997 and making the bridge available for recreational use.
The 2.6-mile long bridge has 274 spans of which 252 are low-level 48ft spans. The current report focuses on the behavior of a typical 48ft span. The elevation of the top of the roadway above the waterline is 11ft 6in for these spans.

The 30ft 7in wide low-level bridge deck is supported by four 3ft 4in deep post-tensioned beams spaced 8ft 6in apart. These act compositely with a 7in thick deck slab. Partial-depth post-tensioned diaphragms tie the beams together at the third points (16ft on center).

The 48ft girders are pre-stressed (post-tensioned) by four, 1.125in diameter, Grade 160, post-tensioning (PT) bars, two straight and two parabolic. With the exception of two 3ft long end zones, no shear steel is provided in the 6in thick webs, over its remaining 42ft length.

The dimensions and details of the bridge deck, from the original plans, are illegible in places and some key information is not clear, e.g., cover at mid-span for the tendons. Additionally, as-built section dimensions, diaphragm, location of the post-tensioned tendons, material strengths of the concrete in the deck slab, and the pre-stressed beams are unknown.

1.3 Consultant’s Repair Recommendations

As noted earlier, consultants selected by the owners inspected the entire bridge and recommended repairs valued at $4.76 million. Of this, $962k was set aside for repairing cracks (1924 linear ft. @$500/ft.) and $15k for repairing a broken pre-stressed bar in span 92. Other recommendations were for painting the structural steel ($500k), repairing pile jackets ($1.96 million) and repairing cracks in the piles and pile caps ($892k).
1.4 Scope of USF Study

This study is limited to the analysis of the superstructure of the 252 low-level 48ft spans. The objective of the study is to determine the probability of superstructure collapse under self-weight and pedestrian loading in the next 20 years. The study considers collapse due to flexural failure.

The accurate prediction of the flexural resistance of a bridge requires analysis of the total structure, rather than analysis of individual girders using simplified AASHTO design guidelines (as is typically done during design). This is because the span consists of four girders interconnected with diaphragms and the deck slab, which allows for significant redistribution of loads amongst the girders.

Variables that impact flexural and shear capacity include:

a. Geometric Dimensions – Variability in dimensions results from construction tolerances. Critical variables include the concrete cover and the location of the post-tensioning ducts.

b. Initial Post-Tensioning (PT) Force – This is expected to vary due to construction process variation.

c. Material Properties – These include strength, modulus of elasticity and density of materials used for the bridge.

d. Post-tensioning (PT) Losses – Creep and shrinkage cause significant reduction in the effective post-tensioning force and redistribute loads from the girder to the composite structure.

e. Loading – Pedestrian loads, other live load (ex. Ambulance) and other concurrent loads (such as dead load).

f. Current Level of PT Corrosion – Loss of post-tensioning steel section changes the service and ultimate capacity of the girders.
g. *Future Corrosion Rate* – This is dependent on the exposure of the structure to chlorides, effectiveness of performed repairs, future exposures in unrepaired area etc.

h. *Effectiveness of Grouting* – PT duct grouting is essential to maintain effective pre-stressing force in the event of corrosion section loss of the steel.

i. *Location of Girder* – Exposure to chlorides (see corrosion)

j. *Current level of Rebar Corrosion* – Rebar corrosion can lead to significant reduction in the capacity of reinforced concrete section (deck and girder end blocks).

k. *Fatigue Damage* – In the event of severe corrosion, fatigue failure of PT or rebar may become critical.

l. *Loss of Concrete Section Due to Spalling* – Corrosion causes loss of concrete section due to spalling.

Items a through e are considered in typical design codes and addressed through appropriate load and resistance factors. These are not considered separately in this study. This study focuses on the impact of PT bar section area loss due to corrosion on the capacity of the bridge while accounting for load redistribution between the four girders of a span through the deck and diaphragm. This situation is not addressed by AASHTO code equations and therefore assessed in this work through fundamental structural analysis.

**1.5 Outline**

This section outlines the contents of the rest of the report. Chapter 2 discusses finite element model development in ANSYS [2]. Since the use of ANSYS for prestressed beam analysis is not very widespread, the modeling approaches was verified by comparing the predicted results against AASHTO equations and also test data from a 1973 report [3] on static load tests performed on similar girders of the old Sunshine Skyway Bridge.
Results of the as-designed condition of the bridge are presented in Chapter 3. The objective of these studies is to compare the current pedestrian loading to the original H20-44 truck loading and estimate the available margin in the ideal case without any consideration for loss of capacity due to corrosion induced deterioration. Chapter 3 also covers some of the fundamental design and behavior of the bridge, such as load balancing approach for post-tensioning design, and the impact of long term creep on load redistribution from the girders in the composite structure.

Chapter 4 focuses on failure analysis of the bridge. The primary objective of these studies is to understand the scenarios under which the bridge might collapse. The studies performed include cases with uniform loss of PT bar area and cases where PT bars are assumed to have broken in some girders.

Probabilistic assessment of the bridge is considered in Chapter 5. The approach used here was to assume statistical distribution for the rate of corrosion from the literature [4-6] and estimate the likelihood of collapse under self-weight and pedestrian loading after a 5-20 years period using Monte Carlo analysis.

Chapter 6 summarizes the findings from the studies and presents conclusions and recommendations.
2. FINITE ELEMENT MODEL

2.1 INTRODUCTION

The objective of this study is to determine the probability of collapse of the Friendship Trail Bridge under self-weight and pedestrian loading in the next 20 years. One approach to determine the likelihood of collapse is to estimate the loss of post tensioning (PT) bar cross-section area due to corrosion and use equations from AASHTO codes to determine the ultimate capacity of the girders [4-6]. This approach is likely to be very conservative since it does not take credit for redistribution of loads between girders occurring through the deck and diaphragms in the presence of distress to some girders. A more realistic estimate of the capacity of the bridge can be obtained by using a structural model capable of accounting for load redistribution.

This chapter presents details of a three dimensional model developed using ANSYS Version 11 [2] to model a typical 48 feet span of the Friendship Trail Bridge. Section 2.2 presents details of the typical bridge span. Section 2.3 describes the ANSYS model in detail. Material properties and boundary conditions are discussed in Sections 2.4 and 2.5 respectively. For verification purposes, ANSYS model results are compared to test data from a 1973 report [3] on the old Sunshine Skyway Bridge girder testing. These comparisons are discussed in Section 2.6.

2.2 TYPICAL SPAN CONFIGURATION

As noted in Section 1.2, the current report focuses on a typical 48ft span of the Friendship Trail Bridge. The typical span is 30ft 7in wide with 7in thick deck and supported by four 3ft 4in deep post-tensioned concrete girders spaced at 8ft 6in. The girders have end diaphragms and post-tensioned partial depth diaphragms at 16ft
spacing. A typical cross-section and side elevation of the bridge from available drawings is shown in Figure 2-1. Since many dimensions are illegible, their values had to be determined by scaling the drawing.

Figure 2-2 shows profile and details of post-tensioning bars used in the concrete girders. There are a total of four 1.125in diameter Grade 160 bars in each girder. Two of the four bars are straight and located at the bottom of the girder, while the other two have a parabolic profile and are located above the two straight bars.

Due to the illegible dimensions, it was initially thought that the post-tensioning bars were of 1.25in diameter, however, subsequently based on subsequent discussions with other engineers inspecting the bridge, it was discovered that the diameter was actually 1.125in. This correction required updating of the analysis presented in the report.

In addition to unknown dimensions, another important cause of uncertainty is the state of grouting of the post-tensioning bars. The post-tensioning bars were assumed to be grouted during an initial assessment of the bridge performed as part of this study. However, further field analysis of the bridge suggested that the grouting may have been ineffective in some locations. This finding is reflected in Chapter 4.
Figure 2-1 Cross section and side elevation of typical 48ft span.
Figure 2-2 Girder Post Tensioning details.
2.3 **Typical Span Finite Element Model**

Figure 2-3 shows the finite element mesh of a typical 48ft simple span (46ft 10in between centerline bearings) found in the Friendship Trial Bridge. This was developed using ANSYS Version 11 [2]. The mesh uses a grid size of approximately 1ft. The model consists of girders and diaphragm modeled with 2 node beam elements (BEAM188). The deck was modeled with 4 node shell elements (SHELL181) with nodes located coincidently with the girder. Both the beam and shell elements have the feature to locate the cross-section offset from the node. This helps in easily modeling composite action without the need for rigid links between the girder and the deck. The model consists of 8914 nodes and 7250 elements.

![Figure 2-3 Three dimensional finite element mesh.](image.png)
For typical design analysis, post-tensioning may be modeled with spar elements (LINK8) or by applying equivalent forces and moments at the nodes of the girder. However, in this model, post-tensioning bars were modeled with BEAM188 elements (see Figure 2-4). The choice of BEAM188 element for post-tensioning was based on the ability of this element to capture non-linear yielding of steel, which is important for determining the ultimate capacity of the bridge. In addition, this approach helps capture losses due to creep and shrinkage more accurately.

![Figure 2-4 Side elevation of typical girder mesh showing post-tensioning elements (Note: Not to scale).](image)

Due to lack of information on continuity of the barrier over a span, the model conservatively ignores the contribution of the barrier to the stiffness of the composite system.

### 2.4 Materials

Concrete was modeled using a plasticity model (UNIAXIAL) [2] that allows differing failure stresses for compression and tension. The material has zero stiffness once the stress exceeds the specified failure stresses. Compression failure was set to the
compressive strength of the concrete (parabolic response was not modeled), and tension failure was set to $7.5 \sqrt{f'c}$ [7]. The compressive strength of the girder was assumed to be 6 ksi, while that of the deck was taken as 4 ksi. This is based on strengths documented in the report regarding testing of similar girders on the old Sunshine skyway bridge [3]. Post-tensioning steel was modeled as an elastic-perfectly plastic model with yield stress of 160 ksi. Although 160 ksi is actually the ultimate strength of the PT bar, the simplified material model still provides a good estimate of the ultimate strength of the structure.

2.5 Boundary Conditions

The typical span was modeled as being simply supported by constraining vertical displacement at both ends of the girder and deck and the longitudinal displacement at one end of the deck (see Figure 2-1). In addition, lateral displacement of all girder ends was restrained to model the effect of end diaphragms.

2.6 Validation Against Sunshine Skyway Girder Test Results

Since the use of ANSYS to model staged construction of post-tensioned concrete structures is not very widespread, the modeling approach was validated against test results. A 1973 report [3] documents the findings of static load tests conducted on girders of the old Sunshine Skyway Bridge. The old Sunshine Skyway Bridge was completed in 1954, two years prior to the Friendship Trial Bridge and used very similar post-tensioned concrete girders. The dimensions of the tested section are shown in Figure 2-5.

The sequence of loading used to simulate the test conditions is as follows:

1. Beams are post-tensioned to $0.81 F_u$
2. Post-tensioning is grouted
3. Self weight of the beam is applied
4. Non-composite dead load (deck load) is applied to the girder
5. Composite section is formed
6. Test load is applied

The report [3] presents results from five girder tests. For validation purposes, results from the test of an undamaged girder (171-S2) were used first. The density of concrete in the finite element model was reduced to 130 pcf to match the dead load measured during the test. The load was applied at a distance for 14ft from the support and the failure load from the test was 112 kip. The finite element model failure load was estimated to be 102 kip based on the load at which the non-linear solution stopped converging due to excessive distortion. The finite element result is within 10% of the measured value and can be considered to be an acceptable comparison.

The difference in computed versus measured result could be due to many factors, including the non-inclusion of creep/shrinkage. Creep/shrinkage strain tends to reduce the compressive stress in the concrete girder and transfer the non-composite load to the composite section, which lowers the stress in the post-tensioning bar and can sometimes increase the ultimate section capacity. Other possible factors contributing to the mismatch include uncertainty associated with material properties and typical construction tolerances (with both the post-tensioning bar location and force).

To ensure the ability of the model to accurately capture the impact of corrosion, a second finite element model was run assuming a loss of 0.125in surface of all the three post-tensioning bars. This resulted in a reduction of the failure load from 102 kip to 61 kip, or a 40% reduction in capacity. This compares to a 37% reduction reported from the tests (Girder 171-S3 in [3]). These comparisons suggest that the finite element model captures the structural behavior of the undamaged and damaged girders quite well.
Based on the findings of this preliminary validation study, models presented in Chapter 3 include creep and shrinkage effects to improve the accuracy of the predictions.

Figure 2-5 Dimension of test girders from the old Sunshine Skyway bridge [3].
3. RESULTS: DESIGN CONDITION

3.1 INTRODUCTION

Chapter 2 presented details of the finite element model used to analyze the typical span of the Friendship Trail Bridge. It also presented results of the validation study performed using test results of girders taken from the old Sunshine Skyway Bridge. This chapter presents results from enhanced finite element models which include the impact of creep and shrinkage determined using the European CEB FIP 1990 code [8].

The objective of this chapter is to present some results using the undamaged bridge model to use as a benchmark to compare against the damaged bridge model results presented in Chapter 4. Some simple design equation based calculations are also presented to compare the original design loading (H20-44) versus the proposed loading (85 psf pedestrian loading).

3.2 DESIGN EQUATION BASED COMPARISON

Appendix A contains design calculations to understand the relative order of magnitude of various loads acting on the as-designed bridge. These calculations suggest that the amount of post-tensioning was selected based on meeting service criteria for maintaining compression at the bottom fiber. As a result, the original design has a factored ultimate moment capacity which is 37% higher than the factored load.

These design calculations also show that the moment due to pedestrian loading is roughly half the moment due to the original design live load of the H20-44 truck. Incidentally, this is practically the load capacity required to accommodate an H10-44
truck, which is representative of an ambulance loading that a pedestrian bridge is required to handle in case of emergencies.

Service assessment design calculations also show that only 25% of the original PT section area is sufficient to carry DL+ pedestrian loading of the bridge.

### 3.3 Finite Element Model Loading Sequence

The following load steps were applied to the finite element model to determine the state of the structure in the designed condition:

1. Beams are post-tensioned to 0.81 $F_u$.
2. Post-tensioning is grouted
3. Self-weight of the beam is applied
4. Creep and shrinkage effects are computed for period between post-tensioning and deck pour (estimated to be 10 days)
5. Non-composite dead load (deck + diaphragm load) is applied to the girder (note composite dead load from barriers is ignored since exterior girders do not govern due to smaller tributary loads from the deck)
6. Composite section is formed
7. Creep and shrinkage effects are computed for 1 year
8. Creep and shrinkage effects are computed for 5 years
9. Creep and shrinkage effects are computed for 10 years
10. Creep and shrinkage effects are computed for 20 years
11. Creep and shrinkage effects are computed for 50 years
12. Pedestrian load $\times 10$ (850 psf) is applied to the deck and run until failure occurs

ANSYS 11 does not permit combination of UNIAXIAL plasticity model (used to model concrete cracking) with creep. To accurately account for creep behavior, Steps 1-11 are run without plasticity since the stresses are expected to be in the linear range. The
creep induced strains at the end of step 11 are applied as initial strains to a new model that uses the UNIAXIAL plasticity model for concrete. The new model also uses the PT forces obtained from step 11.

3.4 Results

Bending moment diagrams, shear force diagrams and the axial force distributions of an interior girder for load cases 1 through 12 are shown in Appendix B. The results agree well with code-based hand calculations (see Table 4-1). Accurate prediction of failure load requires the analytical model to account for load redistribution of the non-composite load from the girder to the composite section due to creep/shrinkage. Figure 3-1 shows the interior girder concrete stress at the neutral axis and the bottom straight PT bar stress for the different load steps. It can be seen that both the concrete and PT bar start with a high stresses (concrete is compressive while PT bar is tensile) right after post-tensioning. However, at load steps modeling creep and shrinkage (4 and 6 thru 11), both concrete stress and PT bar stress reduce due to creep. Creep/shrinkage strain reduces the compressive stress in the post-tensioned concrete and this results in the non-composite load being shed from the girder to the composite section. The reduction of compressive concrete stress in the girder is undesirable since concrete is poor in tension and will crack due to lack of longitudinal reinforcement in the girder.

Figure 3-2 shows the girder axial force right after composite action is formed (Load Step 6) and after 50 years of creep and shrinkage (Load Step 11). It may be seen that the compressive axial force from the post-tensioning at mid-span reduces from about 410 kip to about 290 kip, a 30% reduction.

Girder shear force results for load steps 6 and 11 (see Figure 3-3) show a small shear load when the composite section is formed (load step 6). This means that the post-tensioning force carries the dead load of the structure, which is consistent with the load
balancing approach to post-tensioned concrete design. However, due to creep and shrinkage, a significant part of the dead load is carried by the girders after 50 years (load step 11).

Figure 3-4 shows the interior girder bending moment diagram, shear force diagram and axial force at ultimate load. The failure load for this load step results in a moment of 2018 ft-kip, which compares well with design equation based prediction of 1982 ft-kip. The slight difference in prediction is most likely due to difference in estimated creep and shrinkage loss used for design equation versus that computed based on CEB FIP 1990 [8] in the finite element model.
The next chapter presents results on studies where the area of the post-tensioning bar is reduced after load step 11 and the distressed structure is subsequently subjected to load to determine its ultimate capacity.

Figure 3-2 Effect of creep on girder axial post-tensioning forces (kip).
Figure 3-3  Effect of creep on girder shear forces (kip).
Figure 3-4 Load Case #12 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units – kip-ft, kip).
4. RESULTS: DISTRESSED CONDITION

4.1 INTRODUCTION

Chapter 3 presented results of the as-designed condition of the bridge after 50 years of creep and shrinkage. It was found that the ultimate capacity of the bridge predicted in this state was fairly close to the capacity computed in Appendix A using design code based equations. In this chapter, results of models that incorporate loss of post-tensioning bar area due to corrosion are shown. The results presented here form the basis for the next chapter, where the probability of collapse of any typical span of the Friendship Trail Bridge is computed.

4.2 CORROSION BEHAVIOR

The main cause of deterioration observed in the bridge is corrosion of the post-tensioning (PT) bars. The corrosion of a PT bar in a bridge does not start immediately after construction. It takes several years for the chloride from sea water to diffuse through the concrete cover and reach the post-tensioning bar. Based on average rates found in the literature [6], the initial diffusion period is estimated to be about 5 years based on a cover of 2.25 in (1.5 in diameter duct in a 6 in web).

Once the chlorine reaches the steel surface, corrosion is known to cause two primary types of deterioration to the steel bars. Firstly, there is a uniform loss of steel section. Based on an average rate of corrosion of 0.006 in/year [6], the average loss of area of PT bars is estimated to be 45% (as of 2009). It must be pointed out that the rate of loss is based purely on data found in the literature and can be refined if further data is carefully collected from the field.
The second type of deterioration occurs due to pitting corrosion, where a part of the bar has significant loss of section locally. Pitting corrosion can lead to breakage of the PT bar. It is difficult to predict the average number of locations where pitting corrosions can occur, therefore the capability of the structure to carry girder with multiple broken PT bars must be determined to assess safety of the span.

4.3 Modeling Area Loss

The following load steps were used with the finite element model to analyze the structure in the distressed condition

1. Beams are post-tensioned to 0.81 $F_u$
2. Post-tensioning is grouted
3. Self-weight of the beam is applied
4. Creep and Shrinkage effects are computed for period between post-tensioning and deck pour (estimated to be 10 days)
5. Non-composite dead load (deck + diaphragm load) is applied to the girder (note composite dead load from barriers is ignored since exterior girders do not govern due to smaller tributary loads from the deck)
6. Composite section is formed
7. Creep and shrinkage effects are computed for 1 year
8. Creep and shrinkage effects are computed for 5 years
9. Creep and shrinkage effects are computed for 10 years
10. Creep and shrinkage effects are computed for 20 years
11. Creep and shrinkage effects are computed for 50 years
12. Reduce the cross-section area of affected PT bars
13. Apply Pedestrian Load $\times$ 10 and run until failure occurs

As shown in Step 12 above, uniform loss of area due to corrosion was modeled by reducing the cross-section area of the post-tensioning bars in the bridge. Bar breakage
due to pitting corrosion is modeled by changing the post-tensioning bar cross section to a very small value (0.1% of original area). Based on observations about the grouting quality on the bridge during inspections, it was decided that the broken tendons will be conservatively assumed to be ungrouted. Therefore, a local breakage is modeled by changing the PT bar section area to along the entire span.

Service design equation based analysis in Appendix A show that the PT bar cross-section area needed to prevent collapse is approximately that corresponding to one PT bar. The following nine scenarios were investigated using the finite element model to understand the impact of post-tensioning bar breakage on the ultimate capacity of the bridge. Five of the nine scenarios (1,5,7-9) consider different levels of average PT area loss occurring from 2009 to 2029 in 5 year increments. Three of the cases (2-4) consider scenarios where there is severe level of distress resulting in broken PT bars in addition to uniform area loss. Finally, case 6 looks at the impact of local PT area loss.

Case 1 - All post-tensioning bar areas reduced by 45% (to simulate 2009 level of average post-tensioning bar area loss due to corrosion)

Case 2 – Case 1 and two straight bars broken in all four girders

Case 3 – Case 1 and bottom three bars broken in an interior girder

Case 4 – Case 1 and all bars broken in an interior girder

Case 5 – All post-tensioning bar areas reduced by 59% (to simulate 2029 level of average post-tensioning bar area loss due to corrosion)

Case 6 – Case 5, with PT area reduction applied locally only to 1ft zone at the mid-span (to simulate impact of local area loss and simulate any stress concentration due to sudden section change)

Case 7 – All post-tensioning bar areas reduced by 48% (to simulate 2014 level of average post-tensioning bar area loss due to corrosion)

Case 8 – All post-tensioning bar areas reduced by 52% (to simulate 2019 level of average post-tensioning bar area loss due to corrosion)
Case 9 – All post-tensioning bar areas reduced by 55% (to simulate 2024 level of average post-tensioning bar area loss due to corrosion)

Per computations shown in Appendix A, the target ratio of pedestrian loading needed to meet AASHTO LFD code requirement is 2.7, i.e., if the structure can resist a load of 2.7 x 85 psf on the deck, it meets AASHTO code requirements for strength (moment capacity of 865 ft-kip).

4.4 RESULTS

Figure 4-1 shows the bending moment diagram, the shear force diagram and the axial force distribution for an interior girder at failure load of 5.2 x pedestrian loading (ultimate moment capacity of 1364 ft-kip) for Case 1. It is evident that there is significant loss of moment and shear capacity when compared to Figure 3-4, which shows the ultimate state for PT bar without area loss. Despite the significant loss of area, the structure still exceeds the target ultimate moment of 865 ft-kip, indicating significant margin to carry pedestrian load.

Deflected shapes of the bridge at the failure load are shown in Figure 4-2. The low value of peak deflection of 0.4 inch at mid span indicates that the failure is likely to be a sudden brittle failure, which is suggests behavior similar to unreinforced concrete under force loading. Figure 4-3 shows the stress in an interior girder at failure. It may be seen that a significant portion of the girder in the mid-span has tensile stresses (red contour) indicative of a severely cracked girder. It is interesting to note that the model did not predict failure of the PT bars despite significant loss of area due to corrosion.

To understand why the PT bar does not fail, additional models (Case 5 and 6) were run with more severe PT area loss. Case 5 has uniform area loss of 59%, while Case 6 has this area loss occurring only over 1 ft zone at the mid-span. Figure 4-4 shows interior
girder concrete stress at the neutral axis and bottom straight PT bar stress for different levels of uniform pedestrian loading for these cases. It was shown in Chapter 3 (see Figure 3-1) that creep causes significant reduction in both the compressive stress in concrete and tensile stress in the PT bar. Data at X axis value of -1 shown in Figure 4-4 corresponds to Load Step 11 in Figure 3-1. The change in PT bar stresses and concrete stress from X axis value of -1 to 0 occurs due to reduction in area of the PT bar in the model (Load Step 12). The change in PT bar stress is not very significant since the composite system behavior is essentially strain controlled. This means that the loss of PT bar force results in the non-composite load being shed to the composite sections (which cause a reduction in concrete compressive stress) and the overall strain of the PT bar is not significantly affected. Since stress is proportional to strain prior to yielding, the overall change in PT bar stress is not very significant in both Case 5 and Case 6. In both cases, there is an increase in the concrete tensile stress and PT bar stress for higher levels of applied uniform pedestrian load. At some point, the tensile cracking in concrete causes a significant loss of stiffness and the PT bar sees higher rate of increase in stress. The analysis suggests that the entire girder section cracks prior to the PT bar reaching its yield stress, thus resulting in a brittle failure with minimal deflection. The analysis shows that the case with only local loss of PT bar area has significantly higher capacity due to limited shedding of non-composite load to the composite section over the length of the span.

Results from Case 2, which assumes a 45% section loss in parabolic tendons and 100% loss of straight tendons are shown in Figure 4-5. The results indicate significant reductions in flexural and shear capacity and significant tension is indicated by the axial force. Despite the severe loss of post-tensioning, the structure failed at 2.93 x pedestrian load (M=915 ft-kip), which is above the 865 ft-kip target needed to meet code requirements.
Figure 4-6 shows results from Case 3, where all PT bars are assumed to have 45% section loss and an interior girder (second girder from bottom in the figure), is modeled with complete loss of bottom three PT bars. Despite the extremely severe condition, the failure load was 4.0 x pedestrian loading (M=1127 ft-kip), indicating that the structure meets the code based target of 865 ft-kip. It is clear from the results that the adjacent girders take on the excess load as seen by the difference in the moment and shear of the exterior girder adjacent to the one with PT loss compared to the one at extreme top in the figure. This clearly shows that the structure has a significant level of redundancy due load redistribution occurring through the deck and the diaphragms.

Finally, results from Case 4, which assumes a 45% section loss in all tendons plus complete loss of post tensioning in an interior girder (second girder from bottom in the figure), are shown in Figure 4-7. As with the previous case, the results indicate significant reduction in flexural and shear capacity and significant tension is indicated by the axial force. Despite the severe loss of post-tensioning, the structure failed at 2.78 x pedestrian load (M=885 ft-kip), which is just above the 865 ft-kip target needed to meet code requirements.

Failure loads obtained from all the analyses are summarized in Table 4-1. All the above results indicate that there is significant redundancy in the structure and a collapse is highly unlikely for an average span in the short term. The next chapter looks at computing the probability of failure after additional 5-20 years while accounting for spans that may have more than average level of distress due to corrosion.
Figure 4-1  Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip) – Case 1, Failure Load.
Figure 4-2 Deflections (ft) – Case 1, Failure Load.

Figure 4-3 Interior Beam Concrete Stress (ksf) - Case 1, Failure Load.
Figure 4-4 Interior Girder Concrete Stress at neutral axis and bottom straight PT bar stress at mid-span under uniform pedestrian loading.
Figure 4-5  Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip) – Case 2, Failure Load.
Figure 4-6  All Beams Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip) – Case 3, Failure Load.
Figure 4-7 All Beams Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip) – Case 4, Failure Load.
Table 4-1 Summary of analysis results.

<table>
<thead>
<tr>
<th>Case Num</th>
<th>Description</th>
<th>Ultimate Moment Capacity (ft-kip)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td>Code based ultimate moment (see Appendix A)</td>
<td>1982</td>
</tr>
<tr>
<td>Design</td>
<td>Finite element based ultimate moment</td>
<td>2018</td>
</tr>
<tr>
<td>1</td>
<td>All post-tensioning bar areas reduced by 45% (to simulate 2009 level of post-tensioning bar area loss due to corrosion)</td>
<td>1364</td>
</tr>
<tr>
<td>2</td>
<td>Case 1 and two straight bars broken in all four girders</td>
<td>915</td>
</tr>
<tr>
<td>3</td>
<td>Case 1 and bottom three bars broken in an interior girder</td>
<td>1127</td>
</tr>
<tr>
<td>4</td>
<td>Case 1 and all bars broken in an interior girder</td>
<td>885</td>
</tr>
<tr>
<td>5</td>
<td>All post-tensioning bar areas reduced by 59% (to simulate 2029 level of post-tensioning bar area loss due to corrosion)</td>
<td>1159</td>
</tr>
<tr>
<td>6</td>
<td>All post-tensioning bar areas reduced by 59% only for 1ft zone at the mid-span. (to simulate 2029 level of post-tensioning bar area loss due to corrosion in local area)</td>
<td>2018</td>
</tr>
<tr>
<td>7</td>
<td>All post-tensioning bar areas reduced by 48% (to simulate 2014 level of post-tensioning bar area loss due to corrosion)</td>
<td>1287</td>
</tr>
<tr>
<td>8</td>
<td>All post-tensioning bar areas reduced by 52% (to simulate 2019 level of post-tensioning bar area loss due to corrosion)</td>
<td>1240</td>
</tr>
<tr>
<td>9</td>
<td>All post-tensioning bar areas reduced by 55% (to simulate 2024 level of post-tensioning bar area loss due to corrosion)</td>
<td>1192</td>
</tr>
</tbody>
</table>
5. PROBABILISTIC ASSESSMENT

5.1 INTRODUCTION

The main deliverable from the project is the probability of collapse of the bridge under its own self-weight and pedestrian loading. Deterministic results presented in Chapter 4 provide some insight into the expected structural behavior under severe distress. This chapter uses knowledge of statistical distribution of variables that impact corrosion to compute the probability of collapse within the next 20 years. The probabilistic analysis shown here uses the Monte Carlo method. The values of variables and their distributions are based on available literature on similar analysis performed by other researchers.

5.2 MONTE CARLO ANALYSIS

The likelihood of collapse of a bridge span depends on several variables, such as material strength, geometric dimensions and loads. These are random variables, i.e., their values vary from point to point on the bridge and may vary over time. Such variables can be characterized using statistical distributions, such as normal distribution or log-normal distributions. They are defined using their mean value and coefficient of variation or standard deviation (σ).

A practical method to understand the implication of these variations on probability of failure is to use the Monte Carlo analysis. This method involves generating a very large number of samples (10,000-100,000+) for the variables using the statistical distribution of the variable and evaluating the design at these sampled points. The probability of failure obtained from the large number of samples provides a good indication of expected likelihood of failure.
5.3 Variable Distributions

As discussed in Chapter 1, the key variable and the focus for this study is the loss of post-tensioning (PT) bar section area due to corrosion. Uncertainty of other variables, such as geometry and material properties are addressed by codes and were incorporated using data from [9].

Table 5-1 shows the variables that impact corrosion initiation and rate of corrosion from [6]. Corrosion initiation time, T_i, in years is given by the following expression [6]

\[ T_i = \frac{X^2}{4D_c} \left[ \text{erf}^{-1} \left( \frac{C_0 - C_{cr}}{C_0} \right) \right]^{-2} \]

In this expression, D_c is the chloride diffusion coefficient (in²/year), X is the concrete cover (in), and C_0 and C_{cr} the chloride concentration at the surface and the critical chloride concentration. The effective diameter of PT bar is computed by reducing the original diameter by R_{corr} x (T-T_i), where T is the time from end of construction at which the structure is being assessed.
Table 5-1 Variable distributions used for Monte Carlo Analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution</th>
<th>Mean</th>
<th>Coefficient of variance (% of mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diffusion Coefficient, D (^2/\text{yr})</td>
<td>Lognormal</td>
<td>0.2</td>
<td>0.10</td>
</tr>
<tr>
<td>Surface chloride concentration, C_o (wt % conc.)</td>
<td>Lognormal</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Critical chloride concentration, C_cr (wt % conc.)</td>
<td>Lognormal</td>
<td>0.025</td>
<td>0.10</td>
</tr>
<tr>
<td>Corrosion Rate, R_corr (in/yr)</td>
<td>Lognormal</td>
<td>0.006</td>
<td>0.30</td>
</tr>
<tr>
<td>Cover (in)</td>
<td>Lognormal</td>
<td>2.25</td>
<td>0.05</td>
</tr>
</tbody>
</table>

5.4 Results

A Monte Carlo Analysis was performed using Microsoft Excel and the variable distributions in Table 5-1 to combine the different possible \( T_i \) and \( R_{corr} \) and obtain a distribution of the area of any post-tensioning bar using 10,000 sampling points. Figure 5-1 shows the results of the distribution of PT bar diameter obtained from the Monte Carlo analysis for the year 2029. The results suggest that the average diameter of the PT bar will be around 0.73in, which corresponds to an average loss of 59% of section area. In addition, the results show the worst case diameter to be 0.21in, which corresponds to a loss of 97% of the section area.

Table 5-2 shows results from additional Monte Carlo analysis showing the average and standard deviation of the PT area over 5 year increments from 2009 to 2029.
Table 5-2 PT Area Variation and Flexural Resistance Coefficient of Variation - 2009 to 2029.

<table>
<thead>
<tr>
<th>Year</th>
<th>Avg. PT Area (in²)</th>
<th>Ultimate Moment Capacity (ft-kip)</th>
<th>Std Dev (in²)</th>
<th>-2 σ Reduced Area (in²)</th>
<th>-2 σ Ultimate Moment Capacity (ft-kip)</th>
<th>Coefficient of Variation due to PT Area</th>
<th>Delta from cut strands (ft-kip)</th>
<th>Coefficient of Variation due to broken PT Bar</th>
<th>Coefficient of Variation due to dimensions, materials etc.</th>
<th>Combined Coefficient of Variation for</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>0.566</td>
<td>1377</td>
<td>0.113</td>
<td>0.341</td>
<td>1089</td>
<td>0.105</td>
<td>475</td>
<td>0.112</td>
<td>0.075</td>
<td>0.175</td>
</tr>
<tr>
<td>2014</td>
<td>0.527</td>
<td>1300</td>
<td>0.120</td>
<td>0.288</td>
<td>1026</td>
<td>0.105</td>
<td>475</td>
<td>0.118</td>
<td>0.075</td>
<td>0.184</td>
</tr>
<tr>
<td>2019</td>
<td>0.494</td>
<td>1250</td>
<td>0.127</td>
<td>0.239</td>
<td>950</td>
<td>0.120</td>
<td>475</td>
<td>0.123</td>
<td>0.075</td>
<td>0.192</td>
</tr>
<tr>
<td>2024</td>
<td>0.458</td>
<td>1205</td>
<td>0.131</td>
<td>0.196</td>
<td>859</td>
<td>0.144</td>
<td>475</td>
<td>0.127</td>
<td>0.075</td>
<td>0.198</td>
</tr>
<tr>
<td>2029</td>
<td>0.426</td>
<td>1171</td>
<td>0.137</td>
<td>0.152</td>
<td>738</td>
<td>0.185</td>
<td>475</td>
<td>0.131</td>
<td>0.075</td>
<td>0.204</td>
</tr>
</tbody>
</table>
Table 5-2 also contains predictions on the ultimate moment capacity corresponding to the different PT area. These were estimated by fitting a third order polynomial that relates loss of PT area to the failure load using results obtained from ANSYS for Cases 1, 5 and 7 through 9 shown in Table 4-1.

5.5 Probability of Failure

Probability of failure of a typical span can be computed if the distribution of applied loads (dead load and live load) and resistance (flexural resistance) is known. These were obtained from [9] and are summarized in Table 5-3.

To obtain a baseline probability of failure, Monte Carlo analysis was performed using 100,000 sample using these distributions with original design loads for the typical 48ft span from Friendship Trial Bridge (see Appendix A) and using the nominal resistance specified by the AASHTO Standard Specification. The probability of failure obtained from the analysis was 43 per 100,000. This corresponds to a reliability index of 3.3 and agrees well with the published reliability index for prestressed concrete girder bridges in [9].

The probability of failure in the distressed condition was computed by using the nominal resistance obtained using ANSYS and adjusting the coefficient of variation of resistance to include the expected variation in PT area and likelihood of having broken tendons (see Table 5-2). The inspection report [1] indicated that only 1 of the girders in the 252 spans had a broken PT bar. This corresponds to a probability of 1/252 of having a broken bar. Using results from Chapter 4 which show a reduction of 475 ft-kip ultimate capacity for the case with all straight bars broken, a coefficient of variation was computed to reflect the 1/252 likelihood of having this condition at different time periods (see Table 5-2 for computed Coefficient of Variation due to broken PT Bar). This assumption is conservative since it assumes all straight PT bars are broken in the span.
This conservatism is required since it is very likely that the pitting corrosion deterioration will accelerate over time.

Figure 5-1 Monte Carlo Results of distribution of Friendship Trail Bridge PT Bar diameter in year 2029.
Table 5-3 Distribution of load and resistance [9].

<table>
<thead>
<tr>
<th>Load</th>
<th>Distribution</th>
<th>Bias (ratio of mean to nominal)</th>
<th>Coefficient of variance (% of mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load (factory produced girders)</td>
<td>Normal</td>
<td>1.03</td>
<td>0.08</td>
</tr>
<tr>
<td>Dead Load (cast-in-place)</td>
<td>Normal</td>
<td>1.05</td>
<td>0.10</td>
</tr>
<tr>
<td>Live Load</td>
<td>Normal</td>
<td>1.75</td>
<td>0.18</td>
</tr>
<tr>
<td>Moment Resistance</td>
<td>Normal</td>
<td>1.05</td>
<td>0.075</td>
</tr>
</tbody>
</table>

Table 5-4 Probability of Failure from 2009 to 2029.

<table>
<thead>
<tr>
<th>Year</th>
<th>SELF WT. + PEDESTRIAN LOAD</th>
<th>SELF WT. ONLY</th>
<th>Number of Span Likely to Fail under full Pedestrian Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability of Failure</td>
<td>Reliability Index</td>
<td>Probability of Failure</td>
</tr>
<tr>
<td>2009</td>
<td>0.00128</td>
<td>3.0</td>
<td>0</td>
</tr>
<tr>
<td>2014</td>
<td>0.00373</td>
<td>2.7</td>
<td>0.00001</td>
</tr>
<tr>
<td>2019</td>
<td>0.00624</td>
<td>2.5</td>
<td>0.00003</td>
</tr>
<tr>
<td>2024</td>
<td>0.01075</td>
<td>2.3</td>
<td>0.00006</td>
</tr>
<tr>
<td>2029</td>
<td>0.01569</td>
<td>2.2</td>
<td>0.00019</td>
</tr>
</tbody>
</table>
Table 5-4 shows the probability of failure computed using Monte Carlo analysis using the coefficient of variation for loads shown in Table 5-3 and the coefficient of variation for resistance shown in Table 5-2. Results from the above analysis indicate the probability of failure of the bridge under pedestrian load increases from 128 in 100,000 to 1569 in 100,000 during the period from 2009 to 2029. Given that the original design code would have resulted in a probability of failure is 43 in 100,000, the state of the bridge in 2029 represents a significantly higher risk of failure than is currently found acceptable by design codes. The analysis suggests that of the 252 spans, 1 may fail under full pedestrian loading around 2014, 3 spans around 2024 and 4 spans around 2029. The analysis also shows that the bridge has a very low probability of failure (19 in 100,000) under self-weight alone between 2009 and 2029.
6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The goal of this study was to predict the likelihood collapse of a typical 48ft span of the Friendship Trial Bridge under pedestrian loading and self-weight. A three dimensional non-linear, finite element model was used to capture redistribution of loads and obtain a more realistic prediction of ultimate capacity of the span than typical single girder analysis would provide. The validity of the model was confirmed by comparison with test results from a 1973 report [3] on load test of old Sunshine Skyway Bridge and also by comparison to results obtained using design equations.

Analysis results from the three dimensional finite element model of the bridge for different corrosion scenarios (for years 2009-2029) indicate that due to redundancy in the structure from multiple girders connected through diaphragms and the deck, there is sufficient capacity in the average span to resist self-weight + pedestrian loading. However, the above analysis did not account for variation in the corrosion rate, material properties, geometry and loading. Thus these results are indicative of an average span and not the span that may be more severely distressed. The analysis also indicates the structure failure mode may be sudden brittle collapse due to girder cracking at the mid-span. This is because, the strain controlled behavior of the composite section limits the amount of stress developed in the PT bar even when there is significant reduction in bar area and results in the concrete section developing significant tensile stresses that lead to failure with minimal deflection (0.4in).
The Monte Carlo method was used to compute the probability of failure of a bridge span for the period from 2009-2029 while accounting for the possibility of more distressed spans than those considered in the finite element analysis. The likelihood of the load exceeding the flexural resistance was determined using 100,000 statistical samples. The source of variation considered in the study included loss of post-tensioning bar section area due to corrosion, load and resistance.

Results from the above analysis indicate the probability of failure of the bridge under self-weight + pedestrian load increases from 128 in 100,000 to 1569 in 100,000 during the period from 2009 to 2029. Given that the original design code would have resulted in a probability of failure is 43 in 100,000, the state of the bridge in 2029 represents a significantly higher risk of failure than is currently found acceptable by design codes.

The analysis suggests that of the 252 spans, 1 may fail under full pedestrian loading around 2014, 3 spans around 2024 and 4 spans around 2029. The analysis also show that the bridge has a very low probability of failure under self-weight alone between 2009 and 2029.

### 6.2 Study Limitations

The analysis presented here had to use conservative assumptions where possible to compensate for the high level uncertainty in the state of the bridge. Some of these assumptions were:

1. The PT bars were modeled as ungrouted based on field inspection data.
2. Impact of pitting corrosion was modeled by assuming all straight bars were broken in the span being considered.
3. When computing PT area loss due to corrosion, no credit was taken for corrosion of the zinc PT duct.
4. The PT area loss was assumed to be uniform over the entire length of the bar.
The study presented here focused on PT bar section area loss due to uniform corrosion and only estimated the impact of pitting corrosion by incorporating its impact on the variation of resistance of the span. Pitting corrosion is likely to be a more serious problem since it can cause breakage of PT bars. If the frequency of pitting corrosion is established by performing a bridge survey, the combined result of uniform area loss corrosion and pitting corrosion may be assessed in a more rigorous manner.

The analysis presented here did not address shear strength of the girders. Testing performed on the old Sunshine Skyway Bridge shows that the composite girder plus deck section has significant shear capacity. There would be some impact to shear capacity due to damage to the parabolic tendons, however, in these scenarios the flexural capacity would most likely be the limiting factor.

The study did not assess the impact of loss of concrete section due to spalling. This refinement is not expected to change the conclusions significantly since the ultimate capacity analysis assumed cracked concrete on the tension face, where most of the spalling occurs.

The structural model did not consider deterioration of the diaphragm, anchorages for the PT bars and the deck due to corrosion. Although, the inspection report [1] mentions some form of distress in some diaphragms, they are assumed to have sufficient capacity to help the girder redistribute the load to adjacent girders. This was considered to be a reasonable simplification since the deck also helps load redistribution.

The distributions used for the corrosion rates, load and resistance are based on those found in the literature [6, 9]. Results can be more accurate if they are compared to field data from inspection and updated periodically based on observations.
6.3 Recommendations

The analyses performed in this study show that the bridge is unlikely to meet a service life of additional 20 years at reliability levels required by design codes. In the light of the lower than typical reliability predicted by the analysis, more frequent bridge inspections will be needed to maintain safety in the event the bridge is repaired.

In case the bridge is demolished, it is recommended that a sample of the dimension of PT bars and state of the grouting be studied and documented for potential use in other similar bridges in Florida or elsewhere.

The analysis performed in this study is theoretical and uses data found in the open literature rather than actual measurements from the bridge. As noted in the previous section, many assumptions had to be made, such as the corrosion rate and statistical distributions of load and resistance, which have a critical impact on the failure load predictions. It is therefore recommended that no decision be made solely based on these findings. These results must be used in conjunction with other information based on more detailed inspection of the bridge that document bridge deterioration over time.
REFERENCES


APPENDIX A: DESIGN/CODE

EQUATION BASED CALCULATIONS
**Inputs**

\[ L_{\text{span}} := 46.833 \text{ ft} \quad \text{Span} \]
\[ D_{\text{girders}} := 8.5 \text{ ft} \quad \text{Spacing between girders} \]
\[ T_{\text{deck}} := 7 \text{ in} \quad \text{Deck thickness} \]
\[ \rho_c := 0.150 \frac{\text{kip}}{\text{ft}^3} \quad \text{Density} \]

**Calculate Non Composite DL**

a. Girder

\[ A_{\text{girder}} := 2.4458 \text{ ft}^2 \]

\[ w_{\text{girder}} := A_{\text{girder}} \rho_c \]

\[ w_{\text{girder}} = 0.367 \frac{\text{kip}}{\text{ft}} \]

\[ M_{\text{DL_girder}} := \frac{w_{\text{girder}} L_{\text{span}}^2}{8} \quad V_{\text{DL_girder}} := w_{\text{girder}} \frac{L_{\text{span}}}{2} \]

\[ M_{\text{DL_girder}} = 100.583 \text{ kip-ft} \]

\[ V_{\text{DL_girder}} = 8.591 \text{ kip} \]

b. Slab

\[ A_{\text{slab}} := T_{\text{deck}} D_{\text{girders}} \]

\[ A_{\text{slab}} = 4.958 \text{ ft}^2 \]

\[ w_{\text{slab}} := A_{\text{slab}} \rho_c \]

\[ w_{\text{slab}} = 0.744 \frac{\text{kip}}{\text{ft}} \]

\[ M_{\text{DL_slab}} := \frac{w_{\text{slab}} L_{\text{span}}^2}{8} \]

\[ V_{\text{DL_slab}} := w_{\text{slab}} \frac{L_{\text{span}}}{2} \]

\[ M_{\text{DL_slab}} = 203.911 \text{ kip-ft} \]

\[ V_{\text{DL_slab}} = 17.416 \text{ kip} \]
Calculate Composite DL & Live Load

\[ W_b := (8.5) \text{-ft} \]

\[ W_b = 8.5 \text{-ft} \]

\[ L_b := 46.8 \text{ ft} \]

\[ p := 85 \text{ psf} \]

\[ w := W_b \cdot p \]

\[ w = 0.723 \text{ kip/ft} \]

\[ R_a := w \cdot L_b \]

\[ M_{\text{pedestrian}} = \frac{\left( w \cdot L_b \right)^2}{8} \]

\[ R_a = 4.227 \text{ kip} \]

\[ M_{\text{pedestrian}} = 197.806 \text{ kip-ft} \]

\[ \frac{50 \text{ ft}}{L_b + 125 \text{ ft}} \]

\[ I = 0.291 \]

\[ M_{H20} := 425.6 \text{ ft-kip} \]

\[ M_{H20} \cdot \frac{8.5}{5.5} = \frac{225 \text{ lbf}}{2 \text{ ft}} \]

\[ M_{\text{barrier}} := \frac{L_b^2}{8} \]

\[ M_{\text{barrier}} = 30.8 \text{ kip-ft} \]

\[ V_{\text{barrier}} := \frac{L_b}{2} \]

\[ V_{\text{barrier}} = 2.632 \text{ kip} \]

\[ V_{\text{Pedestrian}} := \frac{L_b}{2} \]

\[ V_{\text{Pedestrian}} = 16.907 \text{ kip} \]

\[ V_{\text{Total}} := V_{\text{DL\_girder}} + V_{\text{DL\_slab}} + V_{\text{Pedestrian}} \]

\[ V_{\text{Total}} = 42.913 \text{ kip} \]

\[ M_{\text{fraction}} := 8.5 \cdot 0.5 \]

\[ M_{\text{fraction}} = \frac{225 \text{ lbf}}{2 \text{ ft}} \]

\[ M_{\text{ult}} := \left[ (1 + I) \cdot 1.3 \cdot 1.67 \cdot M_{H20} \right] \cdot M_{\text{fraction}} + 1.3 \left( M_{\text{DL\_slab}} + M_{\text{DL\_girder}} + M_{\text{barrier}} \right) \]

\[ M_{\text{LL}} := M_{\text{fraction}} \cdot (1 + I) \cdot M_{H20} \]

\[ M_{\text{LL}} = 424.587 \text{ ft-kip} \]

\[ M_{\text{ult}} = 1.358 \times 10^3 \text{ ft-kip} \]
\[
\text{Ratio} := \frac{M_{\text{ult}}}{(M_{DL\_slab} + M_{DL\_girder} + M_{\text{barrier}} + M_{\text{pedestrian}})}
\]

\[\text{Ratio} = 2.547\]

\[
M_{\text{ult\_ped}} := (1.3 \cdot 1.67 \cdot M_{\text{pedestrian}}) + 1.3 (M_{DL\_slab} + M_{DL\_girder} + M_{\text{barrier}})
\]

\[
M_{\text{ult\_ped}} = 865.32 \text{ ft\cdotkip}\quad \text{Needed capacity to meet code with pedestrian loading only}
\]
Original Factored Moment Capacity

\[ b := 8.5\text{ ft} \]
\[ f_u := 160\text{ ksi} \]
\[ a_{ps} := 4\text{ in}^2 \quad \text{Note 1 bar = 1 sq. in (uncorroded)} \]
\[ \beta_1 := 0.85 \]
\[ f_c := 5000\text{ psi} \]
\[ c := \frac{a_{ps} \cdot f_u}{b \cdot f_c} \]
\[ c = 1.255\text{ in} \]
\[ a := \frac{c}{\beta_1} \]
\[ a = 1.476\text{ in} \]
\[ d := 31.148\text{ in} + 7\text{ in} - 0.25\text{ in} \quad \text{Extra 1/4" assuming bar rides top of duct} \]
\[ d = 37.898\text{ in} \]
\[ \Phi := 0.96 \]
\[ M_{\text{capacity}} := a_{ps} \cdot f_u \left( d - \frac{a}{2} \right) \]
\[ M_{\text{capacity}} = 1.982 \times 10^3\text{ ft-kip} \]
\[ d_{\text{orig}} := 3\text{ ft} + 4\text{ in} + 7\text{ in} - 7.5\text{ in} \]
\[ M_{\text{ultimate}} := \Phi \cdot a_{ps} \cdot f_u \left( d_{\text{orig}} - \frac{a}{2} \right) \]
\[ M_{\text{ultimate}} = 1.861 \times 10^3\text{ ft-kip} \]
\[ R_{\text{reserve}} := \frac{M_{\text{ultimate}}}{M_{\text{ult}}} \]
\[ R_{\text{reserve}} = 1.37 \quad \text{Significant excess capacity - design was likely governed by service} \]
Estimate Min PS Area Required for resisting unfactored DL+LL

\[ b := 8.5\text{ ft} \]
\[ f_u := 160\text{ ksi} \]
\[ a_{ps} := 1.03\pi \cdot \frac{(1.125)^2}{4}\text{ in}^2 \]

Note 1 bar = 1 sq. in (uncorroded)

\[ a_{ps} = 1.024\text{ in}^2 \]

\[ \beta_1 := 0.85 \]
\[ f_c := 5000\text{ psi} \]
\[ c := \frac{a_{ps} \cdot f_u}{b \cdot f_c} \]
\[ c = 0.321\text{ in} \]
\[ a := \frac{c}{\beta_1} \]
\[ a = 0.378\text{ in} \]
\[ d := 31.148\text{ in} + 7\text{ in} - 0.25\text{ in} \]

Extra 1/4" assuming bar rides top of duct

\[ d = 37.898\text{ in} \]

\[ M_{\text{capacity}} := a_{ps} \cdot f_u \cdot \left(d - \frac{a}{2}\right) \]

\[ M_{\text{capacity}} = 514.774\text{ ft-kip} \]

\[ d_{\text{orig}} := 3\text{ ft} + 4\text{ in} + 7\text{ in} - 7.5\text{ in} \]

\[ M_{\text{ultimate}} := a_{ps} \cdot f_u \cdot \left(d_{\text{orig}} - \frac{a}{2}\right) \]
$M_{\text{ultimate}} = 536.643 \text{ft} \cdot \text{kip}$

$$R_{\text{ped\_reserve}} = \frac{M_{\text{ultimate}} - (MDL_{\text{slab}} + MDL_{\text{girder}} + M_{\text{barrier}})}{M_{\text{pedestrian}}}$$

$R_{\text{ped\_reserve}} = 1.018$  
Ratio of pedestrian LL to remaining capacity assuming no uncertainty in DL
Estimate Ultimate Load as Multiple of Pedestrian Load

\[
b := 8.5\text{-ft}
\]
\[
f_u := 160\text{ ksi}
\]
\[
a_{ps} := 4\pi \left(\frac{1.125}{4}\right)^2\text{-in}^2
\]
\[
\beta_1 := 0.84
\]
\[
a_{ps} = 3.976\text{in}^2
\]
\[
f'c := 5000\text{ psi}
\]
\[
c := \frac{a_{ps} \cdot f_u}{b \cdot f'_c}
\]
\[
c = 1.247\text{in}
\]
\[
a := \frac{c}{\beta_1}
\]
\[
a = 1.468\text{in}
\]
\[
d := 31.148\text{in} + 7\text{-in} - 0.25\text{in}
\]
\[
d = 37.898\text{in}
\]
\[
M_{\text{capacity}} := a_{ps} \cdot f_u \left(d - \frac{a}{2}\right)
\]
\[
M_{\text{capacity}} = 1.97 \times 10^3\text{ft-kip}
\]
\[
d_{\text{orig}} := 3\text{-ft} + 4\text{-in} + 7\text{-in} - 7.5\text{in}
\]
\[
M_{\text{ultimate}} := a_{ps} \cdot f_u \left(d_{\text{orig}} - \frac{a}{2}\right)
\]
\[
M_{\text{ultimate}} = 2.055 \times 10^3\text{ft-kip}
\]
$$R_{ped\_reserve} := \frac{M_{\text{ultimate}} - (M_{\text{DL\_slab}} + M_{\text{DL\_girder}} + M_{\text{barrier}})}{M_{\text{pedestrian}}}$$

\[ R_{ped\_reserve} = 8.695 \]  

Ratio of pedestrian LL to remaining capacity assuming no uncertainty in DL - used to compare to ANSYS
Service Design Check

\[ I_g := 3.1228 \text{ft}^4 \]

\[ A_g := 2.4458 \text{ft}^2 \]

\[ r := \frac{I_g}{\sqrt{A_g}} \]

\[ M_D := M_{DL\_girder} \]

\[ M_{SD} := M_{DL\_slab} \]

\[ y_t := 1.723 \text{ft} \]

\[ h := 3\text{-ft} + 4\text{-in} \]

\[ y_b := h - y_t \quad y_b = 19.324\text{in} \]

\[ S_b := \frac{I_g}{y_b} \]

\[ M_{CSD} := M_{\text{barrier}} \]

\[ M_L := M_{H20}(1 + I) \cdot M_{\text{fraction}} \]

\[ y_{\text{bar1}} := y_b - 4.5\text{-in} \]

\[ y_{\text{bar2}} := y_b - 4.5\text{-in} \]

\[ y_{\text{bar3}} := y_{\text{bar1}} - 4\text{-in} \]

\[ y_{\text{bar4}} := y_{\text{bar3}} - 4\text{-in} \]

\[ N_{\text{bars}} := 4 \]

\[ e := \frac{(y_{\text{bar1}} + y_{\text{bar2}} + y_{\text{bar3}} + y_{\text{bar4}})}{N_{\text{bars}}} \]
\( e = 11.824 \text{ in} \)

\( c_b := y_b \)

\( A_{\text{bar}} := \pi \frac{(1.125 \text{ in})^2}{4} \)

\( A_{\text{bar}} = 0.994 \text{ in}^2 \)

\( P_{\text{bar}} := 0.8 \times 0.80 \times 160 \text{ ksi} \cdot A_{\text{bar}} \)

\( P_{\text{bar}} = 81.43 \text{ kip} \)

\( P_e := N_{\text{bars}} \cdot P_{\text{bar}} \)

\( P_e = 325.72 \text{ kip} \)

\( l_{\text{comp}} := 187702.84 \text{ in}^4 \)

\( y_{\text{bcom}} := 33.485 \text{ in} \)

\( S_{\text{cb}} := \frac{l_{\text{comp}}}{y_{\text{bcom}}} \)

\( f_b := \frac{-P_e}{A_g} \left(1 + \frac{e \cdot c_b}{r^2}\right) + \frac{M_D + M_{\text{SD}}}{S_b} + \frac{M_{\text{CSD}} + M_L}{S_{\text{cb}}} \)

\( f_b = -8.875 \text{ psi} \quad \text{Okay, bottom fiber in compression} \)

\( f_c = 5 \times 10^3 \cdot \text{ psi} \)

\( f_r := 7.5 \sqrt{\frac{f_c}{\text{ psi}}} \)

\( f_r = 530.33 \text{ psi} \quad \text{Modulus of Rupture} \)
**Needed Ultimate Load as Multiple of Pedestrian Load**

\[ M_{\text{ult_ped}} = 865.32 \text{ftkip} \]

\[ M_{\text{totl_DL}} := M_{\text{DL_slab}} + M_{\text{DL_girder}} + M_{\text{barrier}} \]

\[ M_{\text{totl_DL}} = 335.295 \text{ftkip} \]

\[ M_{\text{req}} := M_{\text{ult_ped}} - M_{\text{totl_DL}} \]

\[ M_{\text{req}} = 530.025 \text{ftkip} \]

\[ R_{\text{reqd}} := \frac{M_{\text{req}}}{M_{\text{pedestrian}}} \]

\[ R_{\text{reqd}} = 2.68 \]

This is the target ratio of pedestrian loading needed from ANSYS for the structure to be consider safe.
APPENDIX B:  FINITE ELEMENT ANALYSIS RESULTS –DESIGN CONDITIONS
Figure B-1  Load Case #1 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-2  Load Case #2 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-3  Load Case #3 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-4  Load Case #4 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-5  Load Case #5 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-6  Load Case #6 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-7  Load Case #7 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-8  Load Case #8 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-9  Load Case #9 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-10  Load Case #10 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-11  Load Case #11 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).
Figure B-12  Load Case #12 - Interior Beam Bending Moment Diagram, Shear Force Diagram and Axial Force (units kip-ft, kip).