ATTENTION:

Your grade for this assignment is based on your answers to the end-of-module questions AND the assessment test that you will take IN CLASS on the next day that class meets after the assignment is due. If you DO NOT take the assessment test you will NOT be eligible for full points.

The module itself will take approximately 2 hours to complete. Please do not leave this until the last minute.

Density and the layers of the Earth

Core Quantitative Literacy Topics
Percentages, ratios, weighted average

Supporting Quantitative Literacy Topics
Unit Conversions

Core Geoscience Subject
Layers of the Earth, density

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Getting started

After completing this module you should be able to:

- Calculate density
- Calculate mass and volume
- Name and describe Earth's layers

In 1772, Nevil Maskelyne was pondering Sir Isaac Newton's theory of gravitation, in which it was proposed that ANY object should exert a gravitational attraction towards any other object. Maskelyne surmised that this should mean that one could measure the gravitational attraction of, for instance, a large mountain and that it should be proportional to the gravitational attraction of the entire Earth. So he set to work measuring the gravitational attraction that a mountain named Schiehallion exerted on his plumb bob (a kind of weight on a string used by surveyors). Six years later a geologist named Charles Hutton used the measurements taken by Maskelyne to estimate the mean density of the Earth at 4.5 g/cm^3, which is pretty close to the modern estimate of 5.5 g/cm^3!

The Earth and its layers

The Earth consists of several distinct layers. The crust is the topmost layer, which we live on. The crust has an average composition similar to granite and consists of both the ocean basins and the continents. Beneath the crust is the mantle, which has the composition of peridotite. It makes up the largest volume of the whole Earth by far. Finally, the core is composed of an outer core and an inner core. Both the inner core and outer core are composed of an iron-nickel alloy but the outer core is liquid while the inner core is solid. The liquid outer core is the source of Earth's magnetic field.

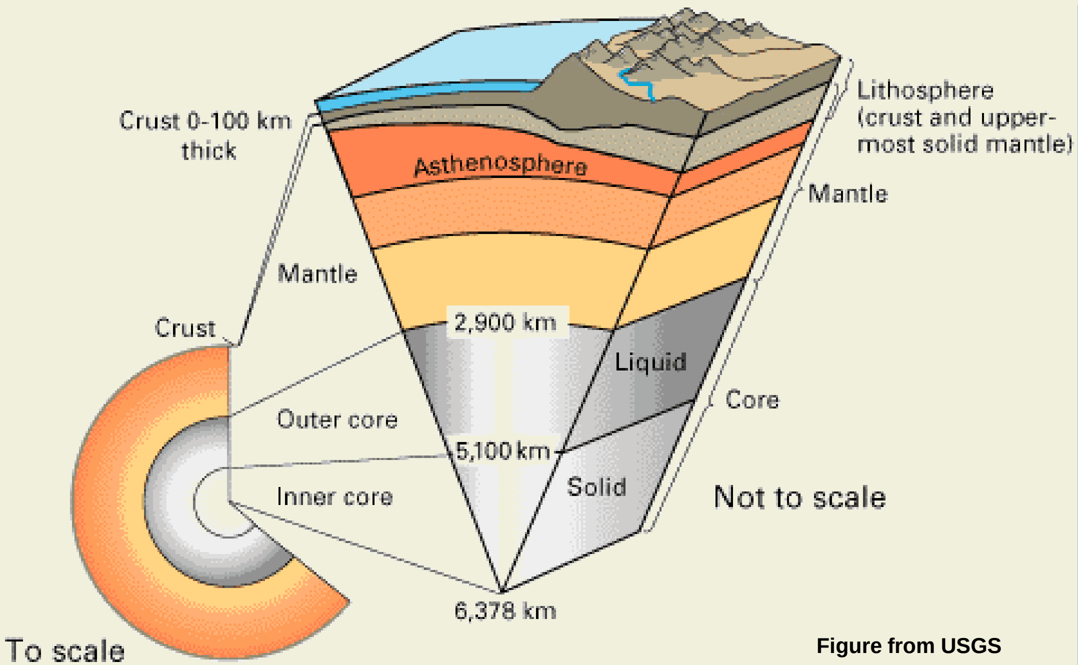


Figure from USGS

Looking closer: The crust and upper mantle also make up two physically distinct layers called the lithosphere and the asthenosphere.

•The lithosphere is composed of the crust and uppermost mantle and is a rigid material which will break when deformed.

•The asthenosphere is composed only of upper mantle and will flow like warm wax when deformed.

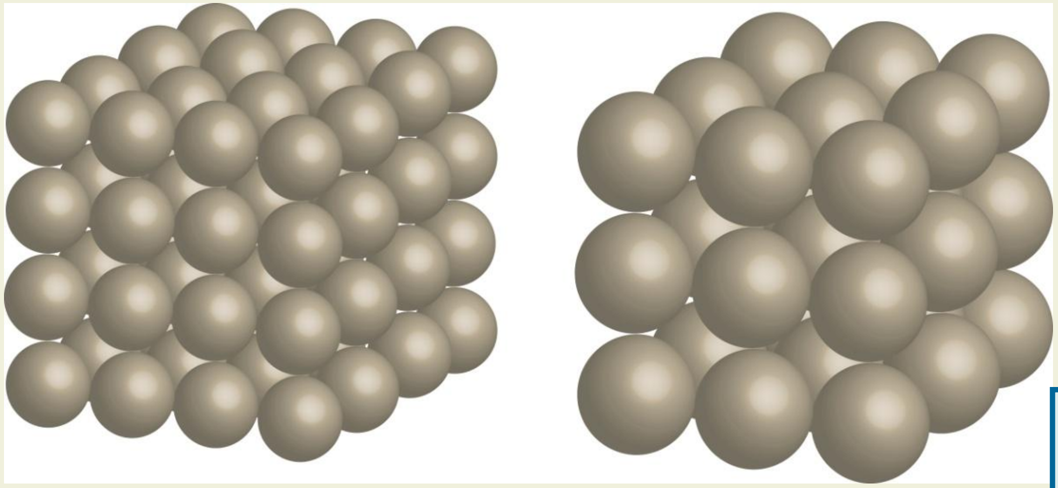
Density

The density of an object is the ratio of its mass to its volume and in general describes how tightly packed the atoms in a material are. The formula for density is:

$$\rho = m / v = \text{mass/volume}$$

A good way to understand density is to think of two cubes with the exact same volume, one cube is made of foam the other of iron as shown in the figure below. Which cube would "feel" heavier?? The iron cube has the larger density (i.e., a larger mass of iron atoms per the same volume cube) and therefore is heavier than the foam cube. If we were to look at those cubes under a microscope we would see that the atoms that make up the iron are much more closely packed into the cube than those of the foam. Therefore we can fit many more iron atoms into the same volume than we can atoms that make up the foam. Since each atom has a certain mass (known as the atomic mass) the more atoms we have in the cube the more mass we have in the cube and thus the larger the density of the cube.

Iron



Foam

Return to Slide 8 or Slide 12.

Density of Earth as a function of depth

Each layer of Earth has a specific density. However, rocks found at the surface of Earth have a much lower density than that of the whole Earth. What does this mean? Where are the high density materials that increase the average density of the whole Earth? The answer is that the interior layers (mantle, outer core and inner core) become denser with depth. This makes sense if we think about what density is.

As we go deeper into the Earth the pressure increases because the amount of material above increases. This increased pressure helps to pack the atoms of Earth's inner materials more tightly together thus increasing the density! So the densest layer of Earth is the inner core.

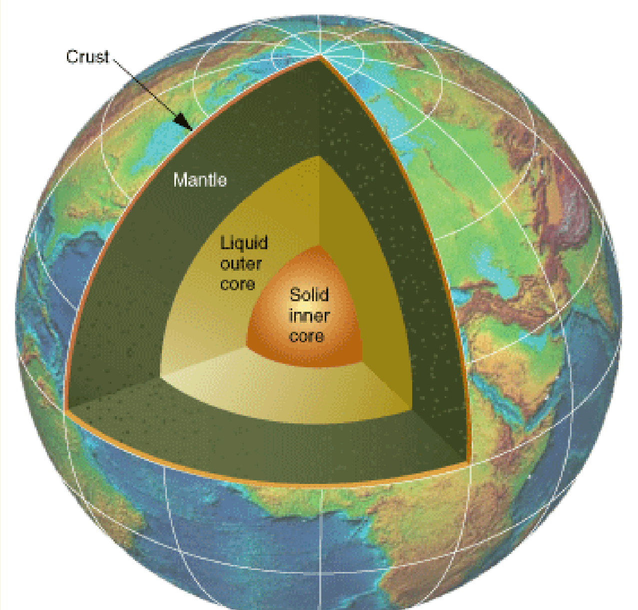


Photo from llnl.gov

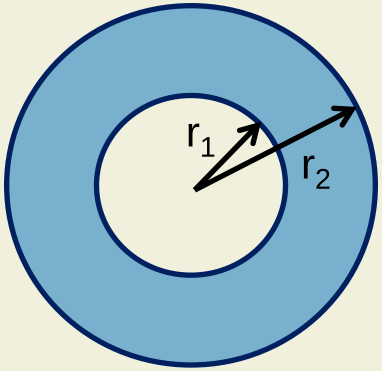
Volume of the Earth

As you should have noticed, the equation for density requires a volume. For the volume of the whole earth this is calculated by using the equation for the volume of a sphere.

$$V_{sphere} = \left(4/3\right)\pi\, r^3,$$

where r is the radius of the sphere. However, for the volumes of the individual layers the calculation is a little bit more complicated as we must now calculate the volumes of spherical shells. Each spherical shell now has an inner (r_1) and outer (r_2) radius, as shown below. The equation for calculating the volume of a spherical shell takes both of these radii into account.

$$V_{shell} = \left(4/3\right)\pi\left(r_2^3 - r_1^3\right).$$



Where r_1 = inner radius

r_2 = outer radius

Return to Slide 8 or Slide 12.

The calculation

Given the values in the tables below you can calculate the density of each layer of the Earth and the density of the whole Earth. You will do your calculations and enter your answers in the Density Worksheet which looks like the one below.

Layer	Thickness (km)	Mass (kg)
Whole Earth	6370	5.95×10^{24}
Layer	Thickness (km)	% Mass
Crust	70	2
Mantle	2900	66
Outer Core	2200	30
Inner Core	1200	2

Cells with given/entered values

Cells in which you will enter formulas

3	1	2			Thickness (km)	Mass (kg)	Volume (km^3)	Density (kg/km^3)	Density (g/cm^3)	
4				Earth	6370	5.95E+24				
5				Radius	Length (km)					
6				r_1						
7	2	4		r_2						
8				r_3						
9				r_4						
10				Layer	Thickness (km)	% Mass	Volume (km^3)	Mass (kg)	Density (kg/km^3)	Density (g/cm^3)
11				Crust	70	2				
12	3	4		Mantle	2900	66				
13				Outer Core	2200	30				
14				Inner Core	1200	2				
15	4	3		Whole Earth						

Density of the whole Earth

First begin by calculating the volume (see Slide 6) and then density (see Slide 4) of the whole Earth in kg/km^3 and then convert the density to g/cm^3. Do your calculations in section 1 of your worksheet.

3	1	2		Thickness (km)	Mass (kg)	Volume (km^3)	Density (kg/km^3)	Density (g/cm^3)	
4			Earth	6370	5.95E+24				
5			Radius	Length (km)					
6			r_1						
7	2	4	r_2						
8			r_3						
9			r_4						
10			Layer	Thickness (km)	% Mass	Volume (km^3)	Mass (kg)	Density (kg/km^3)	Density (g/cm^3)
11			Crust	70	2				
12	3	4	Mantle	2900	66				
13			Outer Core	2200	30				
14			Inner Core	1200	2				
15	4	3	Whole Earth						

Hint: Remember to convert km^3 to cm^3 you will need to cube the conversion factor. For example, to change 1000 km^3 to m^3, there are 1000 m in 1 km so the conversion is as follows:

$$\frac{1000\ km^3}{1} \times \left(\frac{1000\ m}{1\ km}\right)^3 = 1,000,000,000,000\ m^3$$

Return to Slide 14.

Density - Section 1

Question 1 of 2

What is the volume of the whole Earth that you just calculated in Section 1 of your worksheet?

- ☐ 1.1×10^{12} km^3
- ☐ 2.1×10^{12} km^3
- ☐ 1.1×10^{15} km^3

What is the density of the whole Earth that you just calculated in Section 1 of your worksheet?

- ☐ 5.5×10^{12} kg/km^3
- ☐ 5.5×10^{11} kg/km^3
- ☐ 4.0×10^{13} kg/km^3

*Remember that you can always go back to the previous slides to review

PROPERTIES

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User may view slides after quiz:	After passing quiz
User may attempt quiz:	Unlimited times

Properties...

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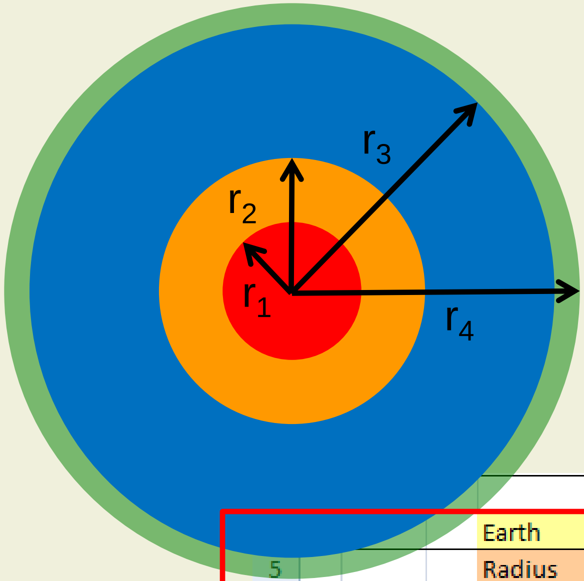
Radii of spherical shells

Use the layer thicknesses to calculate the radii of the spherical shells for each layer of the Earth shown in the picture below. Fill in section 2 of your spreadsheet.



					Thickness (km)	Mass (kg)	Volume (km^3)	Density (kg/km^3)	Density (g/cm^3)	
				Earth	6370	5.95E+24				
5				Radius	Length (km)					
6				r_1						
7	2	4		r_2						
8				r_3						
9				r_4						
10				Layer	Thickness (km)	% Mass	Volume (km^3)	Mass (kg)	Density (kg/km^3)	Density (g/cm^3)
11				Crust	70	2				
12	3	4		Mantle	2900	66				
13				Outer Core	2200	30				
14				Inner Core	1200	2				
15	4	3		Whole Earth						

Density - Section 2

Question 1 of 2 ▾

What is the value for r_1 from Section 2 of your worksheet?

- ☐ 1200 km
- ☐ 1000 km
- ☐ 1300 km

What is the value for r_2 from Section 2 of your worksheet?

- ☐ 3000 km
- ☐ 3200 km
- ☐ 3400 km

PROPERTIES

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On failing, 'Finish' button:	**Goes to Slide**
Allow user to leave quiz:	**At any time**
User may view slides after quiz:	**After passing quiz**
User may attempt quiz:	**Unlimited times**

Properties...

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Volume, mass, and density of Earth's layers

Calculate these values in Section 3 of your worksheet.

A) Calculate the volume of each layer using the formula from Slide 6.

B) Then calculate the mass of each layer using the percentage of mass and the mass of the whole Earth.

C) Finally calculate the density of each layer in kg/km^3 (see Slide 4) and convert it to g/cm^3.

3	1	2			Thickness (km)	Mass (kg)	Volume (km^3)	Density (kg/km^3)	Density (g/cm^3)	
4				Earth	6370	5.95E+24				
5				Radius	Length (km)					
6				r_1						
7	2	4		r_2						
8				r_3						
9				r_4						
10				Layer	Thickness (km)	% Mass	Volume (km^3)	Mass (kg)	Density (kg/km^3)	Density (g/cm^3)
11				Crust	70	2				
12	3	4		Mantle	2900	66				
13				Outer Core	2200	30				
14				Inner Core	1200	2				
15	4	3		Whole Earth						

Density - Section 3

Enter the value for the volume of the crust you just calculated in Section 3.

- ☐ 3.5×10^{10} km^3
- ☐ 3.7×10^{12} km^3
- ☐ 3.5×10^{13} km^3

Enter the value for the volume of the mantle you just calculated in Section 3.

- ☐ 8.8×10^{11} km^3
- ☐ 9.8×10^{11} km^3
- ☐ 8.8×10^{9} km^3

PROPERTIES

On passing, 'Finish' button:	Goes to Next Slide
On failing, 'Finish' button:	Goes to Slide
Allow user to leave quiz:	At any time
User may view slides after quiz:	After passing quiz
User may attempt quiz:	Unlimited times

Properties...

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Density of whole Earth – the check

Now, let's check to make sure we got it right. Sum the thicknesses, % masses, volumes, and masses of the different layers. Then calculate the density of the whole Earth in Section 4 of your worksheet. How does the density of the whole Earth calculated using all the layers compare to the one you calculated in Slide 8 (Section 1)??

3	1	2			Thickness (km)	Mass (kg)	Volume (km^3)	Density (kg/km^3)	Density (g/cm^3)	
4				Earth	6370	5.95E+24				
5				Radius	Length (km)					
6				r_1						
7	2	4		r_2						
8				r_3						
9				r_4						
10				Layer	Thickness (km)	% Mass	Volume (km^3)	Mass (kg)	Density (kg/km^3)	Density (g/cm^3)
11				Crust	70	2				
12	3	4		Mantle	2900	66				
13				Outer Core	2200	30				
14				Inner Core	1200	2				
15	4	3		Whole Earth						

Return to Slide 16.

Density - Section 4

Question 1 of 1

Enter the value for the density of the whole Earth in g/cm^3 from Section 4 of your worksheet.

PROPERTIES

On passing, 'Finish' button:	Goes to Next Slide
On failing, 'Finish' button:	Goes to Slide
Allow user to leave quiz:	At any time
User may view slides after quiz:	After passing quiz
User may attempt quiz:	Unlimited times

Properties...

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End-of-module assignments

1. To calculate the density of the whole Earth in Slide 14 you used the following equation:

$$\rho = \frac{\sum M_{shells}}{\sum V_{shells}}$$

(the sum of the masses of each layer divided by the sum of the volumes of each layer), which is a weighted average. Now go back to your spreadsheet and calculate the average of the layer densities from Column G (excluding the whole Earth) using Excel's AVERAGE() function. How do the two densities you calculated compare? Explain this discrepancy.

2. Now calculate the density of the Moon using the table of information given below. How does the density of the moon compare to that of the Earth? Does this make sense, given how the Moon was formed?

Layer	Thickness (km)	Mass (kg)
Whole Moon	1738	7.35×10^{22}
Layer	Thickness (km)	% Mass
Crust	65	11
Mantle	1673	89

3. Another example of a weighted average is the calculation of a weighted grade. Imagine that in a course the grade is based on 4 exams and 4 assignments. The exams are worth 60% and the assignments are worth 40%. On the exams you receive a 45/50, 47/50, 20/50, and 49/50. On the assignments you receive a 10/20, 20/20, 15/20, and 18/20. Calculate the final grade you will receive in the course using the following equation, where w_i is the weighting factor and x_i is the average grade for either the exams or assignments.

$$Grade = \frac{\sum w_i x_i}{\sum w_i}$$

End notes – Scientific notation

When dealing with very large numbers it is easier to use scientific notation. For example the number 1,000,000,000 in scientific notation is 1.0×10^9, where the 10^9 (ten to the ninth power) represents the 9 zeros. This is **much** easier to deal with than all those zeros!

Examples:

2.546×10^6 = 2,546,000 (take 2.546 and move the decimal place to the right by 6 places and fill in the blank spaces with zeros)

8.5×10^{-12} = 0.0000000000085 (this time move the decimal point 12 spaces to the left and add in zeros as needed. The ten raised to a negative power is a very small number)

Return to Slide 8.