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Young Adolescents’ Opportunity To Develop Concept Images of Polygons in Middle School Mathematics Textbooks

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Young Adolescents’ Opportunity To Develop Concept Images of Polygons in Middle School Mathematics Textbooks

by

Megan N. Cannon

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Curriculum and Instruction with a concentration in Mathematics Education Department of the Teaching and Learning College of Education University of South Florida

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DEDICATION

To my husband, Billy Oropallo, I could not imagine going through this process without you. There are so many times that I wanted to quit and you were always there to bring back my confidence and drive. Watching you go through this process was inspiring and motivating. You are truly the best partner I could have ever imagined and I cannot wait to see what life has in store for us next.

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ABSTRACT

Young adolescents build notions of figures through experiences. When young adolescents begin middle school mathematics, they already have prior assumptions and conceptualizations about shapes (Herbst, Fujita, Halverscheid, & Weiss, 2017). Over years of experiences, interactions, and exposures with a particular concept, these students develop concept images consisting of all the visual information, pictures, mental images, or properties associated with the particular mathematical concept (Vinner, 1983). Seminal works in the field of mathematics education research found that without an opportunity to engage with varied representations, young adolescent students may not develop robust concept images (Aspinwall, Shaw & Presmeg, 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).

Therefore, in this study, I analyzed young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks (grades 6-8). I chose to analyze convex polygons in nine textbooks: Pearson enVisionmath2.0 Grades 6, 7, 8 (Berry et al., 2017), Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014), and McGraw Hill Education Glencoe Math Course 1, 2, 3 (Carter et al., 2015). This sample of textbooks represents one series from each of the three major textbook publishers: Pearson, Houghton Mifflin Harcourt, McGraw Hill Education (Banilower et al., 2018).
I explored young adolescent’s opportunity to develop concept images of polygons in middle school mathematics textbooks in terms of the exposure to a variety of polygons represented, exposure to a variety of polygon orientations, exposure to non-prototypical images, exposure to a variety of contexts, exposure to a variety of image roles, and exposure to developmentally responsive tasks. The results of this study indicate that within three published middle school mathematics textbook series, students do not have adequate opportunities to develop robust concept images of polygons. In all three textbook series, students have more exposure to images of convex polygons in eighth-grade textbooks, followed by sixth-grade textbooks, and the finally seventh-grade textbooks, however, all three textbook series contained little variety in terms of types of convex polygons represented. The three textbook series did not provide ample opportunities for students to engage with images of convex polygons in a variety of orientations and did not provide many opportunities for students to engage with non-prototypical images of convex polygons. Additionally, the three textbook series did not provide much variety in terms of contexts and roles of images with a majority of convex polygons set in purely mathematical contexts and as interpretive images. Further, all three textbook series provided little to no developmentally responsive tasks containing images of convex polygons in terms of young adolescents' physical, psychological, social-emotional, and moral needs.
CHAPTER 1: INTRODUCTION

Young adolescents enter middle school mathematics classrooms as students with prior notions and conceptions of figures based on their previous experiences (Herbst et al., 2017). Over years of experiences, interactions, and exposures with a particular concept, these students develop concept images consisting of all the visual information, pictures, mental images, or properties associated with the particular mathematical concept (Vinner, 1983). Seminal works in the field of mathematics education research found that without an opportunity to engage with varied representations, young adolescent students may not develop robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Therefore, the purpose of this dissertation was to explore young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks.

In this chapter, I present background information about geometry in middle school and recommendations and standards for teaching and learning geometry. I provide the problem statement, research questions, rationale and purpose, and introduce the literature bases guiding my inquiry, specifically, middle school education research and research on geometry teaching and learning. After reviewing the literature bases informing my study, I end the chapter with the significance of my study, delimitations, definitions of terms, and a summary.
Geometry in Middle School

Geometry has existed in work from Egypt, India, and Babylon for over 5000 years, but was organized systematically by Euclid 2000 years ago (Clark, 2012). In geometry, students develop spatial skills and learn how to analyze space and shape (Van de Walle et al., 2016). Geometry is taught in a variety of formats at the middle school level. Some middle school students receive a stand-alone geometry course (often for high school credit) though the majority of middle schools provide geometry content in an integrated format. In middle school, geometry content is vertically aligned and connected to other mathematics domains, for example, measurement and algebra (Association of Mathematics Teacher Education [AMTE], 2017). Geometry content in middle school usually includes real-world problems involving angle measure, area, surface area, and volume, constructions of and relationships of figures, congruence and similarity, and Pythagorean theorem and applications (National Governors Association Center for Best Practices [NGACBP], Council of Chief State School Officers [CCSSO], 2010).

Geometry consists of four major strands: shapes and properties, transformations, location, and visualization (Van de Walle et al., 2016). The strand usually associated with middle school, shapes and properties, involves sorting and classifying, composing and decomposing, categories of 2D and 3D shapes, construction activities, applying definitions and categories, and investigations, conjectures, and the development of proof. Transformations refer to changes in position, size, and shape; particularly, symmetries, translations, congruence, similarity, and dilation. The location strand involves positional descriptors and the use of coordinate planes. The
last strand, visualization, involves mental imagery and manipulation of 2D and 3D images. These four strands overlap in some areas and build on each other (Van de Walle et al., 2016).

Through learning geometry, young adolescent students develop spatial sense applicable to mathematical contexts as well as other contexts (Van de Walle et al., 2016). Basic knowledge of geometry is required for any mathematics or science study but is also necessary for any well-rounded education (Clark, 2012; Van de Walle et al., 2016). However, in the United States (U.S.), students are often underperforming in the geometry domain (Mullis et al., 1997; Mullis et al., 2000; Mullis et al., 2008; Mullis et al., 2012; Mullis et al., 2016).

In general, students in the United States are underperforming in mathematics; only 34% of eighth-grade students tested at a level of proficient or above (National Center for Educational Statistics [NCES], National Assessment of Educational Progress [NAEP], 2019). The Trends in International Mathematics and Science Study (TIMSS) collects data on fourth and eighth-grade students mathematics and science achievement to compare U.S. student achievement to international results. The results from the TIMSS in recent years (2007, 2011, and 2015) have shown that eighth-grade U.S. students are underachieving in geometry (Mullis et al., 2008; Mullis et al., 2012; Mullis et al., 2016). Eighth-graders in the United States scored lower in the content domain of Geometry than fourth-graders (Mullis et al., 2016). Unfortunately, in the U.S., middle school students are not developing an adequate understanding of geometry or visuospatial reasoning (Battista, 1999).

**National Recommendations and Standards**

Young adolescents, 10-15 years of age, are typically enrolled in middle school consisting of grades 6-8 (although other grade level configurations do exist). During early adolescence
while in middle school, many young adolescents are developing their cognitive abilities and
experiencing a transition in mathematics learning from primarily concrete thinking to abstract
understanding (Caskey & Anfara, 2014). This means young adolescents are beginning to
problem solve without the need for physical or tangible visual representations provided for them
and are instead starting to internally visualize and reason (AMTE, 2017). However, this
transition from concrete to abstract thinking is not universal as some students can experience
cognitive development very differently based on areas of expertise, interest, and culture (Chi et
al., 1989; Saxe, 2015; Smagorinsky, 2013).

In middle school, the National Council of Teachers of Mathematics (NCTM) suggests
young adolescents should engage in activities that develop their spatial skills in one, two, and
three dimensions such as composing shapes, creating nets, finding area, perimeter, surface area,
and volume of two and three-dimensional figures, and identifying, classifying and defining
shapes (1989, 2000). Further, NCTM’s Curricular Focal Points for Pre-K through Grade 8
emphasize composing and decomposing, solving problems area and volume through
compositions, similarity, and analyzing spaces using distance and angle measure (Fennell, 2006).
Additionally, the creators of the Common Core State Standards have emphasized solving real-
world and mathematically contextualized problems angle measure, involving area, surface area,
and volume, constructing and describing geometric figures, understanding and applying the
Pythagorean Theorem, and understanding congruence and similarity through tools like models
and dynamic geometry software (NGACBP, CCSSO, 2010). More recently, NCTM (2014)
advocated for the inclusion of more mathematical representations including visual, contextual,
and physical representations in mathematics teaching and learning.
**Problem Statement**

Research on young adolescent development states that middle school is a pivotal time for cognitive development (Caskey & Anfara, 2014; NMSA, 2010). In mathematics, middle school represents the period when many students transition from concrete representations of mathematical concepts to abstract thinking and classification (AMTE, 2017; Caskey & Anfara, 2014; Smith et al., 2018). This cognitive development can vary significantly from student to student among ages, grades, and subjects (Caskey & Anfara, 2014) and some students can experience cognitive development very differently based on areas of expertise, interest, and culture (Chi et al., 1989; Saxe, 2015; Smagorinsky, 2013). As students transition from concrete to abstract thinking, they also progress along geometric learning trajectories (Clark, 2012; Van de Walle et al., 2016). Figures and visual images mediate this transition (Herbst et al., 2017). Thus, if mathematical representations of polygons are not sufficiently varied, students may develop inaccurate concept images of these polygons (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Textbooks often represent the intended curriculum (Lloyd et al., 2016; Stanic & Kilpatrick, 2003) and consequently, provide an opportunity for young adolescent students to develop concept images of polygons.

In middle school mathematics classrooms, textbooks play a role in structuring the sequence of material, determining what concepts to teach, providing examples and explanations, and distributing class time across topics (Banilower et al., 2018; Fan, 2013; Fan et al., 2013; Lloyd et al., 2016; Nicol & Crespo, 2006; Stanic & Kilpatrick, 2003; Reys et al., 2004). Textbooks often represent the intended curriculum, the potentially implemented curriculum, or
the de facto curriculum (Houang & Schmidt, 2008; Lloyd et al., 2016; Törnroos, 2005). The textbook used in a mathematics classroom can impact student learning (Hadar, 2018; Houang & Schmidt, 2008; Nicol & Crespo, 2006; Törnroos, 2005). Students’ opportunity to learn is defined as the encounters, occasions, experiences, and engagement students have with content, teaching and learning strategies, and particular topics (Hadar, 2018; Hiebert & Grouws, 2007; Houang & Schmidt, 2008; Törnroos, 2005), and therefore, textbooks can impact young adolescents’ opportunity to learn mathematics topics. Hence, the images of polygons in middle school mathematics textbooks influence young adolescents’ development of concept images of polygons (Vinner, 1983).

In this dissertation study, I analyzed young adolescents’ opportunities to develop concept images of polygons in three popular middle school mathematics textbook series. I included textbooks in the sample based on market share of publishers. In the 2018 National Survey of Science and Mathematics Education (NSSME), Banilower and colleagues found three U.S. publishing companies provide 80% of the textbooks utilized in middle school mathematics classrooms; in particular, Pearson comprised 17%, Houghton Mifflin Harcourt comprised 37%, and McGraw Hill Education comprised 26% of the market share of middle school mathematics textbooks (Banilower et al., 2018). Therefore, I chose to analyze one popular middle school mathematics textbook series from each of these three publishers at the 6th, 7th, and 8th-grade level. Specifically, in 6th-grade classrooms, the three most commonly used textbook series reported by Banilower and colleagues (2018) were Pearson enVisionmath2.0 (Berry et al., 2017), Houghton Mifflin Harcourt Go Math! (Burger et al., 2014), and McGraw Hill Education Glencoe Math (Carter et al., 2015). Because this selection of textbooks represents one series from each of
the three major textbook publishers (Pearson, Houghton Mifflin Harcourt, McGraw Hill
Education), the textbook sample for this study consists of nine textbooks: Pearson

*enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017), Houghton Mifflin Harcourt *Go Math! Grades 6, 7, 8* (Burger et al., 2014), and McGraw Hill Education *Glencoe Math Course 1, 2, 3* (Carter et al., 2015).

**Research Questions**

The purpose of this study is to analyze young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks (grades 6-8). Specifically, the research question guiding this study was as follows: Within three published middle school mathematics textbook series, in what ways do young adolescents have opportunities to develop concept images of polygons?

1. In what ways do tasks within the textbooks involving visual representations of convex polygons attend to young adolescents’ cognitive-intellectual characteristics? In particular:

   a. How are images of convex polygons oriented in the textbooks?

   b. How are right angles oriented within images of right triangles in the textbooks?

   c. What percentage of convex polygons in the textbooks are prototypical images, in other words, the most common representation of a particular polygon?

   d. In what contexts (i.e., real-world or purely mathematical) are convex polygons represented within the textbooks?

   e. What is the role of the images (i.e., superfluous, illustrative, or interpretive) representing convex polygons within the textbooks?
2. To what extent do tasks within the textbooks involving visual representations of convex polygons attend to other developmental characteristics of young adolescents? In particular:
   a. To what extent do tasks involving visual representations of convex polygons address young adolescents’ physical characteristics?
   b. To what extent do tasks involving visual representations of convex polygons address young adolescents’ psychological characteristics?
   c. To what extent do tasks involving visual representations of convex polygons address young adolescents’ social-emotional characteristics?
   d. To what extent do tasks involving visual representations of convex polygons address young adolescents’ moral characteristics?

**Rationale and Purpose**

The purpose of this study was to analyze young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks. My personal motivation for completing the current study stems from my previous experiences teaching mathematics at the K-12 and college level, teaching and mentoring preservice middle school mathematics teachers, and engaging in previous content analysis research. As a mathematics teacher, I found a common difficulty for K-12 math students, college math students, and preservice teachers was identification and classification of polygons that were represented in atypical ways. As a supervisor for preservice middle school math teachers, I understand that middle school is a crucial period of time in a students’ development and as most young adolescents transition from concrete to abstract thinking, they need exposure and interaction with a variety of
representations (AMTE, 2017; Caskey & Anfara, 2014). However, previous content analysis research projects that I completed confirmed that many textbooks are lacking in variety, especially in visual representations and if teachers also struggle with unusual representations, then they will be unable to supplement the textbook images.

Visual representations are an integral part of mathematics teaching and learning. Mathematical visualizations can support students’ intuition towards solutions, promote conceptual understanding, exhibit algebraic or symbolic connections, or even provide proof or justification for arguments (Arcavi, 2003). Additionally, pictures, diagrams, or other representations are effective tools for mathematical problem solving (Polya, 1945). The NCTM recognizes the importance of mathematical visualization in *Principles to Actions* by including “use and connect mathematical representations” as one of the eight Mathematical Teaching Practices to strengthen the teaching and learning of mathematics (NCTM, 2014). Further, NCTM posits that “effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving” (NCTM, 2014, p. 10).

With the importance of visual representations in mathematics education, AMTE (2017) asserts that well-prepared teachers of mathematics be capable of using appropriate representations. However, merely incorporating visual images in mathematics instruction is not enough. Over years of experiences, interactions, and exposures with a particular concept, students develop concept images consisting of all the visual information, pictures, mental images, or properties associated with the particular mathematical concept (Vinner, 1983). If mathematical representations are not sufficiently varied, students may incorrectly associate
irrelevant properties of images with properties of the mathematical concepts the images represent (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Variety of images is particularly important at the middle school level as students move from a concrete understanding of geometric figures to more abstract representations (AMTE, 2017). Because textbooks function as the “de facto curriculum for teachers” (Stanic & Kilpatrick, 2003, p. 969) and often represent the intended curriculum (Lloyd et al., 2016), the representations of convex polygons presented in textbooks can influence young adolescent students’ development of concept images of polygons.

Guiding Literature

Three literature bases guided my research: middle school education research, research on geometry teaching and learning, and the role of textbooks in teaching and learning. The research questions for this dissertation connect two of the literature bases: middle school education research and research on geometry teaching and learning. While the third base, the role of textbooks in teaching and learning, justified the use of textbook analysis as a research design and details the importance of textbooks in teaching and learning middle school mathematics.

Middle School Education Research

In the United States, there is variation in the definition of ‘middle school’. Some credentialing agencies do not even use the term middle school and instead use middle level or middle grades (Howell et al., 2018). Defining middle school is further complicated by overlapping grade bands in credentialing. Licensing for many states span grades K-6 for elementary school teaching, 5-9 for middle school, 6-12 for secondary school; though some states credential other grade bands under the term ‘middle school’ (AMLE, 2012; Howell et al.,
2018). For example, some states have limited overlap, such as Missouri that certifies 1-6, 5-9, and 8-12 (Howell et al., 2018). Other states have full overlap, such as Georgia that certifies K-5, 4-8, and 6-12 (Howell et al., 2018). Further, other states have no middle certification, such as Colorado which certifies K-6 and 7-12 (Howell et al., 2018). Typically, middle school refers to grades 6-8 (AMTE, 2017), and therefore in this study middle school refers to 6th, 7th, and 8th grade.

The National Middle School Association (NMSA, 2010) defines young adolescence as the developmental period spanning from 10-15 years old, which typically encompasses middle school, though adolescence can last until 18 years of age or longer. This period represents rapid changes in cognitive-intellectual, physical, social, emotional, and moral characteristics, needs, and interests (Caskey & Anfara, 2014). Young adolescents’ cognitive development can vary based on areas of expertise, interest, and culture (Chi et al., 1989; Saxe, 2015; Smagorinsky, 2013), but during this time many students are transitioning from concrete representations of mathematical figures to abstract classification and understanding (AMTE, 2017; Caskey & Anfara, 2014). In middle school geometry lessons, young adolescents are often continuing to learn about shapes and properties, including sorting and classifying shapes, composing and decomposing, categories of 2D and 3D shapes, construction activities, applying definitions and categories, and investigations, conjectures, and the development of proof (Van de Walle et al., 2016). Developmentally responsive geometry teaching would scaffold from concrete examples of figures to abstract discussions of properties and applications.

The goal of middle schools need to provide a developmentally supportive environment for young adolescent learners (Ellerbrock et al., 2018). Responsive middle level mathematics
teaching facilitates students’ cognitive development and explicitly attends to young adolescents’ needs, interests, and characteristics including their physical, psychological, social-emotional, and moral development (Ellerbrock & Vomvoridi-Ivanovic, 2019). As young adolescents experience an increase in cognitive capacity, they begin to solve problems, plan efficiently, and think abstractly (Smith et al., 2018). During this time, many young adolescents are also making the transition from concrete to abstract thinking, meaning they are moving from primarily visual, tangible, and concrete representations to abstract reasoning, and need curricular materials that promote this transition (AMTE, 2017; Caskey & Anfara, 2014). In the current study, I investigated how, and to what extent, tasks involving visual representations of convex polygons attend to young adolescents’ developmental needs.

**Research on Geometry Teaching and Learning**

Baddeley’s (2007) theory of working memory states that students’ visuospatial abilities are a core component of working memory and research on visuospatial ability demonstrates a correlation between visuospatial abilities and both mathematics problem solving and achievement (Booth & Thomas, 1999; Ferguson et al., 2015; van Garderen, 2006). Visuospatial ability is clearly linked to mathematics performance and training students’ visuospatial skills may actually improve their mathematical achievement (Cheng & Mix, 2014). In mathematics education research, the processes and functions related to the visuospatial sketchpad are referred to more generally as *visualization* (Arcavi, 2003; Presmeg, 2006; Zimmermann & Cunningham, 1991). Research on visualization in mathematics education draws on prototype theory (Rosch, 1973) which suggests that there are categories of shapes and prototypic images in each category.
Visualization is the process of using or creating abstract or concrete visual representations of mathematical concepts either physically or mentally. Visualization can support students’ problem solving, provide proof and reasoning, and promote conceptual understanding and mathematical intuition (Arcavi, 2003; Polya, 1945). In mathematics, students are exposed to different representations of concepts. Over time, they develop a concept image of that particular mathematical concept (Vinner, 1983).

Geometry learning is directly tied to visualization in mathematics teaching and learning because geometry as a subject develops students’ spatial sense applicable to mathematical and other contexts (Van de Walle et al., 2016). Geometry consists of four major strands: shapes and properties, transformations, location, and visualization (Van de Walle et al., 2016), though the strand generally associated with middle school learning is shapes and properties. The most widely-accepted theoretical framework for understanding students’ progression through geometric learning is the hierarchical model of the van Hiele levels (Clark, 2012). This hierarchy presents a sequential model of student understanding in five levels; (0) visualization, (1) description/analysis, (2) abstractions/relations, (3) formal deduction, and (4) rigor/metamathematics. The van Hiele levels are connected to other models of student learning such as the structure of observed learning outcomes (SOLO) taxonomy (Biggs & Collis, 1989; Pegg & Davey, 1989). In addition to the van Hiele model, there are other models of geometric learning that are less well known, such as Sfard’s (2008) discursive approach, Wang and Kinzel’s (2014) communication based system, and Duval’s (2006) semiotic representations. Additionally, Herbst and colleagues (2017) represent students’ progression through geometric learning in relation to conceptions of figure. In the current study, my conceptualization of how young adolescents form
concept images was informed by Baddeley’s (2007) theory of working memory, the conceptual framework of visualization (Arcavi, 2003; Presmeg, 2006; Zimmermann & Cunningham, 1991), and the theoretical framework of the van Hiele levels Clark, 2012).

**The Role of Textbooks in Teaching and Learning**

Textbooks influence sequencing of material, determining what concepts to teach, providing examples and explanations, and distributing class time across topics (Banilower et al., 2018; Fan, 2013; Fan et al., 2013; Lloyd et al. 2016; Nicol & Crespo, 2006; Stanic & Kilpatrick, 2003; Reys et al., 2004). The choice of textbook in a mathematics classroom can impact students’ opportunity to learn (Hadar, 2018; Hiebert & Grouws, 2007; Houang & Schmidt, 2008; Törnroos, 2005). Therefore, analyzing the visual representations of convex polygons in a middle school mathematics textbook is a valid approach to explore students’ opportunity to develop concept images of polygons.

Textbook analysis is an accepted research design (Fan, 2013; Fan et al., 2013). Fan, Zhu, and Miao (2013) systematically reviewed literature and mathematics textbook research and developed a framework for classifying the literature as follows: (1) Role of Textbooks, (2) Textbook Analysis and Comparison, (3) Textbook Use, and (4) Other Areas. The researchers state that the second category, Textbook Analysis and Comparison, includes “studies focusing on analysing the concerned features of mathematics textbooks under study and, in the case of textbook comparison, comparing the similarities and differences of two or more series of mathematics textbooks” (Fan et al., 2013, p. 635). In the current study, I analyzed the images of convex polygons present in middle school mathematics textbooks across three popular textbook series published by three major textbook publishers, therefore, this study fits within Fan and

Significance of Study

Geometry is a crucial part of a well-rounded education (Clark, 2012; Van de Walle et al., 2016). However, in the United States (U.S.), students are often underperforming in the geometry domain (Mullis et al., 1997; Mullis et al., 2000; Mullis et al., 2008; Mullis et al., 2012; Mullis et al., 2016). Middle school students often lack geometric reasoning and visuospatial skills required to be successful in geometry (Battista, 1999). In middle school, young adolescent students focus on shapes and properties by sorting and classifying, composing and decomposing, categorizing 2D and 3D shapes, and applying definitions and categories (Van de Walle et al., 2016). As middle school students transition from concrete visual representations of these shapes to abstract categorizations and descriptions (AMTE, 2017; Caskey & Anfara, 2014), it is vital that young adolescent students develop robust concept images of polygons encompassing all of the properties and definitions that are relevant to the shape (Vinner, 1983).

In an effort to increase young adolescent students’ visuality and spatial abilities, middle school mathematics teachers must use appropriate visual representations set in real-world and mathematical contexts (NCTM, 2014). Because many teachers rely heavily on the classroom textbooks (Lloyd et al., 2016; Stanic & Kilpatrick, 2003), the representations of convex polygons in middle school mathematics textbooks may constitute a young adolescent students’ opportunity to develop a concept image. Additionally, young adolescents in middle school have specific developmental needs, so it is important for teachers to have access to developmentally appropriate materials (AMLE, 2012; AMTE, 2017; Caskey & Anfara, 2014; NMSA, 2010);
meaning the tasks including representations of polygons should also address young adolescents' physical, cognitive-intellectual, psychological, social-emotional, and moral characteristics (Ellerbrock & Vomvoridi-Ivanovic, 2019). Teachers could supplement the images in the textbook with other mathematical representations, however Gutierrez and Jaime (1999) found that preservice math teachers had poor concept images of the altitude of a triangle and Cunningham and Roberts (2010) found that preservice math teachers had weak conceptual understanding of geometry concepts even when they memorized definitions. Gutierrez and Jaime’s (1999) findings underline the importance of developmentally responsive curriculum for young adolescents in middle school mathematics programs. The tasks and images in middle school mathematics textbooks should be appropriate for young adolescents and promote the development of robust concept images, even if teachers are unable to supplement resources.

For young adolescent students to develop a robust concept image of a polygon, they need frequent and varied exposure to images of polygons (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Therefore, it is important to understand, categorize, analyze, and document the images of polygons in middle school mathematics textbooks. By completing a content analysis of the tasks involving images of convex polygons in middle school mathematics textbooks, this study contributes to understanding young adolescents’ opportunities to develop robust concept images. The goal of this study was to inform the research community and education stakeholders of the opportunities to develop concept images of polygons in popular middle school mathematics textbooks. Hopefully, the implications will encourage textbook creators to consider
the impact of visual images in mathematics textbooks and teachers to provide a variety of high-quality mathematical representations in the classroom.

**Delimitations of Study**

There are three delimitations associated with the design of this study. First, I chose to only include middle school textbooks in the sample because young adolescence is a period of transition between students’ moving from concrete understandings of figures to abstract representations (AMTE, 2017; Caskey & Anfara, 2007). Second, I included textbooks in the sample based on market share of publishers, meaning while the sample is representative of the majority of textbooks used in middle school math classrooms, it is not exhaustive. And third, the data sources for this study are the images of convex polygons presented within each of the nine textbooks in the sample.

**Definitions of Terms**

In this dissertation, I define the following terms as listed below:

- **Visualization**: Visualization is the process of using or creating abstract or concrete visual representations of mathematical concepts either physically or mentally.
- **Polygon**: A polygon is a geometric plane figure with three or more straight sides.
- **Convex Polygon**: A convex polygon is a polygon in which all interior angles measure less than 180°.
- **Prototypical image**: A prototypical image is the most commonly used, classical representation of a particular concept (Hasegawa, 1997; Presmeg, 1992), typically, a prototypical image is one that contains the most special properties and symmetries. For example, a prototypical right triangle is one with the perpendicular sides vertically and
horizontally aligned (Ward, 2004), a prototypical non-right triangle is an isosceles triangle with a horizontal base (Tsamir et al., 2015), and a prototypical trapezoid is an isosceles trapezoid with the parallel sides aligned horizontally (Turnuklu et al., 2013).

- Middle School: Though there are various definitions of what grade bands constitute middle school (or middle level or middle grades), typically middle school refers to 6th-8th grade, and therefore in this dissertation I will use the term middle school to encompass 6th, 7th, and 8th grades.

- Young Adolescent: In middle level education research, a young adolescent refers to a child of ten to fifteen years old (Caskey & Anfara, 2014; NMSA, 2010)

- Developmental: In middle school education research, the term developmental refers to the specific characteristics of young adolescents, specifically their specific intellectual, physical, social, emotional, and moral characteristics, needs, and interests (AMTE, 2017; Caskey & Anfara, 2014; Ellerbrock & Vomvoridi-Ivanovic, 2019; NMSA, 2010). However, in mathematics education research, the term developmental refers to the specific mathematics learning trajectories in which students progress through mathematics topics sequentially and building on previous learning (Daro et al., 2011; Sarama & Clements, 2009).

**Summary of Introduction**

In middle school, young adolescent students already have notions of mathematical figures based on their previous interactions (Herbst et al., 2017), and through those interactions, students’ develop concept images (Vinner, 1983). In mathematics, it is important for students to have frequent and varied opportunities to engage with visual representations of mathematical
concepts so they can develop robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). This is particularly important in middle school math classes as students transition from concrete to abstract representations (AMTE, 2017; Caskey & Anfara, 2014). Therefore, the purpose of this dissertation was to explore young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks.

In this study, I analyzed the images of convex polygons in middle school mathematics textbooks from three popular textbook series. This study has implications for textbook developers, teacher educators, administrators, and mathematics teachers. Textbook developers are encouraged to provide a variety of representations of mathematical concepts so students can develop robust concept images. Teacher educators and administrators should understand how preservice and inservice teachers’ develop concept images and provide opportunities for these teachers to challenge and improve their concept images. And finally, educators may want to familiarize themselves with the representations in textbooks in order to decide when, and if, they need to supplement with additional representations.

In the following chapter, I present my literature review comprising three areas of research: middle school education, geometry teaching and learning, and the role of textbooks in teaching and learning. Middle school education research informs how many students transition from concrete to abstract representation. Geometric thinking and learning theories undergird visualization in mathematics education, geometric learning theories, and the centrality of figures. Research on the role of textbooks demonstrates how images in textbooks can impact learning.
Each of these literature bases contributes to the understanding of how students develop concept images of polygons.
CHAPTER 2: LITERATURE REVIEW

The purpose of this dissertation was to explore young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks. In this literature review, I examined three areas of research informing this study: middle school education research, research on geometry teaching and learning, and the role of textbooks in teaching and learning. The research questions for this dissertation connects two of the literature bases: middle school education research and research on geometry teaching and learning. Figure 1 shows how the two areas of research informed the study and the research questions. Middle school education research contributes to understanding of the developmental characteristics of young adolescents in middle school, developmentally responsive teaching and responsive middle level mathematics teaching (RMLMT) (Ellerbrock & Vomvoridi-Ivanovic, 2019). Research on geometry teaching and learning provides visuospatial learning theories, geometric learning theories, and existing models and methods for teaching. The third literature base, the role of textbooks in teaching and learning, justifies the use of textbook analysis as a research design and details the importance of textbooks in teaching and learning middle school mathematics.

In addition to informing the research questions and design of the study, each of the three areas of research summarized in this chapter also contribute to young adolescents’ development of concept images. Both middle school education research and research on geometry teaching and learning detail how young adolescents’ progression from concrete to abstract thinking can
impact their development of concept images and the role of textbooks in teaching and learning supports the rationale for considering textbooks as relevant to students’ opportunity to develop concept images of polygons.

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<th>Middle School Education Research</th>
<th>Research on Geometry Teaching and Learning</th>
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<td><strong>Developmentally Responsive Teaching</strong></td>
<td><strong>Characteristics of Young Adolescent Learners</strong></td>
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<td>Middle school classes need to provide a developmentally supportive environment for learning (Ellerbrock, Falbe, &amp; Franz, 2018; NMSA, 2010).</td>
<td>Young adolescent learners have specific intellectual, physical, social, emotional, and moral needs, characteristics, and interests (AMTE, 2017; Caskey &amp; Anfara, 2014; NMSA, 2010).</td>
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Young adolescents experience an increase in cognitive capacity as they mature; they can solve problems, plan efficiently, and think abstractly (Smith et al., 2018). During this time, young adolescents are also making the transition from concrete to abstract thinking and need curricular materials that support this transition (AMTE, 2017; Caskey & Anfara, 2014).

If mathematical representations are not sufficiently varied, students may incorrectly associate irrelevant properties of images with properties of the mathematical concepts the images represent (Aspinwall, Shaw & Presmeg, 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).

Variety of images is particularly important at the middle school level as students move from a concrete understanding of geometric figures to more abstract representations (AMTE, 2017; Caskey & Anfara, 2014). Textbook tasks should present figures and representations in a way that is developmentally responsive to young adolescents’ characteristics.

**Figure 1.** Organization of literature bases guiding the inquiry.
Middle School Education Research

The National Middle School Association (NMSA, 2010) defines young adolescence as the developmental period spanning from 10-15 years old. This period in time is particularly important to young adolescents’ development into functional members of society, as for many of them, this time period represents either opportunities to choose a path towards success or the last chance to avoid a life of failure (Carnegie Council on Adolescent Development [CCAD], 1989). In the following sections, I describe the characteristics of young adolescents, developmentally responsive teaching and related research on middle school mathematics teaching and learning.

Characteristics of Young Adolescents

Many young adolescents in middle school are experiencing a transition period of rapid and profound changes, and therefore have specific intellectual, physical, social, emotional, and moral characteristics, needs, and interests (AMTE, 2017; Brighton, 2007; Caskey & Anfara, 2014; NMSA, 2010). These developmental aspects of young adolescents are often presented chronologically or sequentially; however, development occurs in a complex and intertwined system. In fact, this transition from concrete to abstract thinking is not universal as some students can experience cognitive development very differently based on areas of expertise, interest, and culture (Chi et al., 1989; Saxe, 2015; Smagorinsky, 2013). Thus, development in a particular domain will vary across contexts (Caskey & Anfara, 2014; Smith et al., 2018). Generalizations of development patterns are difficult, because there is high variability, changes are irregular, and each young adolescent starts developing at different times and different paces (Caskey & Anfara, 2014; NMSA, 2010). In the current study, I catalog the ways, and the extent to which, tasks...
including visual representations of convex polygons in middle school mathematics textbooks address young adolescents’ developmental needs

**Physical development.** Young adolescents’ physical development occurs at different paces, but all young adolescents experience bodily changes and physical development (Smith et al., 2018). In particular, young adolescents are typically experiencing puberty and hormonal changes (Eccles, 1999). Young adolescents experience rapid development, surges of physical growth, irregular physical growth, bodily changes, awkward and uncoordinated movements, varying maturity rates, and need physical activity due to increased energy (NMSA, 2010). During this period, young adolescents require access to proper nutrition and health care (CCAD, 1989).

Developmentally responsive teachers should recognize that students are experiencing rapid physical changes and that these changes may also impact their social and emotional development (Brighton, 2007; Caskey & Anfara, 2014). Teachers also need to incorporate opportunities for movement during instruction to accommodate students’ changes in metabolism and energy levels (Caskey & Anfara, 2014). In this study, I explored how tasks in middle school mathematics textbooks attend to young adolescents’ physical characteristics, for example, by asking about walking or biking distances.

**Cognitive development.** For cognitive development, “adolescence is a time of enormous potential for learning” (Halpern et al., 2013, p. 3). Early adolescence (10-12 years) is the last period of significant brain development, especially in the areas that control executive functioning and social cognition (Smith et al., 2018). Cognitive development is not as immediately visible as physical development, but it is still profound (Caskey & Anfara, 2014). Young adolescents
experience an increase in cognitive capacity as they mature; they can solve problems, plan efficiently, and think abstractly (Smith et al., 2018). During this time, many young adolescents are also transitioning from concrete to abstract thinking (AMTE, 2017; Caskey & Anfara, 2014). However, this transition from concrete to abstract thinking is not universal as some students can experience cognitive development very differently based on areas of expertise, interest, and culture (Chi et al., 1989; Saxe, 2015; Smagorinsky, 2013). Many young adolescents in middle school can still benefit from concrete experiential learning during this time and also from active learning strategies, collaborative peer interactions, and real-world connections (NMSA, 2010). Teachers should provide a wide array of differentiated learning activities so students at all levels of cognitive development can participate (Caskey & Anfara, 2014). In this study, I explored how tasks in middle school mathematics textbooks attend to young adolescents’ cognitive-intellectual characteristics by exploring the variety of images presented.

**Emotional development.** As young adolescents develop physically and mentally, they also develop emotionally; however, their emotional development does not necessarily occur at the same pace (Smith et al., 2018). Emotional development in young adolescents is characterized by the development of personal identity and quest for independence (Brighton, 2007; Caskey & Anfara, 2014). Young adolescents experience mood swings. They are concerned about peer acceptance. They seek independence, and they view their experiences, thoughts, and emotions as unique (NMSA, 2010). Young adolescents in middle school are psychologically vulnerable. Therefore, they need access to quality social services and mental health care (CCAD, 1989; NMSA, 2010). Schools and teachers should provide opportunities for students to explore identities, build self-esteem, and bond with adults in an advisory or mentor capacity.
(Caskey & Anfara, 2014). In this study, I explored how tasks in middle school mathematics textbooks attend to young adolescents’ emotional characteristics, for example, by contextualizing a mathematics problem with an emotional situation.

**Social development.** The social development of young adolescents is characterized by the ability to form mature relationships with individuals and groups (Caskey & Anfara, 2014). As young adolescents mature, they begin to focus more on their social relationships, including friendships and romantic relationships, more than their familial relationships (Smith et al., 2018). Typically, young adolescents experience a preoccupation with self and start to challenge authority figures during this developmental period (NMSA, 2010). Young adolescents in middle school seek a sense of belonging in social groups, and they desire peer acceptance (NMSA, 2010). Teachers can capitalize on young adolescents’ social development by incorporating opportunities for productive social interactions in an academic context (Caskey & Anfara, 2014). In this study, I explored how tasks in middle school mathematics textbooks attend to young adolescents’ social characteristics, for example, by contextualizing a mathematics problem with a social situation or directing to discuss a question with a peer.

**Moral development.** Young adolescents tend to be idealistic. They want to make the world a better place, they transition towards showing empathy and sympathy, and they develop their values and moral codes (NMSA, 2010). During this time, young adolescents also begin to form their identities and sense of self (NMSA, 2010). Young adolescents in middle school begin to see ambiguity in moral situations rather than black/white or good/evil (Brighton, 2007; Caskey & Anfara, 2014). Teachers should utilize this increased interest in moral and ethical questions by providing opportunities for higher-order thinking and reasoning about real-world concepts and
situations (Caskey & Anfara, 2014). In this study, I explored how tasks in middle school mathematics textbooks attend to young adolescents’ moral characteristics, for example, by contextualizing a mathematics problem with a social justice, equity, or other real-world situation that gives rise to questions of morality.

**Summary of characteristics of young adolescents in middle school.** Young adolescence is a period of trial and error (CCAD, 1989). Middle-level learners are experiencing a rapid transition period in terms of their physical, cognitive, emotional, social, moral, and behavioral development (AMLE, 2012; AMTE, 2017; Brighton, 2007; NMSA, 2010; Smith et al., 2018). Middle schools should provide developmentally responsive environments that attend to young adolescents' unique needs, interests, and characteristics. (Ellerbrock & Vomvoridi-Ivanovic, 2019). In the next section, I elaborate on developmentally responsive mathematics teaching.

**Developmentally Responsive Teaching**

The history of middle school education spans over a century (NMSA, 2010). A main objective of middle schools is to provide a developmentally supportive environment for young adolescent learners (Ellerbrock et al., 2018). Within this developmentally supportive environment, middle school educators can focus on the main goal: developing students’ intellectual capacities (Jackson & Davis, 2000). Providing a developmentally responsive environment includes providing developmentally responsive curricular materials, including textbooks. In this study, I am analyzing young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks in a developmentally appropriate
way. To contextualize developmentally appropriate mathematics teaching, this section provides an overview of developmentally responsive teaching.

Middle school educators teach young adolescent students in the middle, as they transition from elementary to high school (AMTE, 2017). To ease this transition, many middle schools utilize structures such as advisory programs, flexible scheduling, integrated curriculum, team organization, common planning, and effective teaching practices (AMLE, 2012). These structures can be categorized as structures of people, place, and time (Powell, 2005). To provide young adolescent students with access to a developmentally responsive education, “organizational structures are the key” (Ellerbrock et al., 2018, p. 209). Ellerbrock and Vomvoridi-Ivanovic (2019) describe an emerging framework for Responsive Middle Level Mathematics Teaching (RMLMT) that combines elements of developmentally and culturally responsive teaching. Responsive middle level mathematics teaching facilitates students’ cognitive development while promoting social justice and equity and explicitly attending to young adolescents’ needs, interests, and characteristics.

**Responsive middle level mathematics teaching.** The RMLMT framework consists of three goals and eleven dimensions related to those goals. The three goals are (1) to advance young adolescent learners’ mathematical thinking, (2) to promote equity in young adolescent learners’ mathematics classroom learning experiences, and (3) to attend to young adolescent learners’ characteristics, needs, and interests. Each of the three goals is equally important as well as the dimensions within them, meaning this framework does not represent a hierarchy. The dimensions of goal (1) are cognitive demand, depth of knowledge and student understanding, and mathematical discourse. The dimensions of goal (2) are power and participation, academic
language support for English learners, and cultural and community-based funds of knowledge. The dimensions of goal (3) are physical characteristics, cognitive-intellectual characteristics, psychological characteristics, social-emotional characteristics, and moral characteristics. The dimensions of goal (3) informed the research questions in my study, by providing a framework for analyzing the ways, and extent to which, tasks involving images of convex polygons in middle school mathematics textbooks attend to young adolescents’ developmental needs. By attending to each of the dimensions, middle school educators can provide quality mathematics education to all students that is both culturally and developmentally responsive (Ellerbrock & Vomvoridi-Ivanovic, 2019).

An example of a responsive teacher preparation program. Ellerbrock, Vomvoridi-Ivanovic, and Duran (2018) and Ellerbrock and colleagues (2016) describe a middle school educator preparation program with a focus on developmental responsiveness, called the Helios STEM Middle School Program. The Helios program focuses on middle school education and grants a teaching certification for grades 5-9 specifically, rather than incorporating the program into the existing elementary or secondary programs. The program represents a partnership between the University of South Florida and Hillsborough County Public Schools. This program addresses many of the critical aspects of preparation listed by American Association of Colleges for Teacher Education [AACTE] (2018), Burroughs and colleagues (2020), Council for the Accreditation of Educator Preparation [CAEP] (2013), Howell et al. (2016), and National Council for Accreditation of Teacher Education. [NCATE] (2010). Some of the methods used in the program are early and scaffolded field experiences, a co-teaching model with highly trained collaborating teachers, and a focus on the developmental needs of young adolescents. There are
three major components central to the Helios program: the connection between content and pedagogical knowledge, the importance of young adolescent development, and a focus on rich clinical experiences. The Helios program utilizes a cohort design, so preservice teachers can develop a professional community within their preparation program. By providing specialized courses on middle school education and young adolescent development, scaffolded clinical experiences at each grade level within middle schools, content specific courses with advanced experiences with elementary topics, and collaborative assignments to link content and pedagogy, the Helios program is preparing future middle school educators to provide a developmentally responsive education for young adolescent students (Ellerbrock et al., 2016; Ellerbrock et al., 2018).

**Summary of developmentally responsive teaching.** Developmentally responsive schools attend to young adolescents’ specific needs through the use of structures of people, place, and time. Often, educator preparation programs do not adequately focus on the specific needs of middle school students, and instead group middle school education certification in with elementary or secondary programs (Howell et al., 2016; Howell et al., 2018). A developmentally responsive teacher preparation program focuses on content, pedagogy, young adolescent development, and rich field experiences (Ellerbrock et al., 2018). In addition to providing developmentally responsive learning environments, it is also important for mathematics educators to understand the trajectory of learning that young adolescent students undergo specific to mathematics. In the next section, I summarize relevant research on young adolescents’ learning of geometry and mathematics in general.
Research on Geometry Teaching and Learning

Middle school students enter secondary mathematics classrooms with prior notions and conceptions of figures based on their previous experiences (Herbst et al., 2017). Over years of experiences, interactions, and exposures with a particular concept, students develop concept images consisting of all the visual information, pictures, mental images, or properties associated with the particular mathematical concept (Vinner, 1983). If mathematical representations are not sufficiently varied, young adolescent students may incorrectly associate irrelevant properties of images with properties of the mathematical concepts the images represent (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Variety of images is particularly important at the middle school level as students move from a concrete understanding of geometric figures to more abstract representations (AMTE, 2017; Caskey & Anfara, 2007).

In this section, I provide an overview of how young adolescents in middle school process visual information and learn geometry. I present visuospatial learning theories from cognitive development research as well as an overview of geometric learning theories, including the van Hiele levels and alternative models. Additionally, I review research on young adolescent students' understanding of geometric concepts and methods for improving visuospatial skills.

Visuospatial Learning Theories

In this section, I provide an overview of theoretical and conceptual frameworks related to students’ processing of visual information and development of spatial reasoning. In particular, I discuss one of the theoretical frameworks guiding my research, Baddeley’s theory of working memory. Baddeley’s (2007) theory of working memory states that students’ visuospatial abilities
are a core component of working memory and research on visuospatial ability demonstrates a correlation between visuospatial abilities and both mathematics problem solving and achievement. Additionally, I present a conceptual framework directing this study, visualization in mathematics education along with relevant definitions, concepts, benefits, and challenges. Finally, I provide a summary of the visuospatial learning theories undergirding my research.

**Cognitive theories.** Atkinson and Shiffrin (1968) created the multi-store model of working memory that posits that human memory has three components: a sensory register, short-term store, and long-term store. Baddeley (2007) suggested an alternative to the ‘short-term store’ portion of Atkinson and Shiffrin’s (1968) model that consisted of separated components for specific purposes. Ericsson and Kintsch (1995) present a different theory of working memory in which long term memory functions as short term through a system of retrieval structures, however, this model does not account for a separate visuospatial component so it is not included in this study.

Students’ short-term store, or working memory, is a temporary storage system used for holding information either for storage or manipulation (Baddeley, 2007). Working memory consists of multiple components such as the central executive, the phonological loop, and the visuospatial sketchpad (Baddeley, 2007). There are two components of the visuospatial sketchpad: a passive component that retains visual information like color, shape, and size of images and a spatial system that stores dynamic movement and transformations (Bull & Scerif, 2001; Logie 1995). Working memory, and the individual components of working memory can predict students’ mathematical achievement and problem-solving abilities (De Smedt et al., 2009; Meyer et al., 2009; Swanson, 2004).
Most research on students’ visuospatial ability focuses on correlations between students’ visuospatial capacities (also referred to as visuality or spatial awareness) and their problem-solving competence, mathematics achievement, or math anxiety (Booth & Thomas, 1999; Ferguson et al., 2015; van Garderen, 2006). However, Cheng and Mix (2014) further posit that training students’ visuospatial skills can actually improve their mathematical performance and achievement.

Ferguson et al. (2015) examined the relations between sense of direction and math anxiety, math anxiety and spatial anxiety, and math anxiety and spatial ability of undergraduate students. They established a strong correlation between sense of direction, spatial anxiety, and general math anxiety. Additionally, they showed that individuals with math anxiety performed worse on visuospatial measures and suggested that spatial ability may be a predictor for math anxiety (Ferguson et al., 2015).

Booth and Thomas (1999) investigated the mathematical achievement, ability to solve word problems, and spatial abilities of students ages 11-15. The researchers used a mixture of quantitative data from math achievement and spatial ability measures as well as qualitative data from interviewing students as they solved word problems with varying levels of visual involvement. The results demonstrated that students with high spatial ability performed slightly better on mathematics achievement measures and significantly better on the word problems (Booth & Thomas, 1999).

Similarly, van Garderen (2006) also looked at the relation between visuospatial ability and problem-solving ability of 6th-grade students. Additionally, this study explored the difference in visualization among students with disabilities, average achieving students, and
gifted students. Gifted students demonstrated a significantly higher ability in visualization than students with disabilities and used visual images in problem solving much more frequently. Students with higher visualization ability also scored higher on the mathematics achievement measures and the mathematics word problem-solving measures. In terms of the types of imagery used in problem solving, use of schematic (a drawing showing spatial relationships in the problem) visuals was significantly positively associated with spatial visualization measures, but pictorial (an illustration of the situation in the problem) visuals were not (van Garderen, 2006).

In addition to the studies demonstrating relations between spatial abilities and problem-solving competence, mathematics achievement, or math anxiety (Booth & Thomas, 1999; Ferguson et al., 2015; van Garderen, 2006), Cheng and Mix (2014) found that providing targeted training to improve students’ visuality could actually improve students’ mathematical performance. The participants of the study were students aged six to eight, and the researchers randomly assigned the students to either a control group or an intervention group. The intervention group engaged in training on mental rotations of figures while the control group completed crossword puzzles. The researchers found that just one session of visuospatial training led to an increase in mathematics ability, especially on missing term problems (Cheng & Mix, 2014).

Visuospatial abilities are correlated with problem-solving abilities and mathematics achievement in students (Booth & Thomas, 1999; Cheng & Mix, 2006; Ferguson et al., 2015; van Garderen, 2006). Problem solving in geometry requires storing partial information while processing new information in order to find a solution, and therefore automatically involves the use of working memory components (Raghubar et al., 2010). In teaching and learning geometry,
the visuospatial sketchpad is particularly important, as it is responsible for retaining and processing visual and spatial information (Logie et al., 1994). In mathematics education research and literature, the processes and functions related to the visuospatial sketchpad are referred to more generally as visualization (Arcavi, 2003; Presmeg, 2006; Zimmermann & Cunningham, 1991).

**Visualization in mathematics education.** Visualization in mathematics education is defined in the literature in multiple ways. The term visualization is often used interchangeably as a noun and a verb to describe both visual objects and the process of using or processing visual information. Phillips, Norris, and Macnab’s (2010) review of visualization literature uncovered 28 unique definitions of visualization that consisted of three general categories: visualization objects, introspective visualization, and interpretive visualization. Visualization objects refer to physical and tangible representations such as 3D origami, manipulatives, or dynamic geometry software. Introspective visualizations are mental pictures or representations that students construct or imagine. Interpretive visualization is a process in which students derive meaning from either abstract or concrete images by manipulating or transforming in their minds. For the purposes of this dissertation, I defined visualization as the process of using or creating abstract or concrete visual representations of mathematical concepts either physically or mentally.

**Concept images.** In mathematics teaching and learning, students are exposed to different representations of concepts over time. Throughout these multiple interactions, students develop a concept image that consists of all the mental imagery, pictures, diagrams, properties, or other visual information a student associates with that particular mathematical concept (Vinner, 1983). This concept image is unique to each student as it is created based on their personal exposures.
and engagements with a concept. The concept image differs from the concept definition, which is the formal, rigorous, mathematical definition of a concept shared by students (Vinner, 1983). Figure 2 shows a representation of a possible concept image and concept definition of the altitude of a triangle, and also depicts how the concept image may not always align with the concept definition. Note that since this concept definition relies on an acute triangle, it excludes the possibility of an altitude falling outside of a triangle in the case of an obtuse triangle.

Figure 2. Concept image and concept definition of the altitude of a triangle.

Prototypical images. A particular category of concept images is prototypes. A prototypical image or prototype is the commonly used or classical representation of a particular concept (Hasegawa, 1997; Presmeg, 1992). Typically, a prototypical image is one that contains the special properties and symmetries. Figure 3 demonstrates some common prototypes of convex polygons.
Benefits of visualization. The National Council of Mathematics Teachers (NCTM) recommends using and connecting mathematical representations as an effective mathematics teaching strategy (NCTM, 2014). Visual representations can support students’ intuition towards solutions, promote conceptual understanding, exhibit algebraic or symbolic connections, or even provide proof or justification for arguments (Arcavi, 2003). Pictures, diagrams, or other representations are also effective tools for mathematical problem solving (Polya, 1945). In the teaching and learning of geometry specifically, every idea, concept, and problem begins with a ‘seeing’ process (Gal & Linchevski, 2010) that requires visualization to process the information encoded in figures and to understand the representations presented in diagrams.

Visualization for conceptual understanding. Visual representations can promote students’ understanding of how different mathematics concepts relate to each other (Arcavi, 2003). “As students use and make connections among contextual, physical, visual, verbal, and symbolic representations, they grow in their appreciation of mathematics as a unified, coherent discipline”
(NCTM, 2014, p. 29). Figure 4 shows an example of how a visual representation can help students make connections between two concepts, in particular, the area formulas for rectangles and parallelograms.

![Figure 4. Visual representation of connections between area formulas.](image)

*Visualization for proof and reasoning.* Visual proofs or proofs without words can provide intuitive understanding without rigorous, algebraic proofs. Visualization is not just for illustration purposes but is also a key part of students’ thinking, reasoning, justification, problem solving, and proving processes (Arcavi, 2003; Polya, 1945). Figure 5 shows an algebraic proof of

$$
\sum_{i=1}^{\infty} \frac{1}{4^n} = \frac{1}{3}
$$

while Figures 6 and 7 show visual representations of the same series. A visual representation of infinite series may promote students’ conceptual understanding and develop students’ intuition.
Figure 5. Algebraic proof of infinite series.

An infinite geometric series is the sum of an infinite geometric sequence. The sum with $-1 < r < 1$ is given by:

$$S = \frac{a_1}{1 - r}$$

where $a_1$ is the first term and $r$ is the common ratio.

In this example, $a_1 = \frac{1}{4}$ and $r = \frac{1}{4}$

$$S = \sum_{n=1}^{\infty} \frac{1}{4^n} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{1}{3}}{\frac{3}{4}} = \frac{1}{3}$$
Visualization for problem solving. Drawing a diagram or visual representation is a natural first step in the problem-solving process (Polya, 1945). A visualization object can help students to interpret a mathematical problem and can inspire solutions that would be difficult using procedural, algorithmic, or algebraic methods (Arcavi, 2003; Phillips et al., 2010). Figure 8 demonstrates a visual representation of a mathematical problem.
Challenges of visualization. Many of the challenges associated with visualization in mathematics education are related to one-case-correctness (Presmeg, 1992), generalization (Presmeg, 1992), uncontrollable images (Aspinwall et al., 1997; Presmeg, 1986), and compartmentalization (Vinner & Dreyfus, 1989). One-case-correctness refers to situations where a student incorrectly applies a property that holds true in some cases to a case where the property does not hold true (Presmeg, 1992). For example, if a student notices that in many examples, the altitude of a triangle falls within the triangle, the student may assume this property holds true for all triangles and might inappropriately apply this to obtuse triangles. Generalization is also called inflexible thinking and occurs when a student relies heavily on a prototypical image to represent a category of images and then cannot imagine an atypical image in that category (Presmeg, 1992). The problem of uncontrollable images happens when students are introspectively visualizing with a mental image that is inappropriate for the situation. Visual images tend to be persistent and vivid even when students understand that the image is incorrect in the context of
the problem (Aspinwall et al., 1997). Compartmentalization is a problem that arises from conflicting schemes or concept images (Vinner & Dreyfus, 1989). For example, a student might have the correct concept definition for a parabola, but an incomplete concept definition; their concept definition of a parabola may only include parabolas that open upward. Mathematical representations included in teaching should be sufficiently varied and tied to symbolic or numeric representations to minimize the challenges students encounter while visualizing (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989; Zimmermann & Cunningham, 1991).

**Summary of visuospatial learning theories.** In this section, I presented theoretical and conceptual frameworks guiding my research and related to visuospatial learning. Specifically, I discussed cognitive theories and visualization in mathematics education. Baddeley’s (2007) theoretical framework for working memory states that the visuospatial sketchpad is a core component of working memory. Various studies on working memory show that working memory, and the individual components of working memory, can predict students' mathematical achievement and problem-solving abilities (De Smedt et al., 2009; Meyer et al., 2009; Swanson, 2004). Visualization in mathematics education is the process of using or creating abstract or concrete visual representations of mathematical concepts either physically or mentally and can support students' intuition towards solutions, promote conceptual understanding, exhibit algebraic or symbolic connections, or even provide proof or justification for arguments (Arcavi, 2003). Though, there are challenges associated with visualization: one-case-correctness (Presmeg, 1992), generalization (Presmeg, 1992), uncontrollable images (Aspinwall et al., 1997; Presmeg, 1986), and compartmentalization (Vinner & Dreyfus, 1989). These benefits and
challenges are relevant to students’ geometric thinking, and they show how students progress through geometric learning trajectories.

**Geometric Learning Theories**

In this section, I provide theoretical and conceptual frameworks guiding my research. I start by providing theoretical frameworks on students’ geometric thinking and learning and then progress into conceptual frameworks of geometric reasoning and spatial abilities. I also provide connections to middle-level education research on students’ development from concrete to abstract understanding (AMTE, 2017).

Geometry, as we know it, was organized in a system by Euclid over 2000 years ago but existed in work from Egypt, India, and Babylon for 3000 years prior (Clark, 2012). “Geometry is a way to explore and analyze shape and space” (Van de Walle et al., 2016, p. 501). Through learning geometry, students’ develop spatial sense applicable to mathematical and other contexts (Van de Walle et al., 2016). Basic knowledge of geometry is required for any mathematics or science study but is also necessary for any well-rounded education (Clark, 2012; Van de Walle et al., 2016).

Geometry consists of four major strands: shapes and properties, transformations, location, and visualization (Van de Walle et al., 2016). Shapes and properties is the strand most prevalent in middle school classrooms and involves sorting and classifying, composing and decomposing, categories of 2D and 3D shapes, construction activities, applying definitions and categories, and investigations, conjectures, and the development of proof. Transformations refer to changes in position, size, and shape; particularly, symmetries, translations, congruence, similarity, and dilation. The location strand involves positional descriptors and the use of coordinate planes. The
last strand, visualization involves mental imagery and manipulation of 2D and 3D images. These four strands overlap in some areas and build on each other (Van de Walle et al., 2016).

As geometry is a major component of mathematics education, visuospatial development, and well-rounded education, it is important to understand students’ geometric thinking and learning. The most widely-accepted model of students’ progression through geometric learning is the hierarchical model of the van Hiele levels (Clark, 2012). The van Hiele levels are also understood in relation to other models of learning, such as the structure of observed learning outcomes (SOLO) taxonomy (Biggs & Collis, 1989; Pegg & Davey, 1989). Additionally, other models of students’ geometric learning exist, such as Sfard’s (2008) discursive approach, Wang and Kinzel’s (2014) communication-based system, and Duval’s (2006) semiotic representations. It is important to understand how these learning trajectories relate to young adolescent learners’ specific developmental needs as they move from concrete to abstract understanding (AMTE, 2017). In this section, I present a review of frameworks for students’ geometric thinking and learning along with connections to middle level education research.

**Frameworks for students’ geometric thinking and learning.** The van Hiele hierarchy of student geometric learning is a widely accepted model for students’ geometric thinking and learning (Clark, 2012; Van de Walle et al., 2016). There are many critiques to the van Hiele hierarchy, however, it is still widely used in mathematics teacher education and it influences the geometry curriculum, the NCTM geometry standard, and the Common Core geometry content standards (NCTM, 2000; NGACBP, CCSSO, 2010; Van de Walle et al., 2016). This hierarchy presents a sequential model of student understanding in five levels. The van Hiele levels are connected to other models of student learning such as the structure of observed learning
outcomes (SOLO) taxonomy (Biggs & Collis, 1989; Pegg & Davey, 1989). In addition to the
van Hiele model, there are other models of geometric learning that are less well known, such as
Sfard’s (2008) discursive approach, Wang and Kinzel’s (2014) communication based system, and

**Van Hiele levels.** The van Hiele hierarchy of geometric learning was developed by Pierre
and Dina van Hiele in the 1950s. The hierarchy consists of five levels: (0) visualization, (1)
description/analysis, (2) abstractions/relations, (3) formal deduction, and (4) rigor/
metamathematics. The underlying assumption is that to learn higher levels, a student must first
demonstrate some level of mastery of all the lower-level skills (Clark, 2012).

A student at the visualization level in the van Hiele hierarchy would be able to identify
shapes and other geometric representations based on appearance. At the description/analysis
stage, students would be able to recognize, categorize, and characterize shapes based on
properties. From there, students move to the abstraction phase and form abstract definitions of
concepts with necessary and sufficient conditions. The students then progress to formal
deduction, where they prove theorems within a formal axiomatic system. Finally, students reach
the last level in the hierarchy, rigor/metamathematics, and they are able to describe and
understand the similarities and differences between multiple formal axiomatic systems (Clark,
2012; Van de Walle et al., 2016).

At each level of the van Hiele hierarchy, there are objects of thought and products of
thought. Objects of thought are the geometric ideas students think about, and the products of
thought are what students can do (Van de Walle et al., 2016). The products of thought at each
level become the objects of thought at the next level (Van de Walle et al., 2016). There are four
important characteristics of the van Hiele levels: it is sequential, developmental, age independent, and experience-dependent (Van de Walle et al., 2016). Since the van Hiele levels are sequential, students must progress through prior levels before moving to future levels. The hierarchy is developmental because it progresses students through increasingly more challenging material to improve their understanding. The levels are age-independent because students may progress at various speeds, not necessarily tied to their age. Moreover, the van Hiele levels are experience-dependent because students must advance through the geometric levels of thinking through experiences.

Connections to other models. Pegg and Davey (1989) compare the van Hiele levels to the structure of observed learning outcomes (SOLO) taxonomy. The SOLO taxonomy consists of five levels of understanding, similar to the van Hiele levels, but the levels in the SOLO taxonomy are pre-structural, uni-structural, multi-structural, relational, and extended abstract (Biggs & Collis, 1989; Pegg & Davey, 1989). At the pre-structural level, the student is distracted or mislead and does not understand how to approach the task, then at the uni-structural level students find one relevant aspect of the task to work on, at the multi-structural level students are able to focus on several aspects of the task independently, as students move to the relational level they integrate aspects of the task into a cohesive whole, and finally, at the extended abstract level students conceptualize or generalize (Biggs & Collis, 1989). Pegg and Davey (1989) assert that the SOLO taxonomy descriptors are more accurate for describing a students’ level of thinking, but that within those SOLO levels, there may be categorizations of levels of understanding.

Criticisms of van Hiele levels. Though the van Hiele levels are common throughout North America, they are used less frequently around the rest of the world (Sinclair et al., 2017). There
are several critiques of the traditional van Hiele levels, many of them questioning if the levels are indeed sequential, linear, and discrete and also noting that the van Hiele levels do not sufficiently account for the role of artifacts in geometric development and only addresses the importance of figures at the concrete levels (Battista, 2007; Sinclair et al., 2017). Students’ development can vary based on areas of expertise, interest, and culture (Chi et al., 1989; Saxe, 2015; Smagorinsky, 2013). Furthermore, some researchers are questioning if we can even identify student thinking as occurring ‘at a level’; Gutiérrez, Jaime, and Fortuny (1991) suggested an alternative method to evaluate the van Hiele levels of student thinking that provides continuity between levels.

Additionally, other researchers criticize the lack of attention given to the importance of figures for geometric learning in the van Hiele hierarchy, as the van Hiele hierarchy only addresses figures at the concrete levels (Herbst et al., 2017). Other models of students’ geometric thinking and learning have emerged amidst the criticism of the van Hiele levels.

Other models. Pittalis and Christou (2010) linked 3D geometric reasoning and spatial abilities in elementary and middle school students. They developed a model of 3D geometric thinking with four types of reasoning that could describe students’ thinking: representing 3D objects, spatial structuring, conceptualizing mathematical properties, and measurement reasoning. In addition to creating this model, the researchers also demonstrated a link between all four types of reasoning and students’ spatial abilities (Pittalis & Christou, 2010).

Sfard (2008) developed a communication-based model that builds on the van Hiele levels but addresses some of the weaknesses listed above. This discursive approach contrasts with the van Hiele levels by implying participation in geometric thinking rather than an acquisition. In this model, geometric learning is a form of communication; students interact with experts or
students at a higher level to move forward rather than naturally progressing as a part of development (Sfard, 2008; Sinclair et al., 2017). Similarly, Wang and Kinzel (2014) created a model, but they translated the van Hiele levels into a communication-based system. In their model, each level of the van Hiele hierarchy is a reflection of the discourse from the previous level (Sinclair et al., 2017; Wang & Kinzel, 2014). Duval (2006) developed a model of semiotic representations that connects the communication-based approaches with visual images. Duval emphasizes the importance of visualization in mathematics education and how students derive meaning from images. Duval’s model of geometric learning conflicts with the van Hiele methods in that Duval’s model views all of geometry as a mixture of visualization of shapes and the language used to communicate properties of those shapes and the van Hiele levels only stresses visual representations in the first two levels (Clark, 2012; Duval, 2006; Sinclair et al., 2017; Van de Walle et al., 2016).

Similar to Duval’s model of semiotic representations, Herbst and colleagues (2017) “conceptualize the study of geometry in secondary schools as a process of coming to know figures as mathematical models of the experiential world” (Herbst et al., 2017, p. 48). Before young adolescents start middle school, they already have notions of figures from their real-world experiences even if they do not use that vocabulary to describe their preconceptions. Middle school students have concepts of figures in the macrospace (things that cannot be fully viewed in the line of sight like building interiors and city layouts), mesospace (things that can be fully viewed in the line of sight but not manipulated by hand like cars and refrigerators), and the microspace (things that are tangible and holdable like toys and other small items). In the macrospace, students have experiences travelling around regions or navigating through large
buildings, in the mesospace, students have experience with large, immovable objects they can interact with but not fully hold like playground equipment, and in the microspace, students have experience picking up and grasping objects.

Herbst and colleagues (2017) suggest four categories to describe students’ notions and conceptions of figures as they transition to secondary school mathematics: “(1) figure as navigation of the macrospace, (2) figure as capture of an object in the mesospace, (3) figure as construction of small objects in the microspace, and (4) figure as descriptions and manipulations of small objects in the microspace” (p. 77). They posit that student thinking is mediated by figures at all stages of learning, in contrast to the van Hiele levels which only emphasize concrete representations at the lower levels. With Herbst and colleagues (2017) framework, student learning of geometry can be represented as progressing at different rates based on each of the four conceptions of figure. The four conceptions of figure can be described as actions (moving about a space, visually capturing an object, describing the features of a small object, and constructing a small object), and these actions can be connected to other semiotic representations like language and vocabulary through communication tasks. These communication-based representations connect Herbst and colleagues’ (2017) model of students’ learning of geometry to the discursive models presented by Sfard (2008) and Wang and Kinzel (2014).

**Connection to middle school education.** In terms of cognitive development, “adolescence is a time of enormous potential for learning” (Halpern et al., 2013, p. 3). Young adolescents experience an increase in cognitive capacity as they mature; they are able to solve problems, plan efficiently, and think abstractly (Smith et al., 2018). During this time, many young adolescents are also transitioning from concrete to abstract thinking (AMTE, 2017;
Caskey & Anfara, 2014). However, this transition from concrete to abstract thinking is not universal as some students can experience cognitive development very differently based on areas of expertise, interest, and culture (Chi et al., 1989; Saxe, 2015; Smagorinsky, 2013). Young adolescents in middle school can still benefit from concrete experiential learning contextualized in real world settings (NMSA, 1995). Developmentally responsive geometry teaching would scaffold from concrete examples of figures to abstract discussions of properties and applications.

Young adolescents in middle school are typically learning about shapes and properties, including sorting and classifying shapes, composing and decomposing, categories of 2D and 3D shapes, construction activities, applying definitions and categories, and investigations, conjectures, and the development of proof (Van de Walle et al., 2016). In the van Hiele hierarchy, this means that middle school students are typically doing activities at the visualization, description/analysis, or abstractions/relations levels. At the visualization level, middle-level learners would be able to identify shapes and other geometric representations based on appearance (Clark, 2012). At the description/analysis stage, students would be able to recognize, categorize, and characterize shapes based on properties (Clark, 2012). From there, students move to the abstraction phase and form abstract definitions of concepts with necessary and sufficient conditions (Clark, 2012). This sequential movement through the van Hiele hierarchy aligns with research on the developmental characteristics of young adolescents, which states that middle school students progress from concrete to abstract representations over time (AMTE, 2017; Caskey & Anfara, 2014).

As young adolescents enter middle schools, they already have conceptions of figures from previous experiences (Herbst et al., 2017). These prior notions contribute to students’
concept images of figures (Vinner, 1983). If young adolescents do not have sufficiently varied interactions with concepts, they will not develop a robust abstract understanding (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). In middle school, it is important that educators attend to young adolescents’ cognitive development by providing a variety of images as students move from concrete to abstract representations (AMTE, 2017; Caskey & Anfara, 2014).

**Summary of geometric learning theories.** In this section, I presented theoretical frameworks on students’ geometric thinking and learning, conceptual frameworks of geometric reasoning and spatial abilities, and connections to middle school education research on students’ development from concrete to abstract understanding (AMTE, 2017). A widely accepted theoretical framework for understanding students’ geometric thinking and learning is the van Hiele hierarchy (Clark, 2012; Van de Walle et al., 2016). This hierarchy is also understood in relation to other models of student learning such as the structure of observed learning outcomes (SOLO) taxonomy (Biggs & Collis, 1989; Pegg & Davey, 1989). There are several criticisms of the van Hiele model though, giving rise to other models such as Sfard’s (2008) discursive approach, Wang and Kinzel’s (2014) communication based system, and Duval’s (2006) semiotic representations. Additionally, Herbst and colleagues (2017) provide a model for understanding students’ geometric progression in relation to their conception of figures.

**Existing Models and Methods for Teaching**

Young adolescent learners are experiencing a transitional period of enormous cognitive development (Caskey & Anfara, 2014; Halpern et al., 2013; Smith et al., 2018). During this time, young adolescents start to move from concrete thinking to abstract thinking (AMTE, 2017;
Caskey & Anfara, 2014; NMSA, 2010). In mathematics classrooms during this time period, students are typically focusing on the shapes and properties strand of geometry, which involves sorting and classifying, composing and decomposing, categories of 2D and 3D shapes, construction activities, applying definitions and categories, and investigations, conjectures, and the development of proof (Van de Walle et al., 2016). This means figures, representations, and visualizations are particularly important at the middle level as well as visuospatial learning in general.

Spatial visualization is critical in middle school. In a study on spatial visualization, math learning, and math achievement in 8th-grade, Rabab’h and Veloo (2015) found that spatial visualization mediated the relationship between motivation, math anxiety, and math achievement. Boaler, Chen, Williams, and Cordero (2016) similarly argued that visualization in mathematics can raise mathematics achievement levels in young adolescents. Unfortunately, middle school students often encounter difficulties related to visualization (Ben-Haim Lappan, & Houang, 1985). Erso, Ilhan, and Sevgi (2019) found that 7th-grade students’ van Hiele geometric thinking levels were lower than would be expected.

There are many methods for improving the visuospatial skills of young adolescent learners using physical or virtual manipulatives, dynamic geometry software, or even 3D printing (Ben-Haim et al., 1985; Boakes, 2009; Chan & Leung, 2015; Cheng & Mix, 2009; Cochran et al., 2016; Huleihil, 2017; Leung, 2008). In a meta-analysis of spatial training studies by Uttal and colleagues (2013), the researchers concluded that spatial skills are malleable and can improve with specific visuospatial training.
Physical manipulatives, for example, origami, can be as effective as traditional instruction (Boakes, 2009; Carbonneau et al., 2013), and virtual manipulatives can offer students concrete experiences with mathematical concepts (Durmus & Karakirik, 2006). Dynamic geometry software, such as GeoGebra or Cabri 3D, can be more effective than both manipulatives and traditional instruction in increasing students’ visuality (Baki et al., 2011; Chan & Leung, 2015; Diković, 2009; Güven & Temel, 2008; Leung, 2008). Furthermore, specific training in visuospatial methods can raise students’ spatial skills and lead to success in geometry (Battista et al., 1982; Ben-Haim et al., 1985; Cheng & Mix, 2009).

**Physical and virtual manipulatives.** Carbonneau and colleagues (2013) completed a meta-analysis of the literature on teaching mathematics with concrete manipulatives. The results indicated a small to moderate positive effect for instruction using manipulatives versus abstract symbolic instruction. They suggest that simply incorporating manipulatives is not enough to increase students’ mathematical achievement; educators need to consider other contextual factors such as the level of guidance, the developmental levels of the students, and the richness of objects when planning for instruction using manipulatives (Carbonneau et al., 2013).

Boakes (2009) conducted a study to examine the effects of origami lessons on 7th-grade students’ visuospatial abilities and geometry knowledge. Students participating in the study engaged in origami lessons while learning relevant geometry terms and concepts. The results of the study indicated that the origami lessons were as beneficial as traditional lessons for teaching vocabulary and concepts, but did not significantly impact students’ visuospatial skills (Boakes, 2009).
Virtual manipulatives are similar to physical manipulatives in that they provide students’ with concrete experiences and representations of mathematical concepts, yet virtual manipulatives do not have the same constraints of cost, space, or availability of physical manipulatives (Durmus & Karakirik, 2006).

**Dynamic geometry software.** In a meta-analysis of previous studies, Chan and Leung (2015) found that dynamic geometry software was more effective than traditional instruction in improving students’ mathematical achievement. Specifically, instruction using dynamic geometry software was more effective by one standard deviation and a high statistical significance. The largest effects were in elementary school students, but dynamic geometry software instruction was more successful than traditional instruction for all age groups (Chan & Leung, 2015).

Diković (2009) investigated the impact of using dynamic geometry software on college students’ visuospatial skills. The researcher split the participants into two groups, one of which experienced traditional instruction while the other engaged in lessons using GeoGebra, a dynamic geometry software. Results indicated that the participants who experienced instruction using GeoGebra had significant increases in their visuospatial abilities (Diković, 2009).

Güven and Temel (2008) conducted a similar experiment with preservice teachers using Cabri 3D, a dynamic geometry software. They found that engaging in activities and instruction using Cabri 3D increased the preservice teachers’ spatial skills, especially on tasks that required rotations (Güven & Temel, 2008). Baki and colleagues (2011) also explored interventions to increase preservice teachers’ visuospatial skills; however, they used physical manipulatives in addition to Cabri 3D. They found that both physical manipulatives and dynamic geometry
software were more effective than traditional instruction in increasing visuality, but dynamic geometry software was more successful than the physical manipulatives (Baki et al., 2011).

Leung (2008) also studied the use of dynamic geometry software to teach geometry but focused on elementary school children. The dynamic geometry software was used on the SmartBoard, a digital whiteboard, and students learned geometry concepts through text, narrations, words, pictures, animations, and illustrations. The lessons explicitly focused on the inclusive and transitive properties of quadrilaterals by using on-demand animation of angle measures and side lengths. The researcher noted that after three lessons with the SmartBoard dynamic geometry software, students were more confident in their knowledge of quadrilaterals, and the researcher theorized that this teaching approach might guide students into the van Hiele levels of abstraction (Leung, 2008).

**Visuospatial training.** Cheng and Mix (2009) found that specific training of elementary students’ visuospatial skills could lead to an increase in mathematics ability. They designed a study with two groups of participants, a control group that completed crossword puzzles and a treatment group that engaged in targeted instruction on mental rotations of figures. In just one session of the visuospatial training, the participants in the treatment group showed an increase in mathematical ability (Cheng & Mix, 2009).

Ben-Haim and colleagues (1985) similarly found that visualization training could increase middle school students’ spatial skills. They argued that many middle school students’ experiences with mathematical representations were in static, 2D, textbook images. The researchers implied that middle school students could benefit from concrete experiences with visualization problems (Ben-Haim et al., 1985).
Battista and colleagues (1982) investigated the effect of visuospatial training on preservice teachers’ spatial ability in a geometry course. The researchers found a correlation between the preservice teachers’ visuality score and their overall success in the geometry course. They implied that spatial instruction methods were effective in increasing preservice teachers’ visuality (Battista et al., 1982).

**New methods.** In addition to the existing models for teaching geometry and visuospatial skills, researchers are working on new methods using 3D printing. Cochran, Cochran, Laney, and Dean (2016) developed a curriculum for 4th and 7th-graders using affordable 3D printers and computer-aided design (CAD) software. Through the lessons using the 3D printer and CAD software, students explored the relationship between 2D and 3D shapes, volume, rotations of figures, bases and sides, surface area, cross-sections, and measurement (Cochran et al., 2016). Similarly, Huleihil (2017) found that 6th-grade students who learned geometric concepts through the use of 3D printers and CAD software increased their mathematical achievement.

**Summary of existing models and methods for teaching.** Existing models for teaching geometry and increasing students’ visuospatial skills include using manipulatives (Boakes, 2009; Carbonneau et al., 2013; Durmus & Karakirik, 2006), using dynamic geometry software (Baki et al., 2011; Chan & Leung, 2015; Diković, 2009; Güven & Temel, 2008, Leung, 2008) and teaching specific visualization methods (Battista et al., 1982; Ben-Haim et al., 1985; Cheng & Mix, 2009). Additionally, newer methods of teaching geometry incorporate 3D printing and CAD software (Cochran et al., 2016; Huleihil, 2017). Research on young adolescents’ cognitive development shows that as they move from concrete to abstract representations and develop concept images of polygons, variety in presentation is crucial (AMTE, 2017; Aspinwall et al.,
1997; Caskey & Anfara, 2007; Hasegawa, 1997; Herbst et al., 2017; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983). Since textbooks function as the “de facto curriculum for teachers” (Stanic & Kilpatrick, 2003, p. 969) and often represent the intended curriculum (Lloyd et al., 2016), the representations of polygons presented in textbooks can influence students' development of concept images of polygons. In the next section, I describe the role of textbooks in geometric teaching and learning.

**The Role of Textbooks in Teaching and Learning**

Textbooks function as an important facet of the middle school mathematics curriculum in terms of structuring the sequence of material, determining what concepts to teach, providing examples and explanations, and distributing class time across topics (Banilower et al., 2018; Fan, 2013; Fan et al., 2013; Lloyd, Cai, & Tarr, 2016; Nicol & Crespo, 2006; Stanic & Kilpatrick, 2003; Reys, Reys, & Chávez, 2004). Textbooks often represent the *intended curriculum*, or the expectations for mathematics learning provided by the national, state, or district level and included in curricular materials, textbooks, and standards (Lloyd et al., 2016) as well as the *potentially implemented curriculum* (Houang & Schmidt, 2008; Törnroos, 2005). Due to the pervasiveness of textbooks in middle school mathematics classrooms, the choice of textbook used in instruction can impact student learning (Hadar, 2018; Houang & Schmidt, 2008; Nicol & Crespo, 2006; Törnroos, 2005). Students’ *opportunity to learn* is defined as the encounters, occasions, experiences, and engagement students have with content, teaching and learning strategies, and particular topics (Hadar, 2018; Hiebert & Grouws, 2007; Houang & Schmidt, 2008; Törnroos, 2005). Therefore, the textbook used in a math classroom directly impacts students’ opportunity to learn.
Over the past century, textbooks have transitioned into longer books with more pervasive visual images (Aspinwall et al., 1997; Baker et al., 2010; Dimmel & Herbst, 2015; Stanic & Kilpatrick, 2003). The images of polygons in middle school mathematics textbooks influence students’ development of concept images polygons (Vinner, 1983). If textbooks primarily print prototypical and ‘common’ representations with little variety, students may struggle with issues of one-case correctness, where a student incorrectly applies a property that holds in a particular case to other inappropriate cases, generalization or inflexible thinking, where a student cannot visualize an atypical image that fits the definition of a category of images, uncontrollable images, where a student visualizes mental imagery inappropriate for the context, or compartmentalization, where a student develops a concept image in conflict with the concept definition (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Parzysz, 1988; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Hence, an analysis of the tasks involving visual representations of convex polygons in middle school mathematics textbooks is an appropriate approach in understanding young adolescents’ opportunity to develop concept images of polygons.

**Textbook analysis as a research design.** Fan, Zhu, and Miao (2013) systematically reviewed literature and mathematics textbook research and developed a framework for classifying the literature as follows: (1) Role of Textbooks, (2) Textbook Analysis and Comparison, (3) Textbook Use, and (4) Other Areas. The researchers state that the second category, Textbook Analysis and Comparison, includes “studies focusing on [analyzing] the concerned features of mathematics textbooks under study and, in the case of textbook comparison, comparing the similarities and differences of two or more series of mathematics
In the current study, I analyzed the images of polygons present in middle school mathematics textbooks across three popular textbook series published by three major textbook publishers. Therefore, this study fits within Fan and colleagues’ (2013) second category of textbook research in mathematics education: Textbook Analysis and Comparison.

Fan’s (2013) article on textbook research provides a “conceptual framework about mathematics textbooks as the subject of research” (p. 766) with three areas of research stemming from fundamental issues in the field of mathematics textbook research, textbooks as the subject of research, textbooks as the dependent variable, and textbooks as the independent variable. This framework situates textbooks themselves as an intermediate variable in the context of education. The independent variables are the factors affecting the development of textbooks, the textbooks themselves are the intermediate variables, and the dependent variables are the factors affected by textbooks. Within each of these three areas of research, Fan (2013) posits questions relating to issues about mathematics textbooks. This study will attend to the first area of research within the framework: textbooks as the subject of research. The present study contributes to answering the first question within this area: “What are the features of mathematics textbooks?” (Fan, 2013, p. 772). In particular, this study catalogues the visual representations of convex polygons in middle school mathematics textbooks across three popular textbook publishing companies.

**Textbook analysis research.** Teachers rely heavily on textbooks for designing courses, sequencing, and providing examples and practice (Banilower et al., 2018; Fan, 2013; Fan et al., 2013; Lloyd et al. 2016; Nicol & Crespo, 2006; Stanic & Kilpatrick, 2003; Reys et al., 2004); therefore, analyzing the content in mathematics textbooks is a natural focus of research. Content
analysis is a widely accepted method for analyzing textbooks (Krippendorff, 1980; Neuendorf, 2002; Stemler, 2001; Titscher et al., 2000), and many researchers have completed textbook studies both in the U.S. and internationally. Textbook studies feature a variety of focuses including treatment of a specific mathematical domain, textbooks alignment with standards, context and cognitive demand of content, and comparing textbooks either nationally or internationally.

In the U.S., several researchers have focused on historical changes in math textbooks. Baker and colleagues (2010) used content analysis to understand changes in elementary mathematics textbooks over a century. Jones and Tarr (2007) also examined mathematics textbooks from a historical perspective. The researchers compared popular and alternative middle school textbook series from historical periods noting the cognitive demand of probability and statistics tasks (Jones & Tarr, 2007). Similarly, Arnold and Son (2011) analyzed the pre-algebra content in popular and alternative textbooks over time specifically examining the contexts and cognitive demand of tasks.

Other researchers have focused on current U.S. textbooks and specific content areas or topics. Huntley and Terrell (2014) compared popular and alternative mathematics textbooks and their coverage of linear equations in relation to content, context, cognitive demand, and tools (Huntley & Terrell, 2014). Polikoff (2015) studied current textbooks as well, assessing the alignment of textbooks to Common Core State Standards Recommendations. Hunsader and colleagues (2014) analyzed the assessment tasks in K-12 mathematics textbooks.

Internationally, several studies have compared U.S. textbooks to those of other countries. Alajmi (2012) contrasted the treatment of fractions in elementary textbooks from the U.S.,
Kuwait, and Japan. Li (2000) compared U.S. and Chinese textbooks and their treatment of integer problems. Incikabi and Tjoe (2013) similarly compared U.S. and Turkish middle school math textbooks, specifically, the context and cognitive demand of ratio and proportion problems. Li (2007) completed a study focusing on eighth-grade textbooks from the U.S., China, and Singapore to compare the cognitive demand of mathematical tasks. Hong and Choi (2014) also focused on eighth-grade textbooks, and compared the context. They compared cognitive demand of algebra tasks in textbooks from the U.S. and Korea., then more recently, Hong and Choi (2018) compared cognitive demand of tasks in secondary U.S. and Korean textbooks. Ponte and Marques (2011) similarly compared textbooks from multiple countries examining the cognitive demand, context, and structure of sixth-grade math textbooks from the U.S., Portugal, Spain, and Brazil.


**Summary of the role of textbooks in teaching and learning.** Textbooks influence sequencing of material, determining what concepts to teach, providing examples and explanations, and distributing class time across topics (Banilower et al., 2018; Fan, 2013; Fan et al., 2013; Lloyd et al. 2016; Nicol & Crespo, 2006; Stanic & Kilpatrick, 2003; Reys et al., 2004). The choice of textbook in a mathematics classroom can impact students’ opportunity to learn (Hadar, 2018; Hiebert & Grouws, 2007; Houang & Schmidt, 2008; Törnroos, 2005). Therefore,
analyzing the visual representations of convex polygons in a middle school mathematics

textbook is a valid approach to explore students’ opportunity to develop concept images of
polygons. Textbook analysis is an accepted research design (Fan, 2013; Fan et al., 2013).

Summary of Literature Review

In this chapter, I reviewed the literature guiding my dissertation study. The purpose of my
investigation is to explore young adolescents’ opportunity to develop concept images in middle
school mathematics textbooks. Therefore, each of my three areas of literature, middle school
education research, research on geometry teaching and learning, and the role of textbooks in
teaching and learning, informed my study in some way. Middle school education research
contributes to understanding of the developmental characteristics of young adolescents in middle
school, developmentally responsive teaching and responsive middle level mathematics teaching
(RMLMT). Research on geometry teaching and learning provides visuospatial learning theories,
geometric learning theories, and existing models and methods for teaching. The third literature
base, the role of textbooks in teaching and learning, justifies the use of textbook analysis as a
research design and details the importance of textbooks in teaching and learning middle school
mathematics. In the next chapter, I present my research design and methodology, including my
research questions, rationale and purpose, research design, textbook selection, data sources, data
analysis, and validity criteria.
CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

Middle school is a crucial period in mathematics education because it is when many young adolescents begin to transition from concrete representations to abstract thinking and classification (AMTE, 2017; Caskey & Anfara, 2014; Smith et al., 2018). However, this transition from concrete to abstract thinking is not universal as some students can experience cognitive development very differently based on areas of expertise, interest, and culture (Chi et al., 1989; Saxe, 2015; Smagorinsky, 2013). Students progress through geometric learning trajectories (Clark, 2012; Van de Walle et al., 2016) and figures influence every stage of this transition (Herbst et al., 2017). Therefore, if mathematical representations are not sufficiently varied, students may incorrectly associate irrelevant properties of images with properties of the mathematical concepts the images represent (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).

In this chapter, I describe my research design and methodology. This chapter is divided into nine sections: purpose statement, research questions, rationale and purpose, research design, textbook selection, data sources, data analysis, validity criteria, and summary. Within those sections, I report the textbook series I chose to analyze along with the criteria for inclusion of images. I also detail my a priori coding scheme and how I used the codes to analyze the data.
Finally, I explain my validity criteria and summarize the research design and methodology chapter.

**Purpose Statement**

In this study, I conducted an analysis of the tasks involving images of convex polygons in three middle school mathematics textbook series. Each of the three textbook series is composed of three books (though some textbooks are split into two parts) spanning 6th, 7th, and 8th grade. The mathematics textbooks have geometry lessons infused throughout, but are not geometry specific textbooks. In particular, I examined how the tasks address young adolescents’ cognitive needs as they transition from concrete to abstract representations and develop concept images by noting how images are oriented, how right angles are oriented, how many convex polygons are represented with prototypical images, in what context the image is represented, and the role of the image. Additionally, I cataloged the ways in which tasks involving images of convex polygons address young adolescents’ other developmental characteristics and the extent to which these tasks attend to these characteristics. I also compared the results in relation to the publisher of the textbooks, the grade level, and the location of the image within the textbooks.

**Research Questions**

The purpose of this study was to analyze young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks (grades 6-8). Specifically, the research question guiding this study is as follows: Within three published middle school mathematics textbook series, in what ways do young adolescents have opportunities to develop concept images of polygons?
1. In what ways do tasks within the textbooks involving visual representations of convex polygons attend to young adolescents’ cognitive-intellectual characteristics? In particular:
   a. How are images of convex polygons oriented in the textbooks?
   b. How are right angles oriented within images of right triangles in the textbooks?
   c. What percentage of convex polygons in the textbooks are prototypical images, in other words, the most common representation of a particular convex polygon?
   d. In what contexts (i.e., real-world or purely mathematical) are convex polygons represented within the textbooks?
   e. What is the role of the images (i.e., superfluous, illustrative, or interpretive) representing convex polygons within the textbooks?

2. To what extent do tasks within the textbooks involving visual representations of convex polygons attend to other developmental characteristics of young adolescents? In particular:
   a. To what extent do tasks involving visual representations of convex polygons address young adolescents’ physical characteristics?
   b. To what extent do tasks involving visual representations of convex polygons address young adolescents’ psychological characteristics?
   c. To what extent do tasks involving visual representations of convex polygons address young adolescents’ social-emotional characteristics?
   d. To what extent do tasks involving visual representations of convex polygons address young adolescents’ moral characteristics?
Rationale and Purpose

My interest in completing the current study stems from previous content analysis studies I conducted both individually and as part of a collaborative team. My thesis for my master’s degree in mathematics was a content analysis of high school geometry textbooks entitled “Prevalence of Typical Images in High School Geometry Textbooks.” In this study, I found that across 14 different textbooks, 74.7% of parallelograms and 75.2% of trapezoids were printed aligned horizontally with the text (Cannon, 2017). This discovery alarmed me, because exposure to a variety of images (including variety in alignment and orientation) is critical for students’ formation of robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). I realized the importance of content analysis as a method and textbook analysis as an area of research in mathematics education.

In my work as a Mathematics Education graduate student in the Ph.D. program, I continued engaging in textbook analysis studies. In Spring of 2018, I completed both an individual textbook analysis and a collaborative task analysis as part of course assignments for a class on assessment in mathematics education. In the individual textbook analysis, I coded sections in a high school geometry textbook using the Common Core Standards for Mathematical Practice (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGACBP, CCSSO], 2010). For the collaborative task analysis, we used the Mathematical Processes Assessment Framework to analyze and code test items from various content areas. The Mathematical Processes Assessment Framework is a framework for analyzing the extent to which mathematical tasks address the mathematical processes of
Reasoning and Proof, Opportunity for Mathematical Communication, Connections, Representation: Role of Graphics, and Representation: Translation of Representational Forms (Hunsader et al., 2014). In Fall of 2018, I worked in collaboration on a study entitled “An Analysis of Assessment Tasks in Middle Level Mathematics Textbooks”. In the study, we conducted a content analysis of the assessment tasks within three middle school mathematics textbook series to explore the extent to which the assessment tasks contained provide students with opportunities to engage with varying cognitive domains, visual representations, and to contextualize the mathematical concepts by content and grade level. In the Spring of 2019, I completed an independent study course, in which I developed a preliminary coding scheme for the current study and a pilot study to revise the codes. In the pilot study, I used my preliminary coding scheme to analyze tasks containing images of convex polygons in one section of Houghton Mifflin Harcourt Go Math! Grade 6 (Burger et al., 2014). After completing the pilot study, I was able to revise my coding scheme to clarify some ambiguous codes and add new codes that were relevant.

My personal motivation for exploring this topic stems from my experience teaching mathematics at the K-12 and college level, as well as my experience teaching and mentoring preservice middle school mathematics teachers. I found that a common difficulty for K-12 math students, college math students, and preservice teachers was identification and classification of polygons that were represented in atypical ways. Often, students and preservice teachers I worked with would struggle to answer questions involving visual images, not because they were lacking procedural fluency or conceptual understandings of geometric ideas or formulas, but because they could not properly identify the shape represented in the problem. Additionally, as
an instructor and supervisor in courses and field experiences for preservice middle school math educators, I understand that middle school is a crucial period of time in a students’ development. As many young adolescents transition from concrete to abstract thinking, they need exposure and interaction with a variety of representations (AMTE, 2017; Caskey & Anfara, 2014). Young adolescents in middle school mathematics classrooms are experiencing a transition period of rapid and profound changes. They have specific intellectual, physical, social, emotional, and moral characteristics, needs, and interests (AMLE, 2012; AMTE, 2017; Caskey & Anfara, 2014; NMSA, 2010). Therefore, the curricular materials used in middle school math classrooms need to be developmentally responsive to young adolescents’ physical, psychological, social-emotional, and moral characteristics in addition to their cognitive-intellectual characteristics.

Visual representations are an essential component of mathematics teaching and learning. Mathematical visualizations can promote understanding and intuition, provide proof and reasoning, and support problem solving (Arcavi, 2003; Polya, 1945). NCTM (2014) includes “use and connect mathematical representations” as one of the eight Mathematical Teaching Practices to strengthen the teaching and learning of mathematics and AMTE (2017) posits that mathematics teachers should be capable of using multiple mathematical representations. However, preservice mathematics teachers are often not capable of providing a variety of high-quality visual images since many preservice teachers do not have functioning concept images themselves (Cunningham & Roberts, 2010; Gutierrez & Jaime, 1999).

Effective mathematics teachers use and connect mathematical representations (NCTM, 2014); however, merely using visual representations in mathematics instruction is not enough. Students build concept images of mathematical concepts over time and through repeated
exposures (Vinner, 1983). If teachers include mathematical images that are not sufficiently varied, students may build irrelevant properties of these images into their concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). At the middle school level, this variety in images is particularly important, as students are moving from concrete to abstract representations of geometric figures (AMTE, 2017). Since textbooks often represent the intended curriculum (Lloyd et al., 2016; Stanic & Kilpatrick, 2003) the representations of polygons presented in textbooks can influence students' development of concept images of polygons. Therefore, the purpose of this study was to explore young adolescents’ opportunity to develop concept images of polygons in middle school mathematics textbooks.

**Research Design**

To address the research questions in this study, I employed a content analysis methodology, which is appropriate for a textbook analysis study as it is a well-established technique for analyzing textbooks and other documents across a variety of fields (Krippendorff, 1980; Neuendorf, 2002; Stemler, 2001; Titscher, Meyer, Wodak, & Vetter, 2000). There are a few widely accepted content analysis approaches and designs including the counting strategy, task analysis, and coding and evaluation (Polikoff et al., 2015). The counting method is used to count the number of tasks or pages allocated to a specific topic or standard (Flanders, 1994; Polikoff et al., 2015). A task analysis focuses on the three parts of an academic task as described by Doyle (1983, 1988): the product of the work, the process of the work, and the resources related to the work. In a task analysis, researchers evaluate books based on features like physical characteristics, lesson structure, or nature of the tasks and evaluate tasks based on criteria like
context, manipulative features, computation methods, or whether the material is new or old (Polikoff et al., 2015). Coding and evaluation is a strategy used for specific portions of text, usually to assess alignment (Polikoff et al., 2015). In this study, I utilized the task analysis strategy to evaluate the images of convex polygons in middle school mathematics textbooks.

Content analysis is a flexible methodology that can be applied in quantitative, qualitative, or mixed methods studies (White & Marsh, 2006). This study is a mixed method study as I implemented a descriptive quantitative design in order to catalogue the images in middle school mathematics textbooks (Fan, 2013), but analyzed some of the data qualitatively. A descriptive quantitative design allowed me to count the number of appearances of certain features and types of images. This design complements the task analysis content analysis strategy, as I evaluated the physical characteristics of each image of a convex polygon and the context in which the image appears (Polikoff et al., 2015). The research paradigm I utilized for this study is *pragmatism*. Pragmatism is considered a quintessential American philosophy (Crotty, 1998) concerned with practical applications of methods that are appropriate for the research study. Employing a pragmatic approach means using a method, design, and strategies that are appropriate, practical, and relative to the problem (Badley, 2003; Greene & Caracelli, 2003; Johnson & Onwuegbuzie, 2004).

In a pragmatic study, the researcher makes sampling and data collection decisions based on the research questions (White & Marsh, 2006). Because my research questions were concerned with tasks involving images of convex polygons in middle school mathematics textbooks, the sample consisted of the three most popular middle school mathematics textbook series based on the market share of textbooks used at the 6th-grade level. The unit of analysis in
this study was tasks involving images of convex polygons printed within three popular middle
school mathematics textbook series (White & Marsh, 2006). The data collection units consisted
of the coding categories detailed in the Data Sources section below.

**Textbook Selection**

Textbooks are an essential feature of mathematics teaching and learning (Banilower et al.,
2018; Reys et al., 2004), and the textbooks used in mathematics classrooms “strongly [influence]
both what and how mathematics is taught to middle school mathematics students” (Tarr et al.,
2006, p. 200). Therefore, analyzing the images of convex polygons in middle school
mathematics textbooks is an appropriate approach to understand young adolescents’
opportunities to develop concept images of polygons. In the 2018 National Survey of Science
and Mathematics Education (NSSME), Banilower and colleagues found that three U.S.
publishing companies provide 80% of the textbooks utilized in middle school mathematics
classrooms; in particular, Pearson comprised 17%, Houghton Mifflin Harcourt comprised 37%,
and McGraw Hill Education comprised 26% of the market share of middle school mathematics
textbooks (Banilower et al., 2018). Therefore, I choose to analyze one popular middle school
mathematics textbook series from each of these three publishers at the 6th, 7th, and 8th-grade
level. I did not include any online materials from these textbooks, as many schools do not have
technology tools available for all students and not all students have access to technology or
internet at home.

Banilower and colleagues (2018) report that in 6th-grade classrooms, the three most
commonly used textbook series were Pearson enVisionmath 2.0 (Berry et al., 2017) Houghton
Mifflin Harcourt Go Math! (Burger et al., 2014), and McGraw Hill Education Glencoe Math
(Carter et al., 2015). Since this selection of textbooks represents one series from each of the three major textbook publishers (Pearson, Houghton Mifflin Harcourt, McGraw Hill Education), the textbook sample for this study will consist of nine textbooks: Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017), Houghton Mifflin Harcourt *Go Math! Grades 6, 7, 8* (Burger et al., 2014), and McGraw Hill Education *Glencoe Math Course 1, 2, 3* (Carter et al., 2015). Table 1 presents the textbooks in the sample along with short codes to represent each textbook. The data sources for this study was the images of polygons presented within each of the nine textbooks in the sample.

**Table 1**

*Textbook Sample*

<table>
<thead>
<tr>
<th>Publisher</th>
<th>Title of Series</th>
<th>Grade</th>
<th>Short Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson</td>
<td><em>enVisionmath2.0</em></td>
<td>6</td>
<td>EVM6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>EVM7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>EVM8</td>
</tr>
<tr>
<td>Houghton Mifflin Harcourt</td>
<td><em>Go Math!</em></td>
<td>6</td>
<td>GM6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>GM7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>GM8</td>
</tr>
<tr>
<td>McGraw Hill Education</td>
<td><em>Glencoe Math Course</em></td>
<td>6</td>
<td>GMC6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>GMC7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>GMC8</td>
</tr>
</tbody>
</table>
Description of Textbook Series

**Pearson enVisionmath2.0.** Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017) is a comprehensive curriculum for middle school mathematics that is not aligned with national mathematics content standards, but is aligned with Common Core Standards for Mathematical Practice (NGACBP, CCSSO, 2010). The textbook series covers grades 6-8 and each course is integrated so there are no geometry specific courses. Each grade level is split into two separate volumes, and each volume contains multiple topics. The topics are split into sections dedicated to subtopics and there are additional assessments and activities in the beginning, middle, and end of each topic. Assessments include mid-topic checkpoints and mid-topic performance tasks. Activities include STEM projects, reviewing what you know, math literacy activities, topic openers, building vocabulary, and preparing for reading success, topic reviews, and fluency practice activities.

Each textbook in the Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017) series covers five mathematical domains: Numbers and Computation, Algebra and Functions, Proportionality, Geometry, and Data Analysis and Probability. The 6th-grade textbooks contain eight topics total with two topics related to Numbers and Computation (Use Positive Rational Numbers, Integers and Rational Numbers), two related to Algebra and Functions (Numeric and Algebraic Expressions, Represent and Solve Equations and Inequalities), two related to Proportionality (Understand and Use Ratio and Rate, Understand and Use Percent), one related to Geometry (Solve Area, Surface Area, and Volume Problems), and one related to Data Analysis and Probability (Display, Describe, and Summarize Data). The 7th-grade textbooks contain eight topics total with one topic related to Numbers and Computation (Integers and Rational Numbers, Integers and Rational Numbers), two related to Algebra and Functions (Numeric and Algebraic Expressions, Represent and Solve Equations and Inequalities), two related to Proportionality (Understand and Use Ratio and Rate, Understand and Use Percent), one related to Geometry (Solve Area, Surface Area, and Volume Problems), and one related to Data Analysis and Probability (Display, Describe, and Summarize Data).
Numbers), two related to Algebra and Functions (Generate Equivalent Expressions, Solve Problems Using Equations and Inequalities), two related to Proportionality (Analyze and Use Proportional Relationships, Analyze and Solve Percent Problems), one related to Geometry (Solve Problems Involving Geometry), and two related to Data Analysis and Probability (Use Sampling to Draw Inferences About Populations). The 8th-grade textbooks contain eight topics total with zero topics related to Numbers and Computation, four related to Algebra and Functions (Real Numbers, Analyze and Solve Linear Equations, Use Functions to Model Relationships, Analyze and Solve Systems of Linear Equations), zero related to Proportionality, three related to Geometry (Congruence and Similarity, Understand and Apply the Pythagorean Theorem, Solve Problems Involving Surface Area and Volume), and one related to Data Analysis and Probability (Investigate Bivariate Data).

The authors of the Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017) series are involved in academia and research. Robert Q. Berry III is an Associate Professor of Mathematics Education, Zachary Champagne is an Assistant in Research, Randall I. Charles is a Professor Emeritus in Mathematics, Francis (Skip) Fennell is a Professor of Education and Graduate Studies, Eric Milou is a Professor of Mathematics, Jane F. Schielack is a Professor Emerita in Mathematics, and Johathan A. Wray is a Mathematics Instructional Facilitator. The series was reviewed by Gary Lippman, a Professor Emeritus in Mathematics and Computer Science, and Karen Edwards, a Mathematics Lecturer. The authors represent both mathematics and mathematics education backgrounds, however, there are not any K-12 teachers included on the author or reviewer list.
Houghton Mifflin Harcourt Go Math!. Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014) is an integrated mathematics curriculum for middle school that is aligned with both Common Core math content standards and Common Core Standards for Mathematical Practice (NGACBP, CCSSO, 2010). The textbook series covers grades 6-8 and covers multiple mathematics domains in each grade level, so there is no single geometry textbook. Each textbook is organized into several units and units are further divided into modules and sub-modules. In addition to the sub-modules, there are assessments and other activities built into the modules. Assessments include ready to go on activities and module assessment readiness activities. Other activities included in the modules are careers in math, vocabulary previews, real-world videos, are you ready, reading start-up, and unpacking the standards.

Each textbook in the Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014) series covers a wide variety of mathematics topics. The 6th-grade textbook contains seven units, each divided into modules. The 6th-grade Unit 1 Numbers contains modules Integers, Factors and Multiples, and Rational Numbers. The 6th-grade Unit 2 Number Operations contains modules Operations with Fractions and Operations with Decimals. The 6th-grade Unit 3 Proportionality Ratios and Rates contains modules Representing Ratios and Rates, Applying Ratios and Rates, and Percents. The 6th-grade Unit 4 Equivalent Expressions contains modules Generating Equivalent Numerical Expressions and Generating Equivalent Algebraic Expressions. The 6th-grade Unit 5 Equations and Inequalities contains modules Equations and Relationships and Relationships in Two Variables. The 6th-grade Unit 6 Relationships in Geometry contains modules Area and Polygons, Distance and Area in the Coordinate Plane, and
Surface Area and Volume of Solids. The 6th-grade Unit 7 Measurement and Data contains modules Displaying, Analyzing, and Summarizing Data.

The 7th-grade textbook contains six units, each divided into modules. The 7th-grade Unit 1 The Number System contains modules Adding and Subtracting Integers, Multiplying and Dividing Integer, and Rational Numbers. The 7th-grade Unit 2 Ratios and Proportional Relationships contains Rates and Proportionality and Proportions and Percent. The 7th-grade Unit 3 Expressions, Equations, and Inequalities contains modules Expressions and Equations and Inequalities. The 7th-grade Unit 4 Geometry contains modules Modeling Geometric Figures and Circumference, Area, and Volume. The 7th-grade Unit 5 Statistics contains modules Random Samples and Populations and Analyzing and Comparing Data. The 7th-grade Unit 6 Probability contains modules Experimental Probability and Theoretical Probability and Simulations.

The 8th-grade textbook contains six units, each divided into modules. The 8th-grade Unit 1 Real Numbers, Exponents, and Scientific Notation contains modules Real Numbers and Exponents and Scientific Notation. The 8th-grade Unit 2 Proportional and Nonproportional Relationships and Functions contains modules Proportional Relationships, Nonproportional Relationships, Writing Linear Equations, and Functions. The 8th-grade Unit 3 Solving Equations and Systems of Equations contains modules Solving Linear Equations and Solving Systems of Linear Equations. The 8th-grade Unit 4 Transformational Geometry contains modules Transformations and Congruence and Transformations and Similarity. The 8th-grade Unit 5 Measurement Geometry contains modules Angle Relationships in Parallel Lines and Triangles, The Pythagorean Theorem, and Volume. The 8th-grade Unit 6 Statistics contains modules Scatter Plots and Two-Way Tables.
The authors of the Houghton Mifflin Harcourt *Go Math! Grades 6, 7, 8* (Burger et al., 2014) include individuals involved in academia, research, and K12 education. Edward B. Burger is a University President, Juli K. Dixon is a Professor of Mathematics Education, Timothy D. Kanold is an international educator and author, Matthew R. Larson is a K-12 Mathematics Curriculum Specialist, Steven J. Leinwand is a Principal Research Analyst, and Martha E. Sandoval-Martinez is a Mathematics Instructor. The series was reviewed by 12 individuals representing Instructional Staff Developers, Math Staff Developers, Math Coaches, Secondary Math Coordinators, Math Content Specialists, and both Middle and High School Math Teachers. The authorship and reviewing of this particular textbook series includes people of various backgrounds at all levels of mathematics education and research.

**McGraw Hill Education Glencoe Math Course.** McGraw Hill Education *Glencoe Math Course 1, 2, 3* (Carter et al., 2015) is a middle school mathematics curriculum based around three pillars: rigor, relevance, and results. The textbook series is integrated, meaning there is no specific geometry course. The series is organized by Common Core math content standard domains and embeds Common Core Standards for Mathematical Practice throughout (NGACBP, CCSSO, 2010). Each textbook in the series is divided into units based on the grade level math domains, and units are subdivided into Chapters, and chapters are split into individual lessons. In addition to the lessons, there are also assessments and activities throughout the chapters. Assessments include mid-chapter checks, performance tasks, and unit projects. Activities include what tools do you need, what do you already know, are you ready, inquiry labs, problem-solving investigations, 21st century careers, and chapter reviews.
The McGraw Hill Education *Glencoe Math Course* (Carter et al., 2015) Units are organized around five math domains in each grade level, in 6th and 7th-grade the Units are Ratios and Proportional Relationships, The Number System, Expressions and Equations, Geometry, and Statistics and Probability. In 8th-grade, the Units are The Number System, Expressions and Equations, Functions, Geometry, and Statistics and Probability. The 6th-grade Units are divided into chapters as follows: Ratios and Proportional Relationships (Ratios and Rates, Fractions, Decimals, Percents), The Number System (Compute with Multi-Digit Numbers, Multiply and Divide Fractions, Integers and the Coordinate Plane), Expressions and Equations (Expressions, Equations, Functions and Inequalities), Geometry (Area, Volume and Surface Area), and Statistics and Probability (Statistical Measures, Statistical Displays). The 7th-grade Units are divided into chapters as follows: Ratios and Proportional Relationships (Ratios and Proportional Reasoning, Percents), The Number System (Integers, Rational Numbers), Expressions and Equations (Expressions, Equations and Inequalities), Geometry (Geometric Figures, Measure Figures), and Statistics and Probability (Probability, Statistics). The 8th-grade Units are divided into chapters as follows: The Number System (Real Numbers), Expressions and Equations (Equations in One Variable, Equations in Two Variables), Functions (Functions), Geometry (Triangles and the Pythagorean Theorem, Transformations, Congruence and Similarity, Volume and Surface Area), and Statistics and Probability (Scatter Plots and Data Analysis).

The McGraw Hill Education *Glencoe Math Courses 1, 2, 3* (Carter et al., 2015) textbook series does not include information about the authors beyond their names. The textbooks also contain no mention of the reviewers of the series. This is in contrast to Pearson *enVisionmath2.0*.
Grades 6, 7, 8 (Berry et al., 2017) and Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014) which both give a short overview of the background and expertise of the textbook authors and reviewers.

Data Sources

In order to answer my research questions on young adolescents’ opportunities to develop concept images of polygons, I analyzed the representations of convex polygons present in three popular middle school mathematics textbooks series as my data sources. For each image of a convex polygon in the textbook sample, I analyzed the image with \textit{a priori} codes (Stemler, 2001; White & Marsh, 2006). I initially developed and tested these codes as part of a pilot study conducted in Spring 2019. As part of the pilot study, I created \textit{a priori} codes based on recommendations from literature in the field (Stemler, 2001; White & Marsh, 2006). After multiple iterations and consultation with colleagues, I refined the coding scheme to address polygon type, orientation, prototypicality, context, role of the image, and developmental responsiveness. For some of these categories, I examined the results across grade level, textbook publisher, and location within the textbook.

\textit{A Priori} Coding Scheme

\textbf{Polygon type and orientation.} Each image analyzed in this study is a convex polygon, a geometric plane figure with three or more straight sides. I coded each polygon using the most specific category possible out of the following codes: non-right triangle, right triangle, general quadrilateral, square, rectangle, parallelogram, rhombus, trapezoid, kite, pentagon, hexagon, and octagon. The code of non-right triangle indicates a polygon with exactly three sides and without a right ($90^\circ$) angle, and the code of right triangle specifies a polygon with exactly three sides and
one right angle. For the orientation of the images, I coded each polygon based on how many of the sides are vertically or horizontally aligned with the edges of the textbook pages. The codes for the orientation category cover each possible polygon and denote if there are zero, one, two, three, or four sides aligned with the edges of the text and if they are aligned on the left, right, top, or bottom of the image, as well as if the diagonals are aligned horizontally and vertically.

All quadrilaterals are polygons with exactly four sides, however, the code of general quadrilateral only labels images of quadrilaterals that do not fit into any of the special categories of square, rectangle, rhombus, parallelogram, trapezoid, or kite. Since classifications of special quadrilaterals are hierarchical and overlapping (i.e., a square is also a rectangle, a rhombus, and a parallelogram), I used the following codes to eliminate ambiguity: square (four equal sides, four right angles, and two pairs of parallel sides), rectangle (four right angles, two pairs of parallel sides, and not a square), rhombus (four equal sides, two pairs of parallel sides, and not a square), parallelogram, (two pairs of parallel sides and not a rhombus, rectangle, or square), trapezoid (exactly one pair of parallel sides), and kite (exactly two pairs of adjacent congruent sides). I used the codes for pentagon, hexagon, heptagon, and octagon to designate polygons with exactly five, six, seven, and eight sides, respectively, and I coded polygons with more than eight sides as n-gons with n representing the number of sides of the polygon.

**Prototypical or not prototypical.** I coded each image of a convex polygon contained in the sample of textbooks based on whether or not the image is a prototypical representation. A prototype or prototypical image is the most commonly used classical representation of a particular concept (Hasegawa, 1997; Presmeg, 1992). Typically, a prototypical image is one that contains symmetries and the longest list of special attributes (Tsamir et al., 2015). For the
purposes of this study, I classified polygons as prototypical if they fit the descriptions in the following sentences. A non-right triangle is prototypical if it is oriented with one side horizontally aligned on the bottom of the image with the edges of the textbook and is seemingly isosceles (Tsamir et al., 2015). A right triangle is prototypical if the right angle of the triangle is formed by two sides that are aligned horizontally and vertically with the edges of the textbook (Ward, 2004). For quadrilaterals, a general quadrilateral that does not fit the definition of a special category of quadrilaterals is not prototypical. Squares and rectangles are prototypical if all four sides are aligned horizontally and vertically with the edges of the textbook, and in the case of rectangles, the longer sides are horizontal with the shorter sides vertical. Rhombi and parallelograms are prototypical if one pair of parallel sides is aligned horizontally with the edges of the textbook and in the case of parallelograms, the longer sides are horizontal and the shorter sides are oblique. A prototypical trapezoid is an isosceles trapezoid with the pair of parallel sides aligned horizontally with the edges of the textbook and the longer of the two parallel sides on the bottom rather than the top (Turnuklu et al., 2013). A kite is prototypical if the diagonals of the kite are horizontally and vertically aligned with the edges of the textbook, and the shorter pair of congruent consecutive sides is on the top with the longer pair on the bottom. And finally, polygons with more than five sides are prototypical if they are printed as seemingly regular polygons with one side horizontally aligned with the edges of the textbook on the bottom. Figure 9 presents some examples of prototypical and non-prototypical images.
Figure 9. Examples of prototypical and non-prototypical images.

Context and role. For each image of a convex polygon within the nine textbooks in the textbook sample, I coded the image based on the context and the role. The context of the image
can be either real-world or purely mathematical, depending on how the polygon is represented. The role of the image could be superfluous, illustrative, or interpretive based on an adaption of the Hunsader et al. (2014) Mathematical Processes Assessment Coding Framework (MPAC). Superfluous images are images that are mathematically irrelevant, not necessary, and do not require any interpretation. Illustrative images either illustrate a mathematical concept or are images required for solving a task, but the properties of the image are not represented visually, do not need to be interpreted, and are instead stated within the text. Interpretive images either illustrate a mathematical concept or are images required for solving a task, and properties of the figure are conveyed through the image and require interpretation. Figure 10 shows examples of representations in each category of superfluous, illustrative, and interpretive images.

**Figure 10.** Examples of superfluous, illustrative, and interpretive images.

**Developmental responsiveness.** Young adolescents in middle school are experiencing a transition period of rapid and profound changes, and therefore have specific cognitive-intellectual, physical, psychological, social-emotional, and moral characteristics, needs, and interests (AMLE, 2012; AMTE, 2017; Caskey & Anfara, 2014; Ellerbrock & Vomvoridi-Ivanovic, 2019; NMSA, 2010). The codes for developmental responsiveness were a simple yes/no based on if the task attends to young adolescents’ needs other than their cognitive-intellectual...
development. Figures 11, 12, 13, and 14 show examples of tasks involving convex polygons that attend to young adolescents’ physical, psychological, social-emotional, and moral characteristics.

**Figure 11.** Example of convex polygon task that attends to young adolescents’ physical characteristics.

**Figure 12.** Example of convex polygon task that attends to young adolescents’ psychological characteristics.
Figure 13. Example of convex polygon task that attends to young adolescents’ social-emotional characteristics.

Figure 14. Example of convex polygon task that attends to young adolescents’ moral characteristics.
**Grade level, publisher, and location.** The codes used for the grade level, publisher, and location were the least complicated. The textbook sample included middle school mathematics textbooks spanning 6th, 7th, and 8th grade; therefore, the codes for grade level will denote these three grades. The textbook sample included nine textbooks in total, Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017), Houghton Mifflin Harcourt *Go Math! Grades 6, 7, 8* (Burger et al., 2014), and McGraw Hill Education *Glencoe Math Course 1, 2, 3* (Carter et al., 2015). Hence, the codes for textbooks publisher indicated which of the three publishing companies (Pearson, Houghton Mifflin Harcourt, McGraw Hill Education) published the textbook containing the image. The codes for location specified where the image is located within the textbook (i.e., in a preview or introduction, the main portion of the lesson or section, the exercise or student practice portion, a discovery activity or project, or an assessment or assessment practice).

**Summary of *a priori* coding scheme.** The *a priori* codes I developed for this study assisted me in analyzing the representations of convex polygons in middle school mathematics textbooks in terms of polygon classification, orientation, prototypicality, context, the role of the images, and developmental responsiveness across grade levels, publishers, and location within the textbooks. Table 2 presents the coding scheme for analyzing polygons that I used in this study.
Table 2

**Codes for Data Analysis**

<table>
<thead>
<tr>
<th>Category</th>
<th>Short Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polygon</td>
<td>Non-Right Triangle (NRT), Right Triangle (RT), General Quadrilateral (GQ), Square (S), Rectangle (Re), Parallelogram (Pa), Rhombus (Rh), Trapezoid (T), Kite (K), Pentagon (Pe), Hexagon (Hex), Heptagon (Hept), Octagon (O), n-gon (n)</td>
</tr>
<tr>
<td>Orientation</td>
<td>One Side Vertically Left Aligned (OVL), One Side Vertically Right Aligned (OVR), One Side Horizontally Top Aligned (OHT), One Side Horizontally Bottom Aligned (OHB), Two Sides Vertically Left and Horizontally Top Aligned (TVLHT), Two Sides Vertically Left and Horizontally Bottom Aligned (TVLHB), Two Sides Vertically Right and Horizontally Top Aligned (TVRHT), Two Sides Vertically Right and Horizontally Bottom Aligned (TVRHB), Two Sides Vertically Left and Right Aligned (TVLR), Two Sides Horizontally Top and Bottom Aligned (THTB), All Diagonals Vertically and Horizontally Aligned (ADVH), Three Sides Vertically Left and Right and Horizontally Top Aligned (TVLRHT), Three Sides Vertically Left and Right and Horizontally Bottom Aligned (TVLRHB), Three Sides Vertically Left and Horizontally Top and Bottom Aligned (TVLHTB), Three Sides Vertically Right and Horizontally Top and Bottom Aligned (TVRHTB), All Sides Vertically and Horizontally Aligned (ALL), Other Alignment (O)</td>
</tr>
<tr>
<td>Prototypical</td>
<td>Yes (Y), No (N)</td>
</tr>
<tr>
<td>Context</td>
<td>Real World Context (RW), Purely Mathematical Context (M)</td>
</tr>
<tr>
<td>Role</td>
<td>Superfluous (S), Illustrative (III), Interpretation (Int)</td>
</tr>
<tr>
<td>Physical Characteristics</td>
<td>Yes (Y), No (N)</td>
</tr>
<tr>
<td>Psychological Characteristics</td>
<td>Yes (Y), No (N)</td>
</tr>
<tr>
<td>Social-Emotional Characteristics</td>
<td>Yes (Y), No (N)</td>
</tr>
<tr>
<td>Moral Characteristics</td>
<td>Yes (Y), No (N)</td>
</tr>
<tr>
<td>Grade</td>
<td>6th Grade (6), 7th Grade (7), 8th Grade (8)</td>
</tr>
<tr>
<td>Publisher</td>
<td>Pearson (P), Houghton Mifflin Harcourt (HMH), McGraw Hill Education (MHHE)</td>
</tr>
<tr>
<td>Location</td>
<td>Lesson (L), Practice (P), Activity (A), Review (R), Get Ready/Are you Ready (G), Assessment Practice (AP), Performance Tasks (PT), Assessment (AS), Investigate (I)</td>
</tr>
</tbody>
</table>

**Images of Polygons in Textbooks**

In total, I coded and analyzed 1,698 images of convex polygons. Table 3 presents a summary of the number images of polygons for each textbook and series. I coded and analyzed 454 images of polygons in 6th grade textbooks, 238 images of polygons in 7th grade textbooks,
and 1,006 images of polygons in 8th grade textbooks. Across publishers, I coded and analyzed 664 images of polygons from Pearson’s *enVisionmath2.0* series, 314 images of polygons from Houghton Mifflin Harcourt’s *Go Math!* series, and 720 images of polygons from McGraw Hill Education’s *Glencoe Math Course* series.

Summary of Data Sources

The data sources I analyzed in my study are representations of convex polygons present in three popular middle school mathematics textbooks series. The textbook sample included nine

<table>
<thead>
<tr>
<th>Publisher</th>
<th>Title of Series</th>
<th>Short Code</th>
<th>Grade</th>
<th>Number of Polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson</td>
<td><em>enVisionmath2.0</em></td>
<td>EVM</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EVM6</td>
<td>6</td>
<td>160</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EVM7</td>
<td>7</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EVM8</td>
<td>8</td>
<td>387</td>
<td></td>
</tr>
<tr>
<td>Houghton Mifflin Harcourt</td>
<td><em>Go Math!</em></td>
<td>GM</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>GM6</td>
<td>6</td>
<td>77</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GM7</td>
<td>7</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GM8</td>
<td>8</td>
<td>219</td>
<td></td>
</tr>
<tr>
<td>McGraw Hill Education</td>
<td><em>Glencoe Math Course</em></td>
<td>GMC</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>GMC6</td>
<td>6</td>
<td>217</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GMC7</td>
<td>7</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td></td>
<td>GMC8</td>
<td>8</td>
<td>400</td>
<td></td>
</tr>
</tbody>
</table>
textbooks in total, Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017), Houghton Mifflin Harcourt *Go Math! Grades 6, 7, 8* (Burger et al., 2014), and McGraw Hill Education *Glencoe Math Course 1, 2, 3* (Carter et al., 2015). I analyzed each image of a convex polygon using *a priori* codes created based on recommendations from literature in the field (Stemler, 2001; White & Marsh, 2006). The codes addressed polygon type, orientation, prototypicality, context, the role of the image, and developmental responsiveness. For each of these categories, I examined the results across grade level, textbook publisher, and location within the textbook.

**Data Analysis**

For this study, I analyzed the data using both quantitative and qualitative methods. Most of the data sources are the codes of polygon, orientation, prototypicality, context, role, developmental responsiveness, grade, publisher, and location for the individual polygons. I recorded the codes for each polygon and analyzed the results in terms of polygon, orientation, prototypicality, context, role, and developmental responsiveness across grade, publisher, and location within the textbooks. I analyzed the data using descriptive statistics including frequencies. Then I reported young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks. The remaining data sources were examples of tasks that attend to young adolescents’ developmental characteristics other than cognitive-intellectual. For these examples, I utilized qualitative descriptions of the tasks to demonstrate how the task addresses young adolescents’ specific needs.

Figures 15, 16, and 17 are example images of polygons from textbooks and Table 4 is an example of a coding table with codes for each polygon image.
Figure 15. Example image of a polygon from Pearson *enVisionmath2.0 Grades 6* (Berry et al., 2017).

Figure 16. Example image of a polygon from Houghton Mifflin Harcourt *Go Math! Grades 7* (Burger et al., 2014).
Validity Criteria

White and Marsh (2006) list several methods for establishing validity in a quantitative content analysis study: face validity (correspondence between what is measured and how it is measured), criterion validity (correspondence between code and criterion), content validity (completeness of representation of the concept), and construct validity (the extent to which the measure is related to other measures). Face validity is common in content analysis studies but is
inherently subjective, and therefore, hard to verify (Krippendorff, 1980; White & Marsh, 2006).

In this study, I established criterion validity by having a second researcher code a subset of the images in each textbook included in the sample. This second researcher is another graduate student at my university; they are familiar with middle school mathematics content and definitions of polygons. I provided the second researcher with the coding scheme and examples of coded images and we discussed any questions related to the coding process. We coded a small set of images together to resolve any misunderstandings about the coding scheme. The second researcher then individually coded 10% of the images of polygons in each textbook included in the sample. The coding scheme used for the study should be easy to follow, with clear instructions, obvious definitions, and unambiguous examples so that all researchers using the codes will code items the same way (White & Marsh, 2006). Ideally, researchers would have a 90% or greater agreement on codes, however, an 80% would be acceptable (Neuendorf, 2002). I compared the coding completed by the second researcher to my own coding for the images and we reached above 90% agreement.

**Summary of Research Design and Methodology**

In this chapter, I described my research design and methodology including my research questions, rationale and purpose, research design, textbook selection, data sources, data analysis, and validity criteria. The purpose of this study was to analyze young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks. I employed a content analysis methodology, which is appropriate for a textbook analysis study as it is a well-established technique for analyzing textbooks and other documents across a variety of fields (Krippendorff, 1980; Neuendorf, 2002; Stemler, 2001; Titscher et al., 2000). Content analysis is
a flexible methodology that can be applied in quantitative, qualitative, or mixed methods studies (White & Marsh, 2006). Since my research questions were concerned with images of polygons in middle school mathematics textbooks, the sample for this study consisted of nine textbooks: Pearson enVisionmath2.0 Grades 6, 7, 8 (Berry et al., 2017), Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014), and McGraw Hill Education Glencoe Math Course 1, 2, 3 (Carter et al., 2015). The unit of analysis in this study was tasks involving images of convex polygons printed within three popular middle school mathematics textbook series (White & Marsh, 2006). I analyzed the image with a priori codes based on recommendations from literature in the field (Stemler, 2001; White & Marsh, 2006). The coding scheme addresses polygon type, orientation, prototypicality, context, the role of the image, and developmental responsiveness. For each of these categories, I examined the results across grade level, textbook publisher, and location within the textbook. In the next chapter, I present the results of my data analysis.
CHAPTER 4: RESULTS

The purpose of this study was to analyze young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks (grades 6-8). The textbook sample I chose for this study consisted of nine textbooks: Pearson enVisionmath2.0 Grades 6, 7, 8 (Berry et al., 2017), Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014), and McGraw Hill Education Glencoe Math Course 1, 2, 3 (Carter et al., 2015). These three textbook series represent one series from each of the three major textbook publishers: Pearson, Houghton Mifflin Harcourt, McGraw Hill Education (Banilower et al., 2018).

This chapter presents data to address each of the following research questions. Specifically, the research question addressed this study was as follows: Within three published middle school mathematics textbook series, in what ways do young adolescents have opportunities to develop concept images of polygons?

1. In what ways do tasks within the textbooks involving visual representations of convex polygons attend to young adolescents’ cognitive-intellectual characteristics? In particular:
   a. How are images of convex polygons oriented in the textbooks?
   b. How are right angles oriented within images of right triangles in the textbooks?
   c. What percentage of convex polygons in the textbooks are prototypical images, in other words, the most common representation of a particular convex polygon?
d. In what contexts (i.e., real-world or purely mathematical) are convex polygons represented within the textbooks?

e. What is the role of the images (i.e., superfluous, illustrative, or interpretive) representing convex polygons within the textbooks?

2. To what extent do tasks within the textbooks involving visual representations of convex polygons attend to other developmental characteristics of young adolescents? In particular:

   a. To what extent do tasks involving visual representations of convex polygons address young adolescents’ physical characteristics?

   b. To what extent do tasks involving visual representations of convex polygons address young adolescents’ psychological characteristics?

   c. To what extent do tasks involving visual representations of convex polygons address young adolescents’ social-emotional characteristics?

   d. To what extent do tasks involving visual representations of convex polygons address young adolescents’ moral characteristics?

The remainder of this chapter is divided into sections to address each of these questions individually. The results will cover polygon type, orientation, prototypicality, context, role of the image, and developmental responsiveness across grade level, textbook publisher, and location within each textbook.

**Number of Polygons**

In order to answer my research questions on young adolescents’ opportunities to develop concept images of polygons, I analyzed the representations of convex polygons present in three
popular middle school mathematics textbooks series as my data sources. The textbook sample for this study consisted of nine textbooks: Pearson enVisionmath2.0 Grades 6, 7, 8 (Berry et al., 2017), Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014), and McGraw Hill Education Glencoe Math Course 1, 2, 3 (Carter et al., 2015). Tables 5, 6, and 7 show the number of polygons by publisher, grade level, and location, respectively.

Table 5 shows that the McGraw Hill Education Glencoe Math Course 1, 2, 3 (Carter et al., 2015) textbook series contained the largest number of images of convex polygons with 720 total. Pearson enVisionmath2.0 Grades 6, 7, 8 (Berry et al., 2017) textbook series contains 664 images of convex polygons, which is a similar number to the McGraw Hill Education series. Alternatively, the Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014) textbook series contains less than half the number of images of convex polygons found in the Pearson and McGraw Hill Education series, 314 total images.

<table>
<thead>
<tr>
<th>Publisher</th>
<th>Number of Polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson</td>
<td>664</td>
</tr>
<tr>
<td>Houghton Mifflin Harcourt</td>
<td>314</td>
</tr>
<tr>
<td>McGraw Hill Education</td>
<td>720</td>
</tr>
</tbody>
</table>

In all three textbook series, eight-grade textbooks contained the largest number of images of convex polygons, followed by sixth-grade textbooks, and finally seventh-grade textbooks. As
shown in Table 6, the three eighth-grade textbooks included in the study contained a total of 1,006 images of convex polygons. The sixth-grade textbooks contained a total of 454 images of convex polygons and the seventh-grade textbooks included 238 images of convex polygons.

Table 6

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Number of Polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th Grade</td>
<td>454</td>
</tr>
<tr>
<td>7th Grade</td>
<td>238</td>
</tr>
<tr>
<td>8th Grade</td>
<td>1006</td>
</tr>
</tbody>
</table>

As shown in Table 7, the number of images of convex polygons varied greatly by the location of the textbook. Lessons and practice sections contained the largest number of images of convex polygons with 550 and 663, respectively. Built in activities contained 86, reviews contained 125, get ready or readiness sections contained 48, assessment practices contained 110, performance tasks contained 15, and investigations contained 59 images of convex polygons.

**Types of Polygons**

Each image analyzed in this study was a convex polygon, a geometric plane figure with three or more straight sides. I coded each polygon using the most specific category possible out of the following codes: non-right triangle, right triangle, general quadrilateral, square, rectangle,
parallelogram, rhombus, trapezoid, kite, pentagon, hexagon, and octagon. Figures 18 and 19 show the variety of types of polygons by publisher and grade level, respectively.

Table 7

<table>
<thead>
<tr>
<th>Location</th>
<th>Number of Polygons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson</td>
<td>550</td>
</tr>
<tr>
<td>Practice</td>
<td>663</td>
</tr>
<tr>
<td>Activity</td>
<td>86</td>
</tr>
<tr>
<td>Review</td>
<td>125</td>
</tr>
<tr>
<td>Get Ready</td>
<td>48</td>
</tr>
<tr>
<td>Assessment Practice</td>
<td>110</td>
</tr>
<tr>
<td>Assessment</td>
<td>42</td>
</tr>
<tr>
<td>Performance Task</td>
<td>15</td>
</tr>
<tr>
<td>Investigate</td>
<td>59</td>
</tr>
</tbody>
</table>

*Figure 18. Types of polygons by publisher.*
As shown in Figure 18, all three publishers contained a majority of non-right triangles, right triangles, and to a lesser extent, rectangles. The Pearson series contained little variety of types of polygons with most of the images of convex polygons being non-right triangles, 27%, right triangles, 20%, and rectangles, 22% of polygons. The Pearson series contained no octagons, almost no hexagons and rhombi, and very few pentagons, kites, and general quadrilaterals. The Houghton Mifflin Harcourt series contained little variety in terms of types of polygons with most images of convex polygons being non-right triangles, 32%, and right triangles, 24% polygons. The Houghton Mifflin Harcourt series contained no octagons, almost no hexagons, pentagons, kites, and rhombi, and very few general quadrilaterals, squares, and parallelograms. The McGraw Hill Education series contained little variety in terms of types of polygons with most images of convex polygons being non-right triangles, 26%, and right triangles, 25% of images. The McGraw Hill Education series contained almost no rhombi, kites, and octagons, and very few hexagons, pentagons, and general quadrilaterals.

Figure 19. Types of polygons by grade level.
As demonstrated in Figure 19, the types of polygons represented varied depending on grade level. In sixth grade, textbooks contain a more even distribution of types of convex polygons with 15% non-right triangles, 10% right triangles, 22% rectangles, 19% parallelograms, and 17% trapezoids. In both seventh and eighth-grade, the only types of polygons representing 10% or more are triangles and rectangles. In seventh-grade textbooks, non-right triangles comprised 26%, right triangles comprised 14%, and rectangles comprised 38% of images of convex polygons. In eighth-grade textbooks, non-right triangles comprised 33%, right triangles comprised 31%, and rectangles comprised 10% of images of convex polygons.

**Polygon Orientation**

For the orientation of the images, I will coded each polygon based on how many of the sides are vertically or horizontally aligned with the edges of the textbook pages. The codes for the orientation category cover each possible polygon and denote if there are zero, one, two, three, or four sides aligned with the edges of the text and if they are aligned on the left, right, top, or bottom of the image, as well as if the diagonals are aligned horizontally and vertically.

**Non-Right Triangles**

A non-right triangle indicates a polygon with exactly three sides and without a right (90°) angle. Figures 20 and 21 show the orientations of non-right triangles by publisher and grade level, respectively.
As shown in Figure 20, non-right triangles were most often printed with one side horizontally aligned with the text on the bottom of the triangle, followed by other orientations with no horizontal or vertical side alignments, across all three publishers.

Figure 21. Orientation of non-right triangles by grade level.
Across all three grade levels, non-right triangles were most often printed with one side horizontally aligned with the text on the bottom of the triangle. Though, as shown in Figure 21, in eighth-grade textbooks a large number of non-right triangles were printed in other orientations with no horizontal or vertical side alignments.

Right Triangles

A right triangle specifies a polygon with exactly three sides and one right angle. Figures 22 and 23 show the orientations of right triangles by publisher and grade level, respectively.

**Figure 22.** Orientation of right triangles by publisher.

As shown in Figure 22, across all three publishers right triangles were most often oriented with the perpendicular sides horizontally and vertically aligned with the textbook edges. Right triangles oriented with the perpendicular sides horizontally and vertically aligned comprise four separate categories depending on if the horizontally aligned side was on the top or bottom and the vertically aligned side was on the left or right.
Figure 23. Orientation of right triangles by grade level.

Across all three grade levels right triangles were most often oriented with the perpendicular sides horizontally and vertically aligned with the textbook edges, though there were significantly more right triangles presented in eighth-grade textbooks as shown in Figure 23. Right triangles oriented with the perpendicular sides horizontally and vertically aligned comprise four separate categories depending on if the horizontally aligned side was on the top or bottom and the vertically aligned side was on the left or right.

General Quadrilaterals

All quadrilaterals are polygons with exactly four sides, however, the general quadrilateral is a quadrilateral that does not fit into any of the special categories of square, rectangle, rhombus, parallelogram, trapezoid, or kite. Figures 24 and 25 show the orientations of general quadrilaterals by publisher and grade level, respectively.
General quadrilaterals were printed with a variety of orientations across publishers, as shown in Figure 24. In the Pearson and Houghton Mifflin Harcourt textbook series, most general quadrilaterals were printed with no sides horizontally or vertically aligned with the textbook edges. In the McGraw Hill Education textbook series, most general quadrilaterals were printed with one side horizontally aligned with the text on the bottom of the image, though there were still a large number of general quadrilaterals printed with no sides horizontally or vertically aligned with the textbook edges.

Figure 24. Orientation of general quadrilaterals by publisher.
As shown in Figure 25, there were not many general quadrilaterals printed in sixth and seventh-grade textbooks. In eighth-grade textbooks, most general quadrilaterals were printed with no sides horizontally or vertically aligned with the textbook edges, followed by one side horizontally aligned with the text on the bottom of the image.

**Squares**

Since classifications of special quadrilaterals are hierarchical and overlapping (i.e., a square is also a rectangle, a rhombus, and a parallelogram), to eliminate ambiguity, I define a square as a polygon having four equal sides, four right angles, and two pairs of parallel sides. Figures 26 and 27 show the orientations of squares by publisher and grade level, respectively.
Figure 26. Orientation of squares by publisher.

As shown in Figure 26, across all three publishers the most common orientation for squares was all four sides horizontally and vertically aligned with the textbook edges, though this is more pronounced in the Pearson and McGraw Hill Education textbook series. In the Houghton Mifflin Harcourt textbook series, there were approximately the same number of squares oriented with both diagonals horizontally and vertically aligned with the textbook edges as squares with all four sides aligned this way.
In all three grades, squares were most often printed with all four sides horizontally and vertically aligned with the textbook edges. Though, as shown in Figure 27, there were many more squares represented in eighth-grade textbooks compared to sixth and seventh-grade textbooks.

**Rectangles**

Since classifications of special quadrilaterals are hierarchical and overlapping (i.e., a square is also a rectangle, a rhombus, and a parallelogram), to eliminate ambiguity, I define a rectangle as a quadrilateral having four right angles, two pairs of parallel sides, and not a square. Figures 28 and 29 show the orientations of rectangles by publisher and grade level, respectively.
Figure 28. Orientation of rectangles by publisher.

As shown in Figure 28, across all three publishers the most common orientation for rectangles was all four sides horizontally and vertically aligned with the textbook edges.

Figure 29. Orientation of rectangles by grade level.
In all three grade levels, the most common orientation for rectangles was all four sides horizontally and vertically aligned with the textbook edges. As shown in Figure 29, a small number of rectangles were printed with no sides horizontally and vertically aligned with the textbook edges in all three grade levels.

**Parallelograms**

Since classifications of special quadrilaterals are hierarchical and overlapping (i.e., a square is also a rectangle, a rhombus, and a parallelogram), to eliminate ambiguity, I define a parallelogram as a quadrilateral having two pairs of parallel sides and not a rhombus, rectangle, or square. Figures 30 and 31 show the orientations of parallelograms by publisher and grade level, respectively.

![Figure 30. Orientation of parallelograms by publisher.](image)

As shown in Figure 30, most parallelograms were printed with one pair of parallel sides horizontally aligned with the text on the top and bottom of the parallelogram across all three publishers.
In all three grade levels, most parallelograms were printed with one pair of parallel sides horizontally aligned with the text on the top and bottom of the parallelogram. Though as shown in Figure 31, there were less parallelograms represented in seventh and eighth-grade textbooks.

**Rhombi**

Since classifications of special quadrilaterals are hierarchical and overlapping (i.e., a square is also a rectangle, a rhombus, and a parallelogram), to eliminate ambiguity, I define a rhombus as a polygon having four equal sides, two pairs of parallel sides, and not a square. Figures 32 and 33 show the orientations of rhombi by publisher and grade level, respectively.
Figure 32. Orientation of rhombi by publisher.

Orientations of rhombi varied considerably by publisher. As shown in Figure 32, most rhombi in the Pearson textbook series were printed with a pair of parallel sides horizontally aligned with the text on the top and bottom of the figure. In the Houghton Mifflin Harcourt textbook series, there were an equal number of rhombi represented with a pair of parallel sides horizontally aligned with the text on the top and bottom of the figure and in other alignments that did not include any sides horizontally or vertically aligned with the textbook edges.

Figure 33. Orientation of rhombi by grade level.
As shown in Figure 33, there were no rhombi represented in seventh and eighth-grade textbooks. In sixth-grade textbooks, the most common orientation of rhombi was a pair of parallel sides horizontally aligned with the text on the top and bottom of the figure.

**Trapezoids**

Since classifications of special quadrilaterals are hierarchical and overlapping (i.e., a square is also a rectangle, a rhombus, and a parallelogram), to eliminate ambiguity, I define a trapezoid as a quadrilateral having exactly one pair of parallel sides. Figures 34 and 35 show the orientations of trapezoids by publisher and grade level, respectively.

![Figure 34. Orientation of trapezoids by publisher.](image)

Across all three publishers, the most common orientation for trapezoids was the pair of parallel sides aligned horizontally with the text on the top and bottom of the trapezoid, though as
shown in Figure 34, this is more pronounced in the Pearson and McGraw Hill Education textbook series.

As shown in Figure 35, a large majority of trapezoids at the sixth-grade level were printed with the pair of parallel sides aligned horizontally with the text on the top and bottom of the trapezoid. At the seventh-grade level there were very few trapezoids represented, and at the eight-grade level there was more variety of orientations of trapezoids, though most of the orientations included two or three sides horizontally or vertically aligned with the edges of the textbook.

**Kites**

Since classifications of special quadrilaterals are hierarchical and overlapping (i.e., a square is also a rectangle, a rhombus, and a parallelogram), to eliminate ambiguity, I define a kite
as a quadrilateral having exactly two pairs of adjacent congruent sides. Figures 36 and 37 show the orientations of kites by publisher and grade level, respectively.

Figure 36. Orientation of kites by publisher.

As shown in Figure 36, the Pearson textbook series contained many more images of kites than the Houghton Mifflin Harcourt or McGraw Hill Education textbook series. For all three textbook publishers, the most common orientation for kites was both diagonals vertically and horizontally aligned with the edges of the textbook.

Figure 37. Orientation of kites by grade level.
Most representations of kites were found at the sixth-grade level, followed by the eighth-grade level, with none at the seventh-grade level. As shown in Figure 37, all kites at the sixth-grade level were oriented with both diagonals vertically and horizontally aligned with the edges of the textbook. Most kites at the eighth-grade level were printed with both diagonals vertically and horizontally aligned with the edges of the textbook, though some were printed in other orientations.

**Pentagons**

A pentagon is a polygon with exactly five sides. Figures 38 and 39 show the orientations of pentagons by publisher and grade level, respectively.

![Figure 38](image)

*Figure 38. Orientation of pentagons by publisher.*

Representations of pentagons varied across publishers, as shown in Figure 38. The Pearson textbook series included pentagons with four sides aligned horizontally and vertically with the edges of the textbook, with one side aligned horizontally with the text at the bottom, and
other orientations. The Houghton Mifflin Harcourt series contained pentagons with four sides aligned horizontally and vertically with the edges of the textbook and other orientations. The McGraw Hill Education textbook series included a majority of pentagons with four sides aligned horizontally and vertically with the edges of the textbook.

Figure 39. Orientation of pentagons by grade level.

Orientations of pentagons also varied based on grade level. Sixth and seventh-grade textbooks included pentagons with four sides aligned horizontally and vertically with the edges of the textbook, with one side aligned horizontally with the text at the bottom, and other orientations. Eight-grade textbooks did not contain any pentagons with four sides aligned horizontally and vertically with the edges of the textbook.
Hexagons

A hexagon is a polygon with exactly six sides. Figures 40 and 41 show the orientations of hexagons by publisher and grade level, respectively.

**Figure 40.** Orientation of hexagons by publisher.

Across all three publishers, hexagons were most commonly printed with two sides horizontally aligned with the text on the top and bottom of the hexagon, as shown in Figure 40.

**Figure 41.** Orientation of hexagons by grade level.
As shown in Figure 41, hexagons were most often represented with two sides horizontally aligned with the text on the top and bottom of the hexagon in all three grade levels.

**Polygon Prototypicality**

I coded each image of a convex polygon contained in the sample of textbooks based on whether or not the image is a prototypical representation. A prototype or prototypical image is the most commonly used classical representation of a particular concept (Hasegawa, 1997; Presmeg, 1992). Typically, a prototypical image is one that contains symmetries and the longest list of special attributes (Tsamir et al., 2015). Tables 8, 9, 10, and 11 show the percent of prototypical images by publisher, grade level, location, and polygon type, respectively.

Table 8

<table>
<thead>
<tr>
<th>Publisher</th>
<th>Number of Prototypical Images</th>
<th>Number of Non-Prototypical Images</th>
<th>Percent of Prototypical Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson</td>
<td>373</td>
<td>291</td>
<td>56.2%</td>
</tr>
<tr>
<td>Houghton Mifflin Harcourt</td>
<td>143</td>
<td>171</td>
<td>45.5%</td>
</tr>
<tr>
<td>McGraw Hill Education</td>
<td>402</td>
<td>318</td>
<td>55.8%</td>
</tr>
</tbody>
</table>

As shown in Table 8, the Pearson *enVisionmath2.0* (Berry et al., 2017) and McGraw Hill Education *Glencoe Math* (Carter et al., 2015) series contained a majority of prototypical images of convex polygons with 56.2% and 55.8% respectively. Contrastingly, the Houghton Mifflin
Harcourt *Go Math!* (Burger et al., 2014) series contained slightly more non-prototypical images at 45.5%.

Table 9

*Percent of Prototypical Images by Grade Level*

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Number of Prototypical Images</th>
<th>Number of Non-Prototypical Images</th>
<th>Percent of Prototypical Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th</td>
<td>272</td>
<td>182</td>
<td>59.9%</td>
</tr>
<tr>
<td>7th</td>
<td>161</td>
<td>77</td>
<td>67.6%</td>
</tr>
<tr>
<td>8th</td>
<td>485</td>
<td>521</td>
<td>48.2%</td>
</tr>
</tbody>
</table>

Table 10

*Percent of Prototypical Images by Location*

<table>
<thead>
<tr>
<th>Location</th>
<th>Number of Prototypical Images</th>
<th>Number of Non-Prototypical Images</th>
<th>Percent of Prototypical Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson</td>
<td>304</td>
<td>246</td>
<td>55.3%</td>
</tr>
<tr>
<td>Practice</td>
<td>337</td>
<td>325</td>
<td>50.9%</td>
</tr>
<tr>
<td>Activity</td>
<td>48</td>
<td>38</td>
<td>55.8%</td>
</tr>
<tr>
<td>Review</td>
<td>77</td>
<td>48</td>
<td>61.6%</td>
</tr>
<tr>
<td>Get Ready</td>
<td>24</td>
<td>24</td>
<td>50%</td>
</tr>
<tr>
<td>Assessment Practice</td>
<td>66</td>
<td>45</td>
<td>59.5%</td>
</tr>
<tr>
<td>Assessment</td>
<td>17</td>
<td>25</td>
<td>40.5%</td>
</tr>
<tr>
<td>Performance Task</td>
<td>11</td>
<td>4</td>
<td>73.3%</td>
</tr>
<tr>
<td>Investigate</td>
<td>34</td>
<td>25</td>
<td>57.6%</td>
</tr>
</tbody>
</table>
By grade level, Table 9 shows that sixth and seventh-grade textbooks contained more prototypical images at 59.9% and 67.6% prototypical images respectively, while eighth-grade textbooks contained slightly more non-prototypical images. Seventh-grade contained the largest percentage of prototypical images, even though seventh-grade textbooks contained the fewest images of convex polygons overall.

Table 10 shows the percent of prototypical images by location of the textbooks. All locations of textbooks had a majority of prototypical images of convex polygons aside from readiness sections and assessments, which contained 50% and 40.5% prototypical images, respectively. Lessons and practice sections contained 53.3% and 50.9% prototypical images respectively, and these sections comprise the majority of total images. Performance tasks contained the largest percentage of prototypical images with 73.3%, though these sections do not make up a large portion of the total number of images of convex polygons.

As shown in Table 11, right triangles, 89.4%, squares, 79.0%, rectangles, 76.0%, parallelograms, 71.4%, rhombi, 60%, and hexagons, 66.7%, were printed in prototypical forms more often than non-prototypical forms. Octagons were 100% represented in prototypical form, however, there were only two total images of octagons printed within all three textbook series. Images of non-right triangles, 16.9%, trapezoids, 27.6%, kites, 38.1%, and pentagons, 30.1%, provided slightly more variety in terms of non-prototypical representations. General quadrilaterals are listed as having 0% prototypical images, however, there is not a single prototypical representation of general quadrilaterals so this is not unexpected.
Table 11

*Percent of Prototypical Images by Polygon Type*

<table>
<thead>
<tr>
<th>Polygon Type</th>
<th>Number of Prototypical Images</th>
<th>Number of Non-Prototypical Images</th>
<th>Percent of Prototypical Images</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Right Triangles</td>
<td>79</td>
<td>388</td>
<td>16.9%</td>
</tr>
<tr>
<td>Right Triangles</td>
<td>353</td>
<td>42</td>
<td>89.4%</td>
</tr>
<tr>
<td>General Quadrilaterals</td>
<td>0</td>
<td>56</td>
<td>0%</td>
</tr>
<tr>
<td>Squares</td>
<td>83</td>
<td>22</td>
<td>79.0%</td>
</tr>
<tr>
<td>Rectangles</td>
<td>219</td>
<td>69</td>
<td>76.0%</td>
</tr>
<tr>
<td>Parallelograms</td>
<td>95</td>
<td>38</td>
<td>71.4%</td>
</tr>
<tr>
<td>Rhombi</td>
<td>9</td>
<td>6</td>
<td>60%</td>
</tr>
<tr>
<td>Trapezoids</td>
<td>43</td>
<td>113</td>
<td>27.6%</td>
</tr>
<tr>
<td>Kites</td>
<td>8</td>
<td>13</td>
<td>38.1%</td>
</tr>
<tr>
<td>Pentagons</td>
<td>11</td>
<td>25</td>
<td>30.1%</td>
</tr>
<tr>
<td>Hexagons</td>
<td>16</td>
<td>8</td>
<td>66.7%</td>
</tr>
<tr>
<td>Octagons</td>
<td>2</td>
<td>0</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Polygon Context**

For each image of a convex polygon within the nine textbooks in the textbook sample, I coded the image based on the context. The context of the image can be either real-world or purely mathematical, depending on how the polygon is represented. Figures 42 and 43 show the percent of contexts by publisher and grade level, respectively.
As shown in Figure 42, all three textbook series contained significantly more tasks with images of convex polygons set in purely mathematical contexts as opposed to real world contexts. Pearson contained 76%, Houghton Mifflin Harcourt contained 89%, and McGraw Hill Education contained 83% of images of convex polygons set in purely mathematical settings.

As shown in Figure 43, all grade levels contained significantly more tasks with images of convex polygons set in purely mathematical contexts as opposed to real world contexts. Sixth-
grade contained 76%, seventh-grade contained 66%, and eighth-grade contained 87% of images of convex polygons set in purely mathematical settings.

**Role of the Images**

For each image of a convex polygon within the nine textbooks in the textbook sample, I coded the image based on the role of the image. The role of the image could be superfluous, illustrative, or interpretive based on an adaptation of the Hunsader et al. (2014) Mathematical Processes Assessment Coding Framework (MPAC). Figures 44 and 45 show the percent of roles of images by publisher and grade level, respectively.

![Figure 44. Role of the images by publisher.](image)

As shown in Figure 44, the majority of images of convex polygons were interpretive images, followed by illustrative images, and finally superfluous images in all three textbook series. The Pearson series contained 2% superfluous, 24% illustrative, and 74% interpretive images of convex polygons. The Houghton Mifflin Harcourt series contained 1% superfluous, 13% illustrative, and 86% interpretive images of convex polygons. The McGraw Hill Education
series contained 0% superfluous, 14% illustrative, and 86% interpretive images of convex polygons.

![Diagram showing the role of images by grade level](image)

Figure 45. Role of the images by grade level.

As shown in Figure 45, the majority of images of convex polygons were interpretive images, followed by illustrative images, and finally superfluous images in all three grade levels, similar to the results for publishers. The 6th-grade textbooks contained 2% superfluous, 19% illustrative, and 80% interpretive images of convex polygons. The seventh-grade textbooks contained 2% superfluous, 31% illustrative, and 67% interpretive images of convex polygons. The eighth-grade textbooks contained 0% superfluous, 14% illustrative, and 85% interpretive images of convex polygons.

**Developmental Responsiveness**

Young adolescents in middle school are experiencing a transition period of rapid and profound changes, and therefore have specific cognitive-intellectual, physical, psychological, social-emotional, and moral characteristics, needs, and interests (AMLE, 2012; AMTE, 2017; Caskey & Anfara, 2014; Ellerbrock & Vomvoridi-Ivanovic, 2019; NMSA, 2010). The codes for
developmental responsiveness were a simple yes/no based on if the task attends to young adolescents’ needs other than their cognitive-intellectual development.

**Physical Characteristics**

Young adolescents’ physical development occurs at different paces, but all young adolescents experience bodily changes and physical development (Smith et al., 2018). Young adolescents experience rapid development, surges of physical growth, irregular physical growth, bodily changes, awkward and uncoordinated movements, varying maturity rates, and need physical activity due to increased energy (NMSA, 2010). Tables 12 and 13 show the percent of tasks responsive to physical characteristics by publisher and grade level, respectively.

<table>
<thead>
<tr>
<th>Publisher</th>
<th>Number of Tasks Responsive to Physical Characteristics</th>
<th>Number of Tasks Responsive to Physical Characteristics</th>
<th>Percent of Tasks Responsive to Physical Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson</td>
<td>1</td>
<td>663</td>
<td>0.2%</td>
</tr>
<tr>
<td>Houghton Mifflin Harcourt</td>
<td>7</td>
<td>307</td>
<td>2.2%</td>
</tr>
<tr>
<td>McGraw Hill Education</td>
<td>18</td>
<td>702</td>
<td>2.5%</td>
</tr>
</tbody>
</table>

As shown in Table 12, tasks involving images of convex polygons that were developmentally responsive to students' physical needs were present in all three textbook series, but were by no means prevalent. The Pearson textbook series contained less than 1% of tasks developmentally responsive to young adolescents’ physical needs, and Houghton Mifflin Harcourt and McGraw Hill Education contained less than 3% tasks.
Table 13 shows the percent of tasks involving images of convex polygons that were
developmentally responsive to students' physical needs, all grade levels contained some, but not
many, of these tasks. There were a little over 3% in 6th grade, and a little less than 1% in 7th and
8th-grade of tasks involving images of convex polygons that were developmentally responsive to
students' physical needs. Figure 46 shows an example of a physically responsive task on page
280 in Houghton Mifflin Harcourt *Go Math! Grade 8* (Burger et al., 2014).

Table 13

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Number of Tasks Responsive to Physical Characteristics</th>
<th>Number of Tasks Responsive to Physical Characteristics</th>
<th>Percent of Tasks Responsive to Physical Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th</td>
<td>14</td>
<td>440</td>
<td>3.1%</td>
</tr>
<tr>
<td>7th</td>
<td>2</td>
<td>236</td>
<td>0.8%</td>
</tr>
<tr>
<td>8th</td>
<td>10</td>
<td>996</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

*Figure 46. Example of a physically responsive task.*
This task involves an image of a trapezoid and asks students to complete a set of activities to answer the questions. The activities include tracing, physically translating, sketching, using a ruler to measure, and using a protractor to measure. The activities included in the task promote coordination and motor skills. This task also allows students the opportunity for movement, albeit on a small scale.

**Psychological Characteristics**

Psychological development in young adolescents is characterized by the development of personal identity and quest for independence (Brighton, 2007; Caskey & Anfara, 2014). Young adolescents in middle school are psychologically vulnerable. Therefore, tasks should provide opportunities for students to explore identities, build self-esteem, and bond with adults in an advisory or mentor capacity (Caskey & Anfara, 2014). Unfortunately, no tasks including images of polygons within the textbook sample addressed young adolescents’ psychological needs.

**Social-Emotional Characteristics**

As young adolescents mature, they begin to focus more on their social relationships, including friendships and romantic relationships, more than their familial relationships (Smith et al., 2018). Young adolescents in middle school seek a sense of belonging in social groups, and they desire peer acceptance (NMSA, 2010). Tasks within textbooks can promote social development by incorporating opportunities for productive social interactions in an academic context (Caskey & Anfara, 2014). Tables 14 and 15 show the percent of tasks responsive to social-emotional characteristics by publisher and grade level, respectively.
As shown in Table 14, all of the tasks involving images of convex polygons that were developmentally responsive to students’ social-emotional needs were in the McGraw Hill Education *Glencoe Math Course 1, 2, 3* (Carter et al., 2015) textbook series at 6.3% of tasks. Whereas there were none in the Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017) or Houghton Mifflin Harcourt *Go Math! Grades 6, 7, 8* (Burger et al., 2014) textbook series.

Table 14

<table>
<thead>
<tr>
<th>Publisher</th>
<th>Number of Tasks Responsive to Social-Emotional Characteristics</th>
<th>Number of Tasks Responsive to Social-Emotional Characteristics</th>
<th>Percent of Tasks Responsive to Social-Emotional Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson</td>
<td>0</td>
<td>664</td>
<td>0.0%</td>
</tr>
<tr>
<td>Houghton Mifflin Harcourt</td>
<td>0</td>
<td>314</td>
<td>0.0%</td>
</tr>
<tr>
<td>McGraw Hill Education</td>
<td>45</td>
<td>675</td>
<td>6.3%</td>
</tr>
</tbody>
</table>

Table 15 shows that tasks involving images of convex polygons that were developmentally responsive to students’ social-emotional needs were present in all three grade levels, though there was only a single task in seventh-grade and these types of tasks were not common in the sixth and eighth-grade textbooks either. In 6th-grade textbooks, there were 5.9% of tasks involving images of convex polygons responsive to students’ social-emotional needs. Figure 47 shows an example of a social-emotionally responsive task from page 684 in McGraw Hill Education *Glencoe Math Course 1* (Carter et al., 2015).
Table 15
Percent of Tasks Responsive to Social-Emotional Characteristics by Grade Level

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Number of Tasks Responsive to Social-Emotional Characteristics</th>
<th>Number of Tasks Responsive to Social-Emotional Characteristics</th>
<th>Percent of Tasks Responsive to Social-Emotional Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th</td>
<td>27</td>
<td>427</td>
<td>5.9%</td>
</tr>
<tr>
<td>7th</td>
<td>1</td>
<td>237</td>
<td>0.4%</td>
</tr>
<tr>
<td>8th</td>
<td>17</td>
<td>989</td>
<td>1.7%</td>
</tr>
</tbody>
</table>

Figure 47. Example of a social-emotionally responsive task.

This task involves images of parallelograms and asks students to complete a classification task with a partner. This task promotes mathematical communication and discussion between students and allows for productive, academic socialization. Additionally, these types of partner tasks can also provide students the opportunity to create social relationships.
Moral Characteristics

During young adolescence, students develop their values and moral codes (NMSA, 2010). During this time, young adolescents also begin to form their identities and sense of self (NMSA, 2010). Young adolescents in middle school begin to see ambiguity in moral situations rather than black/white or good/evil (Brighton, 2007; Caskey & Anfara, 2014). Tasks can utilize this increased interest in moral and ethical questions by providing opportunities for higher-order thinking and reasoning about real-world concepts and situations (Caskey & Anfara, 2014). Unfortunately, no tasks including images of polygons within the textbook sample addressed young adolescents’ moral needs.

Summary of Results

In this chapter, I presented the results of this study in terms of polygon type, orientation, prototypicality, context, role of the image, and developmental responsiveness across grade level, textbook publisher, and location within each textbook. The results of this study indicated that within three published middle school mathematics textbook series, students do not have exposure to a variety of representations of convex polygons. In all three textbook series, students have more exposure to images of convex polygons in eighth-grade textbooks, followed by sixth-grade textbooks, and the finally seventh-grade textbooks, however, all three textbook series contained little variety in terms of types of convex polygons represented. The three textbook series did not provide ample opportunities for students to engage with images of convex polygons in a variety of orientations and did not provide many opportunities for students to engage with non-prototypical images of convex polygons. Additionally, the three textbook series did not provide much variety in terms of contexts and roles of images with a majority of convex polygons set in
purely mathematical contexts and as interpretive images. Further, all three textbook series provided little to no developmentally responsive tasks containing images of convex polygons in terms of young adolescents' physical, psychological, social-emotional, and moral needs.
CHAPTER 5: CONCLUSIONS

Young adolescents build notions of figures through experiences. When young adolescents begin middle school mathematics, they already have prior assumptions and conceptualizations about shapes (Herbst et al., 2017). Over years of experiences, interactions, and exposures with a particular concept, these students develop concept images consisting of all the visual information, pictures, mental images, or properties associated with the particular mathematical concept (Vinner, 1983). Seminal works in the field of mathematics education research found that without an opportunity to engage with varied representations, young adolescent students may not develop robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).

Therefore, in this study, I sought to analyze young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks (grades 6-8). I chose to analyze convex polygons in nine textbooks: Pearson enVisionmath2.0 Grades 6, 7, 8 (Berry et al., 2017), Houghton Mifflin Harcourt Go Math! Grades 6, 7, 8 (Burger et al., 2014), and McGraw Hill Education Glencoe Math Course 1, 2, 3 (Carter et al., 2015). This sample of textbooks represents one series from each of the three major textbook publishers: Pearson, Houghton Mifflin Harcourt, McGraw Hill Education (Banilower et al., 2018).

This chapter presents a discussion of the results in relation to the overarching research question and the specific sub-questions. The overarching research question addressed this study
was as follows: Within three published middle school mathematics textbook series, in what ways do young adolescents have opportunities to develop concept images of polygons?

1. In what ways do tasks within the textbooks involving visual representations of convex polygons attend to young adolescents’ cognitive-intellectual characteristics? In particular:
   a. How are images of convex polygons oriented in the textbooks?
   b. How are right angles oriented within images of right triangles in the textbooks?
   c. What percentage of convex polygons in the textbooks are prototypical images, in other words, the most common representation of a particular convex polygon?
   d. In what contexts (i.e., real-world or purely mathematical) are convex polygons represented within the textbooks?
   e. What is the role of the images (i.e., superfluous, illustrative, or interpretive) representing convex polygons within the textbooks?

2. To what extent do tasks within the textbooks involving visual representations of convex polygons attend to other developmental characteristics of young adolescents? In particular:
   a. To what extent do tasks involving visual representations of convex polygons address young adolescents’ physical characteristics?
   b. To what extent do tasks involving visual representations of convex polygons address young adolescents’ psychological characteristics?
   c. To what extent do tasks involving visual representations of convex polygons address young adolescents’ social-emotional characteristics?
d. To what extent do tasks involving visual representations of convex polygons address young adolescents’ moral characteristics?

The remainder of this chapter includes a discussion of results in relation to the research questions listed above, significance of the study, implications, limitations of the study, recommendations for future research, and a summary of conclusions.

Discussion

In the United States, only only 34% of eighth-grade students tested at a level of proficient or above in mathematics (National Center for Educational Statistics [NCES], National Assessment of Educational Progress [NAEP], 2019). Eighth-grade U.S. students are underachieving in geometry (Mullis et al., 2008; Mullis et al., 2012; Mullis et al., 2016) and eighth-graders in the United States scored lower in the content domain of Geometry than fourth-graders (Mullis et al., 2016). Unfortunately, in the U.S., middle school students are not developing an adequate understanding of geometry or visuospatial reasoning (Battista, 1999).

Young adolescents, 10-15 years of age, are typically enrolled in middle school consisting of grades 6-8 (although other grade level configurations do exist). During early adolescence while in middle school, many young adolescents are developing their cognitive abilities and experiencing a transition in mathematics learning from primarily concrete thinking to abstract understanding (Caskey & Anfara, 2014). This means young adolescents are beginning to problem solve without the need for physical or tangible visual representations provided for them and are instead starting to internally visualize and reason (AMTE, 2017). However, this transition from concrete to abstract thinking is not universal as some students can experience cognitive development very differently based on areas of expertise, interest, and culture (Chi et
al., 1989; Saxe, 2015; Smagorinsky, 2013). As students develop cognitively, they also progress along geometric learning trajectories (Clark, 2012; Van de Walle et al., 2016). Figures and visual images mediate this transition (Herbst et al., 2017).

In middle school, the National Council of Teachers of Mathematics (NCTM) suggests young adolescents should engage in activities that develop their spatial skills in one, two, and three dimensions such as composing shapes, creating nets, finding area, perimeter, surface area, and volume of two and three-dimensional figures, and identifying, classifying and defining shapes (1989, 2000). Further, NCTM’s Curricular Focal Points for Pre-K through Grade 8 emphasize composing and decomposing, solving problems area and volume through compositions, similarity, and analyzing spaces using distance and angle measure (Fennell, 2006). Additionally, the creators of the Common Core State Standards have emphasized solving real-world and mathematically contextualized problems angle measure, involving area, surface area, and volume, constructing and describing geometric figures, understanding and applying the Pythagorean Theorem, and understanding congruence and similarity through tools like models and dynamic geometry software (NGACBP, CCSSO, 2010). More recently, NCTM (2014) advocated for the inclusion of more mathematical representations including visual, contextual, and physical representations in mathematics teaching and learning.

If mathematical representations of polygons are not sufficiently varied, students may develop inaccurate concept images of these polygons (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Textbooks often represent the intended curriculum (Lloyd et al., 2016; Stanic & Kilpatrick, 2003). The textbook used in a mathematics classroom can impact student learning
opportunity to learn is defined as the encounters, occasions, experiences, and engagement students have with content, teaching and learning strategies, and particular topics (Hadar, 2018; Hiebert & Grouws, 2007; Houang & Schmidt, 2008; Törnroos, 2005), and therefore, textbooks can impact young adolescents’ opportunity to learn mathematics topics. Hence, the images of polygons in middle school mathematics textbooks influence young adolescents’ development of concept images of polygons (Vinner, 1983).

In this study, I used content analysis to explore the tasks involving images of convex polygons in nine textbooks: Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017), Houghton Mifflin Harcourt *Go Math! Grades 6, 7, 8* (Burger et al., 2014), and McGraw Hill Education *Glencoe Math Course 1, 2, 3* (Carter et al., 2015). This section will discuss young adolescent’s opportunity to develop concept images of polygons in middle school mathematics textbooks in terms of the exposure to a variety of polygons represented, exposure to a variety of polygon orientations, exposure to non-prototypical images, exposure to a variety of contexts, exposure to a variety of image roles, and exposure to developmentally responsive tasks. Without an opportunity to engage with varied representations, young adolescent students may not develop robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).

**Exposure to a Variety of Polygons Represented**

Pearson *enVisionmath2.0 Grade 6* (Berry et al., 2017) contained 507 total pages and one topic devoted to geometry. Pearson *enVisionmath2.0 Grade 7* (Berry et al., 2017) contained 483 total pages and one topic devoted to geometry. Pearson *enVisionmath2.0 Grade 8* (Berry et al.,
2017) contained 451 total pages and three topics devoted to geometry. In total, the Pearson series contained little variety of types of polygons with most of the images of convex polygons being non-right and right triangles and rectangles. The Pearson series contained no octagons, almost no hexagons and rhombi, and very few pentagons, kites, and general quadrilaterals.

Houghton Mifflin Harcourt *Go Math! Grade 6* (Burger et al., 2014) contained 488 total pages and three modules devoted to geometry. Houghton Mifflin Harcourt *Go Math! Grade 7* (Burger et al., 2014) contained 430 total pages and two modules devoted to geometry. Houghton Mifflin Harcourt *Go Math! Grade 8* (Burger et al., 2014) contained 472 total pages and five modules devoted to geometry. In total, the Houghton Mifflin Harcourt series contained little variety in terms of types of polygons with most images of convex polygons being non-right and right triangles. The Houghton Mifflin Harcourt series contained no octagons, almost no hexagons, pentagons, kites, and rhombi, and very few general quadrilaterals, squares, and parallelograms.

McGraw Hill Education *Glencoe Math Course 1* (Carter et al., 2015) contained 926 total pages and two chapters devoted to geometry. McGraw Hill Education *Glencoe Math Course 2* (Carter et al., 2015) contained 854 total pages and two chapters devoted to geometry. McGraw Hill Education *Glencoe Math Course 3* (Carter et al., 2015) contained 732 total pages and four chapters devoted to geometry. In total, the McGraw Hill Education series contained little variety in terms of types of polygons with most images of convex polygons being non-right and right triangles. The McGraw Hill Education series contained almost no rhombi, kites, and octagons, and very few hexagons, pentagons, and general quadrilaterals.
For each textbook series, the eighth-grade textbook contained the highest number of dedicated topics, modules, or chapters on geometry and the highest number of images of polygons. There were 1,006 total images of polygons in the eight-grade textbooks. For each textbook series, the sixth-grade textbook contained the second highest number of images of polygons, 454 total, while seventh-grade contained the least, 238 total. The Pearson enVisionmath2.0 (Berry et al., 2017) series contained a similar number of pages at each grade level as the Houghton Mifflin Harcourt Go Math! (Burger et al., 2014) series, while the McGraw Hill Education Glencoe Math (Carter et al., 2015) series contained significantly more pages overall, nearly double the number of pages in the Pearson and Houghton Mifflin Harcourt textbooks. However, the Pearson series and the Houghton Mifflin Harcourt series did not have a similar number of images of polygons. The Houghton Mifflin Harcourt series contained 314 images of convex polygons, while the Pearson and McGraw Hill Education contained 664 and 720 images of polygons respectively. This means the Pearson textbook series contained the largest number of images of convex polygons per number of pages.

The Pearson enVisionmath2.0 (Berry et al., 2017) and McGraw Hill Education Glencoe Math (Carter et al., 2015) series contained almost double the number of images of convex polygons when compared to the Houghton Mifflin Harcourt Go Math! (Burger et al., 2014). In all three textbook series, students have more exposure to images of convex polygons in eighth-grade textbooks, followed by sixth-grade textbooks, and the finally seventh-grade textbooks. Students using these textbook series would have the most exposure to images of non-right and right triangles and the least exposure to general quadrilaterals, rhombi, kites, pentagons, hexagons, and octagons.
Overall, all three textbook series contained little variety in terms of types of convex polygons represented. Young adolescents develop concept images consisting of all the visual information, pictures, mental images, or properties associated with the particular mathematical concept over years of experiences, interactions, and exposures with a particular concept (Vinner, 1983). Students enter middle school mathematics classrooms as students with prior notions and conceptions of figures based on their previous experiences (Herbst, Fujita, Halverscheid, & Weiss, 2017). In middle school mathematics classrooms, it is crucial that students are exposed to a variety of visual representations of mathematics concepts so they can develop robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).

**Exposure to a Variety of Polygon Orientations**

Across all three textbook series, polygons were printed with a lack of variety in orientations. Non-right triangles were printed most often horizontally aligned with the text on the bottom of the image, especially in sixth and seventh-grade textbooks. In all textbook series and grade levels, right triangles were almost always printed with the perpendicular sides aligned horizontally and vertically with the sides of the textbook pages. General quadrilaterals were printed with a variety of orientations across publisher and grade levels. In all textbook series and grade levels, squares were almost always printed with all four sides aligned horizontally and vertically with the sides of the textbook pages, though in eighth-grade textbooks a small percentage of squares were printed with the diagonals horizontally and vertically with the sides of the textbook pages instead. Rectangles in all three textbook series were similarly represented with all four sides aligned horizontally and vertically with the sides of the textbook pages. In all
three textbook series and all grade levels, parallelograms were printed with one of the pairs of parallel sides horizontally aligned with the text nearly 100% of the time, similarly for rhombi. The majority of trapezoids in all three textbook series and grade levels were printed with the pair of parallel sides horizontally aligned with the text. Kites were represented almost 100% of the time with both diagonals horizontally and vertically with the sides of the textbook pages in all three textbook series and grade levels.

Overall, the three textbook series included in this study did not provide ample opportunities for students to engage with images of convex polygons in a variety of orientations. Mathematics teaching should include varied visual representations (NCTM, 2014). Research on young adolescent development states that middle school is a pivotal time for cognitive development (Caskey & Anfara, 2014; NMSA, 2010). In mathematics, middle school represents the period when many students transition from concrete representations of mathematical concepts to abstract thinking and classification (AMTE, 2017; Caskey & Anfara, 2014; Smith et al., 2018). This cognitive development can vary significantly from student to student among ages, grades, and subjects (Caskey & Anfara, 2014) and some students can experience cognitive development very differently based on areas of expertise, interest, and culture (Chi et al., 1989; Saxe, 2015; Smagorinsky, 2013). As students transition from concrete to abstract thinking, they also progress along geometric learning trajectories (Clark, 2012; Van de Walle et al., 2016). Figures and visual images mediate this transition (Herbst et al., 2017). Thus, if mathematical representations of polygons are not sufficiently varied, students may develop inaccurate concept images of these polygons (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).
Exposure to Non-Prototypical Images

The Pearson *enVisionmath2.0* (Berry et al., 2017) and McGraw Hill Education *Glencoe Math* (Carter et al., 2015) series contained a majority of prototypical images of convex polygons, while the Houghton Mifflin Harcourt *Go Math!* (Burger et al., 2014) series contained slightly more non-prototypical images. By grade level, sixth and seventh-grade textbooks contained more prototypical images while eighth-grade textbooks contained slightly more non-prototypical images. Right triangles, squares, rectangles, parallelograms, rhombi, hexagons, and octagons were printed in prototypical forms more often than non-prototypical forms. Images of non-right triangles, general quadrilaterals, trapezoids, kites, and pentagons provided slightly more variety in terms of non-prototypical representations.

Overall, the three textbook series did not provide many opportunities for students to engage with non-prototypical images of convex polygons, though for certain types of polygons, more variety in prototypicality was present. A prototype or prototypical image is the most commonly used classical representation of a particular concept (Hasegawa, 1997; Presmeg, 1992). Typically, a prototypical image is one that contains symmetries and the longest list of special attributes (Tsamir et al., 2015). If textbooks primarily print prototypical and ‘common’ representations with little variety, students may struggle with issues of one-case correctness, where a student incorrectly applies a property that holds in a particular case to other inappropriate cases, generalization or inflexible thinking, where a student cannot visualize an atypical image that fits the definition of a category of images, uncontrollable images, where a student visualizes mental imagery inappropriate for the context, or compartmentalization, where a student develops a concept image in conflict with the concept definition (Aspinwall et al.,)
Exposure to a Variety of Contexts

All three textbook series contained significantly more tasks with images of convex polygons set in purely mathematical contexts as opposed to real world contexts. Additionally, at every grade level the majority of tasks involving images of convex polygons were set in purely mathematical settings. Overall, the three textbook series did not provide many opportunities for students to engage in problems involving images of convex polygons in real world settings. Young adolescents in middle school can benefit from concrete experiential learning contextualized in real world settings (NMSA, 1995). In an effort to increase young adolescent students’ visuality and spatial abilities, middle school mathematics teachers must use appropriate visual representations set in real-world and mathematical contexts (NCTM, 2014). The creators of the Common Core State Standards have also emphasized solving real-world and mathematically contextualized problems angle measure, involving area, surface area, and volume, constructing and describing geometric figures, understanding and applying the Pythagorean Theorem, and understanding congruence and similarity through tools like models and dynamic geometry software (NGACBP, CCSSO, 2010).

Exposure to a Variety of Image Roles

The majority of images of convex polygons were interpretive images, followed by illustrative images, and finally superfluous images in all three textbook series. Results were similar by grade level. This meant that most of the representations of convex polygons within the
textbook series required that students to interpret visual material present in the image to solve problems or understand concepts.

Overall, the textbook series did not provide an opportunity for students to engage in tasks with variation in the roles of images of convex polygons; a majority of tasks involving convex polygons contained interpretive images. In this study, the codes for the role of the image could be superfluous, illustrative, or interpretive based on an adaption of the Hunsader et al. (2014) Mathematical Processes Assessment Coding Framework (MPAC). Superfluous images are images that are mathematically irrelevant, not necessary, and do not require any interpretation. Illustrative images either illustrate a mathematical concept or are images required for solving a task, but the properties of the image are not represented visually, do not need to be interpreted, and are instead stated within the text. Interpretive images either illustrate a mathematical concept or are images required for solving a task, and properties of the figure are conveyed through the image and require interpretation. Mathematics teaching should include varied visual representations (NCTM, 2014) so that students can develop robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Parzysz, 1988; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989).

**Exposure to Developmentally Responsive Polygon Tasks**

Tasks involving images of convex polygons that were developmentally responsive to students' physical needs were present in all three textbook series and across all three grade levels, but were by no means prevalent. Tasks involving images of convex polygons that were developmentally responsive to students’ social-emotional needs were present in all three grade levels, though there was only a single task in seventh-grade and these types of tasks were not
common in the sixth and eighth-grade textbooks either. All of the tasks involving images of convex polygons that were developmentally responsive to students’ social-emotional needs were in the McGraw Hill Education *Glencoe Math Course 1, 2, 3* (Carter et al., 2015) textbook series, with none in the Pearson *enVisionmath2.0 Grades 6, 7, 8* (Berry et al., 2017) or Houghton Mifflin Harcourt *Go Math! Grades 6, 7, 8* (Burger et al., 2014) textbook series. Across all three textbook series, there were no tasks involving images of convex polygons that were developmentally responsive to students’ psychological or moral needs.

Overall, there was a clear lack of developmentally responsive tasks involving images of convex polygons across all three publishers and grade levels. The goal of middle schools need to provide a developmentally supportive environment for young adolescent learners (Ellerbrock, Falbe, & Franz, 2018). Responsive middle level mathematics teaching facilitates students’ cognitive development and explicitly attends to young adolescents’ needs, interests, and characteristics including their physical, psychological, social-emotional, and moral development (Ellerbrock & Vomvoridi-Ivanovic, 2019). Young adolescents in middle school have specific developmental needs, so it is important for teachers to have access to developmentally appropriate materials (AMLE, 2012; AMTE, 2017; Caskey & Anfara, 2014; NMSA, 2010); meaning the tasks including representations of polygons should also address young adolescents' physical, cognitive-intellectual, psychological, social-emotional, and moral characteristics (Ellerbrock & Vomvoridi-Ivanovic, 2019).

**Summary of Discussion**

The results of this study indicated that within three published middle school mathematics textbook series, students do not have adequate opportunities to develop robust concept images of
polygons. In all three textbook series, students have more exposure to images of convex polygons in eighth-grade textbooks, followed by sixth-grade textbooks, and the finally seventh-grade textbooks, however, all three textbook series contained little variety in terms of types of convex polygons represented. The three textbook series did not provide ample opportunities for students to engage with images of convex polygons in a variety of orientations and did not provide many opportunities for students to engage with non-prototypical images of convex polygons. Additionally, the three textbook series did not provide much variety in terms of contexts and roles of images with a majority of convex polygons set in purely mathematical contexts and as interpretive images. Further, all three textbook series provided little to no developmentally responsive tasks containing images of convex polygons in terms of young adolescents' physical, psychological, social-emotional, and moral needs.

Students can only develop robust concept images if they are provided the opportunity to engage with a variety of representations (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Since textbooks often represent the intended curriculum (Lloyd et al., 2016; Stanic & Kilpatrick, 2003), the textbook used in a mathematics classroom can impact student learning (Hadar, 2018; Houang & Schmidt, 2008; Nicol & Crespo, 2006; Törnroos, 2005). Students’ opportunity to learn is defined as the encounters, occasions, experiences, and engagement students have with content, teaching and learning strategies, and particular topics (Hadar, 2018; Hiebert & Grouws, 2007; Houang & Schmidt, 2008; Törnroos, 2005), and therefore, in these three middle school mathematics textbooks, young adolescents have limited opportunities to develop robust concept images.
Significance of Study

Geometry and visuospatial reasoning are critical to a well-rounded education (Clark, 2012; Van de Walle et al., 2016). However, in the United States (U.S.), students are often underperforming in the geometry domain (Mullis et al., 1997; Mullis et al., 2000; Mullis et al., 2008; Mullis et al., 2012; Mullis et al., 2016). Middle school students often lack geometric reasoning and visuospatial skills required to be successful in geometry (Battista, 1999). In middle school, young adolescent students learn about shapes and properties (Van de Walle et al., 2016). As middle school students transition from concrete visual representations of these shapes to abstract categorizations and descriptions (AMTE, 2017; Caskey & Anfara, 2014), it is vital that young adolescent students develop robust concept images of polygons encompassing all of the properties and definitions that are relevant to the shape (Vinner, 1983).

NCTM (2014) recommends that teachers use appropriate visual representations set in real-world and mathematical contexts to increase students’ visual and spatial abilities. Many teachers rely heavily on the classroom textbooks for instruction (Lloyd et al., 2016; Stanic & Kilpatrick, 2003), so the representations of convex polygons in middle school mathematics textbooks may constitute a young adolescent students’ opportunity to develop a concept image. Additionally, young adolescents in middle school have specific developmental needs, so it is important for teachers to have access to developmentally appropriate materials (AMLE, 2012; AMTE, 2017; Caskey & Anfara, 2014; NMSA, 2010); meaning the tasks including representations of polygons should also address young adolescents' physical, cognitive-intellectual, psychological, social-emotional, and moral characteristics (Ellerbrock & Vomvoridi-Ivanovic, 2019). Teachers could supplement the images in the textbook with other mathematical
representations, however Gutierrez and Jaime (1999) found that preservice math teachers had poor concept images of the altitude of a triangle and Cunningham and Roberts (2010) found that preservice math teachers had weak conceptual understanding of geometry concepts even when they memorized definitions. Gutierrez and Jaime’s (1999) findings underline the importance of developmentally responsive curriculum for young adolescents in middle school mathematics programs. The tasks and images in middle school mathematics textbooks should be appropriate for young adolescents and promote the development of robust concept images, even if teachers are unable to supplement resources.

Previous research on textbooks in the U.S. and internationally has focused on historical changes in textbooks, treatment of specific content areas, and comparative analysis between U.S. and international textbooks. Baker and colleagues (2010) used content analysis to understand changes in elementary mathematics textbooks over a century. Jones and Tarr (2007) also examined mathematics textbooks from a historical perspective. The researchers compared popular and alternative middle school textbook series from historical periods noting the cognitive demand of probability and statistics tasks (Jones & Tarr, 2007). Similarly, Arnold and Son (2011) analyzed the pre-algebra content in popular and alternative textbooks over time specifically examining the contexts and cognitive demand of tasks.

Other researchers have focused on current U.S. textbooks and specific content areas or topics. Huntley and Terrell (2014) compared popular and alternative mathematics textbooks and their coverage of linear equations in relation to content, context, cognitive demand, and tools (Huntley & Terrell, 2014). Polikoff (2015) studied current textbooks as well, assessing the alignment of textbooks to Common Core State Standards Recommendations. Hunsader and

Several studies have compared U.S. textbooks to those of other countries. Alajmi (2012) contrasted the treatment of fractions in elementary textbooks from the U.S., Kuwait, and Japan. Li (2000) compared U.S. and Chinese textbooks and their treatment of integer problems. Incikabi and Tjoe (2013) similarly compared U.S. and Turkish middle school math textbooks, specifically, the context and cognitive demand of ratio and proportion problems. Li (2007) completed a study focusing on eighth-grade textbooks from the U.S., China, and Singapore to compare the cognitive demand of mathematical tasks. Hong and Choi (2014) also focused on eighth-grade textbooks, and compared the context. They compared cognitive demand of algebra tasks in textbooks from the U.S. and Korea., then more recently, Hong and Choi (2018) compared cognitive demand of tasks in secondary U.S. and Korean textbooks. Ponte and Marques (2011) similarly compared textbooks from multiple countries examining the cognitive demand, context, and structure of sixth-grade math textbooks from the U.S., Portugal, Spain, and Brazil.

However, I was unable to find any research focusing on young adolescents' opportunity to develop concept images of polygons in U.S. middle school mathematics textbooks. This study adds to the current body of literature by illuminating the visual representations of convex
polygons in three popular U.S. middle school mathematics textbook series and the developmental responsiveness of the tasks containing images of polygons. For young adolescent students to develop a robust concept image of a polygon, they need frequent and varied exposure to images of polygons (Aspinwall et al. 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Therefore, it is important to understand, categorize, analyze, and document the images of polygons in middle school mathematics textbooks. By completing a content analysis of the tasks involving images of convex polygons in middle school mathematics textbooks, this study will contribute to understanding young adolescents’ opportunities to develop robust concept images. The goal of this study is to inform the research community and education stakeholders of the opportunities to develop concept images of polygons in popular middle school mathematics textbooks. Hopefully, the implications will encourage textbook creators to consider the impact of visual images in mathematics textbooks and teachers to provide a variety of high-quality mathematical representations in the classroom.

Implications for Math Education

Textbooks have transitioned into longer books with more pervasive visual images over the last century (Aspinwall et al., 1997; Baker et al., 2010; Dimmel & Herbst, 2015; Stanic & Kilpatrick, 2003). Because teachers rely heavily on textbooks for designing courses, sequencing, and providing examples and practice (Banilower et al., 2018; Fan, 2013; Fan et al., 2013; Lloyd et al. 2016; Nicol & Crespo, 2006; Stanic & Kilpatrick, 2003; Reys et al., 2004), the images of polygons in middle school mathematics textbooks influence students’ development of concept images polygons (Vinner, 1983). If textbooks primarily print prototypical and ‘common’
representations with little variety, students may struggle with issues of one-case correctness, where a student incorrectly applies a property that holds in a particular case to other inappropriate cases, generalization or inflexible thinking, where a student cannot visualize an atypical image that fits the definition of a category of images, uncontrollable images, where a student visualizes mental imagery inappropriate for the context, or compartmentalization, where a student develops a concept image in conflict with the concept definition (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Parzysz, 1988; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). This study aims to inform the mathematics education community of the opportunities to develop concept images in middle school mathematics textbooks.

The results of this study show a limited exposure to a variety of types of polygons, limited exposure to a variety of orientations of polygons, limited exposure to non-prototypical images, limited exposure to a variety of contexts, limited exposure to a variety of image roles, and limited exposure to developmentally responsive tasks involving images of convex polygons. Seminal works in the field of mathematics education research found that without an opportunity to engage with varied representations, young adolescent students may not develop robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). If textbooks are not providing varied representation of visual imagery, then it is crucial that mathematics teachers are able to supplement the images, however, Gutierrez and Jaime (1999) found that preservice math teachers had poor concept images of the altitude of a triangle and Cunningham and Roberts
(2010) found that preservice math teachers had weak conceptual understanding of geometry concepts even when they memorized definitions.

The implications of this study are three-fold. First, textbook authors should aim to include more varied imagery within all sections of printed textbooks. Second, teacher preparation programs should be aware of preservice math teachers’ potential misunderstandings of concept images of polygons. And third, middle school mathematics teachers should plan on supplementing the textbook with additional images, tasks, and activities. The results of this study have wide implications for the entire math education community and stakeholders.

The results of this study show that three middle school mathematics series from the top three publishers in the U.S. contained limited variety of types of polygons, limited of orientations of polygons, limited non-prototypical images, limited variety of contexts, limited variety of image roles, and limited developmentally responsive tasks involving images of convex polygons. Textbook publishers should familiarize themselves with the results of this study and strive to provide diversification of polygon images and orientations, as well as a plethora of non-prototypical images. The tasks included in textbooks should include appropriate visual representations set in real-world and mathematical contexts to increase students’ visual and spatial abilities. (NCTM, 2014). Additionally, young adolescent students have physical, psychological, social-emotional, and moral characteristics, needs, and interests (AMLE, 2012; AMTE, 2017; Caskey & Anfara, 2014; Ellerbrock & Vomvoridi-Ivanovic, 2019; NMSA, 2010) and textbooks should include tasks are developmentally responsive.

If textbooks are not providing varied representation of visual imagery, then it is crucial that mathematics teachers are able to supplement the images, however, many preservice math
teachers have poor concept images of the altitude of a triangle and weak conceptual understanding of geometry concepts even when they memorized definitions (Cunningham & Roberts, 2010; Guitierrez & Jaime, 1999). This implies that mathematics teacher educators should highlight conceptual understanding and the formation of robust concept images in mathematics teacher preparation programs. Additionally, middle school teacher preparation programs should include a focus on developmentally responsive teaching to train preservice teachers on how to develop and implement developmentally appropriate tasks. It is vital that middle school mathematics teachers are able to supplement the mathematics textbooks with varied visual representations of polygons and developmentally responsive tasks. Mathematics teacher educators and middle school teacher preparation programs should provide specialized courses on middle school education and young adolescent development, scaffolded clinical experiences at each grade level within middle schools, content specific courses with advanced experiences with elementary topics, and collaborative assignments to link content and pedagogy to prepare future middle school teachers to provide a developmentally responsive education for young adolescent students (Ellerbrock et al., 2016; Ellerbrock et al., 2018).

Without an opportunity to engage with varied representations, young adolescent students may not develop robust concept images (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Therefore, based on the results of this study, middle school math teachers should provide supplemental experiences, activities, and visual images in the classroom. Teachers can incorporate more opportunities to engage in varied representations by providing activities that incorporate manipulatives and technology. Physical manipulatives, for example, origami, can be as effective
as traditional instruction (Boakes, 2009; Carbonneau et al., 2013), and virtual manipulatives can offer students concrete experiences with mathematical concepts (Durmus & Karakirik, 2006). Dynamic geometry software, such as GeoGebra or Cabri 3D, can be more effective than both manipulatives and traditional instruction in increasing students’ visuality (Baki, Kosa, & Guven, 2011; Chan & Leung, 2015; Diković, 2009; Güven & Temel, 2008; Leung, 2008). Furthermore, specific training in visuospatial methods can raise students’ spatial skills and lead to success in geometry (Battista et al., 1982; Ben-Haim et al., 1985; Cheng & Mix, 2009). Cochran, Cochran, Laney, and Dean (2016) developed a curriculum for 4th and 7th-graders using affordable 3D printers and computer-aided design (CAD) software. Through the lessons using the 3D printer and CAD software, students explored the relationship between 2D and 3D shapes, volume, rotations of figures, bases and sides, surface area, cross-sections, and measurement (Cochran et al., 2016). Similarly, Huleihil (2017) found that 6th-grade students who learned geometric concepts through the use of 3D printers and CAD software increased their mathematical achievement.

Students’ opportunity to learn is defined as the encounters, occasions, experiences, and engagement students have with content, teaching and learning strategies, and particular topics (Hadar, 2018; Hiebert & Grouws, 2007; Houang & Schmidt, 2008; Törnroos, 2005) and similarly, over years of experiences, interactions, and exposures with a particular concept, students develop concept images consisting of all the visual information, pictures, mental images, or properties associated with the particular mathematical concept (Vinner, 1983). Because teachers rely on textbooks for examples and practice (Banilower et al., 2018; Fan, 2013; Fan et al., 2013; Lloyd et al. 2016; Nicol & Crespo, 2006; Stanic & Kilpatrick, 2003; Reys et al.,
results of this study have implications for textbook authors, teacher educators, and teachers in the math education community.

Limitations of Study

There are two main limitations of this study, textbook sample and data sources. For the textbook sample, I chose to only include middle school textbooks in the sample because young adolescence is a period of transition between students’ moving from concrete understandings of figures to abstract representations (AMTE, 2017; Caskey & Anfara, 2007). Additionally, I included textbooks in the sample based on market share of publishers, meaning while the sample is representative of the majority of textbooks used in middle school math classrooms, it is not exhaustive. The textbook sample also only included physical textbooks and no other virtual or physical curricular materials. I examined only the student edition of the textbooks and did not compare to the teacher editions. Therefore, the results may have been different had I included other textbooks or curricular materials in the sample. For the data sources in this study, I examined images of convex polygons presented within each of the nine textbooks in the sample; the results may have shown more variety in figures had I examined other types of images, such as linear graphs, representations of slope, or properties of circles. Also, I examined the tasks including images of convex polygons in terms of their developmental responsiveness but did not investigate geometry tasks without images. The results may have included more developmentally responsive geometry tasks if I included tasks without images.

Recommendations for Future Research

In this study, I sought to analyze young adolescents’ opportunities to develop concept images of polygons in middle school mathematics textbooks (grades 6-8). The results reveal that
there are limited opportunities for young adolescents to develop robust concept images due to the lack of variety in exposure to a variety of polygons represented, exposure to a variety of polygon orientations, exposure to non-prototypical images, exposure to a variety of contexts, exposure to a variety of image roles, and exposure to developmentally responsive tasks. A natural extension of this study would be to examine the other opportunities young adolescents have to develop concept images of polygons in middle school. One possible method to study students' opportunity to develop concept images of polygons would be to study the enacted curriculum of polygon lessons. For example, a study on whether or not teachers supplement the lessons from the textbook with additional images of convex polygons that include more variety could illuminate students’ opportunity to develop concept images of polygons in the classroom. Additionally, teachers may incorporate developmentally appropriate tasks related to polygons into the classroom even though they are not included in the textbook series. The enacted curriculum in classrooms does not necessarily match the intended curriculum found in textbooks so this area of study could clarify students’ opportunities to develop concept images of polygons in middle school math classrooms.

Another avenue of research would be to analyze the opportunities in all curricular materials for a given textbook series, including the dynamic online materials, practice materials, and teachers’ edition of textbooks. Many textbook series include online materials that allow for movement of and interaction with figures. Online curricular materials could link to dynamic geometry software and allow students to explore what properties of shapes are preserved through movement. Dynamic geometry software, such as GeoGebra or Cabri 3D, can be more effective than both manipulatives and traditional instruction in increasing students’ visuality (Baki, Kosa,
Practice materials for some textbook series may contain more images or topic explorations. Additionally, analyzing the teachers’ edition of textbooks may provide insight into the support that the curricular materials provide in terms enacting the curriculum.

Another natural extension of this study would be to expand the textbook sample. This study focused on the top three publishing companies in the U.S., however, this sample is not exhaustive and does not represent all opportunities of all U.S. middle school students. A similar study could include some middle school textbooks that treat geometry as a stand-alone subject to investigate whether the treatment of polygons is different in a course that is solely focused on geometric topics. An additional possible exploration could compare the images of polygons in free, online, open source middle school math textbooks, less conventional middle school textbooks, textbooks from less popular publishers, or less widely-used textbook series from the three top U.S. textbook publishers. This may provide a better understanding of the opportunities to develop concept images in middle school mathematics textbooks across the entire country.

In addition to examining more textbooks across the United States, another possible study could look at the images of polygons in textbooks from other countries to perform a comparative analysis between the opportunities to develop concept images of polygons in the U.S. and abroad. An analysis of international textbooks alongside U.S. textbooks could be especially valuable since U.S. students are underachieving in geometry (Mullis et al., 2008; Mullis et al., 2012; Mullis et al., 2016). This type of study could provide some cultural context to how students develop concept images of polygons and how different countries approach student learning trajectories in geometry.
This study included an examination of the developmental responsiveness of tasks containing images of polygons. An extension of this type of study would be to analyze the entire middle school mathematics textbook series to all instances of developmental responsiveness in tasks. It is possible that there are more developmentally responsive tasks in sections not devoted to polygons. Additionally, an extension of this research could look at the culturally responsiveness of tasks in middle school mathematics textbooks with a lens of equity and social justice. Further, future work could address middle school mathematics textbooks using the emerging Responsive Middle Level Mathematics Teaching (RMLMT) framework (Ellerbrock & Vomvoridi-Ivanovic, 2019), which includes domains related to both culturally and developmentally responsive mathematics teaching.

Future work in middle school mathematics textbook content analysis could focus on other aspects of the content and alignment. For example, the data sources could be expanded to include not just tasks with an image of a polygon, but also tasks that call for the creation of an image of a polygon by the student. Another possible avenue of research could examine how tasks involving images of polygons connect to different domains of mathematics, i.e. rectangles connecting to distributive property in examples using the area model for multiplication or rectangles connecting to fractions or division using tape diagrams. Additional studies could also investigate how tasks involving images of polygons are aligned with the Common Core State Standards (NGACBP, CCSSO, 2010). Further, future studies could focus on the cognitive complexity of tasks involving polygons and the distribution of low and high -level tasks.
Summary of Conclusions

In this chapter, I discussed the results of this study, the significance of this study, implications for the mathematics education community, limitations, and recommendations for future research. Over years of experiences, interactions, and exposures with a particular concept, these students develop concept images consisting of all the visual information, pictures, mental images, or properties associated with the particular mathematical concept (Vinner, 1983) and students’ opportunity to learn is defined as the encounters, occasions, experiences, and engagement students have with content, teaching and learning strategies, and particular topics (Hadar, 2018; Hiebert & Grouws, 2007; Houang & Schmidt, 2008; Törnroos, 2005). Students can only develop robust concept images if they are provided the opportunity to engage with a variety of representations (Aspinwall et al., 1997; Hasegawa, 1997; Mesa, 2004; Presmeg, 1986; Presmeg, 1992; Tall & Vinner, 1981; Vinner, 1983; Vinner & Dreyfus, 1989). Since textbooks often represent the intended curriculum (Lloyd et al., 2016; Stanic & Kilpatrick, 2003), the textbook used in a mathematics classroom can impact student learning (Hadar, 2018; Houang & Schmidt, 2008; Nicol & Crespo, 2006; Törnroos, 2005). In this study, I sought to explore the following question: Within three published middle school mathematics textbook series, in what ways do young adolescents have opportunities to develop concept images of polygons? Through my analysis of the images of polygons in the three middle school textbook series, I have determined that students do not have adequate opportunities to develop robust concept images of polygons.

In addition to the findings of this study, I would also like to add a plea to textbook authors, teacher preparation programs, district content specialists, and middle school
mathematics teachers. The findings of this study clearly show that students do not have enough varied exposures to different types of convex polygons, alignments of images, non-prototypical representations, roles of images, and contexts of tasks. Therefore, students do not have an adequate opportunity to develop robust concept images of convex polygons. Most of the tasks were set in purely mathematical contexts with no effort made towards developmental responsiveness. Because of the lack of variety in the images present, this means students’ exposure to convex polygons in their middle school mathematics textbooks amount to many repetitive examples over and over again, without any connection to students’ lives, interests, characteristics, or needs. This is simply unacceptable, and not best practices for educating young students. There needs to be a bridge between the mathematics education and the middle school education worlds because it is impossible to prioritize mathematics learning without also prioritizing the relevancy and responsiveness of the material. Textbook authors, teacher preparation programs, district content specialists, and middle school mathematics teachers must shift their thinking from focusing strictly on the subject matter of mathematics to also value students’ lives, interests, characteristics, or needs.
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