Measuring and Utilizing High-Dimensional Information of Optical Fields

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Measuring and Utilizing High-Dimensional Information of Optical Fields

by

Ziyi Zhu

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy Department of Physics College of Arts and Sciences University of South Florida

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Dedication

To my parents,

and

in memory of

my grandparents and maternal grandparents.
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Abstract

Currently, many areas of optical techniques including imaging, inspection and communication emphasize the utilization of the high-dimensional information encoded in optical fields. There is also a requirement for novel measurement techniques to extract this high-dimensional information with high-speed and accuracy. We firstly introduce a scan-free direct measurement technique that is capable of simultaneously characterizing the amplitude and phase of a coherent scalar optical field. Our direct measurement approach is constituted of a weak polarization perturbation which is followed by the recording of a polarization-resolving imaging process. The weak perturbation rotates the linear polarization on the spatial frequency domain of the detected field without noticeably changing the properties of the optical field. Then the high-dimensional Stokes parameter profiles are recorded in a single-shot such that the amplitude and phase profiles of the optical field are presented without some of the common complications from imaging or digital processing methods. Because our approach does not require an additional reference beam, the common-path optical configuration can minimize the effects of vibration and reduce the complication of the optical system. We have also developed our technique to measure the high-dimensional information encoded in an optical vector field, which has spatially varying polarization and phase profiles. Through a sequence of separating polarization components, a weak polarization perturbation, and a polarization-resolving imaging process, the final readout is directly related to the complex amplitude profile of the two polarization components of the vector beam. We experimentally demonstrate that our direct measurement technique can characterize both scalar and optical vector fields in a single-shot proving its use as a high-speed, extremely high-resolution, unambiguous measurement technique.
We next utilize the high-dimensionality of an optical field in applications of three-dimensional (3D) imaging and optical communication. In the 3D imaging application we present a self-interference polarization holographic (Si-Phi) technique which can capture the three-dimensional information of an object illuminated by incoherent light. The light from the object is modulated by a polarization-dependent lens, and a complex-valued polarization hologram is obtained by measuring directly the polarization profile of the light at the detection plane. Using a backward propagating Green’s function, we can numerically retrieve the transverse intensity profile of the object at any desired focus plane. Both 3-D and real-time imaging capabilities are demonstrated experimentally. In optical communication we propose a vector-beam-based communication protocol, namely spatial polarization differential phase shift keying (SPDPSK), which can encode a large number of information levels using orthogonal spatial polarization states of light. We construct a proof-of-principle experiment with a controllable turbulence cell, and we measure a channel capacity of 4.02 bits of information per pulse using 18 vector modes through a highly turbulent channel with a scintillation index of 1.54. Our studies provide direct experimental insights on how the spatial polarization profiles of vector beams are resilient to atmospheric turbulence, and our demonstration paves the way towards practical, high-capacity, free-space communication solutions with robust performance under harsh, turbulent environments.
Chapter 1:

Introduction

1.1 Introduction

The advent of lasers technology in the early 1960s ushered in significant changes in the growth and outlook of optical science, engineering, and technology, affecting almost every aspect of the discipline’s theory and practice. This period also witnessed an unprecedented advancement of the frontiers of technology. Super-resolution microscopy [1–3], combining several techniques, can take images with higher resolution beyond the diffraction limit [4], which brings "optical microscopy into the nanodimension" [5]. This technology has been utilized to investigate the substructure and function of cells and will serves as inspiration for future research across the biological and medical disciplines [6]. The polarization-entangled photon pairs experimentally achieved quantum-mechanical entanglement [7, 8] and demonstrated the extreme contradiction between quantum mechanics and local realism [9–11], which could be exploited for quantum communication and quantum computation [12, 13]. Interaction between very high intensity light and matter leads to new phenomena with non-linear responses such as second harmonic oscillation [14], optical Kerr effect [15], multi-wave mixing [16], etc. The discipline pertaining to these phenomena is known as non-linear optics [17], which can be utilized for wideband, ultrashort pulse fiber laser sources [18], optical amplifiers [19] and nonlinear spectroscopy [20], etc. These technologies have been made possible upon the realization that we are dealing with an integrated body of knowledge that has resulted in a greater need for collecting, deducing, and applying that knowledge to a variety of scientific, industrial, and commercial fields.
Precise measurement techniques are paramount to commanding scientific knowledge. Today’s need for more sophisticated and intricate measurement techniques has arisen from the growth in recent years of a vast collection of scientific knowledge, whose advancing frontiers have made rapid developments in technology, and therefore industry dependent upon them. Optical methods of measurement have provided many of the essential techniques currently in use such as the inspection of material defects, autopilot capabilities, biomedical imaging, and optical communication due to their advantageous qualities of being noninvasive, contactless and fast [21]. The current influx of optical measurement techniques emphasize high-dimensional or whole-field measurement techniques as opposed to point-by-point determinations. The development of computers and charge-coupled devices (CCDs) have promoted a tremendous improvement in the performance of optical measurements, providing potential advantages such as real-time, noninvasive, whole-field, high sensitivity, high accuracy and computer compatible measurements.

Optical measurements can obtain information encoded in various characteristics of a light wave including amplitude, phase, frequency, polarization, etc. Rather than obtaining the value of optical field properties at one spatial point, high-dimensional optical measurements perform a measurement at discrete points or over the whole field with extremely fine spatial resolution. The high-dimensional measurement can extract the entirety of the information contained within the optical field without a scanning process, giving it the tremendous potential for the investigation of a dynamic system [21–24], including the heat and mass transfer dynamics of proteins in living cells as well as chemical reactions. The initial output of these whole-field systems of measurement is often a fringe pattern that can be processed in order to determine the phase information. The resultant fringes are formed by direct interferometry, holographic interferometry, phase-shifting methods, speckle pattern interferometry, and moiré techniques [25, 26]. Meanwhile, the high-dimensional measurement isn’t limited to a singular profile of an optical field. For example, investigating the dynamic behavior of a birefringence biological sample requires simultaneous measurements of both the phase and polarization of a given sample [27–29]. Therefore, it is necessary to develop
a high-dimensional measurement technique for simultaneously characterizing multiple profiles of an optical field with high spatial resolution.

In this dissertation, we propose a series of new direct measurement techniques for measuring the amplitude, phase and polarization of an optical field with a single-shot, that serves as a high-speed, extremely high-resolution, unambiguous method for the complete measurement of the complex field. Before we introduce the detailed concepts of our direct techniques, we here first briefly summarize the basics of interferometry, polarization of light and current methodologies utilized in the measurement of the phase and polarization profiles of light.

1.2 Interference and Phase Measurement

Interference is a basic principle that can turn phase information of a wave into intensity information, which occurs when two or more coherent waves overlap each other in space. Interference is a basic principle that occurs when two or more coherent waves overlap one another in the spatial domain. It allows one to gleam information about the intensity of the waves from their phase information. The superposition principle for optical waves states that, for example, two overlapping fields, $E_1$ and $E_2$, add to give $E_1 + E_2$.

Assume that two optical waves with same frequency and linear polarization in space are described by

$$E_1(r) = A_1(r)e^{i \phi_1(r)},$$  \hspace{1cm} (1.1)
$$E_2(r) = A_2(r)e^{i \phi_2(r)},$$  \hspace{1cm} (1.2)

where $A_1$ and $A_2$ donate the amplitude, and $\phi_1$ and $\phi_2$ are phases of two waves. Notes that here we ignore term of time in wave function. When these two waves are superposed, the quantity
measured by a photodetector is the intensity, which is given by:

\[ I = |E|^2 = |E_1 + E_2|^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2), \]
\[ = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \Delta \phi, \quad (1.3) \]

where \( \Delta \phi = \phi_1 - \phi_2 \) is the phase difference between the two waves.

As can be seen from Eq.1.3, the resulting intensity is not merely the sum of the intensities of the two waves. Here, the two waves interfere and, a interference term, \( 2\sqrt{I_1 I_2} \cos \Delta \phi \), arises. We also see that when

\[ \Delta \phi = (2n + 1)\pi, \quad \text{for} \quad n = 0, 1, 2, ... \quad (1.4) \]

\( \cos \Delta \phi = -1 \) resulting in the intensity \( I \) reaching its minima. Conversely, when

\[ \Delta \phi = 2n\pi, \quad \text{for} \quad n = 0, 1, 2, ... \quad (1.5) \]

\( \cos \Delta \phi = 1 \), leading to the intensity, \( I \), reaching a maximum.

The resulting interference converts the phase information into fringe intensity information, which can be recorded by a camera. To extract the phase information, many methods have been developed including spatial-shear interferometry [30], spiral interferometry [31] and digital holography [32–34], which are all based on interferometry. In addition, some techniques like Shack-Hartman sensors [35] and phase retrieval [36] can obtain the phase information without the use
of an interferometry system. However, some of these techniques including Shack-Hartman sensors and spiral interferometry measure only phase gradients, $\partial \phi / \partial x$ or $\partial^2 \phi / \partial x^2$, and stitch these together to estimate the actual phase, $\phi$.

Digital holography (DH) is a popular technique used to capture the amplitude and phase of an optical field [34, 37]. We here give a brief introduction of digital holography, as it is one of the technique used in our direct measurement method. Digital holography can be generally categorized into two different classes: off-axis DH and on-axis DH. The configuration of off-axis digital holography is shown in Fig. 1.1. In the off-axis DH configuration the optical field, coming from the object, and the reference beam, arrive at the CCD at an angle, while in on-axis DH system, the object field and reference field are parallel.

Assuming the object field at the CCD plane is given by

$$E_o(x, y) = A_o(x, y)e^{i\phi_o(x, y)},$$  \hspace{1cm} (1.6)

where $A_o(x, y)$ is the amplitude and $\phi_o(x, y)$ is the phase, the reference field is assumed to carry an uniform tilt phase as

$$E_r(x, y) = A_r(x, y)e^{i\phi_r(x, y)} = \exp\left[i2\pi \left( \frac{\cos \theta_x}{\lambda} x + \frac{\cos \theta_y}{\lambda} y \right) \right],$$  \hspace{1cm} (1.7)

where $\lambda$ is the wavelength of the light source, $\theta_x$ and $\theta_y$ denote the angles of the wave vector of the reference beam with respect to x and y axis respectively. The object field, $E_o$, and reference field, $E_r$, interfere on the plane of CCD camera, resulting in an intensity distribution of the hologram is given by

$$I_h(x, y) = |E_r + E_o|^2 = |E_r|^2 + |E_o|^2 + E_o^*E_r + E_oE_r^* = 1 + |E_o|^2 + E_o e^{-i\phi_r} + E_o e^{i\phi_r}. \hspace{1cm} (1.8)$$
The first two terms are the noninterfering intensity pattern of the reference and object fields, separately. The last two terms are proportional to the complex object field and its conjugate multiplied by phase tilt.

To retrieve the object field, a Fourier transform is performed on the hologram in Eq. 1.8, resulting in the angular spectrum of the hologram as follows,

\[
AS(f_x, f_y) = FT\{I_h(x, y)\} = E_{as}(f_x, f_y) \ast E_{as}(f_x, f_y) + \delta(f_x, f_y) + E_{as}\left(f_x + \frac{\cos \theta_x}{\lambda}, f_y + \frac{\cos \theta_y}{\lambda}\right) + E_{as}\left(f_x - \frac{\cos \theta_x}{\lambda}, f_y - \frac{\cos \theta_y}{\lambda}\right),
\]

where \(f_x\) and \(f_y\) are coordinates of spatial frequency, \(E_{as}(f_x, f_y)\) denotes the Fourier transform of the object field \(E_o(x, y)\) and \(\ast\) represents a correlation operation. From this angular spectrum, the third term can be filtered out and shifted to the center of the spectral plane to obtain \(E_{as}(f_x, f_y)\), from which the object field can be extracted by performing a Fourier transform operation.

As previously mentioned, in on-axis holography the object field and reference field are parallel. A phase-shifting method \[38\] is utilized to obtain the object field, \(E_o\). An additional spatially-uniform phase shift, \(\alpha\), is applied such that the reference field becomes \(E_r e^{i \alpha}\). The interference intensity is given by

\[
I = |E_r e^{i \alpha} + E_o|^2
= |E_r|^2 + |E_o|^2 + E_r E_o e^{i(\phi - \alpha)} + E_r E_o e^{-i(\phi - \alpha)}
= |E_r|^2 + |E_o|^2 + 2E_r E_o \cos(\phi - \alpha),
\]

where \(\alpha\) is the phase shift and is typically assigned multiple values, and \(\phi\) is the phase of the object field. In the original four-step phase-shifting holography method, phase shifts, \(\alpha = 0, \pi/2, \pi, \) and \(3\pi/2\) are imposed upon the reference field. The interference intensities of the four steps are
expressed as:

\[ I_0 = |E_r|^2 + |E_o|^2 + 2E_rE_o \cos \phi \] (1.11)

\[ I_{\pi/2} = |E_r|^2 + |E_o|^2 - 2E_rE_o \sin \phi \] (1.12)

\[ I_\pi = |E_r|^2 + |E_o|^2 - 2E_rE_o \cos \phi \] (1.13)

\[ I_{3\pi/2} = |E_r|^2 + |E_o|^2 + 2E_rE_o \sin \phi. \] (1.14)

The phase value, \( \phi \), can be unambiguously extracted through the following formula

\[ \phi = \tan^{-1}\left( \frac{I_{\pi/2} - I_{3\pi/2}}{I_0 - I_\pi} \right). \] (1.15)

Note that since the value of sin and cos values are known to be separated, the \( \tan^{-1} \) can give values ranging from 0 to 2\( \pi \). The complex field can be obtained by

\[ u_o = (I_0 - I_\pi) + i(I_{3\pi/2} - I_{\pi/2}). \] (1.16)

The phase shifting method is an effective way to remove the zero-order and twin image terms of the hologram, which also retains the full bandwidth of the optical measurement system.

A Shack–Hartmann wavefront sensor (SHWFS) is an optical instrument used for measuring the wavefront or phase of an optical field. It is often utilized in adaptive optics systems, optical components inspection, atmospheric turbulence measurement and ophthalmologic diagnoses [39–41]. The Shack-Hartmann sensor consists of an array of microlenses in front of a detector array. When a port of an incoming field enters one of the microlens, it is focused into a spot on the detector array plane. As shown in Fig. 1.2, the intensity and position of each spot are analyzed to dynamically measure the complete wavefront of an optical field.

The aforementioned measurement methods have both advantages and disadvantages. Interference based methods like holography can obtain the amplitude and phase information of the optical field with high resolution using single-shot measurement, but they require a reference beam and a
complex optical system. The phase-shifting method is often achieved by using a kinetic reference mirror mounted on a piezoelectric transducer, such that it is not an exact single-shot measurement. The Shack-Hartmann wavefront does not need a reference beam and coherent light source, however, it has limited spatial resolution and as such is unable to reveal large phase variations or complicated phase profiles. Phase retrieval can interpret the phase results from the intensity information of an optical field, but it requires significant more time for executing computer algorithms. Therefore, there is a need to develop an optical measurement technique that can directly obtain the amplitude and phase information of an optical with high resolution in a single-shot.

![Figure 1.2: Schematic of a microlens array focusing a distorted wavefront. Provided by Thorlabs, Inc.](image)

1.3 Polarization and Measurement Method

Light is constituted of both the electric field and magnetic fields oscillating in the phase while propagating in free space. The electric field vector, $\vec{E}$, the magnetic field vector, $\vec{B}$, and the direction of propagation, $\vec{k}$, form an orthogonal triplet. Because a magnetic field vector of the light is unambiguously determined by its electric field vector, the polarization analysis of light considers the direction of the electric field only. The direction of the polarization of light is by convention defined as the direction of the electric field at a given spatial point [42].

Consider a monochromatic plane wave of frequency, $\omega$, traveling in the $+z$ direction

$$\vec{E}(z;t) = \vec{E}_0 \cos(\omega t - kz). \quad (1.17)$$
The vector, \( \vec{E} \), lies in the \( x - y \) plane. We can express \( \vec{E} \) as

\[
\vec{E} = E_x \hat{e}_x + E_y \hat{e}_y.
\]  

(1.18)

The components of \( \vec{E} \) along \( x \)- and \( y \)-direction are described by

\[
E_x = A_x \cos(\omega t - kz + \phi_x),
\]

(1.19)

\[
E_y = A_y \cos(\omega t - kz + \phi_y),
\]

(1.20)

where \( A_x \) and \( A_y \) are the amplitudes, and \( \phi_x \) and \( \phi_y \) are the phases associated with the \( x \)- and \( y \)-components of the electric field, respectively. The vector tip traces an ellipse over each period of the wave. When the phase difference of two components \( \Delta \phi = \phi_x - \phi_y = \pi/2 \) and \( A_x = A_y \), the vector tip describes a circle, and the light is said to be circularly polarized. When \( \Delta \phi = 0 \), the beam is said to be linearly polarized. When the phase difference, \( \Delta \phi \), changes rapidly with time, the light is termed unpolarized. Partially polarized light contains a fraction of the light that is polarized, while the remainder is unpolarized.

For fully polarized light, the polarization state can be described by a two-element column vector, known as the Jones vector [43]. The Jones vector is represented as follows:

\[
\vec{J} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}.
\]  

(1.21)

When light passes through an optical element, the resulting polarization of the light can be obtained by taking the product of the Jones matrix of the optical element and the Jones vector of the incident light. The details of this process can be found in many textbooks such as Anthony Gerrard’s book "Introduction to matrix methods in optics" [44].

For any fully, partially, or randomly polarized light, the polarization state of light can be treated using vectors [42]. This is a four-element column vector, also known as the Stokes vector, which
is defined as follows:

\[
\begin{pmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{pmatrix} = \begin{pmatrix}
|E_x|^2 + |E_y|^2 \\
|E_x|^2 - |E_y|^2 \\
2|E_x||E_y| \cos \phi \\
2|E_x||E_y| \sin \phi
\end{pmatrix} = \begin{pmatrix}
\text{Intensity} \\
I_{0^\circ} - I_{90^\circ} \\
I_{135^\circ} - I_{45^\circ} \\
I_{RCP} - I_{LCP}
\end{pmatrix},
\]

where \(I_\theta\) is the intensity measured when the linear polarizer is oriented at an angle \(\theta\) with respect to the \(x\)-direction, and \(I_{RCP}\) and \(I_{LCP}\) are the intensities measured with the right-circular polarizer and the left-circular polarizer in front of the detector. The effect of an optical element on Stokes vectors can be represented by the Mueller matrix. The details of this is explained in Gerrard’s book "Introduction to matrix methods in optics" [44].

All four elements of the Stokes vector can be experimentally measured. Suppose we have four detectors, three of them having polarizers in front of them. The detector without a polarizer measures the total intensity, \(I_0\), of the wave, the detector with the \(x\)-polarizer measures the intensity, \(I_1\), the detector with the polarizer oriented at \(+45^\circ\) measures the intensity \(I_2\), and the detector with a right-circular polarizer in front measures the intensity \(I_3\). The Stokes parameters are related to the measured intensities as \(S_0 = I_0, S_1 = 2I_1 - I_0, S_2 = 2I_2 - I_0,\) and \(S_3 = 2I_3 - I_0\).

A polarimeter is a basic scientific instrument used to measure the polarization state of light. A common polarimeter is comprised of a retarder, a polarizer, and an intensity detector [45, 46]. The retarder is a polarization element designed to produce a specified phase difference between two orthogonal incident polarization states of the exit beam. A typical retarder is produced by a quarter-wave plate or photoelastic modulator. According to the definition of the Stokes vector in Eq.1.22, several intensity signals should be obtained simultaneously in order to determine all four elements of the Stokes vector. Therefore, the following two methods have been developed [47]. In one, a quarter-wave plate retarder, or a polarizer rotates with high speed allowing for rotating element polarimetry. In the other, the photoelastic modulator varies the phase difference rapidly allowing for a phase modulation polarimetry.
To measure the spatial polarization profile of light, a photoelastic modulator is used with a CDD to construct imaging polarimeters [48–50]. One challenge of imaging polarimeters is to obtain the entire set of Stokes parameters simultaneously, due to the slow readout of the CCD and phase retardation modification. Several techniques have been developed to overcome this challenge. One technique is that precisely control the exposure of CCD in synchrony with the modulation of two photoelastic two modulator on phase retardation of the detected light [51]. Another technique is that integrate nanowire optical filters on the CCD imaging array [52] of polarimeter. However, these imaging polarimetry techniques have to sacrifice aspatial resolution or utilize a more complex optical system with multiple retarders or detectors.

Furthermore, the key limitation of an imaging polarimeter is that it does not provide any phase information, and therefore the polarization profile at the measurement plane cannot be used to predict the polarization profile at a different plane. For example, to predict the evolution of a vector beam, which has spatially varying polarization and phase profiles, a characterization method, which can obtain the spatial phase and polarization distribution simultaneously, is desired. Our research aim to overcome this limitation.

1.4 Dissertation View

In this dissertation we introduce a novel technique that can directly measure the high-dimensional information of an optical field including its amplitude, phase, and polarization profiles with high resolution and accuracy through a single-shot measurement. We then demonstrate that the high-dimensional information in some pratical applications such as the imaging of three dimensional object and optical communication under the condition of atmospheric turbulence. We firstly give a brief introduction about our motivation, the foundational principle and some methods of optical measurement in Chapter 1.

Chapter 2 describes a scan-free, direct measurement approach that is capable of simultaneously measuring the amplitude and phase of a coherent scalar optical field. Through the introduction of a small polarization perturbation on the spatial frequency domain, while recording the conventional
polarization image, the complex values of the entire field become measurable directly. We also demonstrate it as a high-speed, extremely high-resolution, unambiguous measurement technique.

In chapter 3 we present an improvement of our direct measurement technique that can characterize both the spatial polarization and vector beam, which has spatially varying polarization and phase profiles. Through a sequence of separating polarization components, a weak polarization perturbation, and a polarization-resolving imaging process, the final readout is directly related to the complex amplitude profile of the two polarization components of the vector beam. We experimentally demonstrate our direct measurement protocol on a variety of commonly used vector beams, including vector vortex beams and full Poincaré beams.

Chapter 4 describes a new three dimensional imaging technique using the same concept of direct measurement, called a self-interference polarization holographic imaging technique, that can capture the three-dimensional information of an incoherent scene. Using an on-axis polarization holography configuration and a polarization-resolving detector array, a complex-valued hologram is captured in a single-shot while utilizing the full spatial bandwidth of the optical imaging system. Using a backward propagating Green’s function, we can numerically retrieve the transverse intensity profile of the scene at any desired focus plane. Both 3-D and real-time imaging capability are demonstrated experimentally.

In chapter 5, we introduce a new high-information-capacity optical communication protocol utilizing vector beams. The communication system is designed based on vector beams. We experimentally generate atmospheric turbulence of various strength in a lab setting. We then compare the communication system using both vector and orbital angular momentum (OAM) beams. Our result shows that the vector beams are much more resilient to turbulence for free-space communication. A summary and the outline of future work on the basis of this dissertation is presented in Chapter 6.
References


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Chapter 2:

Direct Measurements of the High-Dimensional Information in a Transverse Optical Fields

2.1 Introduction

Light plays an important role in modern physics as it possess both a well-understood classical wave picture and a particle quanta picture. Classically, Maxwell’s work provided a clear understanding of light. It is a fundamental tool used in observing the universe from a macro to micro scale, and plays a profound role in scientific, industrial, and commercial endeavors. In quantum region, packets of light, also called photons, have been used as a unique platform for the studies of quantum science and applied technology including precision measurement [1], quantum entanglement [2–5], parallel information processing [4] and secure communication [6]. The complex amplitude of the wave function is often used to describe the characteristics of light. used to describe light. Measuring the wavefunction is a fundamentally significant issue needed to manipulate and utilize light for practical applications.

Quantum state tomography is an established method used to determinate the wavefunction, in which a complete set of measurements in incompatible bases on identically prepared copies of photon states are performed [7–10]. The time required to scan all bases of interest of the system limits this method in its ability to characterize high-dimensional states of light. In classical region, there are also a variety of techniques that have been developed to measure the optical field, particularly the phase. These include spatial-shear interferometry [11], Shack-Hartman sensors [12], spiral interferometry [13], phase retrieval [14], and digital holography [15, 16]. However, none of these techniques are direct. Shack-Hartman sensors and spiral interferometry measure only phase gradients, $\partial \phi/\partial x$ or $\partial^2 \phi/\partial x^2$, which are then stitched together to estimate the actual phase, $\phi$. 
Phase retrieval methods require computer algorithms to interpret the phase results. Holography needs an additional reference beam to interfere with the detected field.

Recently, direct measurement [17] has attracted a tremendous amount of research interest as it offers an alternative metrology technique that can greatly reduce the experimental complexity involved in characterizing a quantum system [18–21]. Here we extend this technique as a scan-free approach for characterizing high dimensional information of light. It also serves as a high-speed, extremely high-resolution method for measuring the amplitude and phase profiles of the scalar transverse optical field.

2.2 Principle

We here describe the direct measurement technique using classical language. The scalar transverse light field is a spatially coherent beam, which we assume to be vertically polarized. The transverse profile of the beam at the input plane can be expressed as

$$\vec{E}_{in} = \hat{e}_y E_{in}(x, y) = \hat{e}_y A(x, y)e^{i\phi(x,y)}.$$  (2.1)

The beam passes through a 4-f system. The transverse field distribution at the mutual focal plane of the two lenses is the Fourier transform of the input field as follows:

$$\vec{U}_p(\xi, \eta) = \hat{e}_y U_p(\xi, \eta) = \hat{e}_y F\{A(x, y)e^{i\phi(x,y)}\},$$  (2.2)

where $\xi$ and $\eta$ denote the transverse coordinates on the focal plane, and $f$ is the focal length of the lens.

A small polarization rotation is applied on the field component passing through a small area near the center of the focal $(\xi, \eta)$ plane. The angle, $\alpha$, of the polarization rotation is sufficiently small such that, to the first order of approximation, the vertically polarized field at the center of the
focal plane is nearly left in its original state as follows:

\[ U_y'(\xi = 0, \eta = 0) = \cos \theta U_p(0, 0) \approx (1 - a^2) U_p(0, 0) \approx U_p(0, 0). \] (2.3)

Since the field passing through other areas on the focal plane is not changed at all, the total transmitted field in the vertical polarization can be approximated as the input field \( U_x'(\xi, \eta) \). Meanwhile, we have generated a point source in the horizontal polarization

\[ U_x'(\xi, \eta) = \sin \alpha U_p(\xi, \eta) \approx \alpha U_p(\xi, \eta) \delta(\xi, \eta). \] (2.4)

The transverse field profile, \( \vec{E}(x, y) \), at the image plane of the 4-\( f \) system is the inverse Fourier transform of the field at the mutual focal plane. Its \( y \)-polarization component is given by

\[
E_y(x', y') = \mathcal{F}^{-1} \{ U_y'(\xi, \eta) \} \approx \mathcal{F}^{-1} \{ U_y(\xi, \eta) \} \\
= \mathcal{F}^{-1} \{ \mathcal{F} \{ A(x, y)e^{i\phi(x, y)} \} \} \\
\approx A(-x', -y')e^{i\phi(-x', -y')} \\
\approx E_{in}(-x, -y). \quad (2.5)
\]

\( E_y(x', y') \) is approximately identical to the flipped version of input field, \( E_{in}(-x, -y) \). In addition, we obtain a \( x \)-polarized field component through the polarization rotation as follows:

\[
E_x(x', y') = \mathcal{F}^{-1} \{ U_x'(\xi, \eta) \} \\
= \mathcal{F}^{-1} \{ a U_p(\xi, \eta) \delta(\xi, \eta) \} \equiv B. \quad (2.6)
\]

One sees that the \( y \)-polarized field is a plane wave with a constant, \( B \). We further convert the horizontal and vertical (\( x \) and \( y \)) polarization components into right and left-hand circular polarization components, respectively. Combining the results of two polarization components, we can obtain
the expression of the detected transverse field at the image plane as follows:

\[
\hat{E}'_{\text{det}}(x', y') = \hat{e}_l \mathcal{F}^{-1} U_p(\xi, \eta) + \hat{e}_r \mathcal{F}^{-1} \left\{ \alpha U_p(\xi, \eta) \delta(\xi, \eta) \right\}
\]

\[
= \hat{e}_l A(-x', -y') e^{i\phi(-x', -y')} + \hat{e}_r B
\]

\[
= \hat{e}_l E_{\text{in}}(x', y') + \hat{e}_r E_{\text{ref}}(x', y'),
\]

(2.7)

where \( l \) and \( r \) donate the left and right-hand circular polarization components, respectively. Here \( E'_{\text{in}} \) is the flipped version of the input beam, and \( E_{\text{ref}} \) is an orthogonally polarized reference field generated through the weak polarization perturbation process, and is essentially a plane wave of constant amplitude.

The polarization state of the field depends on the transverse coordinates, \((x', y')\). We can describe the position dependent polarization state in terms of Stokes parameters. Specifically, we have

\[
S_{1,\text{det}}(x, y) = I_{h,\text{det}}(x, y) - I_{v,\text{det}}(x, y),
\]

(2.8)

\[
S_{2,\text{det}}(x, y) = I_{d,\text{det}}(x, y) - I_{a,\text{det}}(x, t),
\]

(2.9)

where \( I_h, I_v, I_d \) and \( I_a \) are the intensity profiles of the field components in the horizontal, vertical, diagonal and anti-diagonal linear polarization states, respectively.

The linear polarization profiles of the left- and right-hand circular polarized field, \( E_{l,\theta} \) and \( E_{r,\theta} \), at a particular angle, \( \theta \), with respect to the vertical direction, can be obtained using Jones matrix analysis [22] as follows:

\[
\hat{e}_\theta E_{l,\theta}(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \frac{E_l}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}
\]

\[
= \frac{\hat{e}_\theta E_l}{\sqrt{2}} (\cos \theta + i \sin \theta) = \frac{\hat{e}_\theta E_l}{\sqrt{2}} e^{i\theta},
\]

(2.10)
\[
\hat{e}_\theta E_{r,\theta}(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \frac{E_I}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} = \hat{e}_\theta \frac{E_r}{\sqrt{2}} (\cos \theta - i \sin \theta) = \hat{e}_\theta \frac{E_r}{\sqrt{2}} e^{-i\theta}.
\]

(2.11)

Therefore, the intensity of the detected field on a linear polarization component at angle \(\theta\) is expressed as:

\[
I(\theta) = \frac{1}{2} |E'_{\text{in}} e^{i\theta} + E_{\text{ref}} e^{-i\theta}|^2 = \frac{I_{\text{in}}}{2} + \frac{I_{\text{ref}}}{2} + \sqrt{I_{\text{in}} I_{\text{ref}}} \cos(\phi + 2\theta),
\]

(2.12)

where \(I_{\text{in}}\) and \(I_{\text{ref}}\) are the intensity of the input beam and reference field, respectively, and \(\phi\) is the spatial phase profile of the input field. Note that the linear polarization direction angle, \(\theta\), produces a variable uniform phase difference between the input field and the reference field. Thus, the four intensity profiles of the detected field components in horizontal, vertical, diagonal and anti-diagonal states, respectively, are given by:

\[
I_{v,\text{det}} = \frac{1}{2} \left( I_{\text{in}} + I_{\text{ref}} + 2\sqrt{I_{\text{in}} I_{\text{ref}}} \cos \phi \right) = \frac{1}{2} |E'_{\text{in}}|^2 + \frac{1}{2} |E_{\text{ref}}|^2 + \Re\{E'_{\text{in}} E'_{\text{ref}}^*\},
\]

(2.13)

\[
I_{h,\text{det}} = \frac{1}{2} \left( I_{\text{in}} + I_{\text{ref}} - 2\sqrt{I_{\text{in}} I_{\text{ref}}} \cos \phi \right) = \frac{1}{2} |E'_{\text{in}}|^2 + \frac{1}{2} |E_{\text{ref}}|^2 - \Re\{E'_{\text{in}} E'_{\text{ref}}^*\},
\]

(2.14)

\[
I_{d,\text{det}} = \frac{1}{2} \left( I_{\text{in}} + I_{\text{ref}} - 2\sqrt{I_{\text{in}} I_{\text{ref}}} \sin \phi \right) = \frac{1}{2} |E'_{\text{in}}|^2 + \frac{1}{2} |E_{\text{ref}}|^2 - \Im\{E'_{\text{in}} E'_{\text{ref}}^*\},
\]

(2.15)

\[
I_{a,\text{det}} = \frac{1}{2} \left( I_{\text{in}} + I_{\text{ref}} + 2\sqrt{I_{\text{in}} I_{\text{ref}}} \sin \phi \right) = \frac{1}{2} |E'_{\text{in}}|^2 + \frac{1}{2} |E_{\text{ref}}|^2 + \Im\{E'_{\text{in}} E'_{\text{ref}}^*\},
\]

(2.16)

where \(\Re(x)\) and \(\Im(x)\) denote the real and imaginary parts of the complex quantity, \(x\), respectively.

Here, the spatial dependence of these quantities is not explicitly shown for simplicity. One can obtain the following relation between the Stokes parameters and the optical field profile, \(E'_{\text{in}}\):

\[
S_{1,\text{det}}(x', y') = I_{v,\text{det}} - I_{v,\text{det}} = 2\Re\{E'_{\text{in}}(x', y') E_{\text{ref}}^*(x', y')\},
\]

(2.17)

\[
S_{2,\text{det}}(x', y') = I_{d,\text{det}} - I_{a,\text{det}} = -2\Im\{E'_{\text{in}}(x', y') E_{\text{ref}}^*(x', y')\}.
\]

(2.18)
In other words, the transverse complex amplitude profile of the beam is given by

\[
E'_{\text{in}}(x, y) = \frac{S_{1,\text{det}}(-x, -y) - iS_{2,\text{det}}(-x, -y)}{2E^*_\text{ref}}
\]  

(2.19)

The above expression shows that the rotation of the polarization state of the field in the horizontal-vertical linear bases and diagonal–anti-diagonal linear bases are directly proportional to the real and imaginary part of the inverse of the transverse complex amplitude profile of the beam, respectively.

2.3 Experiment

2.3.1 Experimental Setup of Direct Measurement

To demonstrate our scan-free approach, we apply our method to measure the scalar transverse field of light. Our experimental procedure is is outlined in the remainder of this section. A single-longitudinal-mode 532 nm laser (Coherent Compass M315) is used as the photon source. The linearly polarized beam is expanded using a 100X beam expander before shining on a phase-only spatial light modulator (SLM; CambridgeCorrelaters SDE1024). Computer generated holograms are imposed on the SLM, and the photons in the first diffraction order are identically prepared with the desired complex optical field to be measured [23]. The details for generating an arbitrary complex field is described in the following section.

The optical field than passes through a 4-\(f\) imaging system (see Fig. 3.1), during which the transverse field is modified by a weak perturbation and a strong measurement in the mutual focal plane and imaging plane, respectively [17, 18]. The weak perturbation, also referred to weak measurement [24–26], is performed in the mutual focal plane of the two lenses to generate a reference field by rotating the linear polarization of the photons in the zero-momentum state through a small angle, \(\alpha\). The weak perturbation apparatus is comprised of a half-wave plate (HWP), a second phase-only SLM (Hamamatsu X10468) and a quarter-wave plate (QWP), as shown in Fig. 2.2. The SLM works in reflection mode, and it changes the phase of the reflected field in horizontal
polarization with negligible influence on the vertically polarized field. The HWP is positioned to rotate the orientation of the linear polarization before the light is launched onto the SLM. The birefringent response of the SLM changes the polarization of the reflected light, and the two waveplates are adjusted such that the linear polarization of the reflected light remains in the vertical direction when the SLM is set at zero phase shift, but is rotated by approximately 19 degree, when the entire SLM is set with a phase shift of approximately 0.74 radians. To apply the weak measurement, only the polarization state of the reflected optical field within an area of 40-by-40 $\mu$m$^2$ near the zero momentum state is rotated (see the phase profile of the SLM illustrated in Fig. S1). Such an area is comparable to the diffraction limited spot size of the incident photons with the imaging
system used, and therefore we are effectively rotating only the polarization of photons in a single momentum state of $p = 0$.

A strong measurement is performed at the image plane of the 4-$f$ system using a camera. Before the field is captured by the camera, a polarization separation apparatus is used. A beam splitter (BS) is firstly used to split the transverse light into two paths. The first path goes through a quarter wave plate (QWP) and a polarized beam splitter (PBS). The QWP is adjusted to convert the $x$- and $y$-component bases of the disturbed field into left- and right-hand circular polarized bases, such that the two exiting beams of the PBS correspond to the intensity profiles of the detected field in the vertical (V) and horizontal (H) linear polarization bases. The difference between these two images, i.e., $S_{1,\text{det}}$, is proportional to the real part of the transverse field. The second path goes through a QWP, a HWP, and a PBS. The HWP and QWP are adjusted such that the two exiting beams of the PBS correspond to the detected field in the diagonal (D) and anti-diagonal (A) components. The difference between the two images, in this case, i.e., $S_{2,\text{det}}$, is proportional to the imaginary part of the transverse field. Note that the strong measurement can be perform by a polarization resolving camera with QWP will be introduced in the following section.

Figure 2.2: Experimental realization of weak measurements using a phase-only SLM and two waveplates.
2.3.2 Utilization of a Polarization-Resolving Camera (PolarCam) to Measure a Complex Optical Field

The polarization-resolving camera (4D technology polarcam) provides a quick and convenient method to directly determine the complex-value of the optical field, which could replace the complex multiple optical element setup of strong measurements. There is an array of pixel-pitch matched aluminum nanowire polarization filters covering the CCD array of photoelements in the camera, as shown in Fig 2.3. This micropolarization filter array contains four distinct directions of linear polarization filter as 0°, 45°, 90° and 135°, corresponding to the vertical, horizontal, diagonal and anti-diagonal linear polarization components, respectively, which are combined as a super-pixel [27, 28]. Thus, a single image captured by the polarization-resolving camera actually contains the four desired polarized images $I_{v,\text{det}}$, $I_{h,\text{det}}$, $I_{d,\text{det}}$ and $I_{a,\text{det}}$ simultaneously, which could provide the complex amplitude profile of a transverse field.

To accurately obtain the complex values from the intensity profiles of the detected field on four linear polarized components, the input and reference profiles of the detected field $E'_{\text{in}}$ and $E'_{\text{ref}}$ should be adjusted as exactly circularly polarized. In addition, due to the birefringent response of the second SLM there is a phase difference between the reflected light on horizontal and vertical polarization components. The reflected light from the second SLM (Hamamatsu X10468) of a linear polarized input field at 45 degree is elliptically polarized light. Therefore, adjusted wave plates
including a quarter-wave plate (QWP1) and a half-wave plate (HWP1) are placed in front of the second SLM. Then a quarter-wave plate (QWP2), a half-wave place (HWP2) and another quarter-wave plate (QWP3) are placed in this order after the second SLM. Vertically linear polarized light is launched to the second SLM, and the HWP1, QWP1, and QWP2 are carefully adjusted in particular directions to remove the phase difference between the horizontal and vertical polarization of the SLM. Therefore, the reflected output field passing through QWP2 is linear polarized, and its linear polarization direction can be rotated by setting a uniform value of phase shift on the entire SLM. HWP2 is used to adjust the linear polarization direction of the output field to vertical when the SLM is set at zero value of phase shift. QWP3 is set at $45^\circ$ to convert the linearly polarized field into the circularly polarized field. As a result, the input field profile, $E_{in}'$, is set exactly to the left-hand circular polarization, and the reference field profile, $E_{in}''$, which is created by polarization rotation, is set exactly to right-hand circular polarization.

2.3.3 Encoding Complex Optical Fields with a Phase-Only Spatial Light Modulator

A spatial light modulator is a transmissive or reflective optical device that is used to spatially modulate the amplitude and phase of an optical wavefront in two dimensions. In general, a spatial light modulator can’t modulate arbitrarily both the amplitude and phase profile of an optical field. In our experiment, we use a reflective, phase-only, liquid crystal on silicon spatial light modulator (LCOS-SLM) to create an arbitrary, complex optical field for evaluating our measurement method. Here we provide a brief introduction about the method which was first described in the reference "Encoding amplitude information onto phase-only filters". [29]

On a LCOS-SLM, a liquid crystal layer severing as the light modulation component is arranged on a silicon substrate with a formed electrical addressing circuit by CMOS technology [30]. The incident light is transmitted through the liquid crystal layer with almost zero absorption. The integration of high-performance driving circuitry allows the applied voltage to be changed on each pixel, thereby controlling the phase retardation of the incident wavefront across the device. By imposing a programmable, grayscale, two-dimensional pattern with the same resolution as liquid
crystal layer, the phase delay can be controlled on a pixel by pixel basis corresponding to the
grayscale values of the pattern.

To explain the method for encoding both phase and amplitude information, we consider the
function of a continuous, one dimensional, phase-only diffraction grating, which is expressed as

$$ T(u) = \exp(i2\pi Mud), \quad (2.20) $$

where \( u \) is the spatial frequency variable, and \( 1/d \) is the spatial period of the grating. The parameter
\( M \) is defined as the extent of the phase shift over each period of the grating from 0 to 1. The phase
shift is increased from a minimum value of 0 to maximum value of \( 2\pi \). This periodic function can
be expanded in a Fourier series as

$$ T(u) = \sum_{n=-\infty}^{\infty} T_n \exp(i2\pi n), \quad (2.21) $$

where the coefficients \( T_n \) are functions of the parameter \( M \) and are given by

$$ T_n = \exp[i(n-M)\pi] \frac{\sin[\pi(n-M)]}{\pi(n-M)}, \quad (2.22) $$

We can obtain the diffraction pattern for this grating by taking the Fourier transform of Eq.2.21
as:

$$ t(x) = \sum_{n=-\infty}^{\infty} T_n \delta(x - nd), \quad (2.23) $$

Note that the diffraction function consists of a series of delta functions whose amplitudes are given
by the coefficients in Eq. 2.22. The light on different diffraction orders could be separated by a
spatial filter.

The grating phase patterns are shown in Fig. 2.4(a) and (b). When \( M = 1 \), all of the light is
diffracted into the first order \( (n = 1) \). As the value of \( M \) decreases, the fraction of light diffracted
into the first order decreases while the fraction of light in reflective orders increase. When \( M = 0 \),
almost all the light is reflected into the \( n = 0 \) order.
Figure 2.4: Schematic drawing showing the diffraction of light by a phase grating: (a) phase depth of $2\pi$ rad over each period; (b) phase depth of less than $2\pi$ rad over each period; (c) phase depth decreasing from the center of the grating; (d) phase depth increasing away from the center of the grating.

To generate a field with spatial amplitude distribution, the parameter $M$ can be spatially varied as a function of position as $M(u)$. Here $M(u)$ is also limited to a value from 0 to 1. The grating function is expressed as

$$T(u) = \exp \left[ i2\pi M(u)ud \right]. \quad (2.24)$$

The percentage of the light diffracted into the first order varies spatially. For example, as shown in Fig. 2.4(c), the distribution of $M(u)$ decreases away from the center of the grating such that the light incident near the center of the grating will be more efficiently diffracted into the +1 order. By contrast, light near the edges will be more efficiently diffracted into the zeroth order. Conversely, the case when $M(u)$ increases toward the edges is shown in Fig. 2.4(d). In this situation, the light incident will be more efficiently diffracted into the +1 order near the edges, while the light near the center will be more efficiently diffracted into the zeroth order.
Now assume that we plan to encode a general complex function

\[ E(u) = A(u) \exp[i\phi(u)], \]

(2.25)

where \( \phi(u) \) is the phase distribution as a function of spatial frequency, \( u \), and \( A(u) \) is the amplitude distribution that we want to encode.

---

Figure 2.5: (a) Schematic diagram of the experimental setup for generating complex field. (Note: the angle between the zeroth and first order is very small) (b) The relationship between the amplitude of diffracted light into +1 order, \( A \), and the parameter, \( M \), on phase pattern. (c) Grayscale phase pattern on the SLM for creating a Gaussian beam.

To generate any arbitrary complex field, the light field, which is modulated by a phase grating pattern on the SLM, will pass through a \( 4 - f \) system as shown in Fig. 2.5(a). A spatial filter is placed on the focal plane between the two lenses so that only the diffracted light into this +1 order can pass through. Note that the amplitude of diffracted light into +1 order is not proportional to the parameter \( M \), which is shown in Fig. 2.5(b) through simulation. According to Eq. 2.22, the relationship between the coefficient parameter \( M \) and encoded phase, \( A \), on +1 order can be described by

\[ A = \frac{\sin \pi[1 - M]}{\pi[1 - M]}, \]

(2.26)
where \( \text{sinc}(x) = \sin \frac{\pi x}{\pi x} \) is a sinc function of \( x \). Therefore, the spatially varied parameter, \( M(u) \), used to encode the amplitude, \( A(u) \), is written as

\[
M(u) = 1 - \text{sinc}^{-1}[A(u)],
\]

where the function \( \text{sinc}^{-1}(x) \) is the inverse function of \( \text{sinc}(x) \).

In addition, a linear phase term, \( \phi_L(u) = 2\pi ud \), that increases the separation of different diffraction orders on the Fourier plane is added on the original phase, \( \phi(u) \), to reduce the noise from unwanted orders. We can then impose a phase pattern on the SLM, which is expressed as

\[
T(u) = 1 - \text{sinc}^{-1}[A(u)] \exp[\phi(u) + \phi_L(u)].
\]

Thus, the complex field \( E(u) = A(u) \exp[i\phi(u)] \) will be generated on the image plane. In our experiment, the phase grating term, \( \phi_L(u) \), is set along the diagonal direction to reduce the diffraction noise from the boundary of the SLM. An example of a gray scale phase pattern for generating a Gaussian beam is shown in Fig. 2.5(c).

### 2.4 Results and Discussion

In our experimental demonstration, we first characterize the transverse field carrying an orbital angular momentum (OAM) phase [31], which has recently been the subject of many fundamental studies in quantum mechanics [2, 6, 32, 33]. The created optical field can carry different values of the OAM quantum number, \( l \), using the SLM technique described above. The real and imaginary parts of the measured transverse field with \( l = 3 \) are plotted in Figs. 2.6(a) and 2.6(b), respectively. The corresponding phase and amplitude of, \( E_{in} \), are shown in Figs. 2.6(c) and 2.6(d), which accurately reveal the azimuthal phase structure and the central-null feature of the amplitude.
Figure 2.6: Measured (a) real and (b) imaginary parts of the two dimensional transverse field and the corresponding (c) phase and (d) amplitude profiles carrying orbital angular momentum (OAM) with $l = 3$. The measured complex values have very large magnitudes toward the center of the mode and therefore are truncated for the purpose of better visualization. (e)-(h) Extracted phase profiles of an optical field carrying an OAM mode with $l$ ranging from -2 to 2.
We further quantify the fidelity of the resultant measurements result using the standard definition of fidelity [34] as:

\[
\mathcal{F} \equiv \frac{\left| \int E_{\text{in,exp}}(x,y) E_{\text{in,ide}}^*(x,y) \, dx \, dy \right|}{\sqrt{\int |E_{\text{in,exp}}(x,y)|^2 \, dx \, dy} \sqrt{\int |E_{\text{in,ide}}(x,y)|^2 \, dx \, dy}},
\]

(2.29)

where \(E_{\text{in,exp}}\) and \(E_{\text{in,ide}}\) denote the experimental and ideal, y-measured transverse fields, respectively. The fidelity of the shown \(l = 3\) OAM mode is calculated to be approximately 0.93. Note that this less-than-unity fidelity can be partially attributed to the nonideal optical field preparations in our experiment. Nonetheless, the high fidelity of our result demonstrates that our direct measurement technique is indeed capable of measuring the complex optical field with very high accuracy. Similar high-fidelity results are obtained for optical fields carrying other OAM modes, along with the measured phase profiles of OAM modes with \(l\) ranging from -2 to 2 are shown in Figs. 2.6(e)-(h). The effective number of dimensions of the measured optical field, or the resolution of the image, due to the space–bandwidth product of our imaging system. Yet, the effective retrieved information from the high dimensionality of light can be arbitrarily enlarged by optimizing the measurement apparatus, such as using larger optical components and a detector array that covers a larger area.

We perform additional tests to our method on the light with more arbitrary amplitude and phase profiles. First, we impose a bull-shaped letter “U” pattern on the amplitude profile of the optical field with various Zernike phase profiles. The obtained amplitude of the optical field, \(|E_{\text{bull}}|\), is shown in Fig. 2.7(a), which is in good agreement with the result obtained using conventional intensity measurements in Fig. 2.7(b)], i.e., the square root of a direct image captured by the camera.

We also measure the optical field, \(|E_{\text{Gauss}}|\), with a gradually varying amplitude profile carrying various Zernike polynomial phase structures. One measured amplitude using our direct approach is shown in Fig. 2.7(c), and a cross section of \(|E_{\text{Gauss}}|\) (thick red line) is plotted in Fig. 2.7(d) in comparison with the conventional strong measurement result (thin blue line). Meanwhile, we also
Figure 2.7: Upper row: measured amplitude profiles, $|E_{\text{in}}|$, of the transverse field with an amplitude profile that incorporates a University of South Florida “Bull” logo using (a) our direct measurement approach and (b) conventional intensity measurement. Lower row: (c) measured amplitude, $|E_{\text{in}}|$, of the transverse field with a truncated Gaussian amplitude profile using our scan-free direct measurement approach and (d) one cross-section of the directly measured result (thick red line) in comparison with the result of the conventional intensity measurement (thin blue line). The measured Zernike polynomial phase profiles (e) $Z_0^0$, (f) $Z_2^0$ and (g) $Z_2^2$, along with the ideal phase profiles are illustrated in the upper-left corner of the figure.
obtain the various carried Zernike polynomial structure phase profiles including $Z_{0}^{0}$, $Z_{2}^{0}$ and $Z_{2}^{2}$, as shown in Fig. 2.7(e)-(g), which are in good agreement with the relative ideal Zernike polynomials. Note that the theory of our approach assumes that the perturbation due to the weak measurement in the momentum space is sufficiently weak such that the rotation of the polarization state of light in each position is small relative to the initial polarization. This imposes a practical limit on the minimum intensity ($|E|^2$) that can be accurately measured, which is experimentally determined by the accuracy of the polarization measurement in our case.

To further demonstrate the capability of phase measurement, we measure a transverse field carrying several square uniform amplitude profiles with difference values of uniform phase in each square. These values of phase profiles on the nine squares range from 0 to $1.8\pi$ with a $0.2\pi$ interval step. The measured intensity and phase profiles are shown in Fig. 2.8(b) and (c). Comparing with the imposed ideal phases in Fig. 2.8(a), the similar color on relative squares indicates that our method has the capability of measuring the phase of transverse fields quantitatively.

2.5 Conclusion

In this chapter, we demonstrate that our direct measurement approach is capable of determining the entire amplitude and phase profiles of a transverse optical field with high resolution and accu-
acy in a single-shot. It is a promising new technology for classical wavefront sensing applications in fields as diverse as observational astronomy, free-space optical communication, and biomedical imaging. This technique also opens up the possibility of characterizing a high-dimensional quantum system in real time for which a state-by-state scanning process would become impractically time-consuming or even infeasible.
References


Chapter 3:

Direct Measurement of the Spatial Polarization States of Light and Vector Beams

3.1 Introduction

Vector beams [1], characterized by their spatially varying polarization states, have garnered tremendous interest recently due to their potential applications in optical microscopy [2, 3], optical tweezers [4], optical metrology [5], laser material processing [6], and optical communication [7–10]. Over the past few years, many methods have been investigated to generate vector beams using e.g., spatial light modulators (SLM) [11, 12], Q-plates [13–15], optical fibers [16, 17], and metamaterials [18, 19].

However, the characterization of vector beams still largely relies on imaging polarimetry [20], where intensity images of vector beams are obtained after the beam passes through different polarizers. While such a method conveniently reveals the spatial polarization profile of vector beams, it does not provide any information about its transverse phase profile. Some methods have characterized vector beams composed of a limited number of selected spatial polarization modes [21, 22], but since the limited number of modes typically does not span a complete mode basis set, these methods are also incapable of fully describing the transverse profile of a vector beam. In addition, most available phase measurement techniques [23–28], including Shack-Hartmann microlens arrays and interferometric techniques, are designed for scalar beams and cannot be used directly for revealing the phase structure of vector beams nor their polarization profiles.

From an information retrieval point of view, both the transverse polarization and phase profile of a vector beam carry information, and therefore a characterization method that can reveal information encoded in all of the degrees of freedom available in a vector beam is naturally de-
sired. Furthermore, in many applications, including imaging and communication, a vector beam typically needs to propagate through an optical system or interact with various optical elements. With the knowledge of both polarization and phase profiles, one can predict the evolution of vector beams upon propagating through an optical system or free space. With the current surge of fundamental studies and applications, there is a huge demand for the development of a high-efficiency characterization method with the capability to fully characterize vector beams.

Here we describe a scan-free direct tomography protocol that, for the first time to our knowledge, is capable of characterizing the complete field structure of a fully polarized vector beam through a single measurement. The concept of "direct measurement" was first introduced in the context of quantum states [29] but can be described equally well in both quantum and classical languages. The direct measurement refers to metrology protocols in which the measurement readouts directly correspond to the complex-valued quantities that describe a quantum system or a classical optical field [30–35].

3.2 Theory

We here choose to describe our direct measurement protocol using classical physical optics terminology. A spatially coherent vector beam can be described by the superposition of two scalar beams with orthogonal polarizations. In the circular polarization basis, for example, the transverse vectorial field profile, $\vec{E}(u,v)$, at the initial $(u,v)$ plane can be written as follows:

$$\vec{E}(u,v) = \hat{e}_\ell E_\ell(u,v) + \hat{e}_r E_r(u,v), \quad (3.1)$$

where $\hat{e}_\ell$ and $\hat{e}_r$ denote the unit vectors in the left- and right-circular polarization (LCP and RCP) basis, respectively, and $E_\ell(u,v)$ and $E_r(u,v)$ denote the transverse complex-amplitude profile of the two circular polarization components, respectively.

To fully characterize the transverse profile of a vector beam defined by Eq. 3.1, we first introduce a relative transverse shift, $2\delta u$, between the two polarization components of the vector
beam. Here the value of $\delta u$ is chosen to be slightly larger than the radius of the beam such that the two polarization components are non-overlapping. At the same time, we adjust the polarization of the two components into the same horizontal linear polarization state. Since the total beam now has two spatially-separated parts, we herein refer to it as the "twin-beam". The field profile of the twin-beam, $\hat{e}_x E_s(u, v)$, after such a polarization separation and adjustment can be written as follows:

$$\hat{e}_x E_s(u, v) = \hat{e}_x [E_{\ell}(u + \delta u, v) + E_r(u - \delta u, v)].$$  \hspace{1cm} (3.2)

Since the twin-beam has now become a spatially coherent scalar beam of a single polarization, we may apply the recently-developed scan-free direct measurement technique [34] to characterize its total transverse beam profile. Specifically, the experimental apparatus is based on a 4-$f$ imaging system, where $f$ is the focal length of the lenses. For a twin-beam, $\hat{e}_x E_s(u, v)$, at the input plane of the 4-$f$ system, the field at the focal plane between the two lenses is the Fourier transform of $E_s(u, v)$ as follows:

$$\vec{E}_p(\xi, \eta) = \hat{e}_x E_{p}(\xi, \eta) = \hat{e}_x \mathcal{F}\{E_s(u, v)\},$$  \hspace{1cm} (3.3)

where $\xi$ and $\eta$ denote the transverse coordinates on the focal plane. A weak perturbation, in the form of a small polarization rotation of angle, $\alpha$, is applied to the field over a diffraction-limited area in the vicinity of the center of $E_p(\xi, \eta)$. After such a weak polarization perturbation, the total field exiting the focal plane has two polarization components, which can be expressed as:

$$\vec{E}_p'(\xi, \eta) = \hat{e}_x E_{p}(\xi, \eta) [1 + (\cos \alpha - 1)\delta(\xi - \xi_0, \eta - \eta_0)] + \hat{e}_y E_p(\xi, \eta) [\sin \alpha \delta(\xi - \xi_0, \eta - \eta_0)]$$

$$\approx \hat{e}_x E_{p}(\xi, \eta) + \hat{e}_y \alpha E_p(\xi, \eta) \delta(\xi - \xi_0, \eta - \eta_0),$$  \hspace{1cm} (3.4)

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where $\alpha$ is the angle of polarization rotation, and $\delta(\zeta - \zeta_0, \eta - \eta_0)$ is Dirac delta function centered at $(\zeta_0, \eta_0)$. One sees that when the angle of polarization rotation, $\alpha$, is sufficiently small, the $x$-polarized component at the Fourier plane can be approximated as the original unperturbed field $E_p(\zeta, \eta)$, and the generated $y$-polarized field is essentially a point source located at $(\tilde{\zeta}_0, \eta_0)$.

The field at the image $(x, y)$ plane of the 4-$f$ system is the Fourier transform of the weakly-perturbed field at the focal plane, and we further convert the horizontal and vertical (H and V) polarization components into RCP and LCP, respectively. The final detected field at the image plane can be written as follows:

$$\vec{E}_{\text{det}}'(x, y) = \hat{e}_\ell F\{E_p(\zeta, \eta)\}$$

$$+ \hat{e}_r F\{\alpha E_p(\zeta, \eta)\delta(\zeta - \zeta_0, \eta - \eta_0)\}$$

$$\approx \hat{e}_\ell E_s'(x, y) + \hat{e}_r E_{\text{ref}}(x, y)$$

$$= \hat{e}_\ell E_s(-x, -y) + \hat{e}_r B\exp\left[i2\pi(\tilde{\zeta}_0 x + \eta_0 y)/\lambda f\right]$$

(3.5)

(3.6)

where $E_s'(x, y)$ is the flipped version of the twin-beam, and $E_{\text{ref}}(x, y)$ is an orthogonally polarized reference field generated through the weak polarization perturbation process, and is essentially a plane wave of constant amplitude $B$ and a well-defined linear phase profile.

One sees that the polarization state of the detected field varies across the image $(x, y)$ plane, which can be expressed in terms of Stokes parameters as follows:

$$S_{1, \text{det}}(x, y) = I_h, \text{det}(x, y) - I_v, \text{det}(x, y),$$

$$S_{2, \text{det}}(x, y) = I_d, \text{det}(x, y) - I_a, \text{det}(x, y),$$

(3.7)

(3.8)

where $I_h$, $I_v$, $I_d$ and $I_a$ are the intensity profiles of the field component in the horizontal, vertical, diagonal and anti-diagonal linear polarization states, respectively. The intensity profiles are the
interference between the signal and reference beams, which are given by:

\[ I_{h, \text{det}} = \frac{1}{2} |E'_s|^2 + \frac{1}{2} |E_{\text{ref}}|^2 + \Re \{ E'_s E^*_\text{ref} \}, \quad (3.9) \]

\[ I_{v, \text{det}} = \frac{1}{2} |E'_s|^2 + \frac{1}{2} |E_{\text{ref}}|^2 - \Re \{ E'_s E^*_\text{ref} \}, \quad (3.10) \]

\[ I_{d, \text{det}} = \frac{1}{2} |E'_s|^2 + \frac{1}{2} |E_{\text{ref}}|^2 - \Im \{ E'_s E^*_\text{ref} \}, \quad (3.11) \]

\[ I_{a, \text{det}} = \frac{1}{2} |E'_s|^2 + \frac{1}{2} |E_{\text{ref}}|^2 + \Im \{ E'_s E^*_\text{ref} \}, \quad (3.12) \]

where \( \Re(x) \) and \( \Im(x) \) denote the real and imaginary parts of the complex quantity \( x \), respectively.

Here, the spatial dependence of these quantities is not explicitly shown for simplicity. Using these results, one can obtain the following relation between the Stokes parameters and the transverse field profile of the twin-beam, \( E'_s \):

\[ S_{1, \text{det}}(x, y) = 2\Re \{ E'_s(x, y) E^*_\text{ref}(x, y) \}, \]

\[ S_{2, \text{det}}(x, y) = -2\Im \{ E'_s(x, y) E^*_\text{ref}(x, y) \}. \quad (3.13) \]

The transverse complex amplitude profile of the twin-beam is therefore given by:

\[ E'_s(x, y) = \frac{S_{1, \text{det}}(x, y) - iS_{2, \text{det}}(x, y)}{2E^*_\text{ref}(x, y)}. \quad (3.14) \]

The above expression shows that after the weak polarization perturbation, the polarization state of the final detected field, expressed in the linear polarization basis, is directly proportional to the real and imaginary part, respectively, of the the transverse complex amplitude profile of the twin-beam.

According to Eq. (3.2), the left- and right- parts of the field profile of the twin-beam, \( E_s(u, v) \), after coordinate flipping are exactly the transverse profile of the two polarization components, \( E_l(u, v) \) and \( E_r(u, v) \), respectively, of the vector beam to be measured. Furthermore, since the two polarization components are measured simultaneously, the relative phase information between them is retained, which is essential for revealing its polarization profile. The Stokes parameters of
the vector beam under test can then be obtained through the following relations:

\[ S_0(u,v) = |E_\ell(u,v)|^2 + |E_r(u,v)|^2, \]  
\[ S_1(u,v) = 2\Re\{E_\ell^*(u,v)E_r(u,v)\}, \]  
\[ S_2(u,v) = -2\Im\{E_\ell^*(u,v)E_r(u,v)\}, \]  
\[ S_3(u,v) = |E_\ell(u,v)|^2 - |E_r(u,v)|^2. \] 

3.3 Experimental Configuration

Figure 3.1: (a) Schematic diagram of the experimental setup which includes a vector beam generation module and a direct measurement module. (b) Detailed illustration of the Sagnac interferometer for generating a vector beam from a \(45^\circ\) linearly-polarized twin-beam input. (c) Diagram explaining the weak polarization perturbation using a polarization-sensitive phase-only spatial light modulator. A diffracted-limited area on SLM-2 is set at a \(42^\circ\) phase such that the reflected field (the green beam) from that small area has an approximately \(19^\circ\) rotated polarization as compared to that of the twin-beam.

To demonstrate our direct measurement protocol for vector beams (see supplementary material), we constructed the experimental setup illustrated in Fig. 3.1, which includes both a vector beam generation module and a direct measurement characterization module.

Our method for generating the vector beam is adapted from H. Moradi’s work [36]. A beam from a 532-nm laser (Coherent Compass M315) with horizontal polarization is expanded and
launched onto a spatial light modulator (SLM-1; CambridgeCorrelaters SDE1024). A computer-generated hologram (CGH) is imprinted on SLM-1, and the diffracted light passes through a 4-\(f\) imaging system with spatial filtering at the focal plane. Such a setup can generate a field with any desired spatial profile with a high degree of control [37, 38] at the output of the 4-\(f\) system. Here we set the desired spatial field to be two transversely separated coherent beams, corresponding to the LCP and RCP components of the desired vector beam.

A Sagnac interferometer is placed between the second lens and the image plane of the generation 4-\(f\) system, which is composed of a polarizing beam splitter (PBS) and two mirrors. Before the twin-beam enters the Sagnac interferometer, its polarization is adjusted to 45° using a polarizer. As the twin-beam enters the Sagnac interferometer, it is split by the PBS into horizontally and vertically polarized components which then pass through the interferometer in opposite directions. The Sagnac interferometer is adjusted such that the two polarization components experience a transverse shift at the output. Specifically, the left side of the H-polarized output overlaps with the right side of the V-polarized output. A quarter-wave plate (QWP) is used to convert the H- and V-polarized components into LCP and RCP components, respectively. An iris is then used to only allow the generated vector beam to pass. As a result, the vector beam produced by the generation module has its two circular polarization components determined by the left and right part of the CGH on SLM-1, respectively.

The direct measurement module is also built based on a 4-\(f\) imaging system, whose object plane overlaps with the output image plane of the beam generation module. A second Sagnac interferometer is inserted before the first lens to transform the vector beam into a twin-beam with a transverse shift of \(2\delta u\) between the horizontal and vertical polarization components. When a QWP is used before the Sagnac interferometer, the vector beam characterization is effectively performed in the circular-polarization basis. When this QWP is absent, the beam characterization is performed in the horizontal and vertical linear polarization basis. A polarizer is placed after the Sagnac interferometer to make the twin-beam uniformly polarized in the diagonal direction. A phase-only SLM (SLM-2; Hamamatsu X10468) is placed at the focal plane of the characterization 4-\(f\) system.
to perform the weak polarization perturbation. SLM-2 responds only to horizontally polarized light and is operating in the reflection mode. The birefringent response of SLM-2 effectively alters the polarization of the reflected light. The phase on SLM-2 is set to zero everywhere except for a small area near the center of the focused beam, which is given a small phase value of $42^\circ$, which leads to a polarization rotation of approximately $\alpha = 19^\circ$. The size of the small area (80 $\mu$m by 80 $\mu$m) is comparable to the diffraction-limited spot size, and therefore the generated anti-diagonally-polarized reference field at the detection plane can be expressed analytically.

A polarization-resolving camera (4D Technology PolarCam) is placed at the detection plane with a QWP in front of it. The QWP converts the diagonally and anti-diagonally polarized signal and reference fields into left- and right-handed circular polarization, respectively. The camera includes a micropolarizer array that contains a pattern of linear polarizers (oriented at 0°, 45°, 90°, and 135°), capable of resolving $I_h$, $I_v$, $I_d$, and $I_a$. Since all four polarizations can be measured simultaneously, our direct measurement of a vector beam can be performed in a single shot. Note that the polarization-resolving camera can be replaced by a combination of beam splitters, polarization optics, and regular cameras [34].

### 3.4 Results and Discussion

To demonstrate the capability of our direct measurement protocol, we test a variety of vector beams, including several that are commonly used in applications. Firstly, we generate a vector beam that has uniform amplitude over a circular aperture and Zernike polynomial phase profiles, $Z_4^2$ and $Z_2^{-2}$, encoded into the LCP and RCP components, respectively. The directly-measured real and imaginary parts of the two circular polarization components are shown in Figs. 3.2(a)-(d). The corresponding phase profile of the two components as well as the profiles of three normalized Stokes parameters, are shown in Figs. 3.2(e)-(i), respectively. One sees that our experimental results match well with the theoretical expectations, shown as insets in the upper-right corner of each figure.
Figure 3.2: (a)-(d): The directly measured real and imaginary parts of the left- and right-handed circular polarization components of a vector beam that has uniform amplitude over a circular aperture and Zernike polynomial $Z_4^2$ and $Z_2^{-2}$ phase profiles; (e) and (f): the corresponding phase profiles of the two polarization components; (g)-(i) the corresponding normalized Stokes parameters of the vector beam. Insets on the upper-right corner are theoretical predictions.
Figure 3.3: The Stokes parameters (a) $S_1$, (b) $S_2$, (c) the phase of left- and (d) right-hand polarized components of vector beam by the pair of encoded phase profiles \{Z_3^1, Z_3^{-1}\} [a-d], \{Z_3^1, Z_2^{-2}\} [e-h] and \{Z_3^1, Z_4^{-2}\} [g-l]. Insets on the upper-right corner are the theoretical predictions.
To quantitatively evaluate our direct measurement results, we use beam fidelity as the figure of merit, which is defined as follows:

$$F \equiv \frac{\left| \sum_p \int E_{p,\text{exp}}(x,y)E_{p,\text{the}}^*(x,y)dx\,dy \right|}{\sqrt{\sum_p \int |E_{p,\text{exp}}(x,y)|^2dx\,dy} \sqrt{\sum_p \int \left|E_{p,\text{the}}(x,y)\right|^2dx\,dy}}$$  \hspace{1cm} (3.19)

where the subscript $p$ denotes the polarization components for the chosen basis, and $E_{p,\text{exp}}$ and $E_{p,\text{the}}$ denote the experimental results and theoretical predictions, respectively. The fidelity of the circular vector beam with uniform amplitude and Zernike polynomial phase profiles shown in Fig. 3.2 is calculated to be approximately 0.95, and similar high fidelity is observed for a variety of tested vector beams with different Zernike phase profiles. In addition, a variety of vector beams with different encoded phase profile pairs of $\{Z_3^1, Z_3^{-1}\}$, $\{Z_3^1, Z_2^{-2}\}$ and $\{Z_3^1, Z_4^{-2}\}$, are detected. The measured phase profiles and Stokes parameters, $S_1$ and $S_2$, are shown in Fig. 3.3, respectively. These measured complex profiles and polarization distributions also meet the theoretical spatial distribution as expected. The high fidelity of our results demonstrates that our technique is capable of accurately measuring the complex field profiles as well as the polarization profile of vector beams. The resolution of our experimental result is approximately 100,000 pixels, which is limited by the numerical aperture of the imaging system and by the total pixel count of the camera used in the experiment.

Secondly, we utilize our method to measure the states of a spiraling cylindrical vector beam. The expression is given by:

$$\vec{E}(r,\theta) = \hat{e}_r A(r)e^{il\theta} + \hat{e}_\theta A(r)e^{(il\theta + \phi)}.$$  \hspace{1cm} (3.20)

where $l$ is the order of vector beam, $\phi$ is the phase difference and $A(r)$ is the flat amplitude of the cylinder. The cylinder vector beam is composed of two opposite orbital angular momentum phase profiles on the left- and right-hand circular polarized components, respectively. According to the expression, the azimuthal or radial mode vector beam is determined by the relative phase differ-
Figure 3.4: From left to right: the detected phase on left-, right-hand polarized components, the Stokes parameters, $S_1$ and $S_2$, of spiral vector beam ($l = 1$) with a phase difference between two polarized components at (a) 0, (b) $\pi/2$, (c) $\pi$ and (d) $3\pi/2$, respectively; (e) the detected phase difference between two polarized components at 0(a), $\pi/2$, $\pi$(c) and $3\pi/2$. The expected theoretical relative distributions are inset at the top right corner of each figure.
Figure 3.5: From left to right: the detected phase on left-, right-hand polarized components, the Stokes parameters, $S_1$ and $S_2$, of a spiral vector beam ($l = 2$) with a phase difference between two polarized components at (a) 0, (b) $\pi/2$, (c) $\pi$ and (d) $3\pi/2$, respectively; (e) the detected phase difference between two polarized components at 0, $\pi/2$, $\pi$ and $3\pi/2$. The expected theoretical relative distributions are inset at the top right corner of each figure.
ence, $\phi$, between the two components. Note that our method has the capability of qualitatively measuring the phase profiles of each polarized component. Therefore, we could measure the exact value of the phase difference between the two polarized profiles of a vector beam and then discern the azimuthal or radial modes. To demonstrate, the vector beams of order $l = 1$ and $l = 2$ are generated and detected, where the relative phase, $\phi$, is set as $0$, $\pi/2$, $\pi$ and $3\pi/2$, respectively. We know that the radial mode of the beam is at $\phi = 0$ and the azimuthal mode is at $\phi = \pi$. Fig. 3.4 shows the detected Stoke parameters, $S_1$ and $S_2$, and their corresponding phase profiles on two polarized components. The detected polarization states of the vector beams with $l = 1$ are rotated $0^\circ$, $90^\circ$, $180^\circ$, and $270^\circ$ as expected with relative phase difference at $0$, $\pi/2$, $\pi$ and $3\pi/2$, respectively. The polarization states of the higher order beams with $l = 2$ exhibit a four-petal flower pattern as shown in Fig. 3.5, which are also in good agreement with the theoretical polarization distribution. These results indicate that our technique could not only distinguish the azimuthal or radial modes of a vector beam, but could also measure the spatial polarization distribution of an optical beam quantitatively.

Thirdly, we measure a family of four vector vortex beams [1] that have been used for high-dimensional secure quantum communication [7, 8, 10]. These four vector vortex beams use $LG_{0,1}$ and $LG_{0,-1}$ Laguerre-Gaussian (LG) modes as the two circular polarization components with an additional $0$ or $\pi$ phase difference between the two polarization components. Here $LG_{p,l}$ denotes the Laguerre-Gaussian mode with radial index, $p$, and azimuthal index, $l$. As a result, these four vector beams have the same intensity profile but very different spatial polarization profiles as illustrated in the first row of Fig. 3.6. Since these four vector modes are orthogonal to each other, they can be used to represent 2 bits of information in a spatial-mode-encoding protocol. As shown in the second and third rows of Fig. 3.6, our direct measurement technique distinctly reveals the azimuthal phase profile of each LG mode as well as the donut-shaped amplitude profile (illustrated by the saturation of each plot). Moreover, mode 1 and mode 2 (similarly for mode 3 and mode 4) have identical transverse phase profiles for the LCP component, while the two RCP components have the same spiral phase structure but with an additional “0” and “$\pi$” phase difference with re-
Figure 3.6: Measured phase of the two polarization components and the normalized Stokes parameters of a series of four vector vortex beams that are commonly used for high-dimensional optical communication. These vector vortex beams are comprised of $LG_{0,1}$ and $LG_{0,-1}$ Laguerre-Gaussian beams donating the polarization components and with different phase difference between the two polarization components. The color saturation reflects the experimentally-measured intensity of the beams. Insets in the upper-right corner are the theoretical predictions.
Figure 3.7: The measured phase of the two polarization components and the normalized Stokes parameters of (a) a vector vortex beam comprised of $LG_{0,1}$ and $LG_{0,-1}$ Laguerre-Gaussian modes as two circular polarization components; (b) the same beam measured in the horizontal and vertical polarization bases; and (c) a different beam that has identical polarization profiles but different phase profiles for the two polarization components. The color saturation reflects the experimentally-measured intensity of the beams. Insets in the upper-right corner are the theoretical predictions.

spect to the LCP component, respectively. This relative phase difference determines that mode 1 is radially polarized and mode 2 is azimuthally polarized. Thus, our direct measurement method correctly measures the relative phase difference between the two polarization components for each mode, which can lead to the correct spatial distribution of Stokes parameters. The accurate mapping of the polarization profile would not have been possible if the complex field profiles of the two polarization components were measured separately. The fidelity of our measurement results for the four modes are 0.91, 0.92, 0.92, and 0.94, respectively.

To further emphasize the advantage of our direct measurement method over conventional imaging polarimetry, we next demonstrate its ability to distinguish between two different vector beams with identical transverse polarization profiles. The first beam (mode 1 in Fig. 3.6) is comprised of
LG_{0,1} and \(LG_{0,-1}\) Laguerre-Gaussian modes as the two circular polarization components. We first characterize this vector vortex beam in the circular polarization bases. As shown in Fig. 3.7(a), our measurement correctly reveals the amplitude and phase of the \(LG_{0,1}\) and \(LG_{0,-1}\) modes in the circular polarization bases with the correct relative phase difference, which leads to the expected Stokes parameter profiles as well. It is well known that such a radially polarized beam can also be constructed by the superposition of \(HG_{1,0}\) and \(HG_{0,1}\) Hermite-Gaussian (HG) modes in the linear polarization basis [39]. When we remove the QWP at the very front of the characterization module, we can measure the radially-polarized beam in the H-V polarization basis. As shown in Fig. 3.7(b), our experimental results match well with the theoretical prediction of the two HG modes, illustrating the versatility of our direct measurement method in characterizing vector beams in arbitrary polarization bases. The second vector beam is created using the same amplitude profile of the first beam, but we remove the spiral phase from the LCP component and double the spiral phase on the RCP component. Since the relative phase difference between the LCP and RCP components at each point remains the same, the two vector beams have identical polarization profiles and are therefore indistinguishable if measured by conventional imaging polarimetry. While our measured Stokes parameter profiles show such indistinguishability in the polarization profile, our direct measurement method also reveals the different phase profiles of the two vector beams [see Figs. 3.7(a) and (c)]. Such a capability of resolving the complex field profile of individual polarization components makes our direct measurement technique a more valuable tool when compared to conventional imaging polarimetry.

Finally, we demonstrate the generation and characterization of a full Poincaré beam, which has attracted much research interest for its richness in fundamental physics as well as its potential applications in imaging and particle tracking [40]. Our full-Poincaré beam is generated by superposing an LCP fundamental Gaussian mode and an RCP \(LG_{0,1}\) Laguerre-Gaussian mode. As shown in Fig. 3.8, our experimental results match well with the theoretical predictions (shown as insets in the upper-right corner of each plot), which indicates that the polarization state across the beam indeed spans the entire surface of the polarization Poincaré sphere. The fidelity of our mea-
Figure 3.8: The measured phase of the two circular polarization components and the normalized Stokes parameters of a full Poincaré beam. The color saturation reflects the experimentally-measured intensity of the beams. Insets in the upper-right corner are the theoretical predictions, and the theoretical polarization profile of this beam is illustrated in the upper-left corner of the figure.

measurement result is 0.95, which further demonstrates that our direct measurement method is capable of measuring any possible fully-polarized vector beam.

3.5 Conclusion

To conclude this chapter, we have introduced a direct measurement protocol that is capable of characterizing the full transverse field profile of fully polarized vector beams. Our direct measurement process involves a separation of orthogonal polarization components, a weak polarization perturbation, and a polarization-resolving imaging process. The measured polarization of the final detected field is directly related to the real and imaginary parts of the complex-amplitude profile of each polarization component of the vector beam. We have demonstrated our direct measurement protocol by measuring a variety of vector beams that are relevant to optical information science, including vector vortex beams and full Poincaré beams. Our experimental measurement
results have shown consistently high data fidelity, and the unique capability of revealing both the complex-amplitude and polarization information provides a robust and versatile metrology tool for fundamental studies of vector beams and a wide spectrum of applications utilizing vector beams.
References


Chapter 4:

Direct Measurement Protocol for Imaging a Three-Dimensional Incoherent Scene

4.1 Introduction

The ability to obtain the three-dimensional (3-D) information of a scene or a volume object has diverse applications in biological imaging [1, 2], single-molecule imaging [3, 4], surveillance [5], shape measurement [6, 7], and so on. Many approaches [8–16] have already been developed to achieve this. For example, confocal microscopy [17, 18] is a powerful technique to provide sensitive 3-D information of an object scene. However, confocal microscopy often involves a scanning process of optical sectioning to construct the 3-D data. The requirement of such a lengthy scanning process limits the image acquisition rate, and consequently imposes a huge challenge for real time imaging or the observation of dynamically-evolving samples.

In contrast, digital holography [19–21] can record sufficient information from a 3-D coherent scene with only one or a limited number of exposures. In particular, parallel phase shifting techniques [22–24] have been successfully demonstrated to acquire the full 3D image information in a single shot for coherent illumination, and therefore can greatly enhance the image acquisition speed.

However, the signal light from many imaging modalities, such as fluorescence imaging, is spatially incoherent, and therefore imposes an additional challenge for holographic imaging. Recently, the concept of digital holography has been extended to the imaging of an incoherent scene through the use of a self-interference technique [25, 26]. With the capability of achieving resolutions beyond the conventional Rayleigh limit [27], incoherent digital holography has been successfully integrated into various applications [28–31]. Nevertheless, it still remains a challenge to obtain the
3-D information of an incoherent scene in a single shot while fully utilizing the spatial bandwidth of the optical imaging system [32]. In this chapter, we introduce a direct measurement technique also called self-interference polarization holographic imaging (Si-Phi) that can capture the 3-D information of an incoherent scene with only a single exposure, while also utilizing the full spatial bandwidth of the imaging system.

4.2 Theory

![Conceptual construction of a self-interference polarization holographic imaging system](image)

Figure 4.1: Conceptual construction of a self-interference polarization holographic imaging system.

The conceptual construction (see Fig. 4.1) of our direct measurement system constitutes an object scene, a chiral lens, and a polarization analyzing detector-array. Florescent or natural light from the object scene is first filtered by a linear polarizer and propagates a distance $z_0$ before reaching a chiral lens, which has different focal lengths, $f_L$ and $f_R$, for the left- and right-handed circular polarization components, respectively. The two separately-modulated polarization components then co-propagate a distance $z_c$ before they reach a detector array that sits behind a linear polarizer. We here denote $x$, $\zeta$ and $u$ to be the transverse coordinates in the object plane, lens plane and detection plane, respectively. We start by investigating the case of a single point object with an intensity $I_0(x) = |E_0|^2 \delta(x_0)$ located at $x_0$ in the object plane. Here $\delta(x_0)$ denotes the Dirac delta function centered at the transverse location $x_0$. Under the Fresnel approximation, the left-
and right-handed circularly polarized field components at the detection plane are given by:

\[ E_j(u) \propto \int E_0 e^{\frac{ik}{2} \left( \frac{(x_0-u)^2}{z_0} f_j^2 + \frac{(u-z)^2}{z_c f_j} \right)} d\xi = E_0 e^{\frac{ik}{2} \left( \frac{z_c f_j + z_0 f_j - z_0 z_c}{z_0 z_c f_j} \right)} \int e^{\frac{ik}{2} \left( \frac{z_c f_j + z_0 f_j - z_0 z_c}{z_0 z_c f_j} \right)} \left[ \xi - \left( \frac{z_0 + u}{z_c} \right) \right] d\xi \]

\[ = A_j E_0 e^{\frac{ik}{2} \left( \frac{(x_0-u)^2}{z_0} f_j^2 + \frac{x_0^2 f_j - x_0 z_c - u^2 z_0}{z_0 z_c f_j} \right)}, \quad (4.1) \]

where the subscript \( j \) can be either “L” or “R”, corresponding to left- or right-handed polarization, respectively, and \( A_j \) is a constant independent of \( u \), whose value is given by the following integral:

\[ A_j = \int e^{\frac{ik}{2} \left( \frac{z_c f_j + z_0 f_j - z_0 z_c}{z_0 z_c f_j} \right)} \left[ \xi - \left( \frac{z_0 + u}{z_c} \right) \right] d\xi \]

\[ = (1 + i) \sqrt{\frac{\pi (z_0 z_c f_j)}{k(z_c f_j + z_0 f_j - z_0 z_c)}}, \quad (4.2) \]

For simplicity we have here dropped some constant propagation factors that are independent of \( x_0 \) and \( u \) [33]. When a linear polarizer, set at an angle \( \theta \) with respect to the horizontal direction, is placed before the detector array, each of the two circularly-polarized components will partially pass through with an added geometrical phase determined by the angle \( \theta \) [34]. The two transmitted parts then interfere with each other, and the corresponding selected polarization intensity of the field at the detection plane is given by:

\[ I_\theta(u; x_0) = |E_L e^{i\theta} + E_R e^{-i\theta}|^2 \]

\[ = |A_L|^2 I_0 + |A_R|^2 I_0 + 2|A_L A_R| I_0 \cos \left( \frac{k}{2} \left[ \frac{(f_L - f_R)(z_0 u + z_c x_0)^2}{(f_L z_0 + f_L z_c - z_0 z_c)(f_R z_0 + f_R z_c - z_0 z_c)} \right] + 2\theta \right) \]
\begin{align*}
&= |A_L|^2 I_0 + |A_R|^2 I_0 \\
&+ 2 |A_L A_R| I_0 \cos \left( \frac{k}{2} \left[ \frac{(f_L - f_R)(u/\alpha - x_0)^2}{(f_L z_0 + f_L z_c - z_0 z_c)(f_R z_0 + f_R z_c - z_0 z_c)} \right] + 2\theta \right) \\
&= |A_L|^2 I_0 + |A_R|^2 I_0 + 2 |A_L A_R| I_0 \cos \left[ \Psi(u/\alpha - x_0) + 2\theta \right] \\
&= I_0 \{ B + \cos \left[ \Psi(u/\alpha - x_0) + 2\theta \right] \},  \\
\end{align*}

where we have absorbed a factor of \( C = 2 |A_L A_R| \) into \( I_0 \), \( \alpha \equiv -z_c/z_0 \) is a magnification factor, 
\( B = (|A_L|/|A_R| + |A_R|/|A_L|)/2 \) is a quantity independent of \( x_0 \) and \( u \), and

\begin{equation}
\Psi(x) = \frac{kx^2}{2z_h},
\end{equation}

and where \( z_h \) is an effective hologram propagation distance determined by \( z_0 \) and other configuration parameters, such as \( z_c \), \( f_L \) and \( f_R \), of the holographic imaging system through the following definition:

\begin{equation}
z_h(z_0) = \frac{(f_L z_0 + f_L z_c - z_0 z_c)(f_R z_0 + f_R z_c - z_0 z_c)}{(f_L - f_R) z_c^2}. \quad (4.5)
\end{equation}

For an extended incoherent object scene, the polarization intensity profile recorded by the camera can be considered as the summation of all the resultant intensity patterns from each individual point in the object plane, i.e.,

\begin{align*}
I_\theta(u) &= \int I_\theta(u; x) dx \\
&= \int I_0(x) \{ B + \cos \left[ \Psi(u/\alpha - x) + 2\theta \right] \} dx \\
&= \int I_0(x_0) B dx_0 + \int I_0(x_0) \cos \left[ \Psi(u/\alpha - x_0) + 2\theta \right] dx_0 \\
&= \text{Const} + I_0(u/\alpha) \ast \cos \left[ \Psi(u/\alpha) + 2\theta \right],
\end{align*}

where “\( \ast \)” denotes the convolution operator. By selecting different values for the polarizer angle, \( \theta \), the spatial polarization profile of the light field at the detection plane can be measured directly.
Specifically, the spatial profile of the Stokes parameters, $S_1(u)$ and $S_2(u)$, are directly related to the real and imaginary parts of the complex-valued hologram, which are written as:

$$S_1(u) = I_0(u) - I_{\frac{\pi}{2}}(u) = \Re \left\{ I_0(u/\alpha) * e^{i\Psi(u/\alpha)} \right\}, \quad (4.7)$$

$$S_2(u) = I_{\frac{3\pi}{4}}(u) - I_{\frac{\pi}{4}}(u) = \Im \left\{ I_0(u/\alpha) * e^{i\Psi(u/\alpha)} \right\}, \quad (4.8)$$

where $\Re$ and $\Im$ are the real and imaginary part notations, respectively. By combining $S_1(u)$ and $S_2(u)$, a complex-valued polarization hologram can be obtained as follows:

$$H_p(u) = S_1(u) + iS_2(u) = I_0(u/\alpha) * e^{i\Psi(u/\alpha)}. \quad (4.9)$$

As shown in the equation, the polarization hologram, $H_p(u)$, is the convolution of a magnified version of the intensity profile of the scene $I'_0(x) = I_0(u/\alpha)$ at the given $z_0$ plane with a complex-valued Green’s function:

$$G(u; z_h) = e^{i\Psi(u/\alpha)} = e^{\frac{ik(u/\alpha)^2}{2z_h}}. \quad (4.10)$$

Consequently, the transverse intensity profile of the scene, $I'_r(x)$, at any desired focus plane, $z_r$, can be retrieved using a backward-propagation process [26, 35] as follows:

$$I'_r(x) = H_p(ax) * G(ax; -z_h(z_r)), \quad (4.11)$$

where $z_r$ denotes the distance between the focus plane and the chiral lens, and the corresponding effective hologram propagation distance, $z_h(z_r)$, is calculated according to Eq. (4.5). When we choose the focus plane to be the object plane, i.e., $z_r = z_0$, the retrieved scene would be identical to the original transverse profile of the object. In addition, we can numerically refocus the retrieved scene at any desired focus plane and consequently obtain the three-dimensional information of the object scene.
4.3 Experiment

Our Si-Phi method utilizes a concept similar to the polarization-based static phase shifting technique developed for coherent holography [22–24]. However, our method utilizes chiral optical elements and a self-referencing on-axis holography configuration in order to make sure that the two polarization components pass through the same optical path. Using such a common-path arrangement is crucial to achieving the polarization hologram for incoherent light, and it makes our Si-Phi system robust against vibrations.

![Si-Phi Imaging System](image)

Figure 4.2: The schematic diagram of the Si-Phi experiment setup. SLM: spatial light modulator; HWP: half-wave plate; QWP: quarter-wave plate; BS: beam splitter.

To demonstrate our proposed Si-Phi method, we construct a proof-of-principle experiment setup as illustrated in Fig. 4.2. A 532-nm laser beam is first expanded by a telescope and launched onto a fast-reconfigurable spatial light modulator (SLM1 in Fig. 4.2). The modulated beam then passes through a spatially-filtered 6-f system to form a desired 3-D object scene within a certain volume. To mimic an incoherent object scene using a coherent light source, we use a fast-reconfigurable binary-amplitude SLM (TI LightCrafter DLP3000) as SLM1. SLM1 is imposed with a binary amplitude pattern that can produce a circular beam with some tilted and quadratic phase at the first diffraction order through the spatially-filtered 4-f system [36]. The generated cir-
cular beam then passes through another focusing lens and leads to a single spot, whose longitudinal and transverse position is determined by the quadratic and tilted phase components of the circular beam. Since SLM1 is capable of switching at a refresh rate of 4 kHz through different patterns, each corresponding to a single spot at different locations, the different spots of the created scene are therefore lit at different times. When the integration time of the camera is sufficiently longer than the time SLM1 takes to sweep through all the points of the scene, the camera effectively sees the entire object scene rather than individual spots. Consequently, since the light fields from any two different spots of the scene are generated at different times and therefore do not interfere, the scene is effectively spatially incoherent [37]. Note that the use of a SLM-based setup allows full control of the location and brightness of each point of the created 3-D scenes. The flexibility and precision of this setup allow us to quantitatively evaluate the real-time 3-D imaging performance of our Si-Phi method.

The light from the created scene continues to propagate through the holographic imaging system. A front lens with a focal length $f_a$ is placed at a distance $z_1$ from the object plane. Here the entire object scene volume is chosen to be on one side of the focal plane, i.e., $z_1 < f_a$, such that the object scene can be reconstructed without any ambiguity [35]. A phase-only SLM (SLM2 in Fig. 4.2; Hamamatsu X10468) is placed at a distance $d$ from the front lens. The effective propagation distance from the object plane to SLM2 is therefore given by:

$$z_0 = -\frac{z_1 f_a}{z_1 - f_a} + d. \quad (4.12)$$

Thus, the over-all magnification factor of the reconstructed image with respect to the object scene is given by:

$$M = \left[\left(\frac{z_1 f_a}{f_a - z_1}\right) / z_1\right] \times \left[-z_c / \left(\frac{z_1 f_a}{f_a - z_1} + d\right)\right] = -\frac{z_c}{z_1} \left[\frac{z_1 f_a}{z_1 f_a + d (f_a - z_1)}\right]. \quad (4.13)$$
A half-wave plate (HWP) is used to adjust the light field to be 45° linearly polarized before it reaches SLM2. SLM2 responds only to horizontally polarized light, and therefore a quadratic phase profile is imposed only onto the horizontally polarized light through SLM2 [38]. On the other hand, SLM2 acts like a plane mirror for the vertically polarized light. As a result, SLM2 is effectively a lens that has two different focal lengths for the two orthogonally polarized field components. A quarter-wave plate (QWP) is placed after SLM2 to convert vertical and horizontal polarizations into left-handed and right-handed circular polarizations, respectively.

A polarization-resolving camera (4D Technology; PolarCam) is placed a distance $z_c$ from SLM2 to measure the polarization profile of the final field. The polarization-resolving camera has an array of pixel-pitch matched aluminum nanowire polarization filters covering on the CCD array of photo elements. This micropolarization filter array contains four distinct directions of linear polarization filter at 0, 45, 90, and 135 degrees [39, 40], respectively, which are combined as a super pixel. A single image captured by such a camera actually contains the four desired linearly-polarized images, $I_{0^\circ}$, $I_{45^\circ}$, $I_{90^\circ}$ and $I_{135^\circ}$, simultaneously. The obtained four intensity images can be used to combine the real and imaginary parts of the complex-valued hologram by Eq. 4.8.

4.4 Experimental Results

We first demonstrate our Si-Phi system by measuring a scene containing only a single spot. The experimental parameters, $f_a = 100$ cm, $d = 40$ cm and $z_1 = 96$ cm, lead to an effective value of $z_0$ to be 24.40 m. The focal length corresponding to the parabolic phase set on SLM2 is chosen to be $f_L = 45$ cm, and the camera is placed at a distance $z_c = 90$ cm from the SLM2. The four linearly polarized images, shown in Fig. 4.3(a)-(d), are extracted from a single-shot image by the PolarCam. The real and imaginary parts of complex hologram, $H_c(x)$, are shown in Fig. 4.3(e) and 4.3(d). One sees that the complex hologram of a single point is a set of concentric circular rings, whose spacing is determined by the actual distance between the point source and the chiral mirror. By taking different values of, $z_0$, which are yielded by Eq. (4.12), we use the deconvolution process in Eq. (4.11) to reconstruct the incoherent scene at any transverse plane of the object scene.
Figure 4.3: The polarcam captured the four desired polarized images of a single point: (a)\(I_0^\circ\), (b)\(I_{0^\circ}\), (c)\(I_{90^\circ}\), and \(I_{135^\circ}\). (e) The real and (f) the imaginary parts of complex hologram, \(H_c\), for a single point. The reconstruct image of single point at focus plane (g) \(z_1 = 86\) cm, (h) \(z_1 = 96\) cm and (i) \(z_1 = 111\) cm.

When the chosen distance value, \(z_0\), equal to the actual distance, 96 cm, the reconstructed images in Fig. 4.3(h) show a sharp point. If we choose another distance value, the image is defocused, revealing the 3D information of the scene as Fig. 4.3 (g) and (i).

In Fig. 4.4(c), we display the intensity distribution profile along the x-axis over the reconstructed single point at different focused planes. One sees that the reconstructed spot spread and the intensity on center of spot decrease when the reconstructed focused plane move far away from the focused plane located at \(z_1 = 46\) cm. Consequently, the depth position, \(z_1\), of a single spot can be determined when the full width at half maximum (FWHM) of the reconstructed intensity profile is minimum. To demonstrate the ability of depth position measurement, we have performed our experiment with the single point object at different distances. The FWHM and peak intensity of reconstructed intensity profiles are plotted in Fig. 4.4(a) and (b), in which the difference of distance between each plot is set to 2 cm. The depth positions, \(z_1\), of the single point object are indicated clearly, according to plots of FWHM or peak intensity toward \(z_1\). As shown in Fig. 4.4(d), the measured distances from all the plots scale linearly.

Secondly, we utilize the same imaging system for reconstructing more complicated scenes, in which \(f_a = 100\) cm, \(d = 40\) cm, \(z_1 = 96\) cm, and \(z_c = 90\) cm. Nine points at different
Figure 4.4: (a) The FWHM and (b) peak intensity of reconstructed intensity distribution profile versus the transverse plane at $z_1$. Each plot is based on the single point object located at a particular position. The circular masks indicate the positions of the object. (c) The reconstructed intensity distribution at a variety of transverse planes along $z_1$, in which the object is located at $z_1 = 46$ cm. (d) The measured positions of singular points at $z_1$ according the marks on Fig.4.4(a).

locations, which constitute a cross pattern scene, are displayed after the 6-$f$ system at the object plane, and then are captured. The intensity image recorded by a general imaging system is shown in Fig 4.5(g). The intensity images on four linear polarizations captured by PolarCam are shown in Fig. 4.5(a)-(d). The Fig 4.5(e) and (f) reveal the real and imaginary parts of the complex hologram, which is more complicated than the hologram of the single point in Fig 4.3. The reconstructed image is shown in Fig. 4.5(h). When comparing the object scene in Fig 4.5g, the reconstructed image reveals the object scene very accurately at exactly the same location.

From Fig. 4.5(g) and (f), we can also observe that the points in the reconstructed image are sharper than the points in object scene, which indicate the self-interference digital polarization holography has the capability of super resolution. To examine the resolution of our imaging system, we create the two point object scenes in which two points are very close to each other with a distance of 0.8 mm and 1.0 mm. The image of the object scenes are directly captured by camera on the object plane, which are shown in Fig 4.6(a) and (c). Fig 4.6 (b) and (d) show the reconstructed images. One sees that the retrieved images are much easier to distinguish as compared to the images taken from the object plane, which can be considered as coming from a conventional
Figure 4.5: The polarcam captured the four desired polarized images of a multiple points images: (a) $I_{0^\circ}$, (b) $I_{45^\circ}$, (c) $I_{90^\circ}$ and (d) $I_{135^\circ}$. (e) The real and (f) the imaginary parts of complex hologram $H_c$ for the image of nine points scene. (g) The directly taken object scene of multiple points. (h) The reconstructed image of multiple points scene.

4-f imaging system. The FWHM of the retrieved intensity distribution of the points images are also shorter than the intensity distribution from the object image, as shown by the intensity cross-section of both the object scene and the reconstructed image along the x-axis in Fig 4.6 (e) and (f). Consequently, our polarization holography can exceed the Rayleigh criterion, because of the specific modulation transfer function (MTF), which has the shape of a coherent imaging system and the cutoff frequency of an incoherent imaging system, as is explained in Ronsen’s previous research [27].

The capability of our polarization holography to obtain 3-D information of objects is based on the ability to retrieve the transverse field on different focus planes. To prove this capability, we next illustrate the 3-D imaging capability of our Si-Phi method by observing a 3-D scene comprised of the patterns of two letters, “P” and “L” at two different object planes 20 cm apart (see the middle
Figure 4.6: (a) The directly taken object image, (c) the reconstructed image and (e) the cross-section along the x-axis of the object scene and the reconstructed cross-section of the two point scene at a distance of 0.8 mm. (b) The directly taken object image, (d) the reconstructed image and (f) the cross-section along the x-axis of object scene and the reconstructed cross-section of two point scene at a distance of 1.0 mm.

As shown in Figs. 4.7(a)-(d), each letter pattern is sharply resolvable only when the focus plane chosen for the image reconstruction matches the actual pattern plane; otherwise the letter pattern becomes out of focus and appears blurry. Such 3-D information of the scene can be better perceived by continuously scanning the focus plane through the object scene volume as shown in the multimedia view of Fig. 4.7.

Since all of the necessary information of the 3-D scene is taken in a single exposure, Si-Phi is capable of measuring and monitoring a 3-D scene continuously in real time. To demonstrate this capability, we create an artificial moving object, specifically, a single spot moving along a desired path over approximately three seconds. We perform continuous imaging of this moving object at
Figure 4.7: The reconstructed scene on different object planes at (a) $z_1 = 65$ cm, (b) $z_1 = 75$ cm, (c) $z_1 = 85$ cm and (d) $z_1 = 95$ cm of a three-dimensional object containing letters “P” and “L” at two different planes as illustrated in the middle inset (Multimedia view).

a frame rate of 30 frames per second. Figures 4.8(a)–(d) show the reconstructed scene at a fixed focus plane ($z_1 = 96$ cm) at four different instants of time. The spot moves along a circular path in this focus plane for the first second, and then moves along a 3-D spiraling path, travelling gradually away from the focus plane. The increasing distance between the object and the focus plane can be seen from Figs. 4.8(c) and (d) as the spot becomes more and more blurry. A continuous monitoring of the scene at this fixed focus plane can be seen in the multimedia view of Figs. 4.8(a)–(d). We further perform object tracking using a peak searching algorithm [41, 42]. The 3-D trajectory of the moving object is shown in Fig. 4.8(e). Note that the image acquisition rate of our Si-Phi system is only limited by the frame rate of the CCD camera and can be easily improved with a high-speed camera.

4.5 Conclusion

In summary, we have presented a self-interference polarization holographic imaging (Si-Phi) method that can perform real-time, 3-D imaging of an incoherent scene. Using a on-axis polarization holography configuration and a polarization-resolving detector array, a complex-valued
Figure 4.8: The reconstructed scene in a fixed focus plane at four different instants of time [(a)-(d)] for a single point object moving along a 3-D path (Multimedia view), and (e) the 3-D trajectory of the single point object over a period of 3 seconds (Multimedia view).

hologram is captured in a single shot while utilizing the full spatial bandwidth of the optical imaging system. Both 3-D and real-time imaging capabilities have been demonstrated experimentally by imaging mimicked incoherent scenes generated with a fast-reconfigurable spatial light modulator. Our Si-Phi method can be useful for diverse 3-D imaging applications such as time-lapse microscopy of biological samples in vivo, study of dynamically-evolving objects and surveillance in harsh environments.
References


Chapter 5:

Compensation-Free, High-Capacity, Free-Space Optical Communication Using
Turbulence-Resilient Vector Beams

5.1 Introduction

In previous chapters, we have developed a method to characterize the high-dimensional information of light such as its amplitude, phase and polarization. Here we introduce a novel method that utilizes the high-dimensional information of light for optical communication applications. Free-space optical communication offers flexibility, security and large signal bandwidth as compared to other means of communication [1–3]. Recently, there has been a great amount of research interest in using spatially structured light for optical communication [4–6] as the spatial modes provide a new degree of freedom to encode information, thereby greatly increasing the system capacity and spectral efficiency within a finite spatial-bandwidth of an optical channel. Among various families of spatial modes that have been investigated, the orbital angular momentum (OAM) modes of light have been used most widely and successfully to increase the information capacity of a free-space optical link [7–11]. However, atmospheric turbulence can impose serious practical limitations on the utilization of OAM modes as the fluctuation in the refractive index of air can alter both the transverse amplitude and phase structures of OAM modes. This, in turn, leads to crosstalk between neighboring OAM modes and degradation of the information capacity of a free-space link [12–16]. Adaptive optics has been the standard approach to compensate for thin turbulence [17–19], but it still remains a challenge to compensate beam distortions caused by strong volumetric turbulence. Other approaches such as image recognition based on artificial intelligence and machine learning [20–23] have also been demonstrated to resolve the information
encoded in distorted signals through turbulence. However, these approaches require both the acquisition of images and significant computing resources, making them not suitable for high-speed operation in real time.

Meanwhile, vector beams [24] are optical fields that can carry high-dimensional information by non-uniform spatial profiles in both complex-amplitude and polarization. The diversity in degrees of freedom within the vectorial optical fields has brought new dimensions for fundamental studies [25–27] and led to new optical applications [28–36] with performances surpassing conventional approaches. In particular, it has been shown that atmospheric turbulence is mostly anisotropic, and therefore the spatial polarization profiles of vector beams are more resilient to atmospheric turbulence as compared to the transverse phase profiles [37, 38]. Interestingly, many proposals of using vector beams for free-space communication [34, 39–43] show that vector-beam-based protocols do not outperform their scalar-beam-based counterparts, and both are equally vulnerable to atmospheric turbulence. Thus, it remains a challenge to effectively utilize a large number of spatial modes to transmit information through a turbid channel.

In this chapter, we propose a new information encoding protocol, namely, spatial polarization differential phase shift keying (SPDPSK), that encodes high-dimensional information on orthogonal spatial polarization states of a family of vector vortex beams. We observe experimentally that the spatial polarization profile of vector vortex beams is resilient against atmospheric turbulence. By utilizing such advantages, our SPDPSK protocol can transmit high-dimensional information reliably through a moderately strong turbulence cell without the need of any beam compensation mechanism. We demonstrate a proof-of-principle, high-dimensional communication system by transmitting 34 information levels (5.09 bits of information) per pulse through a free-space channel in the moderately strong turbulence regime with small information loss.

5.2 Principles of Vector Beam Communication

We here propose to use a family of orthogonal vector vortex beams to represent a large number of information levels. For example, each vector vortex beam can be formed by the superposition of
two Laguerre-Gaussian (LG) beams with opposite OAM charges in the circular polarization bases and with a relative phase difference of 0 or $\pi$. Such a LG vector vortex beam can be expressed as follows:

$$\vec{E}_{m,\pm}(r, \theta, z) = \hat{e}_r\text{LG}_{0,m}(r, \theta, z) \pm \hat{e}_r\text{LG}_{0,-m}(r, \theta, z),$$  

(5.1)

where \(\text{LG}_{p,l}\) denotes the Lagurre-Gaussian beam

$$\text{LG}_{p,l}(r, \theta, z) = \sqrt{\frac{2p!}{\pi (p+|l|)! w(z)}} \frac{1}{w(z)} \left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} e^{-r^2/2w(z)^2} L_p^{|l|} \left(\frac{2r^2}{w(z)^2}\right) e^{-ik\frac{r^2}{2w(z)^2}} e^{il\theta} e^{-ikz} e^{i\phi(z)},$$  

(5.2)

and where \(p\) and \(l\) denote the radial and azimuthal index, \(L_p^{|l|}\) is the generalized Laguerre polynomial, \(w(z) = w_0 \sqrt{1 + (z/z_R)^2}\) is the beam waist, \(z_R = \pi w_0^2 / \lambda\) is the Rayleigh range and \(R(z) = z[1 + (z_R/z)^2]\) is the radius of curvature of the beam. If \(m\) takes \(N\) different values, we have a total number of \(2N\) orthogonal vector vortex beams to represent \(2N\) information levels. Figure 5.1 shows ten examples of such vector vortex beams with mode orders \(m = -2, -1, 0, 1, 2\) and with a relative phase relation of 0 and \(\pi\) between the two circular polarization components.

Since the spatial polarization profile is determined by the relative phase (and amplitude) of the two orthogonal polarization components, the information encoded in the spatial polarization profiles is essentially encoded as the differential between the spatially-varying phase profile of the two polarization components. Thus, analogously to the well-known differential phase shift keying (DPSK) protocol in which the information is encoded in the relative phase between neighboring pulses in the time domain, we name our protocol spatial polarization differential phase shift keying (SPDPSK), indicating that the information is encoded in the relative phase between the two polarization components of a single vectorial beam pulse. In theory, the spatial polarization profiles can span a Hilbert space of infinitely large dimensions, indicating the number of information
levels that can be encoded is infinitely large. In practice, the number of usable information levels
is determined by the spatial-bandwidth product (Frensel number) of the free-space optical link.

When the free-space optical channel is turbulence-free, the two polarization components of
each vector vortex beam would experience exactly the same field evolution as the beam propagates
through the channel. As a result, the spatial-dependent phase difference between the two polariza-
tion components is always well-defined and independent of the propagation distance, which can
be expressed as follows:

$$\Delta \phi_{m,\pm}(r, \theta, z) = \arg [LG_{0,m}(r, \theta, z)] - \arg [LG_{0,-m}(r, \theta, z)] = 2m\theta + \beta, \quad (5.3)$$

where $\beta$ is 0 or $\pi$.

To decode the information at the receiver end, we first split the received vector vortex beam
into $N$ equal copies (see Fig. 5.2). Each copy then passes through a decoding phase plate that
has a polarization-dependent transmission function. The transmission function of the \( \text{n} \)th-order decoding phase plate can be written in the left- and right-handed circular polarization bases as follows:

\[
T_n(r, \theta) = e^{i\phi_l(r, \theta)} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\Delta \phi_n(r, \theta)} \end{bmatrix},
\]

where \( \Delta \phi_n(r, \theta) = 2n\theta \) is the difference of the phase between the right- and left-handed circular polarization components, and \( e^{i\phi_l(r, \theta)} \) is the transmission function for the left-handed circularly polarized light. When a \( m \)th order vector vortex beam passes through such a \( n \)th order decoding chiral phase plate, the transmitted field becomes

\[
\vec{E}^{n,\text{out}}_{m,\pm} = T_n \vec{E}^{\text{in}}_{m,\pm}
= \hat{e}_l \text{LG}_{0,m}(r, \theta, z) e^{i\phi_l} \pm \hat{e}_r \text{LG}_{0,-m}(r, \theta, z) e^{i\phi_l} e^{i2m\theta}.
\]

When the output beam passes through a polarizing beam splitter (PBS), the intensity of the field exiting the two output ports becomes:

\[
i^{n,h}_{m,\pm}(r, \theta) = |\text{LG}_{0,m}(r, \theta, z)|^2 \cos^2[(m - n)\theta - (1 \pm 1)\frac{\pi}{4}],
\]
\[
i^{n,v}_{m,\pm}(r, \theta) = |\text{LG}_{0,m}(r, \theta, z)|^2 \sin^2[(m - n)\theta - (1 \pm 1)\frac{\pi}{4}].
\]

We then integrate these distributions to obtain the power of the beam as follows:

\[
P^{n,h}_{m,\pm} = \begin{cases} \frac{1}{2} \pm \frac{1}{2} & n = m \\ 0.5 & n \neq m \end{cases},
\]
\[
P^{n,v}_{m,\pm} = \begin{cases} \frac{1}{2} \pm \frac{1}{2} & n = m \\ 0.5 & n \neq m \end{cases}.
\]

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The final \( n \)th order signal is the difference between the above two outputs, which is given by:

\[
P_{m,\pm}^{n} \equiv P_{m,\pm}^{n,h} - P_{m,\pm}^{n,v} = \begin{cases} 
\pm 1 & n = m \\
0 & n \neq m 
\end{cases}.
\]

As one sees, for the \( n \)th order decoding channel, an incoming \( n \)th order vector beam would result in a detection signal of 1 or -1 depending upon the relative phase between the LCP and RCP components. On the other hand, an incoming \( m \)th order vector vortex beam would result in detection signal of 0, if \( m \neq n \) under no turbulence condition.

![Figure 5.2](image_url)  
Figure 5.2: The principles of signal detection. The incoming optical signal is divided into \( N \) copies, each passing through a \( n \)th order decoding chiral phase mask before the differential power between the H and V polarizations are measured. All \( N \) decoded signals are then compared to determine the final detected information level (vector beam modes).

When the vector beam propagates through a realistic turbulence channel, both of the two circular polarization components of the beam inevitably experience phase and amplitude distortions. However, there has been both theoretical and experimental evidence [37, 38] showing that the polarization of light is not sensitive to atmospheric turbulence distortion, and therefore the two different components of a vector beam would experience similar distortions with a relative difference much smaller than the distortion of itself. The information is encoded in the relative phase and amplitudes of the two polarization components, making our spatial polarization differential phase...
shift keying (SPDPSK) protocol much more resilient to atmospheric turbulence as compared to protocols in which the information is encoded directly in the transverse complex beam profile (spatial modes) of a scalar beam. Our SPDPSK protocol can also be understood analogously to the well-known differential phase shift keying (DPSK) in which the information is encoded in the relative phase between neighboring pulses in the time domain. Although the phase of the optical signals through an optical fiber would experience uncontrollable drift and fluctuations in time, the relative phase drift between two neighboring time slots are very small, and therefore the information encoded in the DPSK protocol is almost immune to the phase fluctuations of fibers.

5.3 Experimental Protocol

5.3.1 Experiment Setup

To demonstrate the performance of our proposed SPDPSK protocol, we construct a proof-of-principle experiment in a laboratory setting. As shown in Fig. 5.3, our experimental setup is constituted of a vector beam generation module, a controllable turbulence cell, and a signal detection module. To generate the desired vector vortex beams, a beam from a 532-nm laser (Coherent Compass M315) with horizontal polarization is first expanded, collimated and then launched onto a spatial light modulator (SLM-1; CambridgeCorrelaters SDE1024). A computer-generated hologram (CGH) for $LG_{0,m}$ is imprinted onto SLM1 [44], and the diffracted light passes through a 4-$f$ imaging system with spatial filtering at the focal plane. The light is adjusted to be 45 degree polarized before it reaches a second phase-only SLM (SLM2; Hamamatsu) placed at the image plane, which is also responsive only to horizontal polarization. SLM2 is imposed with a phase profile of $-2m\theta + \beta$ in which $\beta$ is equal to 0 or $\pi$ for the + and $-$ modes, respectively. As a result, the horizontal and vertical components of the exiting beam share an identical amplitude profile to that of a Laguerre-Gauss mode but with opposite OAM charges and with an overall relative phase shift of 0 or $\pi$. The beam further passes through a quarter-wave plate to convert the H and V polarizations into left- and right-handed circular polarizations, respectively, in order to become the desired $m$th order vector vortex beams, \( \vec{E}_{m,\pm} \).
Figure 5.3: The schematic diagram of the proof-of-principle experiment. SLM: spatial light modulator; HWP: half-waveplate; QWP: quarter-waveplate; PBS: polarizing beam splitter; Det: detector.

The generated vector vortex beam is then expanded by a 3.3x telescope before it propagates through a hotplate-based turbulence cell that is approximately 60 cm in length. The strength of the turbulence is controlled by adjusting the temperature of the hotplate. The transmitted beam then passes through a receiving telescope with 3.3x demagnification and a quarter-waveplate before it is launched onto a decoding spatial light modulator (SLM3). The decoding SLM3 responds only to horizontal polarization, and in imposed a spiral phase profile of $2n\theta$ to decode a $n$th-order of encoded phase. The decoded beam then passes through a quarter-waveplate and a polarizing beam splitter, and the two outputs are focused onto two photodetectors.

Note that in a full-scale system the vector beam reaching the receiving end should firstly pass through a $1 - N$ beam splitter into $N$ identical copies, where $N$ is the total order number of vector modes used. Each copy would then pass through a decoding module designed for the $n$th-order vector mode, and a total of $N$ final detection signals would be obtained simultaneously for each incoming pulse. The information (spatial polarization mode) of the received signal is then determined by comparing these $N$ signals. However, our experimental setup only collects one decoded signal at a time, which leads to similar data fidelity and error rate when compared to a
5.3.2 Turbulence Characterization

We characterize the turbulence strength of our free-space channel by propagating a large Gaussian beam through the channel and measure its scintillation at the beam center when the beam reaches the decoding SLM3 plane. The scintillation is defined as follows [45, 46]:

\[
\sigma^2_I = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1,
\]  

(5.9)

where \( \langle \cdot \rangle \) denotes ensemble average, and \( I \) is the measured beam intensity at the statistical center of the beam. The measured scintillation as a function of control temperature is shown in Fig. 5.4. One sees that the measured scintillation value ranges from approximately 0 to 1.54, which indicates that such a controllable turbulence channel can mimic medium to moderately-strong turbulence. Note that the measured scintillation value is largely independent of the beam size so long as it is approximately larger than 1 cm in the turbid channel, which is also confirmed in the numerical simulation.
5.3.3 Phase Calibration of Liquid Crystal SLMs for Spatial Polarization Profile Modulation

The coherent polarization profile of an optical field can be described by the combination of complex profiles on two orthogonal polarization bases. The spatial light modulator (SLM) has been regarded as a promising device for its flexibility and programmability on phase modulations. In our experiment, the liquid crystal spatial light modulator is utilized to modify the spatial polarization profile by varying the spatial phase difference between the two orthogonal polarization bases. However, the initial phase and its associated response distortions influence the modulation accuracy heavily. Generally, the phase modulation accuracy of a LC-SLM is mainly influenced by three aspects of aberration: (1) the static phase distortion; (2) the spatially nonuniform phase response; (3) the nonlinear phase response. To improve the modulation accuracy, many researchers have proposed methods to calibrate the aberration of a SLM.

Here, we explain our method to calibrate the phase difference of the reflective light field from the horizontal and vertical polarization components of a SLM. A beam of light linearly polarized at 45° emitted from a laser is launched on the SLM, which we propose to calibrate. The reflected light, in which $E_H$ and $E_V$ denote the horizontally and vertically linearly polarized components respectively, from the SLM passes through a 4−f imaging system and a quarter-wave plate (QWP), and then it is captured by the polarization-resolving camera (4D Technology; PolarCam) at the image plane. The QWP converts the horizontally and vertically polarized components into left- and right-hand circular components such that the PolarCam can obtain the intensity of four linearly polarized profiles, $I_h$, $I_v$, $I_d$ and $I_a$, as following:

$$I_v = \frac{1}{2}|E_H|^2 + \frac{1}{2}|E_V|^2 + |E_H E_V| \cos \phi,$$  \hspace{1cm} (5.10)

$$I_h = \frac{1}{2}|E_H|^2 + \frac{1}{2}|E_V|^2 - |E_H E_V| \cos \phi,$$  \hspace{1cm} (5.11)

$$I_d = \frac{1}{2}|E_H|^2 + \frac{1}{2}|E_V|^2 + |E_H E_V| \sin \phi,$$  \hspace{1cm} (5.12)

$$I_a = \frac{1}{2}|E_H|^2 + \frac{1}{2}|E_V|^2 - |E_H E_V| \sin \phi,$$  \hspace{1cm} (5.13)
where $\phi$ is the phase difference between the complex profiles on two polarization basis. Thus, we can obtain the phase difference between two polarized components, $\phi$, based on the intensity profile recorded by polarcam:

$$
\phi = \arctan \left( \frac{I_d - I_a}{I_v - I_h} \right).
$$

(5.14)

Figure 5.5: (a) The phase response curve at three locations. (b) The initial residual phase difference between the reflected field on the horizontally and vertically polarized components. (c) The calibrated phase difference.

The phase difference between two polarized components on each pixel is determined by a gray scale value ranging from 0 to 255. To make the SLM accurate, we need to figure out the phase corresponding to the gray scale value on all pixels. Here we impose uniform values from 0 to 255 on the entire screen of the SLM, and then measure the corresponding phase difference is measured. Fig. 5.5(a) shows the gray scale phase value curve at three locations. It can be seen that the response curves are not linear and, in fact, are varied on different location. Due to the phase response being very similar for neighboring pixels, we are able to interpolate the measured phase as the resolution of the SLM with 800x600 pixels. As a result we obtain a 800 by 600 by 256, three dimensional look-up-table (LUT), which describes the relationship between the phase and the gray scale value of all pixels on the SLM. The correct compensation phase can be generated based on the new LUT.

Next, to minimize the size of the LUT and increase the speed of the SLM display, the least-square algorithm is used to fit a curve of the gray scale phase response on each pixel. For a required
phase value, $\phi$, the set gray level is expressed:

$$L(\phi) = \text{round}(p_1\phi^5 + p_2\phi^4 + p_3\phi^3 + p_4\phi^2 + p_5\phi^1 + p_6), \quad (5.15)$$

where $L(\phi)$ is the new set of gray levels, round(.) denotes the rounding operation, and $p_i$ ($i = 1, 2, 3, 4, 5$ and $6$) is parameter from the fitting model. Using the new LUT, the gray level distribution of a designed phase image is remapped as:

$$I_g = L(\phi). \quad (5.16)$$

The initial residual phase difference between the horizontally and vertically polarized components is shown in Fig.5.5(a). The calibrated phase difference is displayed in Fig.5.5(b). The root mean square error was chosen as the evaluation function, and the RMSE is reduced from 0.39 rad to 0.1 rad. The results prove that our method works very well for two component phase difference calibration. This method can also be used for calibrating reflected wavefronts from SLMs by utilizing a reference beam and a Michelson interferometer setup.

5.4 Results and Discussion

In our experiment, we use $m = -8$ to 8 for a total number, $N = 17$, mode orders, which leads to a total of $2N = 34$ information levels, i.e., 5.08 bit of information per pulse. For each transmitted mode at various turbulence strengths, we measure the detected signals using each decoding mode in sequence. A total of 2500 measurements are taken for each encoding-decoding configuration, and the averaged detected signal matrices for all the $(+)$ and $(-)$ input modes are shown in Fig. 5.6. As shown on the figure, when the channel is turbulence-free, the correctly decoded signals (the diagonal elements) are very close to 1 (and -1) for all of the 17 input modes and for both $(+)$ and $(-)$ information levels, while the incorrect detection signal is approximately zero as expected. As the temperature of the turbulence cell increases such that the scintillation exceeds approximately 0.7, the average correctly decoded signals start to drop, which becomes more severe.
for vector beams with larger mode orders. This drop at higher mode orders can be understood as the difference between the wavefronts of the two polarization components becoming larger, which leads to a great difference in beam evolution as they propagate through the volumetric turbulence. The average incorrectly decoded signal remains more or less the same, centered around zero. In general, the standard deviation of all the detected signals increases at higher turbulence strength and for larger mode orders.

To gain a better understanding of the preservation and eventual degradation of the spatial polarization profiles through turbulence, we have captured the polarization profiles of two representative vector vortex beams, \((m = 4, +)\) and \((m = 8, +)\), after transmission through the free-space channel with various turbulence strengths and with no decoding, correct decoding, and incorrect decoding, respectively. One sees from the no-decoding results that while the intensity profile of the vector beams experience significant distortion as the turbulence becomes stronger, the polarization profile actually is much better preserved. As a result, when the beam is decoded with the correct chiral phase mask, the vector beam becomes a scalar beam that can lead to a large detection signal. Note that the detected signal (shown below each \(S_1\) profile in Fig. 5.7) is calculated by summing over the \(S_1\) profiles directly measured by our polarization resolving camera, which has a lower polarization extinction ratio. This leads to a lower detection rate of a signal as compared to the results shown in Fig. 5.6 that are obtained by a polarizing beam splitter and two photodetectors. As the channel enters the strong turbulence regime, i.e., \(T > 150°C\) with a scintillation index larger than one, the polarization profiles of the transmitted vector beams experience more significant distortions such that the decoding becomes less effective. Furthermore, higher order vector modes have finer spatially-varying polarization features, which become more distorted. Consequently, the average detected signal drops more severely for larger values of \(m\) at stronger turbulence. On the other hand, when the beam is decoded with the non-matching polarization phase mask, the power of the beam is still well split between the two polarizations, which is quite independent of the vector mode order. Such a power split leads to a close-to-zero average detection signal with slightly more fluctuation as the turbulence gets stronger.
Figure 5.6: The average values of the directly measured signal of each input mode through each decoding (output) channel at various turbulence strengths (at temperature of 25 °C, 50 °C, 100 °C, 125 °C, 150 °C, 175 °C, and 200 °C on the hotplate). The top and bottom rows are for (+) ($\beta = 0$) and (−) ($\beta = \pi$) input signals as described in Eq. (5.1). Note that the z-axis of the matrix for $\beta = \pi$ is reversed.
Figure 5.7: The intensity and $S_1$ Stokes parameter profiles of $(m = 4, +)$ (upper) and $(m = 8, +)$ (lower) vector vortex modes propagating through the free-space channel at various turbulence settings with no decoding, correct decoding and incorrect decoding, respectively.
Figure 5.8: The detection probability matrix for each transmitted and received information level at different turbulence strengths. The upper and lower rows are results from the experimental data, and the lower rows are modeled based on the experimental data. The horizontal axes denote the transmitted and received information levels.
Figure 5.9: The experimental (a) signal error rate and (b) mutual information as functions of the number of vector modes used in the system at various temperatures. Here all levels are considered equally probable in the data stream.

The final received information level is determined by selecting the largest positive (or negative) signal among all of the 17 signals, each through a different decoding channel. Note that in a full-scale system, the vector beam that reaches the receiving end should pass through a $1 \times N$ beam splitter into $N$ identical copies, where $N$ is the total number of vector modes used. Each copy would then pass through a decoding module for each $n$-th order mode, and a total of $N$ final detection signals would be obtained simultaneously for each incoming signal pulse. However, our numerical simulation indicates that the fluctuations in all $N$ detected signals for any particular input mode due to turbulence can be considered uncorrelated. Thus, as a proof-of-principle demonstration, we perform the signal detection with different decoding modules in sequence and then process all $N$ detected signals to determine the final read out signal as if they were collected simultaneously. The information detection probability matrices based on experimentally measured results for all of the 34 information levels are shown in the upper row of Fig. 5.8. As one sees, the
probability of correctly detecting the information levels remains approximately unity even when the temperature of the turbulence cell is 175 °C, corresponding to scintillation index of 1.1.

Furthermore, if we assume the fluctuations in the detected signals of each encoding-decoding configuration obey Gaussian statistics and signals for different encoding-decoding configurations are mutually independent, we can build a statistical model based upon the experimental data and consequently calculate the detection probability. This may might be more accurate than the results obtained based on a limited number of experimental measurements. The information detection probability matrices based on such statistical models are shown in the lower row of Fig. 5.8, which only exhibits a slight difference from the results obtained directly from the measurement data.

Based on the detection probabilities for each information level, we can compute the average signal error rate and mutual information according to the following formulas:

\[
\text{ER}(2N) = \sum_{m=1}^{2N} P_m \sum_{n \neq m} P_{n|m},
\]

\[
\text{MI}(2N) = - \sum_{m=1}^{2N} P_m \log_2(P_m) + \sum_{m=1}^{2N} P_m \sum_{n \neq m} P_{n|m} \log_2(P_{n|m}),
\]

where \(2N\) is the total number of vector modes used, and \(P_m\) is the probability of information level, \(m\), in the data stream, all levels are assumed to be equally probable such that \(P_m = 1/(2N)\), and \(P_{n|m}\) is the conditional probability of detecting transmitted information level \(m\) as information level \(n\).

Figure 5.9 shows the experimentally measured average optical signal error rate, ER, and mutual information, MI, as functions of the number of vector modes used in the protocol at various temperatures. As one sees, the system is almost error free for temperatures under 150 °C, with an average bit error rate less than 0.35%, and the mutual information transmitted from the sender to the receiver follows the theoretical upper bound of \(\log_2(N)\) bits per pulse, which is 5.08 bits per pulse for 34 vector modes. As the turbulence strength enters the strong turbulence regime, the er-
ror rate starts to increase, this is largely due to the rapid degradation in the detection of high-order vector modes. The average error rate is approximately 4.1\% and 19.7\% at temperatures of 175 °C and 200 °C, respectively. However, if one reduces the total number of vector modes to 18, the average error rate is reduced to 0.3\% and 2.6\% for the two strongest turbulence strengths, respectively. Consequently, one sees that a 18-level protocol can effectively carry mutual information above 4 bits-per-pulse for all turbulence strengths used in our experiment, which is very close to the theoretical upper bound of 4.17 bits-per-pulse.

![Intensity Phase Intensity Phase OAM mode \(l = 4\) Intensity Phase OAM mode \(l = 8\) (a) (b)](image)

Figure 5.10: The intensity and phase profiles of an OAM beam carrying mode (a) \(l = 4\) and (b) \(l = 8\) that pass through turbulence generated by the hotplate at 20 °C, 50 °C, and 100 °C.

To demonstrate that the spatial polarization profiles of vector beams are more resilient to atmospheric turbulence than the spatial phase profiles of scalar beams used for communication, we here generate scalar Laguerre-Gaussian mode beams with encoded OAM modes that pass through our turbulence cell. The encoded OAM beams interfere with a reference beam without turbulence distortion. The intensity and phase profiles of the OAM beams are characterized by digital holography. The intensity and phase profiles of these beams carrying helical phase \(l = 4\) and \(l = 8\) are shown in Fig 5.10. Comparing to the polarization profiles of the vector beams in Fig 5.7, even
The cross-talk matrices for OAM beams with each input and detection modes at various turbulence strengths (at temperature of 25 °C, 50 °C, and 100 °C). The signal value indicates the energy distribution on the received mode of an OAM beam with transmitted mode, \( l \). Note that the summation of all the received modes of the transmitted modes is equal to 1.

Weak turbulence created by the 50 °C hotplate can bring significant distortion to the phase profiles of scalar OAM beams. When the scintillation is equal to 0.4 at 100 °C, the phase profile of the OAM beam is distorted so much that we are unable to directly discern the number of transmitted helical modes from the phase image. Conversely, we can easily point out the mode number of a vector beam on the spatial polarization profile images. The cross-talk matrices of OAM beams at various turbulence strengths are obtained by a digitally simulated OAM mode sorter [47, 48], as shown in Fig 5.11. The signal value is corresponds to the received energy of a particular helical mode, \( l \). Due to the mode sorter providing orthogonal decoding, the superposition of all the received mode signals from one encoded beam is equal to 1. One sees that the received energy on corresponding modes along the diagonal of the cross-talk matrix of the OAM beams decreases and spreads to other modes rapidly as the strength of turbulence increases. Meanwhile, the received signals and probability of vector beam along the diagonal of the matrix just drops marginally, this indicates the capability for vector beams to be resilient to atmospheric turbulence.

For the OAM beam, the turbulence not only effects the spatial distribution after propagation but also alters the phase value directly. In this case of vector beams, the spatial polarization only has its spatial distribution modified due to the isotropic interaction between the atmosphere and polarization profiles. We calculate the correlation coefficient between \( S_1 \) of a vector beam at 20 °C and 100 °C as well as the correlation coefficient between the complex field of an OAM beam
at 20 °C and 100 °C. The correlation coefficient of the vector beam at $m = 4$ is 0.57, which is larger than the value of the coefficient, 0.22, of the OAM beam calculated for an OAM beam with mode $l = 4$. This result quantitatively shown why vector beams are more robust than scalar OAM beams for free-space communication in a turbulent environment.

Lastly, we demonstrate the transfer of a data packet through the turbid channel using our high-dimensional vector beam communication protocol. The data packet used here is a 5-bit gray scale image with $128 \times 128$ pixels. We use all of the 32 non-zeroth-order vector modes such that the 5-

\begin{table}[h]
\centering
\caption{5-bit data encoding look-up table}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
\text{gray level} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
\text{mode index} & (8,+) & (-8,+) & (8,-) & (-8,-) & (7,+) & (-7,+) & (7,-) & (-7,-) \\
\hline
\text{gray level} & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\hline
\text{mode index} & (6,+) & (-6,+) & (6,-) & (-6,-) & (5,+) & (-5,+) & (5,-) & (-5,-) \\
\hline
\text{gray level} & 16 & 17 & 18 & 19 & 20 & 21 & 22 & 23 \\
\hline
\text{mode index} & (4,+) & (-4,+) & (4,-) & (-4,-) & (3,+) & (-3,+) & (3,-) & (-3,-) \\
\hline
\text{gray level} & 24 & 25 & 26 & 27 & 28 & 29 & 30 & 31 \\
\hline
\text{mode index} & (2,+) & (-2,+) & (2,-) & (-2,-) & (1,+) & (-1,+) & (1,-) & (-1,-) \\
\hline
\end{tabular}
\end{table}
bit gray scale information of each pixel is fully encoded into one pulse. The relationship between the encoded modes and the gray scale value is expressed in Table 5.1, where the corresponding gray scale value is displayed in each cell, and mode \( l = 0 \) is not used for encoding. The received images at various turbulence strengths are shown in Fig. 5.12. One sees that our high-dimensional communication system can reliably transmit the image through moderate to strong turbulence, and the error rate measured in the images matches well with our previous level-by-level measurements.

### 5.5 Conclusion

In this chapter, we proposed a vector-beam-based communication protocol, namely spatial polarization differential phase shift keying (SPDPSK), that can encode a large number of information levels using orthogonal spatial polarization states. We show experimentally that the spatial polarization profiles are much more resilient to atmospheric turbulence when compared to scalar modes, and therefore can reliably transmit high-dimensional information through turbulence even without the use of any adaptive optics for beam compensation. We construct a proof-of-principle experiment with a controllable realistic turbulence cell, and we show that our system can transmit 5.08 bits (34-levels of information) per pulse through a strongly turbulent channel that has a scintillation index around 1.0 with a very small data error. Our demonstration paves the way towards high-capacity, free-space communication solutions with robust performance under harsh turbulent environments.
References


Chapter 6:

Summary and Future Work

6.1 Summary

In this dissertation, we firstly presented the approach of a new direct measurement technique for characterizing the complex valves of scalar optical fields. The approach is performed by a weak measurement that is followed by a strong measurement. The weak measurement produces a reference field from a weak perturbation by rotating the linear polarization on the Fourier transform of the input field, which does not obviously change the properties of the optical field. The strong measurement then determines the high dimensional Stokes parameter profiles in a single-shot which yields the amplitude and phase profiles of the entire optical field without some of the complications of imaging or digital processing methods. We have experimentally demonstrated our method which is able to obtain a variety of amplitude and phase profiles of coherent scalar optical fields with high resolution and accuracy. Because our method does not need an additional reference beam, the common path of the optical configuration can minimize the effect of vibration and reduce the complexity of the optical system. As a scan-free single-shot method, the phase structure switches between various rotating Zernike polynomial functions, and the dynamic evolution of complex fields is recorded continuously by our direct measurement method.

Next, our direct measurement technique is developed for measuring the spatial polarization profile of the light field. A fully polarized light beam can be described by the superposition of two scalar waves with orthogonal polarization components. For characterizing the polarization profiles, a Sagnac interferometer is utilized to separate two orthogonal polarization components of fully polarized vector beams. Through a sequence of polarization component separation, a
weak polarization perturbation, and a polarization-resolving imaging process, the final readout is directly related to the complex amplitude profiles of the two polarization components of the vector beam. We have demonstrated our direct measurement protocol by measuring a variety of vector beams that are relevant to optical information science, including vector vortex beams and full Poincaré beams. With the complete information of the amplitude, phase and polarization of the light field, we can identify and evaluate the evolution of any fully polarized vector beams. Our experimental results have shown consistently high data fidelity. Additionally, our ability to reveal both the complex-amplitude and polarization information of a light field provides a robust and versatile metrology tool for fundamental studies of vector beams and a wide spectrum of applications utilizing vector beams.

The ability to obtain the three-dimensional (3-D) information of a scene or a volume object illuminated by incoherent light has diverse applications. To obtain the 3-D incoherent scene, we have developed a self-interference polarization holographic imaging (Si-Phi) technique that combines our modified direct measurement method and an incoherent holography technique. Differing from the direct measurement technique that only modulates polarization in a small area, our new method modulates the entire field of light from an incoherent scene on one of the two orthogonal polarization components. Then the complex hologram is composed of the unmodulated field interfering with another component. This is captured by a polarization-resolving detector array in a single shot while utilizing the full spatial bandwidth of the optical imaging system. Using the back-propagation of the hologram, we can numerically retrieve the transverse intensity profile of the scene at any desired object plane and reconstruct the 3-D information. We also experimentally prove the real-time imaging capability of our Si-Phi technique by tracking an object’s position in 3-D space.

Our direct measurement technique provides an effective method to determine the high dimensional information of optical fields or photons. One of the most popular examples of high-dimensional photons is a vector beam. With a better understanding of vector beams from our novel detection method, we design an optical communication system based on them. Our communication
system has the capability of multiplexing, encoding, and decoding high-dimensional information, and precisely transferring the data for free-space communication through atmospheric turbulence. Here we utilize a controllable-temperature hotplate to generate different strengths of turbulence. Comparing with OAM beam free-space communication, our experimental results present that vector beams are more robust than OAM ones for free-space communication under strong turbulence conditions.

6.2 Future Work

In this dissertation, we have demonstrated that our direct measurement technique can measure the phase profile of a light field. We are interested in extending our direct measurement method into a real application namely in quantitative phase imaging microscopy. In our proposal, a specially designed laser source microscopy setup will be followed by a spatial light modulator in order to perform weak perturbations and resolve polarization states to record imaging profiles. Our proposed direct measurement microscopy will be utilized to detect a micro-fabricated glass sample along with some biological samples to demonstrate the capability of quantitative phase imaging.

When using a coherent light source, speckle noise is inevitable in wide-field imaging. We will apply a partially coherence light source such as a high power LED to perform the direct measurement. Due to the direct measurement only being performed with a a small polarization rotation, the reference field components may be so weak that the signal to noise ratio of the imaging result is limited. To overcome this limit, we would like to use an LED light source with a ring aperture for illumination similarly to phase contrast microscopy. Additionally, a ring structure polarization rotation will replace the weak polarization perturbation in the direct measurement. The new optical configuration of microscopy will more than likely produce an additional phase aberration on the observed phase image, we plan to develop an algorithm to eliminate the phase aberration.

In the realm of optical communication, we are interested in the performance of free-space, vector beam communication in real atmospheric turbulence. We will improve our communication
system for free-space communication at a distance above 100 m under real atmospheric conditions. We are also interested in the performance of the vector beams in oceanic turbulence. We plan to add Maalox liquid in a sink with pumping equipment to simulate the ocean turbulence to evaluate beam vector optical communication in the oceanic condition.
Appendix: List of Publications

Peer-reviewed Journal Articles:


Refereed Conference Papers:

About the Author

Ziyi Zhu was born in 1986 in Beijing, China. He received a Bachelor of Science degree in materials science and engineering from University of Science and Technology of China in 2009. He came to United States in 2010, and obtained a Master of degree from Stevens Institute of Technology in 2012, majored in materials engineering. He then joined the University of South Florida in the fall of 2012, and switched his interests in physical optics.