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Students’ Opportunity To Learn Surface Area And Volume In Middle Grades Mathematics Textbooks

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Students’ Opportunity To Learn Surface Area And Volume

In Middle Grades Mathematics Textbooks

by

Sofia Hatziminadakis

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Curriculum and Instruction with an emphasis in Mathematics Education Department of the Teaching and Learning College of Education University of South Florida

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November 2, 2018

Keywords: Geometry, Opportunity to Learn, Mathematics Textbooks, Content Analysis

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DEDICATION

I dedicate this dissertation to my Lord and Savior Jesus Christ for granting me the strength and wisdom to complete this work. I also dedicate this work to my departed and dearly loved and missed father, Antonios, for always believing in me.
ACKNOWLEDGEMENTS

First of all, I would like to extend my sincere gratitude and appreciation to my major professor, Dr. Eugenia Vomvoridi-Ivanovic, for her consistent guidance, support, and encouragement. I would also like to thank my committee members, Dr. Jennifer Wolgemuth, Dr. Sarah VanIngen, and Dr. Mile Krajcevski, for their invaluable assistance and constructive comments that made it possible for me to complete this dissertation.

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ABSTRACT

I conducted a content analysis to examine the treatment of the surface area and volume concepts within four published middle-grades mathematics textbook series. In particular, I examined the treatment of the surface area and volume concepts in terms of the location of surface area and volume lessons in the textbook and the number of pages and lessons devoted to these concepts. I also investigated the sequence of the instructional blocks of surface area and volume lessons. In addition, I evaluated the tasks included in these lessons in regards to the performance expectations of students, the types of visual representations of 3D objects, and the level of mathematical complexity. At last, I examined the extent to which the content of surface area and volume lessons address the Common Core Content Standards (CCCS) for 6-8 geometry that are aligned with these topics.

I used content analysis to analyze relevant content in a total of twelve middle-grades student edition mathematics textbooks from two popular textbook series, Go Math!(GM) and Glencoe Math (GMC); and two alternative textbook series, Connected Mathematics 3 (CM) and University of Chicago School Project (UCSMP). First, I used Flanders’ (1994) counting method to examine the physical characteristics of textbooks, such as the location of the surface area and volume lessons in the textbook, the number of pages and lessons devoted to these concepts. Second, I analyzed the sequence of the instructional blocks of surface area and volume lessons by using content analysis. Third, I adapted the Trends in International Mathematics and Science Study [TIMSS] (2002) Performance Expectations for Mathematics Framework to examine the performance expectations of students within tasks. Fourth, I developed and used the Visual
Representations of 3D Objects Framework to examine the types of visual representations of 3D objects included in the tasks. Fifth, I employed the Mathematics Framework for the 2007 National Assessment of Educational Progress (NAEP) to examine the level of mathematical complexity of tasks. Finally, I created the CCCS for 6-8 Geometry Components guideline to examine to what extend the surface area and volume lessons address the geometry content standards.

Results indicated that the majority of textbooks place the concepts of surface area and volume towards the end of the textbook. Small percentages of instructional pages and lessons are devoted to these concepts in all textbooks. Findings also revealed great similarities among the instructional blocks of lessons within three textbook series (GM, GMC, and UCSMP). The majority of tasks within all textbook series contain miniscule amounts of important performance expectations such as justifying and proving and visual representations of 3D objects such as nets and pictures. A significant amount of tasks are of moderate complexity across all textbook series. Analysis also showed that the CM textbook series offers the greatest opportunity for students to generate visual representations of 3D objects and contains the largest amount of high complexity tasks. At last, nearly all lessons address the appropriate geometry content standard among all textbook series. Limitations of the study, implications for mathematics education, as well as recommendations for future research are also presented.
CHAPTER 1: INTRODUCTION

Geometry is an essential part of the mathematics curriculum (Battista & Clements, 1988; Choi & Park, 2013; Mistretta, 2000; National Council of Teachers of Mathematics [NCTM], 2000). It is the study of shapes, motions, and relationships in a spatial space (Clements & Battista, 1986; Clements, 1998). Through the study of geometry, students make sense of the space around them (NCTM, 1989, 2000; Sherard, 1981). Both the *Curriculum and Evaluation* (1989) and *Principles and Standards for School Mathematics* (2000) documents have advocated that geometry is more than definitions; it is a place where students should observe, explore, and reason the structure, characteristics and relationships of geometric shapes in order to interpret and describe their physical environments. Geometry should be a place that allows students to develop their geometric reasoning and spatial abilities (NCTM, 2000). Similarly, the recently adopted Common Core State Standards in Mathematics (CCSSM) have emphasized the importance of geometry in helping students understand, describe, and reason about real-world situations involving geometrical concepts (National Governors Association, 2010).

**Spatial Geometry**

One important aspect of geometry is spatial geometry. Spatial geometry concentrates on examining the form, shape, size, pattern, and design of shapes (NCTM, 2000). The study of spatial geometry is important for several reasons. Spatial geometry provides students with knowledge to understand, represent, and solve problems in other areas of mathematics such as measurement and algebra (Dindyal, 2007) and in real-world situations (NCTM, 2000). It also helps students build understanding of basic mathematical concepts needed to move to higher
mathematics (NCTM, 2000; Seng & Chan, 2000). In addition, spatial geometry it is necessary for the study of other subjects such as science, engineering, and computer science (Clements, 1998; NCTM, 2000). Finally, it offers opportunities to develop students’ logical thinking abilities needed in problem solving (NCTM, 2000).

In the study of spatial geometry, spatial reasoning also called spatial thinking is fundamental (NCTM, 2000). Spatial reasoning focuses on the mental representation and manipulation of spatial shapes. Both NCTM (1989, 2000) documents have emphasized the importance of developing students’ spatial reasoning. For instance, spatial reasoning can help students learn how to use maps, planning routes, designing floor plans, and creating art (NCTM, 2000). Researchers have also noted about the importance of spatial reasoning in the teaching and learning of mathematics. For instance, Clements (1998) noted that spatial reasoning forms the foundation for learning mathematics.

Clements (1998) defined spatial reasoning as the ability to see, build, manipulate, and reflect on spatial images, objects, relationships, and transformations. Clements also stated that spatial reasoning includes two major spatial abilities: spatial orientation and spatial visualization. Spatial visualization is described as the ability to understand and manipulate two-dimensional (2D) and three-dimensional (3D) shapes. Spatial orientation is defined as the ability to understand and navigate on relationships between positions based on the observer’s position. The main difference between spatial orientation and spatial visualization is that spatial visualization involves creating mental images and manipulating them and spatial orientation involves the comprehension of these manipulations but do not necessarily need to create them mentally. Both spatial visualization and spatial orientation are essential components of spatial geometry (Clements, 1998; NCTM, 2000).
Researchers have claimed that spatial abilities are related to students’ success in geometry (Battista, 1990; Guzel & Sener, 2009; Pittalis & Christou, 2010; Pitta-Pantazi & Christou, 2010; Tarte, 1990) and mathematics in general (Cheng & Mix, 2014; Fennema & Sherman, 1977; Fennema & Tarte, 1985; Guay & McDaniel, 1977; Hegarty & Waller, 2005; Newcombe, 2010; Seng & Chan, 2000). As stated by Clements (1998), “spatial ability and mathematics achievement are related” (p. 10). Spatial abilities are also closely connected to the study of 3D shapes (Pittalis & Christou, 2010). More specifically, spatial abilities are related to students’ ability to solve 3D geometrical tasks such as computing the surface area and volume of 3D shapes (Pittalis & Christou, 2010).

Through the study of spatial geometry students can enhance their spatial abilities (Pittalis & Christou, 2010). The NCTM (1989, 2000) documents recommend that the middle-grades mathematics curriculum should include the study of the geometry of one, two, three-dimensions in a variety of tasks such as constructing nets, creating 3D shapes using 2D shapes, identifying and comparing 3D shapes and their properties, structuring arrays of cubes, and computing surface area and volume of 3D shapes in order for students to develop their spatial abilities (NCTM, 1989, 2000). Furthermore, the creators of the CCSSM have emphasized the importance of providing middle school students with the opportunity to reason and solve real-world problems involving constructing nets, drawing 3D shapes, and calculating the surface area and volume of 3D shapes (National Governors Association, 2010).

The study of spatial geometry starts by allowing students to investigate, analyze, and compare the characteristics and properties of 2D and 3D shapes by visualizing, drawing, and measuring them (Battista & Clements, 1988; NCTM, 1989, 2000). Next, students need to be provided with opportunities to use 2D representations of 3D shapes to visualize and understand
tasks involving surface area and volume (NCTM, 2000). Finally, students need to be provided with opportunities to examine, build, compose, and decompose 3D shapes by using paper sketches, geometric models, or dynamic geometry software in order to be able to solve surface area and volume tasks (NCTM, 2000, 2006).

The geometric concepts of surface area and volume are important components of the middle-grades mathematics curriculum and standards (CCSSI, 2010; NCTM 2000). However, both international and national studies have shown that United States (U.S.) students are underperforming in the area of geometry and in particular on geometric tasks involving geometric reasoning and spatial abilities that are essential abilities required to solve surface area and volume tasks.

**Geometry Achievement**

During the past three decades, results from international comparative studies have indicated that students from other nations are outperforming U.S. students in the content area of geometry (Beaton et al., 1996; Fleischman et al., 2010; Ginsburg et al., 2005; Lemke et al., 2004; Mullis et al., 1997; Mullis et al., 2000; Mullis et al., 2008; Mullis et al., 2012; Mullis et al., 2016). More recently, results from TIMSS (2007, 2011, 2015) revealed that the performance of U.S. eighth grade students in the content domain of geometry was relatively weak (Mullis et al., 2008; Mullis et al., 2012; Mullis et al., 2016). In fact, there was no significant increase in the geometry performance of U.S. eighth grade students from 2007 to 2011 (Mullis et al., 2008; Mullis et al., 2012). As stated by Battista (1999), “as numerous studies have shown, U.S. elementary and middle school students are failing to develop and adequate understanding of geometric concepts, geometric reasoning, and geometric problem solving” (p. 368). These results
suggest that U.S. students’ geometric reasoning and spatial abilities might not be properly developed in the geometry classroom.

Findings from national studies have also indicated that students have difficulties with solving geometric tasks that require geometric reasoning and spatial abilities. Over the past four decades, studies have shown that students exhibit low levels of geometrical thinking on geometrical tasks that require reasoning (Carroll, 1998; Mistretta, 2000, 2003; Senk, 1989). Furthermore, students have difficulties with solving tasks involving the use of spatial abilities such as constructing nets (Mariotti, 1989; Stylianou, Leikin, & Silver, 1999), drawing 3D shapes (Johar & Aklimawati, 2015; Mitchelmore, 1978, 1980), mentally manipulating 3D representations (Fujita et al., 2017), and computing the surface area and volume of solids (Battista & Clements, 1996; Ben-Chaim, Lappan, & Houang, 1985; Isiksal, Koc, & Osmanoglu, 2010; Tekin-Sitrava & Isiksal-Bostan, 2014).

In response to these discouraging findings mathematics reform movements have argued about the importance of providing students with increased opportunities to develop their geometric reasoning and spatial abilities that are essential skills required to solve surface area and volume tasks. An important step in understanding the opportunities provided to students to learn mathematics is by examining the curriculum materials such as curriculum guides and textbooks (Schmidt et al., 1996; Valverde et al., 2002).

The Importance of Geometry Curriculum

The most fundamental component in teaching and learning mathematics is the intended curriculum (Schmidt, Houang, & Cogan, 2002). Lloyd, Cai, and Tarr (2017) divided the curricula into three levels: intended curriculum, enacted curriculum, and attained curriculum. The intended curriculum is defined as the national, state, or district expectations for mathematics
learning as reflected in curricular materials such as textbooks. All components of the curriculum are important but special attention should be given to the intended curriculum because it influences students’ opportunity to learn mathematics (Begle, 1973; Schmidt et al., 2002). Students’ opportunity to learn is described as students’ opportunity to encounter, experience, and learn particular topics (Houang & Schmidt, 2008). Students’ opportunity to learn is directly affected by educational policies and by curricular materials such as curriculum guides and textbooks (Houang & Schmidt, 2008).

Many educational researchers have criticized the U.S. intended mathematics curriculum as reflected in textbooks and state standards for its lack of coherence, consistency, and rigor (Houang & Schmidt, 2008; Schmidt et al., 2002; Valverde et al., 2002). The U.S. intended mathematics curriculum has been described as “mile wide and an inch deep” (Schmidt et al., 2002). For example, the analysis of the TIMSS (1999) data showed that the U.S. intended mathematics curriculum is unfocused, incoherent, and lacks rigor compared to the curriculum of top achieving TIMSS countries (Schmidt et al., 2002). The authors concluded that the U.S. intended mathematics curriculum is unfocused, incoherent, and unchallenging because of the poorly designed standards and textbooks.

**National Recommendations and Mathematics Standards**

This lack of a focused, coherent, and rigor national curriculum and U.S. students’ continuous underperformance in international and national studies led to the development of various documents from the NCTM such as *The Curriculum and Evaluation Standards* (NCTM, 1989), *The Principles and Standards for School Mathematics* (NCTM, 2000), and *The Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006). The NCTM (1989, 2000, 2006) documents have called for curricular
change to provide students with the opportunity to learn mathematics (Reys, Reys, & Rubenstein, 2010; Stein, Remillard, & Smith, 2007). These documents have also called for the development of more rigorous and challenging mathematics for all students.

The three documents published by the NCTM (1989, 2000, 2006) to address these long standing concerns about student achievement have strongly influenced the K-12 mathematics curriculum materials (Choi & Park, 2013). All three documents have also emphasized the importance of providing students with the opportunity to explore 3D geometrical concepts in order to develop their geometric reasoning and spatial abilities required to solve surface area and volume tasks. The Curriculum and Evaluation Standards (NCTM, 1989) offers recommendations regarding the development of students’ geometric reasoning and spatial abilities. In this document it is suggested that students must be provided with various opportunities to investigate the characteristics of 2D and 3D shapes. In this document, the NCTM also recommends that students must be provided with opportunities to discover and explore the relationships between 2D and 3D shapes.

The Principles and Standards for School Mathematics (NCTM, 2000) includes a set of standards about the geometrical knowledge and skills that all students should acquire from Kindergarten through grade 12. It also contains standards for evaluating the quality of both the curriculum and student achievement. In this document it is suggested that students must be provided with the opportunity to develop their geometric reasoning and spatial abilities through the use of physical and visual representations. For instance, students need to experience and explore a variety of geometric shapes by drawing, composing, and decomposing them. Students must also be exposed to activities that require them to build and move from 2D to 3D shapes and their representations. In addition, students need to be exposed to activities that allow them to
create and interpret the top and side views of 3D shapes. Students can also develop their geometric skills by being challenged to find the minimum number of blocks needed to build the structure.

The *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM, 2006) includes recommendations for the teaching and learning of mathematics. This document aims to provide common mathematical focus points for each grade level. It was also created to address and make connections among important K-8 mathematical topics. This document emphasizes the importance of providing students with opportunities to develop an understanding of 3D shapes. It emphasizes that students need to be provided with opportunities to compose and decompose 3D shapes in order to develop their geometric reasoning and spatial abilities.

The recently adopted CCSSM standards were also designed to provide common learning goals and ensure student opportunity to learn at the national and state level. The developers of the CCSSM aimed to include higher levels of cognitive demand than the previous state standards. They also “strove for coherence as well as focus” (Cobb & Jackson, 2011, p. 184). The ultimate goal of the development of the CCSSM was to create coherent, focus, and rigorous standards in order to increase students’ international and national performance (National Governors Association, 2010).

Forty-two states, the District of Columbia, four territories, and the Department of Defense Education Activity (DoDEA) have adopted the CCSSM. The adopted CCSSM standards promote the implementation of geometric tasks that demand reasoning, explanation, justification, and application, and can be presented in real-world contexts (National Governors Association, 2010). The CCSSM emphasizes the importance of exposing students to meaningful, rigorous,
and worthwhile geometric tasks that can help them develop their geometric reasoning and spatial abilities.

**The Importance of Textbooks**

National curriculum documents and standards influence the design of textbooks (Ponte & Marques, 2011; Schmidt et al., 2002; Zhu & Fan, 2006). Most publishers use the national recommendations and standards to design mathematics textbooks (Houang & Schmidt, 2008; Reys et al., 2010; Stein et al., 2007). As stated by Houang and Schmidt (2008), “textbook authors write to support implementations of national intentions” (p. 3). Also, textbooks may include the education philosophies and pedagogical values of the textbook developers (Zhu & Fan, 2006).

Over the past four decades, researchers have repeatedly reported that textbooks influence students’ opportunity to learn because teachers use textbooks as their primary instructional tool for the teaching and learning of mathematics (Begle, 1973; Johansson, 2005; Thompson, Senk, & Johnson, 2012). In fact, textbooks play an essential role in mathematics education all around the world, because teachers use textbooks as a main resource for planning instruction, and for structuring the course (Reys, Reys, & Chavez, 2004). At the national level, Tyson-Bernstein and Woodward (1991) reported, “textbooks are a ubiquitous part of schooling in the United States” (p. 91). At the international level, Robitaille and Travers (1992) stated:

Teachers of mathematics in all countries rely heavily on textbooks in their day-to-day teaching, and this is perhaps more characteristic of the teaching of mathematics than of any other subject in the curriculum. Teachers decide what to teach, how to teach it, and what sorts of exercises to assign to their students.
largely on the basis of what is contained in the textbook authorized for their course (p. 706).

As part of the TIMSS (2011) study, mathematics teachers were surveyed about the classroom materials used for teaching mathematics at the fourth and eighth grades. It was found that 75% of the fourth grade teachers and 77% of the eighth grade teachers used textbooks as their basis for mathematics instruction (Mullis et al., 2012).

Textbooks influence what students learn (Begle, 1973; Schmidt et al., 2002; Stein et al., 2007; Zhu & Fan, 2006). In school systems, textbooks serve as a link between the intended and attained curriculum (Johansson, 2005; Thompson et al., 2012). As noted by Stein and colleagues (2007) all types of curriculum influence students learning but a direct link exist between the intended curriculum (e.g., textbooks) and students’ learning. Begle (1973) noted, “most students learning is directed by the text rather than the teacher” (p. 209). Therefore, if a topic it is not included in the text, most likely it will not be taught (Begle, 1973; Flanders, 1994; Stein et al., 2007; Thompson et al., 2012; Tornroos, 2005). However, others have argued that the presence of a topic in the text is not enough. The way the topic is presented in the text is equally important (Stein et al., 2007; Thompson et al., 2012).

Based on the central role that textbooks play in students’ learning of geometry and evidence that students are underperforming in solving geometric tasks, it’s important to examine the opportunities textbooks offer to students to develop their geometric reasoning and spatial abilities. During this study, I examined the physical characteristics of the textbooks, the structure of the lessons, the performance expectations of students within tasks, the types of visual representations of 3D objects included in tasks, the level of mathematical complexity of tasks,
and the content of the lessons within middle-grades mathematics textbooks in order to determine students’ opportunity to learn the geometric concepts of surface area and volume.

Theoretical Considerations

Textbooks have been recognized as the primary source of mathematics instruction (Li, 2000). There are two types of mathematics textbooks: the conventional curriculum materials also referred to as popular textbooks and the standard-based curriculum materials also called alternative textbook (Stein et al., 2007). The popular textbooks are commercially created textbooks usually not influenced by reform documents (Stein et al., 2007). These types of textbooks focus on the development of procedural skills rather than conceptual skills (Polikoff, 2015). In contrast, the alternative textbooks are designed based on the NCTM recommendations and supported by the National Science Foundation (NSF) (Choi & Park, 2013; Lloyd et al., 2017; Reys et al., 2004; Stein et al., 2007). The ultimate goal of the alternative textbooks is to develop students’ mathematical thinking by exposing them to rigorous tasks in order to provide all students with the opportunity to learn mathematics (Reys et al., 2004; Stein et al., 2007). Given the different opportunities textbooks offer in the teaching and learning of mathematics, it is important to examine both popular and alternative middle-grades mathematics textbooks.

According to Valverde and colleagues (2002) an examination of the structural and pedagogical features of textbooks can help us understand educational opportunities in the classroom. Therefore, I examined both the structural and pedagogical features of middle-grades mathematics textbook in order to determine students’ opportunity to learn the geometric concepts of surface area and volume. In terms of structural features, I examined the physical characteristics of the textbooks such as the location of surface area and volume lessons in the textbook and the number of pages and lessons devoted to these concepts. I also investigated the
sequence of the instructional blocks of surface area and volume lessons. In regards to pedagogical features, I examined the tasks contained in the surface area and volume lessons in terms of their performance expectations, types of visual representations of 3D objects, and level of mathematical complexity.

One important component of the structural features of the textbooks are the physical characteristics of the textbooks such as the number of pages and number of lessons devoted to a concept, and the location of lessons within the text (Valverde et al., 2002). These types of physical characteristics of textbooks can provide us with important information about the possibilities and limitations of students’ opportunity to learn mathematics (Valverde et al., 2002). Chavez (2003) stated that the amount of pages that the textbook devotes to a topic influence the amount of instructional time that topic receives. Grouws and Smith (2000) reported that many teachers do not “cover” the entire book. Therefore, it is vital that the physical characteristics of textbooks are examined.

Many researchers have argued the importance of examining both the structure and the content of the lessons within mathematics textbooks (Alajmi, 2012; Begle, 1973; Huntley & Terrell, 2014; Lo, Cai, & Wafanabe, 2001; Valverde et al., 2002). Valverde and colleagues (2002) noted, “how content is presented in textbooks (with what expectations for performance) is how it will likely be taught in the classroom” (p. 125). Alajmi (2012) also stated that what topics are covered and how these topics are presented influence students’ opportunity to learn mathematics. Other researchers have argued that the pedagogical approaches used to present mathematical concepts provide different opportunities for students’ learning (Stein et al., 2007). Begle (1973) reported that students that used mathematics textbooks that emphasized the development of conceptual skills outperformed students that used mathematics textbooks that
focused on the development of procedural skills. For these reasons, it is important to investigate the sequence of instructional blocks of surface area and volume lessons. It is also imperative to examine the performance expectations of students within tasks.

The use of visual representations has long been recognized as a necessary component for the teaching and learning of geometry (Gutierrez, 1996). The NCTM (2000) has emphasized the importance of the use of visual representations to help students develop their geometric reasoning and spatial abilities needed to solve surface area and volume tasks. Students can develop their spatial visualization skills by being provided with opportunities to visualize and deform 2D and 3D shapes (NCTM, 2000). Studies have also indicated that visual representations can help develop students’ conceptual understanding (Xin, 2007; Zhu & Fan, 2006). Therefore, it is vital to investigate the types of visual representations of 3D objects used to help students understand and solve surface area and volume tasks.

Studies have shown that the nature of tasks can influence the way students think and it can limit or broaden their views of the subject matter with which they are engaged (Boston, 2012; Henningsen & Stein, 1997). Mathematical tasks can provide students with the opportunity to engage in high-level cognitive processes or low-level cognitive processes (Boston, 2012). Others have argued that the analyses of mathematical tasks can provide valuable information about students’ opportunity to learn mathematics (Doyle, 1983,1988; Li, 2000). However, a task as presented in the curriculum provides an opportunity to influence students’ mathematical thinking (Charalambous, 2010; Choppin, 2011). Thus, it is imperative to examine and document the level of mathematical complexity exhibited among the surface area and volume tasks within and across published mathematics textbooks.
Several scholars also believe that the implementation of the CCCS can help improve instruction and thus increase students’ opportunity to learn various mathematical concepts (Polikoff, 2015; Porter et al., 2011; Schmidt & Houang, 2012). Therefore, I also examined the extend to which the surface area and volume lessons address the CCCS for 6-8 geometry that are aligned with these topics.

The study of geometry in middle-grades is important because it provides a link between the informal explorations of geometric topics in elementary grades and the more formalized study of abstract geometric concepts in high school (NCTM, 1989, 2000). For instance, investigation of 3D shapes involving surface area and volume fosters understanding of other areas of mathematics such as measurement. Therefore, it is crucial to study the treatment of the geometric concepts of surface area and volume in middle-grades mathematics textbooks.

**Problem Statement**

International and national studies have indicated that U.S. students are underperforming on geometric tasks that require geometric reasoning and spatial abilities. For example, results from TIMSS (2011, 2015) studies revealed that the geometry performance of U.S. eighth grade students was weak compared to the performance of eighth grade students in top achieving countries (Mullis et al., 2012; Mullis et al., 2016). Indeed, U.S. eighth grade students performed worst in the content area of geometry compared to the content areas of number, algebra, and data and chance (Mullis et al., 2012; Mullis et al., 2016). These findings demonstrate the need to provide students with increased opportunities to develop their geometric reasoning and spatial abilities required to solve surface area and volume tasks.

Textbooks represent the most important feature of the teaching and learning of mathematics (Johansson, 2005) because teachers rely heavily on them for their daily instruction
Textbooks have a strong impact on what and how mathematics is taught (Huntley & Terrell, 2014; Schmidt et al., 2002; Thompson et al., 2012), thus textbooks influence what students learn (Reys et al., 2004). Indeed, textbooks indicate students’ opportunity to learn mathematics (Johansson, 2005; Yang, Tseng, & Wang, 2017).

Based on the role and influence of textbooks on students’ learning, textbook analysis is the first step in understanding students’ opportunity to learn mathematics (Huntley & Terrell, 2014; Thompson et al., 2012). Further, the geometric concepts of surface area and volume are important components of the middle school mathematics curriculum (NCTM, 2000). However, no previous content analysis study on the treatment and opportunity to learn the geometric concepts of surface area and volume in U.S. middle-grades mathematics textbooks has been published to date. This dissertation study examined the treatment of the geometric concepts of surface area and volume within published popular and alternative middle-grades mathematics textbooks in the U.S.

**Purpose of the Study**

The purpose of this content analysis was to examine the treatment of the surface area and volume topics in popular and alternative middle-grades mathematics textbooks published within the past ten years. Content analysis is a research method that is used to systematically evaluate the symbolic content of all forms of written communications (Kolbe & Burnett, 1991). This study had five foci: a) to examine the physical characteristics of textbooks such as the number of pages and lessons devoted to surface area and volume concepts and the location of these concepts within the middle-grades mathematics textbooks to understand the possibilities and limitations of learning these concepts b) to investigate and describe the sequence of instructional
blocks of surface area and volume lessons in order to determine the opportunity to learn these concepts as recommended in the curriculum standards, c) to examine the performance expectations of students within tasks in order to understand the different performance requirements contain in these tasks, d) to analyze the types of visual representations of 3D objects within tasks used to help students understand the geometric concepts of surface area and volume, e) to examine the tasks in terms of their level of mathematical complexity to determine the extent to which these tasks follow the national recommendations and standards, and f) to evaluate the content of the surface area and volume lessons to determine the extent to which these lessons address the CCCS for 6-8 geometry that are aligned with these topics.

**Research Questions**

The purpose of this study is to examine the treatment of the surface area and volume concepts in student editions of middle-grades mathematics textbooks. This study was guided by the following three research questions:

1) Within published mathematics textbook series and across different publishers, what are the structural features devoted to the concepts of surface area and volume? In particular,
   a. Where are the surface area and volume lessons located and how many pages and lessons are devoted to surface area and volume?
   b. How are the instructional blocks of surface area and volume lessons sequenced?

2) What are the pedagogical features of the tasks included in the surface area and volume lessons within a published mathematics textbook series, and across different publishers? Specifically,
   a. What are the performance expectations of students within these tasks?
   b. What types of visual representations of 3D objects are included in these tasks?
c. What is the level of the mathematical complexity of these tasks?

3) To what extent do the content of surface area and volume lessons address the Common Core Content Standards for 6-8 geometry that are aligned with these topics?

**Significance of Study**

The role of the curriculum is to specify goals, topics, sequences, instructional activities, and assessment methods (NCTM, 1989). However, the mathematics curriculum in the U.S. has been defined as highly repetitive (Flanders, 1994), unfocused, unchallenging, and incoherent (Houang & Schmidt, 2008; Schmidt et al., 2002). Given the role of the curriculum in students’ learning of mathematics and concerns with its structure and content, the NCTM (1989, 2000, 2006) documents were created to response to the call for reform in the teaching and learning of mathematics. These documents content principles and standards designed to help improve mathematics education. In addition, the CCSSM were created to provide common goals and expectations for the mathematical knowledge and skills students need to develop at each grade level (National Governors Association, 2010).

International studies have indicated that U.S. eighth grade students tend to underperform on geometric tasks, especially on tasks that are related to students’ geometric reasoning and spatial abilities (Ginsburg et al., 2005; Mullis et al., 2012; Mullis et al., 2016). National studies have also indicated that students have difficulties with visualizing 3D shapes (Carpenter et al., 1975; Hirstein, 1981), and solving volume tasks (Battista & Clements, 1996; Ben-Chaim et al., 1985). Many researchers have recognized the need to improve students’ geometric reasoning and spatial abilities that are essential skills required to solve surface area and volume tasks (Battista, 1999; Clements & Battista, 1992; Hoffer, 1981; Pittalis & Christou, 2010).

Many teachers use textbooks as their main resource for planning instruction
(Reys et al., 2004; Robitaille & Travers, 1992). Studies have shown that middle–grades mathematics teachers heavily rely on the use of published textbooks (Banilower et al., 2013; Weiss, 1978, 1987; Weiss, Matti, & Smith, 1994; Weiss et al., 2001). Weiss and colleagues (1978) revealed that more than ninety percent of the middle-grade mathematics classes used commercially published textbooks, a finding supported by their later research about the use of textbooks (Weiss, 1987; Weiss et al., 1994; Weiss et al., 2001). More recently, Banilower and colleagues (2013) found that more than eighty percent of the middle-grade mathematics classrooms rely on a single textbook. In addition, the majority of mathematics teachers consider their textbooks to be of relatively high quality (Banilower et al., 2013; Weiss, 1987; Weiss et al., 1994; Weiss et al., 2001).

The mathematics curriculum is reflected and delivered by the use of textbooks in the mathematics classroom (Lloyd et al., 2017; Schmidt et al., 2002; Thompson et al., 2012; Zhu & Fan, 2006). Therefore, it’s important to examine and document the opportunities to learn mathematics textbooks offer to students. A content analysis of textbooks helps understand the process of the teaching and learning mathematics (Johansson, 2005; Lo et al., 2001; Thompson et al., 2012). The ultimate goal of this study was to inform the research community and policymakers regarding the learning opportunities presented to students to learn and understand the geometric concepts of surface area and volume in popular and alternative middle-grades mathematics textbooks.

**Delimitations**

There are three delimitations associated with this study. First, I only examined middle-grades mathematics textbooks because the concepts of surface area and volume are mainly introduced and developed in grades 6-8 (CCSSI, 2010; NCTM, 2000). Second, I included
textbooks with the largest market share in this sample. I selected and examined two popular and two alternative middle-grades mathematics textbook series from three main publishing companies. In particular, I chose the GM (Grades 6-8) and GMC (Course 1,2, and 3) popular middle-grades mathematics textbook series for this study. I also selected the CM (Grades 6-8) developed from the Connected Mathematics Project [CMP] (Lappan et al., 1996) and UCSMP (Pre-Transition Mathematics, Transition Mathematics, Algebra) alternative middle-grades mathematics textbook series.

Lastly, I adopted two existing frameworks: the TIMSS 2002 Performance Expectations for Mathematics (Valverde et al., 2002) and the Mathematics Framework for the 2007 NAEP. I selected these well-known frameworks because they have been previously used in studies to detect differences among tasks. I also developed and used the Visual Representations of 3D Objects framework based on the national recommendations and standards (CCSSI, 2010; NCTM, 2000) and the CCCS for 6-8 Geometry Components guideline based on the CCSSM (CCSSI, 2010).

**Definition of Terms**

Alternative Textbooks- are mathematics textbooks designed based on the national recommendations and standards to provide greater emphasis on the development of conceptual understanding through problem solving (Stein et al., 2007).

Curriculum- for the purpose of this study, curriculum is defined as the intended curriculum that is replicated in textbooks.

Middle Grades- for this study consists of grades 6,7, and 8.

Opportunity to learn- for this study, students’ opportunity to learn is defined as students’ opportunity to encounter, experience, and learn particular topics (Houang & Schmidt, 2008).
Popular Textbooks- are commercially and widely used textbooks that usually focus on the development of procedural skills rather than conceptual skills (Stein et al., 2007).

Visualization-is the ability to view and interpret objects such as pictures, 3D representations, schematic representations, and animations in order to understand something other than the object itself. These objects may appear on different types of media such as paper, computer screens, and slides (Phillips, Norris, & Macnab, 2010).

Surface Area- is the total area of the exterior faces of a three-dimensional figure (Miles & Williams, 2016, p. 154).

Task- is defined as a single complex problem that focuses students’ attention on a specific mathematical concept (Stein, Grover, & Henningsen, 1996).

Three-dimensional- is a term used to represent a shape that has length, width and height.

Two-dimensional- is a term used to represent a shape that has width and length but not depth.

Volume- is the amount of space contained in a solid (Miles & Williams, 2016, p. 154).

Summary

Textbooks are used to represent the national or state curriculum and standards (Schmidt et al., 2002). Research has shown that teachers use textbooks as their primary resource to teach mathematics (Reys et al., 2004; Robitaille & Travers, 1992). Many researchers have argued that students’ opportunity to learn mathematical concepts depends on the materials they are taught (Begle, 1973; Schmidt et al., 2002). Therefore, textbooks play a critical role in students’ opportunity to learn mathematical concepts.

NCTM has emphasized the importance of helping middle-grade students develop their geometric reasoning and spatial abilities required to solve surface area and volume tasks. For instance, the NCTM (2000) document recommends that middle-grade students should be
provided with increased opportunities to explore and solve problems involving surface area and volume. It also recommends that the majority of instructional time in middle-grades should be devoted to address algebraic and geometric concepts. However, researchers have claimed that U.S. students are underperforming in the area of geometry, especially in spatial geometry. Therefore, it is important to examine the opportunities middle-grades mathematics textbooks offer students to learn the geometric concepts of surface area and volume.

In this study, I examined the treatment of the geometric concepts of surface area and volume in popular and alternative middle-grades mathematics textbooks. The results of this study can help curriculum developers make improvements on the treatment of the geometric concepts of surface area and volume in middle-grades mathematics textbooks. Additionally, the information provided by this study regarding the strengths and weaknesses of various textbooks can help teachers make instructional modifications to meet their students’ needs.
CHAPTER 2: LITERATURE REVIEW

The purpose of this dissertation study was to examine the treatment of the geometric concepts of surface area and volume in middle-grades mathematics textbooks. In the previous chapter, I discussed the importance of these concepts and significance of this study. In this chapter, I provide a review of the literature on several topics that guided this study. I divided the literature review into three major sections. In the first section, I discuss the different types of mathematics curriculum, the role and use of textbooks. In the second section, I present several national and international mathematics textbook content analysis studies. In the third section, I review various theoretical considerations regarding students’ 3D geometric thinking and learning and research related to the investigation of students’ difficulties with 3D thinking in geometry and other fields. I conclude this literature review with a brief summary.

For this study, I conducted the literature selection in two phases. During the first phase, I located articles, conference reports, dissertations, and books using Google Scholar, Education Resources Information Center (ERIC), ProQuest, and JSTOR Education. I used the following subject headings and key terms to find related articles, conference reports, dissertations, and books for inclusion: mathematics textbooks, textbook use, textbook research, textbook analysis, volume and middle school students, and surface area and middle school students. During the second phase, I conducted additional research to locate important resources such as the Handbook of Research on Mathematics Teaching and Learning and Second Handbook of Research on Mathematics Teaching and Learning.
Types of Curriculum

Many educators have defined the term *curriculum* for different purposes (Houang & Schmidt, 2008; Lloyd et al., 2017; Robitaille et al., 1993; Schmidt et al., 1997; Schmidt et al., 2002; Stein et al., 2007; Valverde et al., 2002). In this study, I used the term *curriculum* to refer to as the substance or content of teaching and learning (Stein et al., 2007). Several educators have also attempted to describe the different levels of curriculum and their characteristics (Crosswhite et al., 1986; Lloyd et al., 2017; Robitaille et al., 1993; Schmidt et al., 1997; Stein et al., 2007; Valverde et al., 2002).

Lloyd and colleagues (2017) separated the curricula into three levels: *intended curriculum*, *enacted curriculum*, and *attained curriculum*. The *intended curriculum* is the mathematics content as prescribed by the national, state, or school district’s educational system. The *intended curriculum* is reflected in curricular materials such as textbooks. The *enacted curriculum* is the teaching and learning of mathematics that occurs as teachers and students interact with curricular materials. The *attained curriculum* is the outcome of students’ learning. That is, the *attained curriculum* describes and measures students’ learning and achievement in regards to mathematics.

In this study, I examined the *intended curriculum*. Based on Lloyd and colleagues (2017) work, I defined the *intended curriculum* as the national or state recommendations and standards replicated in textbooks. I also defined the textbook as the printed and published materials used for mathematics instruction. In addition, the textbook serves as the link between what should be taught and what is taught in the mathematics classroom (Valverde et al., 2002). Therefore, it is important to examine the role of the textbooks in the mathematics classroom. In the following paragraphs, I discuss the role of textbooks in the mathematics classroom.
The Role of Textbooks

Textbooks are documents that reflect the national, state, or school district’s curricular expectations, goals, and visions (Barr, 1988; Lloyd et al., 2017; Robitaille et al., 1993; Schmidt et al., 1996; Valverde et al., 2002). In particular, textbooks reflect the national, state, or school district’s curriculum regarding the scope and sequence of content, methods of instruction, and students’ performance expectations (Schmidt et al., 1996; Tyson-Bernstein & Woodward, 1991; Valverde et al., 2002). In the U.S., there is no national curriculum guide (Alajmi, 2012; Lloyd et al., 2017; Reys et al., 2004; Schmidt et al., 1996). The NCTM (1989, 2000) documents have influenced the design of U.S. textbooks (Lloyd et al., 2017; Ponte & Marques, 2011; Reys et al., 2004; Schmidt et al., 2002; Zhu & Fan, 2006).

Many researchers have suggested that textbooks are the most important feature of the teaching and learning of mathematics (Chang & Silalahi, 2017; Harris & Sutherland, 1999; Reys et al., 2004; Robitaille & Travers, 1992). The content and structure in textbooks define the scope and sequence of instruction (Barr 1988; Chang & Silalahi, 2017; Sosniak & Stodolsky, 1993; Tyson & Woodward, 1989; Tyson-Bernstein & Woodward, 1991). According to Li (2000), “the textbook provides a blueprint for content coverage and instructional sequences” (p. 236). Many teachers of mathematics use textbooks to decide what to teach and how to teach different mathematical concepts (Alajmi, 2012; Barr, 1988; Chang & Silalahi, 2017; Chavez, 2003; Fan & Kaeley, 2000; Fan et al., 2004; Reys et al., 2004; Robitaille & Travers, 1992; Tyson-Bernstein & Woodward, 1991). Educators view mathematics textbooks as the most important resource for students’ learning (Fan et al., 2004).

Researchers have also acknowledged the impact of textbooks on students’ opportunity to learn and achievement in mathematics (Robitaille et al., 1993; Schmidt et al., 1996; Tornroos,
2005; Valverde et al., 2002; Xin, 2007; Yang et al., 2017). Schmidt and colleagues (1996) stated that textbooks “provide a transition from curriculum intentions to learning opportunities” (p. 38). Studies have also shown that the textbooks influence students’ learning in terms of the quality and types of opportunities made available to them (Robitaille et al., 1993; Schmidt et al., 1996; Tornroos, 2005; Valverde et al., 2002; Xin, 2007). Therefore, differences in students’ curricular experiences can mean differences in opportunity to learn and achievement in mathematics. Other studies have also indicated that teachers and students rely heavily on their textbooks for the teaching and learning of mathematics (Bagley, 1931; Banilower et al., 2013; Braswell et al., 2001; Chavez, 2003; Grouws & Smith, 2000; Mullis et al., 2004; Mullis et al., 2008; Mullis et al., 2012; Tyson & Woodward, 1989; Tyson-Bernstein & Woodward, 1991). In the next paragraphs, I report the findings of national and international studies regarding the use of textbooks in the mathematics classroom.

The Use of Textbooks

Since the early 1930s, many researchers have documented the extensive use of textbooks in the classroom (Bagley, 1931; Banilower et al., 2013; Braswell et al., 2001; Chavez, 2003; Grouws & Smith, 2000; Mullis et al., 2004; Mullis et al., 2008; Mullis et al., 2012; Tyson & Woodward, 1989; Tyson-Bernstein & Woodward, 1991). These researchers have examined the frequency with which both teachers and students use textbooks on a regular basis. In this section, I report the findings of both national and international studies in regards to the use of textbooks in the teaching and learning of mathematics.

At the national level, Bagley (1931) first reported that textbooks hold a prominent position in the classroom. Other researchers have also claimed that 75% to 90% of the instructional time was structured around textbooks (Tyson & Woodward, 1989). In a later study,
Tyson-Bernstein and Woodward (1991) reported similar findings. Tyson-Bernstein and Woodward noted that approximately 90% of the instructional time was structured by instructional materials, such as textbooks.

More recently, analysis of the NAEP (2000) survey indicated that more than 70% of eighth grade teachers reported using textbooks as their main source for instruction on a daily basis (Grouws & Smith, 2000). Furthermore, 72% of the eighth grade students reported doing math problems from textbooks on a daily basis (Braswell et al., 2001). Interestingly, Grouws and Smith (2000) also noted that there was an 11% decrease from 1992 to 1996 in the use of textbooks on a daily basis in grade 8. Chavez (2003) also reported that approximately 70% of the middle-school teachers used their mathematics textbooks in more than 75% of their lessons.

At the international level, analysis of the TIMSS (2003) report showed that on average 65% of the eighth grade students had teachers that used textbooks as their primary source to teach mathematics (Mullis et al., 2004). In addition, findings from the TIMSS (2007) report indicated no significance difference in the use of textbooks by eighth grade teachers since 2003. Approximately, two-thirds of the eighth grade students had teachers that used textbooks for instruction on a daily basis (Mullis et al., 2008). However, analysis of the TIMSS (2011) report signified an increase on the international average of textbook use since 2007. At the eighth grade level, 77% of the students had teachers that based their instruction on mathematics textbooks (Mullis et al., 2012). Analysis of the TIMSS (2012) survey also indicated that more than 80% of the elementary, middle, and high school classes used published textbooks (Banilower et al., 2013).

Both at the national and international level, textbooks are heavily relied on despite the rapid technological advances and the use of Internet in more recent years. Therefore, the strong
influence and extensive use of textbooks can help improve or hinder the teaching and learning of mathematics based on the quality of their content (Sosniak & Stodolsky, 1993). In short, textbooks have a direct impact on students’ learning (Reys et al., 2004). The quality of the content and structure of textbooks can determine students’ opportunity to learn and achievement in mathematics (Chang & Silalahi, 2017; Robitaille & Travers, 1992; Tornroos, 2005; Xin 2007). As stated by Reys and colleagues (2004), “the choice of textbook often determines what teachers will teach, how they will teach it, and how their students will learn” (p. 61). Hence, the quality of textbooks has a strong impact on students’ opportunity to learn mathematics.

Researchers have expressed their concerns about U.S. mathematics textbooks’ quality. Some researchers have criticized U.S. textbooks for being too large, including too many topics, and repeating topics (Alajmi, 2012; Choi & Park, 2013; Reys et al., 2004; Schimdt et al., 1996; Schimdt et al., 1997; Valverde et al. 2002), while others have claimed that U.S. textbooks lack rigorous content (Incikabi & Tjoe, 2013; Reys et al., 2004; Yang et al., 2017; Zhu & Fan, 2006). Given the concerns about U.S. mathematics textbooks’ quality and the positive relationship between the quality of textbooks, opportunity to learn, and students’ achievement in mathematics, it is important to examine the quality of different mathematics textbooks (Tornroos, 2005; Xin, 2007). In the next section, I present large- and small-scale national and international textbook content analysis studies.

**Research on Mathematics Textbook Content Analysis**

Content analysis begins with the detailed examination of textbook’s lessons, activities, exercises, and other learning opportunities (American Association for the Advancement of Science [AAAS], 2000). Textbook content analysis studies are conducted to examine textbooks alignment with the national recommendations, standards, and the quality of their text (Polikoff,
2015). Confrey (2006) stated, “content analyses are necessary to document the coverage of a curriculum in relation to standards, and to assess the quality of the content and presentations” (p. 199). The National Research Council [NRC] (2004) also calls for content analysis of textbooks to examine the treatment of standards and investigate the depth of mathematical reasoning in the curriculum materials. In this respect, curriculum materials such as textbooks should provide opportunities for students to learn mathematical concepts in-depth (Thompson et al., 2012).

In the past three decades, several major textbook content analysis studies have been conducted to evaluate the quality of conventional curriculum materials, also called popular textbooks, and standard-based curriculum materials, also referred to alternative textbooks. The NCR (2004) and Project 2016 (AAAS, 2000) studies both evaluated the quality of content in popular and alternative textbook series. In addition to evaluating the textbook content, Jones and Tarr (2007) and Arnold and Son (2011) both examined the content in popular and alternative textbook series from a historical perspective. More recently, Huntley and Terrell (2014) analyzed the content in popular and alternative textbook series and Polikoff (2015) examined the content in popular textbooks. The ultimate goal of these studies was to evaluate the quality of the content in both popular and alternative textbook series regarding students’ opportunity to learn various mathematical concepts.

In 1999, the U.S. Department of Education reviewed the quality of both popular and alternative textbook series used in grades K-12. This evaluation was guided by eight criteria. These criteria were designed based on the Curriculum and Evaluation Standards (NCTM, 1989) document (NRC, 2004). The criteria were created to assess the quality of each curriculum and evaluate its alignment to the NCTM (1989) mathematics standards. Results revealed that alternative textbook series were closely aligned to the NCTM (1989) mathematics standards
compared to popular textbook series. Overall, alternative textbook series were rated better under these criteria than most popular textbook series.

Another large-scale study, Project 2016 (AAAS, 2000) analyzed both popular and alternative textbook series to assess the degree of alignments to the benchmarks and NCTM (1989) mathematics standards. Their sample included eight popular and four alternative middle-grades mathematics textbook series. Both teacher and student textbook editions of each textbook series were examined. The content of the textbook series was evaluated and rated based on six benchmarks: number concepts, number skills, geometry concepts, geometry skills, algebra graph concepts, and algebra equation concepts. In addition, twenty-four instructional criteria were considered. Findings revealed that three alternative and one popular textbook series addressed four or more benchmarks in depth. Only four textbook series were rated satisfactory in terms of their quality of instruction. Interestingly, none of the popular textbook series were rated as satisfactory. Their results supported the previous findings by the U.S. Department of Education (NRC, 2004) that alternative textbook series were aligned to NCTM (1989) mathematics standards.

Jones and Tarr (2007) examined the probability content in popular and alternative textbook series from a historical perspective. Jones and Tarr selected a total of twelve middle-grades textbook series from four recent eras of mathematics education. One popular and one alternative textbook series were chosen for each mathematics era. Jones and Tarr evaluated the mathematical problems by using the Mathematics Tasks Framework [MTF] (Stein et al., 2000). According to the MTF, there are four levels of mathematical complexity: memorization (low-level), procedures without connections (low-level), procedures with connections (high-level), and doing mathematics (high-level).
By analyzing the probability problems, Jones and Tarr (2007) found that there was a significant increase in the number of probability problems included in most textbook series between 1994-2000. Furthermore, the majority of the problems were coded as low cognitive demand in both popular and alternative textbook series in most eras. However, Jones and Tarr noted that there was a significant increase of high cognitive demand problems contained in the latest alternative textbook series. Jones and Tarr concluded that the most recent alternative textbook series supported the NCTM (1989, 2000) recommendations for the development of rigorous materials that can promote conceptual understanding. Jones and Tarr also stated that this alternative textbook series received the highest quality rating and was aligned to the NCTM (1989) mathematics standards in the Project 2061 (AAAS, 2000) study. Jones’s and Tarr’s findings coincide with results from the NCR (2004) and Project 2016 (AAAS, 2000) studies that alternative textbook series were aligned to NCTM (1989, 2000) recommendations of providing more rigorous materials.

Arnold and Son (2011) also sought to investigate potential differences in the quality of the pre-algebra content in popular and alternative textbooks from a historical prospective. For their analysis, Arnold and Son selected two historical textbooks, one alternative textbook, and two popular textbooks. Arnold and Son used a two-dimensional framework to examine the content and problems in each textbook. Arnold and Son evaluated the content based on allocation and topic, and problems in regards to context, response type, and cognitive level.

Analysis of the content revealed that the percent of materials devoted to pre-algebra content has increased from 1965-2005 across all textbooks. Analysis of the problems revealed that all problems in the alternative textbook were presented in the context of a real-world situation. In contrast, less than one-fifth of the problems in popular textbooks were presented in
the context of a real-world situation. In addition, all problems in the alternative textbook were coded as level 4 (extended thinking) for cognitive requirements. However, approximately two-thirds of the problems in popular textbooks were coded as level 1 (recall) for cognitive requirements.

Arnold’s and Son’s (2011) supported Jones’s and Tarr’s (2007) findings that alternative textbooks included higher percentage of high level cognitive demand problems compared to popular textbooks. However, it is important to point out that Jones and Tarr stated that both popular and alternative textbook series included a large percentage of low cognitive demand problems. In contrast, Arnold and Son noted that all problems in the alternative textbook were high cognitive demand.

More recently, Huntley and Terrell (2014) conducted a comparison study to examine the treatment of line equations in five textbook series. Their sample included one popular and four alternative textbook series. Huntley and Terrell analyzed each textbook series using four curriculum variables: content, cognitive behavior, real-world context, and tools. Based on Garden’s and colleagues (2006) taxonomy, the cognitive behavior of mathematical problems was classified as knowing, applying, and reasoning.

Content analysis revealed that popular textbook series included the highest percentage of linear equation problems. In terms of cognitive requirements and context, half of the problems in popular textbook series were classified as knowing and less than a quarter of the problems were set in real-world context. In general, alternative textbook series included higher percentage of problems that required reasoning but not all alternative textbook series included high percentage of problems set in real-world context. Huntley and Terrell (2014) also supported Arnold’s and Son’s (2011) and Jones’s and Tarr’s (2007) findings that alternative textbooks included higher
A proportion of high cognitive demand problems than popular textbooks. However, Huntley and Terrell did not confirm Arnold’s and Son’s findings that alternative textbooks place more emphasis on real-world situations. Huntley and Terrell noted that there was a variation regarding the percentage of problems set in real-world context across popular and alternative textbook series.

In contrast, Polikoff (2015) conducted a systematic examination of the alignment of popular textbooks to the CCSSM. In particular, Polikoff examined how well the content of eight popular fourth grade textbooks were aligned to the CCSSM in terms of topics and cognitive demand. Findings indicated that popular textbooks included an overwhelming amount of low-level cognitive demand problems. More than 85% of the total textbook content emphasized memorization and procedures. Four out of the eight textbooks included almost zero high-level cognitive demand problems. Also, all textbooks included topics not addressed in the standards. Despite the fact that Polikoff only examined the quality of content in popular textbooks, the results of her study coincided with the findings of previous studies (Arnold & Son, 2011; Huntley & Terrell, 2014; Jones & Tarr, 2007) regarding the lack of high-level cognitive demand problems in popular textbooks.

In sum, researchers have indicated that alternative textbooks are aligned to the national recommendations. The results from both NCR (2004) and Project 2016 (AAAS, 2000) studies were similar; their findings indicated that alternative textbook series were aligned to NCTM (1989) mathematics standards. As a result, the content in alternative textbooks was rated as higher quality than the content in popular textbooks. Notably, Jones’s and Tarr’s (2007) findings coincide with results from the NCR (2004) and Project 2016 (AAAS, 2000) studies that alternative textbook series were aligned to NCTM (1989, 2000) recommendations of providing
more rigorous materials. Furthermore, Jones’s and Tarr’s, Arnold’s and Son’s (2011), Huntley’s and Terrell’s (2014), and Polikoff’s (2015) results coincide regarding the lack of high-level cognitive demand problems in popular textbooks.

Arnold’s and Son’s (2011) findings indicated a higher percentage of high-level cognitive demand problems included in alternative textbook series compared to Jones’s and Tarr’s (2007) and Huntley’s and Terrell’s (2014) findings. Additionally, some of the results from Arnold and Son and Huntley and Terrell studies were different. On the one hand, Arnold and Son reported that all problems in the alternative textbook and approximately less than one-fifth of the problems in popular textbooks were set in real-world context. On the other hand, Huntley and Terrell reported mixed findings regarding the percentages of problems set in real-world context included in alternative and popular textbook series.

The studies reviewed in this section analyzed various mathematical concepts within textbooks. However, none of these studies examined the treatment of surface area and volume in middle-grades mathematics textbooks. This suggests that further research needs to be conducted to analyze the treatment of these geometric concepts in middle-grades mathematics textbooks. In the following section, I review several studies that have analyzed and compared U.S. popular and alternative mathematics textbooks to mathematics textbooks of other countries.

**International Mathematics Textbook Content Analysis Studies**

During the past three decades, another line of research has undertaken the task to evaluate the quality of content in U.S. mathematics textbooks to mathematics textbooks of other countries because of the continuous underperformance of U.S. students on international comparison mathematics studies (Choi & Park, 2013; Hong & Choi, 2014; Incikabi & Tjoe, 2013; Li, 2000, 2007; Ponte & Marques, 2011; Yang et al., 2017; Zhu & Fan, 2006). These cross-national
mathematics textbook analysis studies have been conducted to examine how different countries structure learning opportunities for their students. Some researchers have examined the content and problems in U.S. popular mathematics textbooks to mathematics textbooks of other countries (Incikabi & Tjoe, 2013; Li, 2000, 2007; Ponte & Marques, 2011). While others have analyzed the content and problems between U.S. alternative mathematics textbooks and mathematics textbooks of other countries (Choi & Park, 2013; Hong & Choi, 2014; Yang et al., 2017; Zhu & Fan, 2006). In this section, I report the results of these international mathematics textbook content analysis studies.

Li (2000) examined the content of integer problems in several middle-grades mathematics textbooks from U.S. and China. The textbooks under analysis were five U.S. popular textbooks and four Chinese textbooks. For the analysis, Li developed and used a framework that included three dimensions of problem requirements: mathematical features, contextual features, and performance requirements. In terms of mathematical features, the analysis revealed that U.S. and Chinese textbooks had an overwhelming amount of problems that required a single computation procedure. In regards to contextual features, both U.S. and Chinese textbooks had a large percentage of problems that were presented in a purely mathematical context. However, Li found a significant difference between U.S. and Chinese textbooks regarding problems’ performance requirements. Li noted that U.S. textbooks contained a larger number of high cognitive demand problems.

In a later study, Li (2007) investigated the content of mathematical problems in eighth grade mathematics textbooks regarding their cognitive expectations for students in three different educational systems. Li selected five U.S. popular textbooks, one Chinese textbook, and one Singaporean textbook. Li also developed a framework to code the mathematical problems in
each textbook. The framework included three dimensions: mathematics, context, and performance requirements.

Results showed that all textbooks included a large amount of problems that did not require explanation and were set in a purely mathematical context. However, further analysis indicated that U.S. textbooks contained more problems that required explanations and were set in different contexts than the Chinese and Singaporean textbooks. The U.S. textbooks also included fewer problems that required performing routine problems. Li (2007) also noted that the differences for mathematics and context were not significant but differences in cognitive requirements were significant between U.S. and Asian textbooks. Li’s findings were similar to the result of Li’s (2000) previous study in regards to U.S. textbooks including problems that did not require explanation and were set in a purely mathematical context. In both studies, Li also found that U.S. textbooks contained a larger percentage of high cognitive demand problems.

In a more recent study, Ponte and Marques (2011) sought to investigate the quality of sixth grade mathematics textbooks content of four different countries. Ponte and Marques selected one popular textbook from Portugal, Brazil, Spain, and U.S. Ponte and Marques also developed and used a framework to analyze the mathematical problems. The framework consisted of three categories: cognitive demand, structure, and context. Ponte and Marques also examined the structure and content of chapters within the four textbooks. Analysis of the mathematical problems showed that the majority of tasks were closed-ended in all textbooks. Interestingly, the U.S. textbook contained the largest number of problems presented in a purely mathematical context and reflection problems. Results also indicated that the lessons within the four textbooks followed a similar pattern. The lessons contain an introductory task, worked example with solution, explanation of concepts, application tasks, and practice problems. All
textbooks also included review problems in the beginning of the chapter. Ponte and Marques supported the results of Li (2000, 2007) about the U.S. textbook including a higher percentage of high-cognitive demand problems. However, Ponte and Marques also noted that the U.S. textbook included the highest percentage of reproduction problems.

Incikabi and Tjoe (2013) examined the content presentation of ratio and proportion problems in U.S. and Turkish middle-grades mathematics textbooks. Their sample included three U.S. popular textbooks and two Turkish textbooks. Incikabi and Tjoe classified mathematical problems by using a three-dimensional framework. The mathematical problems were coded in terms of their mathematical features, contextual features, and performance requirements.

Findings indicated that more than half of the problems in U.S. textbooks and less than 10% of the problems in the Turkish textbooks required single computation. The U.S. textbooks also included a larger proportion of problems that required a numerical answer but a smaller proportion of problems set in purely mathematical context. In terms of contextual features, U.S. textbooks contained miniscule amounts of visual representations (e.g., figures, pictures, or models) within problems. Incikabi and Tjoe (2013) also noted that U.S. textbooks included fewer high-cognitive demand problems than the Turkish textbooks. These trends regarding the mathematical features of problems in U.S. textbooks coincide with the findings of previous studies by Li (2000, 2007) and Ponte and Marques (2011). However, the results of this study regarding the contextual features and cognitive requirements of problems in U.S. popular textbooks do not support Li’s and Ponte’s and Marques’s findings.

In contrast, Zhu and Fan (2006) conducted an exploratory case study to examine the algebraic and geometric problem types in two U.S. alternative textbooks and five Chinese textbooks. The mathematical problems were coded based on seven features. The coding results
indicated that the majority of problems in all textbooks were routine, traditional, and closed-ended. Approximately two-thirds of the problems in U.S. textbooks and more than half of the problems in Chinese textbooks required one-step computation. Zhu and Fan noted that U.S. textbooks included more authentic and real-world problems and fewer challenging problems. Zhu and Fan also reported that small proportions of problems within the U.S. and Chinese textbooks included some type of visual representation such as figures, pictures, graphs or diagrams, 8.6% and 3.3% respectively.


In another recent study, Yang and colleagues (2017) also examined geometry problems in middle-grades mathematics textbook series from Taiwan, Singapore, and U.S. Yang and colleagues analyzed the geometry problems in terms of their representation forms, contextual features, and response type. Results showed that the majority of the problems in U.S. textbooks were not contextualized in real-world situations. The U.S. textbooks also had a larger proportion
of problems that contained some type of visual representation (e.g., figures, pictures, graphs, or diagrams) than the other textbooks. All textbooks included an overwhelming amount of closed-ended problems. More specifically, more than four-fifths of the problems in U.S. textbooks were classified as closed-ended. Despite these findings, Yang and colleagues reported that U.S. textbook series had the highest percentage of open-ended problems compared to other textbook series. Yang and colleagues did not support Choi’s and Park’s (2013) and Zhu’s and Fan’s (2006) findings regarding U.S. alternative textbooks containing more real-world problems but fewer challenging problems.

In an earlier study, Hong and Choi (2014) analyzed the topics, content, and mathematical problems in Korean and U.S. textbooks. The sample included one U.S. 8th grade alternative textbook and one Korean 8th grade textbook. Hong and Choi analyzed the algebraic problems in terms of their context and cognitive demand. Analysis of the problems indicated that the U.S. textbook included more problems with higher level of cognitive demand and set in real-world context. Hong and Choi further noted that the U.S. textbook contained larger amounts of problems that required explanation. The Hong and Choi stated that the U.S. textbook emphasized real-life applications and included more challenging problems. Hong and Choi supported Zhu’s and Fan’s (2006) and Choi’s and Park’s (2013) findings that U.S. alternative textbooks included more problems that emphasize real-life applications but did not support their results related to the level of cognitive demand of problems.

In conclusion, a significant amount of research has been conducted to examine the quality of the content in both U.S. popular and alternative mathematics textbooks to mathematics textbooks of other countries. It is worth noting that several studies indicated that U.S. popular textbooks contained a larger amount of problems set in a purely mathematical context (Incikabi
& Tjoe, 2013; Li 2000, 2007; Ponte & Marques; 2011). These findings contradict NCTM’s (1989, 2000) and CCSSI (2010) recommendations of exposing students to problems set in real-world situations in order to promote students’ conceptual understanding of different mathematical concepts. However, the findings from other studies showed that U.S. alternative textbooks included a higher percentage of real-world application problems (Choi & Park, 2013; Hong & Choi, 2014; Zhu & Fan, 2006).

Another important observation is that Zhu and Fan (2006) reported that U.S. alternative textbooks have small amounts of problems with visual representations while Yang and colleagues (2017) stated that U.S. alternative textbooks have large amounts of problems with visual representations compared to textbooks of other countries. Incikabi and Tjoe (2013) also found that popular textbooks contained a small proportion of problems with some type of visual representation. Mixed findings have also been reported regarding the cognitive requirements of mathematical problems in both U.S. popular and alternative textbooks.

The studies reported in this section either evaluated the quality of content in popular or alternative mathematics textbooks to mathematics textbooks of other countries. Given this limitation, extended studies of textbooks including both popular and alternative mathematics textbooks need to be conducted to assess the quality of content in both popular and alternative mathematics textbooks to mathematics textbooks of other countries. Furthermore, none of these studies examined how geometric concepts such as surface area and volume are introduced and developed in U.S. middle-grades mathematics textbooks and mathematics textbooks of other countries. Thus, additional research needs to be conducted to analyze more textbooks with different mathematical topics.
Besides evaluating the textbook content, other researchers have attempted to investigate students’ 3D geometric thinking and learning to address students’ underperformance in geometry (Gutierrez, Jaime, & Fortuny, 1991; Gutierrez, 1992; Pittalis, Mousoulides & Christou, 2009; Pittalis & Christou, 2010). In the following section, I present several theoretical considerations on students’ 3D geometric thinking and learning.

**Theoretical Considerations on Students’ 3D Geometric Thinking**

There is no widely established theory on 3D geometric learning and teaching (Pittalis et al., 2009). Some researchers have analyzed students’ 3D geometric thinking in terms of their cognitive progress by extending the original van Hiele levels and Bloom’s taxonomy to 3D geometry (Denenberg, 2011; Gutierrez et al., 1991; Gutierrez, 1992). Others have examined students’ 3D geometric thinking in terms of their geometric 3D abilities also called spatial abilities by extending the original van Hiele levels or developing new models (Gutierrez, 1992; Pittalis et al., 2009; Pittalis & Christou, 2010). In this section, I present several theoretical considerations regarding students’ 3D geometric thinking.

The van Hiele theory was originally developed to understand and explain students’ geometric thinking in Euclidean (two-dimensional flat) geometry (Gutierrez, 1992; Senk, 1989; van Hiele, 1986). Gutierrez and colleagues (1991) extended the original van Hiele levels to examine and understand students’ thinking in 3D geometry. Gutierrez and colleagues identified four levels of 3D geometric thinking as described by Clements and Battista (1992). At level 1, students can visually and holistically identify solids without considering their components or properties. At level 2, students can recognize the components of solids and informally describe their properties but they cannot draw relationships between properties. At level 3, students can
logically categorize solids and understand their definitions. At level 4, students can prove theorems about solids.

Denenberg (2011) designed a unit on teaching and learning the concepts of surface area and volume of 3D shapes and proposed that teachers should use Bloom’s taxonomy to evaluate their students’ level of 3D geometric thinking. Denenberg described the five levels of 3D geometric thinking as follows: at level 1- students can define 2D and 3D shapes, at level 2-students can classify shapes by their properties, compare shapes’ characteristics and measurement, and select formulas to make calculations, at level 3- students can construct shapes and apply appropriate formulas to find the surface area and volume of 3D shapes, at level 4-students can analyze the geometric formulas to justify their work, examine the connections between surface area and volume, and solve for unknown dimensions, and at level 5- students can develop their own formulas for making measurements of more complex shapes.

Gutierrez (1992) also used the van Hiele levels and proposed four levels to examine and analyze students’ acquisition of spatial abilities in 3D geometry. At level 1, students can compare solids based on the shape of the solids or their elements (e.g., face, edges, vertices). At this level, students are not able to visualize solids, position of solids, or their movements. At level 2, students can compare solids by observation. Students can visualize and analyze the components, properties, and movements of solids. At level 3, students can compare solids by mathematically analyzing their elements. At this level, when a solid needs to be moved, students are able to visualize and reason about the initial and final position of the solid. At level 4, students can also analyze the solid prior to any manipulation. Students’ reasoning is based on the mathematical structure of solids, or their elements and properties. At this level, students have high visualization skills.
Pittalis and colleagues (2009) identified four categories of students’ 3D geometric thinking based on their 3D geometric abilities. At category 1, students can recognize solids. At category 2, students can recognize solids, construct nets, and represent solids. At category 3, students can recognize solids, construct nets, represent solids, structure 3D arrays of cubes, and calculate the area and volume of solids. At category 4, students can complete all tasks from previous categories and compare properties of solids. Based on these findings, Pittalis and colleagues suggested that these categories might represent four levels of 3D geometric thinking.

Pittalis and colleagues (2009) also proposed a model to illustrate the structure of students’ 3D geometric abilities. This model included six 3D geometric abilities strongly related and interrelated to 3D geometric thinking: recognizing and constructing nets, representing 3D solids from one representational format to another, structuring 3D arrays of cubes by enumerating the cubes in 3D arrays, recognizing 3D solids’ properties and their structural elements in 3D format or in 2D drawings, calculating the area and volume of 3D solids, and comparing the properties of 3D solids by comparing their parts (e.g., vertices, faces, and edges), and 3D solids’ properties.

In a later study, Pittalis and Christou (2010) identified four types of 3D geometric abilities that support the development of students’ 3D geometric thinking. These 3D geometric abilities were defined as: understanding 3D representations that includes manipulating 3D representations and constructing nets, spatial structuring that consist of structuring 3D arrays of cubes, conceptualization of mathematical properties that includes recognizing and comparing 3D solids’ properties, and measurement that consist of calculating the area and volume of solids.

Gutierrez and colleagues (1991) and Denenberg (2011) extended existing frameworks to 3D geometry in order to examine the cognitive progress of students’ 3D geometric thinking. Yet,
others have extended existing frameworks or developed new models to interpret students’ 3D geometric thinking in terms of their 3D geometric abilities also referred to as spatial abilities (Gutierrez, 1992; Pittalis et al., 2009; Pittalis & Christou, 2010). These researchers have argued that spatial ability is related to students’ 3D geometric thinking. Further, spatial ability is closely connected to students’ understanding of the concepts of surface area and volume of 3D shapes (Obara, 2009; Revina et al., 2011). In the next section, I report on recommendations related to students’ learning of the concepts of surface area and volume.

**Students’ Learning of Surface Area and Volume**

In middle grades, students are expected to develop conceptual understanding of measuring the surface area and volume of solids (CCSSI, 2010; NCTM, 2000). Battista (2003) suggested that students need to develop two skills in order to be able calculate the surface area and volume of solids. These two skills included: an understanding of the numerical operations and connections to the formulas with the structure of solids and an understanding and visualization of the structure of solids. More specifically, Battista described the ability to enumerate the arrays of squares or cubes as essential to the development of students’ conceptual understanding of measuring area and volume.

Battista (2003) also claimed that four mental processes are important for enumerating arrays of squares or cubes: forming and using mental models, spatial structuring, units-locating, and organizing-by-composites. In the forming and using mental models process, students can create and use mental representations to visualize, understand, and reason about situations. In the spatial structuring process, students can understand the object’s composition. For example, students can enumerate arrays of squares or cubes. In the units-locating process, students can locate squares or cubes by coordinating their position along the dimensions of an array. In the
organizing-by-composites process, students can group and repeat an array’s unit into more composite units to create the whole array.

More recently, Clements and Sarama (2009) identified learning trajectories of volume measurement that are similar with those Battista (2003) proposed for surface area and volume measure. At the first level, students recognize volume as an attribute of an object. At the second level, students can use cubes and other pourable materials to structure, build, or fill objects in order to determine their volume. At the third level, students develop understanding of the single unit structure before composing rows and columns as composite units. Next, students can compose and decompose rows and columns, structuring layers as 2D arrays that can be stacked and counted. At the final level, students can use their understanding of structuring space in 3D arrays to support their numerical reasoning of addition and multiplication in order to understand and enumerate more complex 3D spaces.

Researchers have also argued that students need to be provided with different kinds of tasks such as predicting, drawing units, and computing to support the development of their spatial structuring abilities required to solve surface area and volume tasks (Outhred & Mitchelmore, 2000). For primary students to conceptualize the concept of volume, they should be exposed to activities such as filling, packing, building, and comparing (Sarama et al., 2011). That is, students can develop their understanding of volume as measurable quantity as they engage in attempts to measure it.

At the secondary level, students should first be provided with the opportunity to establish a relationship between the area of a 2D net and the surface area of a solid by allowing them to move between 2D and 3D shapes and their representations (NCTM, 2000). Stylianou and colleagues (1999) stated, “translating between three-dimensional solids and their two-
dimensional representations in one kind of processing that is particular important to mathematics” (p. 241). Duval (1999) also noted that recognizing and understanding 3D representations plays an important role to students’ 3D geometric thinking. While others have argued that it is impossible for students to understand the concepts of surface area and volume without knowledge of 2D and 3D shapes (Smith & Barrett, 2017).

Middle-grades students should also be given the opportunity to explore and understand the concept of volume by visually and physically building the structure of solids using unit cubes (Battista, 2007; Carpenter et al., 1975; NCTM, 2000). Hirstein (1981) argued that the role of unit cubes is critical to the development of the concept of volume. Battista (2003) also noted that students should be exposed to activities that require them to enumerate the arrays of cubes. These types of activities can support students understanding of prismoidal spaces as array of cubic units (Battista & Clements, 1996).

Revina and colleagues (2011) proposed four consecutive activities to support students’ learning of the volume concept. The four activities consisted of isometric drawings, building 3D objects by using cube blocks, counting and comparing the cube blocks in their drawings and construction, and estimating the number of cube blocks needed to cover up a box. Obara (2009) also proposed that visual objects such as nets and models should be used to help students develop conceptual understanding of measuring the surface area and volume of solids. Yet, others have argued that the use of 2D diagrams to represent 3D objects in textbooks place heavy demands on students’ visualization skills (Smith & Barrett, 2017). Thus, students’ difficulties with learning the concept of volume might be due to their curricular experiences (Smith & Barrett, 2017).

Research has also indicated that students’ difficulties with finding the surface area and volume of solids is linked to their inability to interpret 2D and 3D shapes and their
representations and visualizing 3D shapes and their hidden faces (Ben-Chaim et al., 1985; Obara, 2009; Revina et al., 2011). Therefore, it is important to discuss students’ difficulties with understanding the concepts of surface area and volume. In the following section, I discuss the findings of studies related to students’ 3D geometric thinking and difficulties with solving surface area and volume tasks.

**Research on Students’ Difficulties with 3D Geometric Thinking**

Studies have showed that students’ 3D geometric thinking is related to their ability to understand the concepts of volume and surface area (Pittalis & Christou, 2010). Surprisingly, few studies have been conducted to document students’ 3D geometric thinking and difficulties with solving surface area and volume tasks. Some researchers have examined students’ difficulties with drawing 2D and 3D shapes and their representations (Mariotti, 1989; Stylianou et al., 1999). Others have investigated students’ difficulties with finding the volume of rectangular prisms using unit cubes (Battista & Clements, 1996; Ben-Chaim et al., 1985; Carpenter et al. 1975; Curry, Mitchelmore, & Outhred, 2006; Hirstein, 1981; Tekin-Strava & Isiksal-Bostan, 2014).

Mariotti (1989) interviewed elementary and middle school students to examine their understanding of solids and their representations. This study included two tasks. For the first task, students had to identify the characteristics (e.g., faces, vertices, and edges) of the solid. For the second task, students had to draw the different types of nets of the solid. Analysis of the first task revealed that students’ precision on counting the characteristics of the solid was related to students’ ability to mentally manipulate the solid. Analysis of the second task indicated that students were able to draw some types of nets of the solid but had greater difficulty or even denied the possibility of the existence of other types of nets of the solid. Mariotti concluded that
students had difficulties with recognizing and constructing the types of nets of the solid that required more complex transformations from the solid to the net.

Similarly, Stylianou and colleagues (1999) analyzed the work of 8 eighth grade students on problems related to drawing 2D net-representations of 3D shapes. Each student was given a picture demonstrating the process of opening up a closed cube in order to produce its net. Then the students were asked to draw all possible nets for a cube. Analysis of the students’ work showed that students were able to draw the 2D net-representations of the solid that required fewer transformations from the solid to the net. Stylianou and colleagues noted that some nets were produced by trial-and-error. Both Stylianou and colleagues and Mariotti (1989) reported students’ difficulties with drawing 2D net-representations of the solid that required more complex transformations from the solid to the net. However, Stylianou and colleagues did not support Mariotti’s findings in regards to students’ difficulties with producing certain types of nets of the solid.

Carpenter and colleagues (1975) analyzed the results of area and volume tasks among elementary, middle, and high school students on the NAEP. The volume tasks included a picture of a unit cube. Analysis of the volume responses revealed that only 6% of the 9-year-olds, 21% of the 13-year-olds, and 43% of the 17-year-olds correctly computed the volume of rectangular prisms. Carpenter and colleagues also reported two common errors with computing the volume of rectangular prisms. Younger students counted the number of visible cube faces and older students counted and doubled the number of visible cube faces. Carpenter and colleagues concluded that students had difficulties with visualizing 3D solids. That is, students could not mentally take apart the solid in order to count the units inside. Carpenter and colleagues also noted that students confused the concepts of surface area and volume.
Hirstein (1981) also sought to examine students’ responses on calculating the volume of rectangular prisms using unit cubes on the NAEP. Findings indicated that only 24% of the 13-year-olds, and 39% of the 17-year-olds accurately calculated the volume of rectangular prisms. Hirstein stated that the most common errors related to calculating the volume of rectangular prisms were counting the number of visible cube faces and counting and doubling the number of visible cube faces. Hirstein claimed that these errors were related to students’ inability to visualize solids. Hirstein supported the results of Carpenter and colleagues (1975) regarding students’ difficulties with computing the volume of rectangular prisms. Hirstein also reported that students confused the concepts of surface area and volume.

Ben-Chaim and colleagues (1985) reported similar findings. Ben-Chaim and colleagues investigated students’ difficulties with computing the volume of rectangular prisms using isometric type drawings. Approximately 1,000 students were tested on computing the volume of rectangular prisms using pictures of unit cubes. Results indicated that 75% of the fifth graders, 60%-55% of the sixth and seventh graders, and 50% of the eighth graders were not able to correctly compute the volume of rectangular prisms. In addition, Ben-Chaim and colleagues reported four common types of errors that students made when solving the volume tasks: counted the number of visible cubes, counted and doubled the number of visible cubes, counted the number of visible cube faces, and counted and doubled the number of visible cube faces. Ben-Chaim and colleagues concluded that the majority of students were unable to visualize the hidden portion of the figure. In fact, less than 50% of middle-grade students correctly computed the volume of rectangular prisms. Ben-Chaim and colleagues findings coincide with results from previous studies (Carpenter et al., 1975; Hirstein, 1981). However, Ben-Chaim and colleagues reported more common types of errors that students made when solving the volume tasks.
Battista and Clements (1996) sought to extend the work of Carpenter and colleagues (1975), Hirstein (1981), and Ben-Chaim and colleagues (1985) by further examining students’ understanding and difficulties with finding the volume of rectangular prisms using unit cubes. Battista and Clements reported that the majority of students had difficulties with visualizing and manipulating mental images because of lack of spatial structuring abilities. Further, Battista and Clements reported that 64% of the third graders and 21% of the fifth grades exhibited lack of coordination of views. These students counted and doubled the visible number of cubes. In a later study, Curry and colleagues (2006) also claimed that students could not create a mental picture of the unit structure. Curry and colleagues noted that approximately 50% of the students correctly calculated the volume of a solid using repeated units.

More recently, Tekin-Strava and Isiksal-Bostan (2014) investigated middle school students’ performance, strategies, and difficulties with calculating the volume of rectangular prisms. Thirty-five middle school students were given a rectangular prism volume questionnaire that included a picture of a 10 x 10 x 10 large cube. Analysis of students’ responses indicated that approximately 60% of the students correctly calculated the volume of the solid. Notably, the eighth graders performed better than the sixth and seventh graders. Tekin-Strava and Isiksal-Bostan also claimed that sixth and seventh graders exhibited lack of spatial structuring abilities. These results coincide with the findings of previous studies regarding students’ difficulties with finding the volume of rectangular prisms and students’ performance being related to age and experiences.

In sum, research has documented students’ difficulties with drawing the nets of solids and finding the volume of rectangular prisms due to students’ inability to visualize and mentally manipulate solids (Battista & Clements, 1996; Ben-Chaim et al., 1985; Carpenter et al., 1975;
Hirstein, 1981; Mariotti, 1989; Stylianou et al., 1999; Tekin-Strava & Isiksal-Bostan, 2014). For drawing the nets of solids, students had difficulties with recognizing and constructing the types of nets of the solid that required more complex transformations from the solid to the net. For calculating the volume of rectangular prisms, students tend to count the number of visible cube faces, count and double the number of visible of cube faces, count the number of visible cubes, and count and double the number of visible of cubes due to lack of spatial structuring abilities. As noted by Hart (1981), “the problem of finding the volume of a cuboid by counting cubes is that many cannot be seen…children often count what they see” (p. 18). Research has also shown that students tend to confuse the concepts of surface area and volume (Carpenter et al., 1975; Hirstein, 1981).

Based on the literature review, these studies were limited to investigating students’ thinking and difficulties with drawing the nets and finding the volume of cubes and rectangular prisms. Further research needs to be undertaken to investigate students’ thinking and difficulties regarding drawing the nets and findings the volume of other solids. In addition, the mathematics education literature related to students’ difficulties with 3D geometric thinking are scant. Therefore, it is imperative to examine students’ difficulties with 3D thinking in other fields. In the final section, I present relevant research in regards to students’ difficulties with 3D thinking in engineering and science.

**Research on Students’ Difficulties with 3D Thinking in other Fields**

Spatial ability is a fundamental skill in the acquiring of knowledge in other fields such as engineering and science. Research has reported that students’ spatial ability influence their performance in engineering drawing and designing courses (Garmendia, Guisasola, & Sierra, 2007; Kadam & Iyer, 2015; Potter & Merwe Van Der, 2003). Studies have also been conducted
to investigate students’ difficulties with 3D thinking in the fields of engineering and science. Several researchers have examined engineering students’ difficulties with visualization and drawing tasks (Akasah & Alias, 2010; Garmendia et al., 2007; Nagy-Kondor, 2007). While others have investigated students’ difficulties with 3D representations, rotations, and reflections of molecular structures (Ferk et al., 2003; Tuckey, Selvarathan, & Bradley, 1991).

Garmendia and colleagues (2007) interviewed 12 first year engineering students to identify their difficulties with solving spatial visualization tasks. All participants were enrolled in an engineering technical drawing course. The participants were asked to solve a three-part visualization problem, analyze the views of the tasks, and draw their solutions in perspective. Analyses of the students’ responses showed serious conceptual and procedural deficiencies and difficulties with analyzing and drawing spatial visualization tasks. In particular, students exhibited difficulties with analyzing the shape of surfaces, interpreting and analyzing the orthographic and isometric views, and identifying multiple views.

Nagy-Kondor (2007) evaluated the spatial ability of 80 first year mechanical engineering students. All participants were administered a test that contained five tasks: imaginary manipulation of the solid, imaginary rotation of the solid, representation of the solid, reading of the projection of the solid, and reconstruction of the solid. Analyses of the students’ work indicated that the majority of students were successful with mentally manipulating and rotating the solids and reading the projection of the solids. However, most students had difficulties with representing and reconstructing the solids. Nagy-Kondor concluded that the students had major difficulties with performing the transformations between 2D and 3D.

Akasah and Alias (2010) reported similar findings regarding students’ difficulties with solving spatial visualization tasks. Twenty-nine engineering students were divided into two
groups. The novice group that included students with no experience in engineering drawing and the expert group that included students with experience in engineering drawing. Both groups were administered the spatial visualization ability test (SVATI) that contained tasks such as drawing, constructing, and rotating solids. Results indicated that both groups performed poorly on the SVATI tasks. Akasah and Alias supported the results of Nagy-Kondor (2007) regarding students’ difficulties with performing the transformations between 2D and 3D.

Tuckey and colleagues (1991) identified students’ difficulties with 3D thinking in chemistry courses. Thirty-one 2nd year undergraduate students were administered a pre-test and post-test that included spatial visualization tasks. Findings revealed that students had difficulties with visualizing the 3D structure of molecules. The majority of students also had difficulties with visualizing the positions of the atoms after rotation or reflection. Tuckey and colleagues concluded that many university students have difficulties with 3D thinking due to lack of spatial skills.

More recently, Ferk and colleagues (2003) examined primary, secondary, and university students’ understanding of molecular structure representations. Participants were administered the chemistry visualization test (CVT) that included five different types of tasks: 1) perception, 2) perception and rotation, 3) perception and reflection, perception, rotation, and reflection, 5) perception and mental transfer of information from 2D representations of molecular structures to constructing 3D molecular models. Results showed that students’ performance significantly decreased when a combination of two or more mental processes was required for solving a task. Both Tuckey and colleagues (1991) and Ferk and colleagues (2003) reported that students had difficulties with mentally manipulating, rotating, and reflecting solids.
Most of the participants in these studies were university students enrolled in engineering or science courses. Findings indicated that university students are exhibited serious difficulties with 3D thinking required to understand concepts of graphics. According to Garmendia and colleagues (2007), many students have not developed their spatial ability prior to university entry. Thus, further research needs to be conduct to investigate students’ difficulties with solving spatial visualization tasks at the secondary level in these fields.

**Summary of Literature Review**

The ultimate goal of this chapter was to provide a review of the literature on several topics that guided this study. I divided the literature review into three major sections. In the first section, I discussed the different types of mathematics curriculum, the role and use of textbooks. In the second section, I presented several national and international mathematics textbook content analysis studies. In the third section, I reviewed various theoretical considerations regarding students’ 3D geometric thinking and learning, and research related to the investigation of students’ difficulties with 3D thinking in geometry and other fields.

Lloyd and colleagues (2017) divided the curricula into three levels: intended curriculum, enacted curriculum, and attained curriculum. The intended curriculum is defined as the national, state, or school district’s expectations for mathematics learning as replicated in textbooks. Both teachers and students rely heavily on the use of textbooks. In fact, textbooks are important components of daily instruction that impact students’ opportunity to learn various mathematical concepts and achievement in mathematics. Therefore, textbook content analysis studies can help us better understand student’s opportunity to learn and achievement in mathematics.

Both at the national and international level, findings from studies showed that alternative textbooks were better aligned to NCTM’s (1989, 2000) recommendations (AAAS, 2000; Choi &
Park, 2013; Hong & Choi, 2014; Jones & Tarr, 2007; NCR, 2004) compared to popular textbooks (Incikabi & Tjoe, 2013; Li 2000, 2007; Ponte & Marques; 2011). However, none of these studies examined the treatment of surface area and volume in popular and alternative middle-grades mathematics textbooks.

Some researchers have also investigated students’ 3D geometric thinking in regards to their cognitive progress by extending the original van Hiele levels and Bloom’s taxonomy to 3D geometry (Denenberg, 2011; Gutierrez et al.,1991; Gutierrez, 1992). Others have analyzed students’ 3D geometric thinking in terms of their geometric 3D abilities also called spatial abilities by extending the original van Hiele levels or developing new models (Gutierrez, 1992; Pittalis et al., 2009; Pittalis & Christou, 2010). In broader terms, there is no widely established theory on 3D geometric learning and teaching (Pittalis et al., 2009). Therefore, additional research needs to be undertaken to establish a common theoretical framework on 3D geometric learning and teaching.

Studies have also indicated students’ difficulties with drawing the nets of solids and finding the volume of rectangular prisms are related to students’ inability to visualize and mentally manipulate solids (Battista & Clements, 1996; Ben-Chaim et al., 1985; Carpenter et al., 1975; Hirstein, 1981; Mariotti, 1989; Stylianou et al., 1999; Tekin-Strava & Isiksal-Bostan, 2014). However, these studies investigated students’ thinking and difficulties with drawing the nets or finding the volume of cubes and rectangular prisms. Thus, a logical extension is to examine students’ understanding and difficulties with drawing nets and finding the volume of other solids.

In the field of engineering, research has reported that students have conceptual and procedural difficulties with drawing and solving spatial visualization tasks (Garmendia et al.,
2007) and performing the transformations between 2D and 3D (Akasah & Alias, 2010; Nagy-Kondor, 2007). In the field of chemistry, students have difficulties with visualizing the 3D structure and position of molecules after rotation or reflection (Ferk et al., 2003; Tuckey et al., 1991). Nevertheless, these studies mostly investigated university students’ difficulties with 3D thinking. Hence, further research needs to be conducted to examine secondary students’ difficulties with 3D thinking in these fields.

This review of relevant literature demonstrates the importance and the need for textbook content analysis on geometric concepts such as surface area and volume. Therefore, I conducted this study to examine students’ opportunities to learn the geometric concepts of surface area and volume by examining the structural, pedagogical, and content features in popular and alternative middle-grades mathematics textbook series. In the next chapter, I discuss the methods that I used to conduct this content analysis.
CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

In this study, I conducted an analysis of the treatment of surface area and volume concepts in middle-grades student edition mathematics textbooks from 2008 to the present. In particular, I examined the treatment of surface area and volume concepts in terms of the location of surface area and volume lessons in the textbook and the number of pages and lessons devoted to these concepts. I also investigated the sequence of the instructional blocks of surface area and volume lessons. In addition, I examined the tasks included in these lessons in regards to the performance expectations of students, the types of visual representations of 3D objects, and the level of mathematical complexity. Finally, I examined the extent to which the content of surface area and volume lessons address the CCCS for 6-8 geometry that are aligned with these topics.

In this chapter, I describe the research design and methods that I used in this study. I have divided the content of this chapter into five sections. First, I present the three research questions that I addressed in this study. Second, I present the sample of textbooks that I analyzed. Then, I discuss the research design methods that I used to examine the treatment of surface area and volume concepts. Next, I describe the coding scheme, data collection, and procedures that I utilized to analyze the mathematics textbook series. Lastly, I present the reliability and validity measures, and summary of the research design and methods that I used.

Research Questions

The purpose of this study was to examine the treatment of surface area and volume concepts in student editions of middle-grades mathematics textbooks. This study was guided by the following three research questions:
1) Within published mathematics textbook series and across different publishers, what are the structural features devoted to the concepts of surface area and volume? In particular,
   a. Where are the surface area and volume lessons located and how many pages and lessons are devoted to surface area and volume?
   b. How are the instructional blocks of surface area and volume lessons sequenced?
2) What are the pedagogical features of the tasks included in the surface area and volume lessons within a published mathematics textbook series, and across different publishers? Specifically,
   a. What are the performance expectations of students within these tasks?
   b. What types of visual representations of 3D objects are included in these tasks?
   c. What is the level of the mathematical complexity of these tasks?
3) To what extent do the content of surface area and volume lessons address the Common Core Content Standards for 6-8 geometry that are aligned with these topics?

Sample Selection

The two major and opposite types of textbooks that I included in this study were popular and alternative textbooks (Stein et al., 2007). The popular textbooks focus more on the development of procedural skills (Polikoff, 2015; Reys et al., 2004; Senk & Thompson, 2003; Stein et al., 2007). Conversely, the alternative textbooks were developed in response to the Curriculum and Evaluation Standards (NCTM, 1989) to place greater emphasis on conceptual understanding through investigation and problem solving (Cai et al., 2011; Choi & Park, 2013; Reys et al., 2004; Senk & Thompson, 2003; Stein et al., 2007). Therefore, it’s important to include both types of textbooks in the sample in order to obtain a better understanding of the
treatment of surface area and volume concepts in different textbook series. In the following paragraphs, I present the selection criteria and information about each textbook series.

**Textbook Selection Criteria**

I used four criteria to select the middle-grades mathematics textbook series for this study. First, I selected widely used popular and alternative middle-grades mathematics textbook series. I used Weiss and colleagues (1994, 2001) and Banilower and colleagues (2013) reports to select the most widely used middle-grades mathematics textbook series and commercial publishers. I also used other sources to help me select the most widely used textbook series (Cai et al., 2011; Huntley, 2008; Polikoff, 2015; Rivette et al., 2003). Second, I only examined the latest student edition textbooks because the primarily focus of this study is to investigate students’ opportunities to learn the concepts of surface area and volume. Students normally do not interact with teacher’s edition textbooks.

I also selected textbook series that included at least one student edition textbook for each grade 6, 7, and 8 because I am interested in examining middle-grades mathematics textbooks. According to AAAS (2000), the mathematics curriculum materials such as textbooks influence student learning at all educational levels but the quality of middle school curriculum materials in particular, are in need of urgent attention due to middle grades students’ underperformance in the area of mathematics. Lastly, I selected textbook series that are written for the “average-level” student, that is, I did not include any remedial or accelerated materials in this sample.

Based on these criteria, I selected the **GM (Grades 6-8)** and **GMC** (Course 1, 2, and 3) popular middle-grades mathematics textbook series and **CM (Grades 6-8)** and **UCSMP** textbooks titled *Pre-Transition Mathematics, Transition Mathematic, Algebra* alternative middle-grades mathematics textbook series for this analysis.
Table 1. Textbooks Selected for Analysis

<table>
<thead>
<tr>
<th>Publisher</th>
<th>Title</th>
<th>Grade</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Houghton Mifflin Harcourt</td>
<td>Go Math</td>
<td>6</td>
<td>GM6</td>
</tr>
<tr>
<td></td>
<td>Go Math</td>
<td>7</td>
<td>GM7</td>
</tr>
<tr>
<td></td>
<td>Go Math</td>
<td>8</td>
<td>GM8</td>
</tr>
<tr>
<td>McGraw Hill</td>
<td>Glencoe Math</td>
<td>6</td>
<td>GMC6</td>
</tr>
<tr>
<td></td>
<td>Course 1</td>
<td>7</td>
<td>GMC7</td>
</tr>
<tr>
<td></td>
<td>Course 3</td>
<td>8</td>
<td>GMC8</td>
</tr>
<tr>
<td>Pearson</td>
<td>Connected Mathematics 3</td>
<td>6</td>
<td>CM6</td>
</tr>
<tr>
<td></td>
<td>Grade 7</td>
<td>7</td>
<td>CM7</td>
</tr>
<tr>
<td></td>
<td>Grade 8</td>
<td>8</td>
<td>CM8</td>
</tr>
<tr>
<td>McGraw Hill</td>
<td>University of Chicago School Mathematics Project</td>
<td>6</td>
<td>U6</td>
</tr>
<tr>
<td></td>
<td>Pre-Transition Mathematics</td>
<td>7</td>
<td>U7</td>
</tr>
<tr>
<td></td>
<td>Transition Mathematics</td>
<td>8</td>
<td>U8</td>
</tr>
</tbody>
</table>

During this study, I examined a total of 12 middle-grades student edition mathematics textbooks. Table 1 presents the set of textbooks that I selected for this analysis. I examined the Table of Contents to determine the number of lessons included in each textbook. I analyzed a total of 49 lessons (17 surface area, 24 volume, and 8 surface area & volume) during this study. Lessons that address both concepts I labeled them as surface area and volume lessons. It is important to note that the UCSMP Algebra textbook does not contain any lessons that are devoted to the concepts of surface area and volume. Furthermore, I examined all tasks within these lessons in terms of their performance expectations, types of visual representations of 3D objects, and level of mathematical complexity. Table 2 illustrates the number of surface area, volume, and surface area and volume lessons included in the sample textbooks.
Table 2. Number of Surface Area (SA), Volume (V), and Surface Area and Volume (SA&V) Lessons in Each Textbook

<table>
<thead>
<tr>
<th>Type</th>
<th>SA Lessons</th>
<th>V Lessons</th>
<th>SA&amp;V Lessons</th>
<th>Total Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM6</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>GM7</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>GM8</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>GMC6</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>GMC7</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>GMC8</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>CM6</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>CM7</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>CM8</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>U6</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>U7</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>U8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>TOTAL</td>
<td>17</td>
<td>24</td>
<td>8</td>
<td>49</td>
</tr>
</tbody>
</table>

Description of Textbook Series

I selected the first set of textbooks from the commercial publisher *Houghton Mifflin Harcourt: Go Math! Grade 6, Grade 7, and Grade 8*. The *Go Math!* curriculum was written to provide opportunities for students to access the content in depth and rigor at the appropriate level (Houghton Mifflin Harcourt, 2017). According to Houghton Mifflin Harcourt, the *Go Math!* curriculum was also developed to provide engaging and dynamic materials that build procedural fluency and conceptual understanding. Houghton Mifflin Harcourt further notes that the ultimate goal of this curriculum is to provide focused, coherent, and rigorous materials that cover and support the CCSSM.

I drew the second set of textbooks from the commercial publisher *McGraw Hill: Glencoe Math, Course 1, Course 2, and Course 3*. The *Glencoe Math* curriculum was developed to make math relevant, rigorous, and focused (McGraw Hill, 2017). This curriculum was also created to help increase learning through engaging and effective experiences (McGraw Hill, 2017).

According to McGraw Hill, the *Glencoe Math* curriculum materials were designed to emphasize
procedural skills, conceptual understanding, and application. McGraw Hill further states that the *Glencoe Math* curriculum materials are aligned to the CCSSM.

The third set of textbooks that I selected was the *CM, Grade 6, Grade 7, and Grade 8* funded by the NSF. In the early 1990s, a team of scholars developed the *CMP* which is a problem-centered curriculum which focus is to help students develop their critical thinking and mathematical reasoning by exploring and solving rich mathematical problems (Lappan, Phillips, & Fey, 2007). The *CMP* curriculum materials are also designed to help students develop conceptual understanding and meaningful skills through problem solving and investigation (Lappan et al., 2007). The *CMP* curriculum also follows the NCTM (2000) recommendations (Choi & Park, 2013) and is aligned to the CCSSM (*CMP*, n.d.).

The fourth set of textbooks that I chose is another NSF funded curriculum, the *UCSMP*, *Pre-Transition Mathematics* (sixth grade), *Transition Mathematics* (seventh grade), and *Algebra* (eighth grade). The development process of the *UCSMP* textbooks started in 1983 (Thompson & Senk, 2001; Usiskin 1986). The *UCSMP* curriculum includes educational materials that promote a more sophisticated understanding and wide experience in problem solving (Usiskin, 1986). The *UCSMP* curriculum materials also emphasize reading, everyday applications, and the use of calculators and computers (Usiskin, 1986). The *UCSMP* curriculum materials are aligned to the CCSSM (UCSMP, n.d.) and reflect the NCTM (1989, 2000) recommendations (Thompspon & Senk, 2001; Usiskin, 1986).

**Research Design**

In this study, I used content analysis to analyze the textbook in order to address the three research questions. Many researchers have defined content analysis as a systematic research method that is used to identify and analyze characteristics of written, verbal, and visual
communication messages (e.g., Cole 1988; Kolbe & Burnett, 1991; Kondracki, Wellman, & Amundson, 2002; Williams, 2007). Holsti (1969) also defined content analysis as, “any technique for making inferences by objectively and systematically identifying specified characteristics of messages” (p. 14). Similarly, Weber (1990) described content analysis as “a research method that uses a set of procedures to make valid inferences from text” (p. 9).

For this study, I chose to use content analysis because it is an established research method used to analyze documents and textbooks (Cole 1988; Elo & Kyngas, 2008; Kondracki et al., 2002; Stenler, 2001; Williams, 2007) in diverse fields, including education (Elo & Kyngas, 2008; Krippendorff, 1980; Titscher et al., 2000). Generally, researchers use content analysis to analyze data by using pre-established key words, categories, or variables (Elo & Kyngas, 2008; Kondracki et al., 2002; Thomas, 2006).

Elo and Kyngas (2008) identified three phases of the content analysis process: preparation, organizing, and reporting. At the preparation phase, the researcher starts the analysis processes by selecting the unit of analysis. The unit of analysis is selected based on the research question. At the organizing phase, the researcher uses a matrix or framework with pre-established categories to code the data. At the reporting phase, the researcher reports the findings by using visual graphs or tables.

During the conduct of this study, I followed the three phases of content analysis process described above to examine the treatment of the concepts of surface area and volume in middle-grades mathematics textbooks. In particular, I used content analysis to analyze the data of the present study addressed in the research questions. First, I identified the unit of analysis for each research question. I then used pre-established frameworks to code the data for research questions 1 (part a) and 2 (parts a and c). I developed and used frameworks to code the data for research
questions 2 (part b) and 3. I also generated the data for research question 1 (part b) by using content analysis. Finally, I used simple descriptive statistical measures and visual graphs/tables to analyze the display findings. In the next section, I present detail information about the coding scheme, data collection, and procedures.

**Coding Scheme, Data Collection, and Procedures**

According to Valverde and colleagues (2002) both the structural and pedagogical features of textbooks influence students’ opportunity to learn mathematics. Therefore, I examined both the structural and pedagogical features of textbooks devoted to the concepts of surface area and volume. I also examined the content features of the surface area and volume lessons in regards to what extend do these lessons address the CCSS for 6-8 geometry. This is imperative because the CCSS must be supported with aligned textbooks and curriculum materials in order for the standards to be effective (Polikoff, 2015). In the following paragraphs, I present and describe the coding scheme, data collection, and procedures that I employed to address each research question.

**Physical Characteristics of Textbooks**

I examined the physical characteristics of the textbooks using Flanders’ (1994) counting method. More specifically, I used Flanders’ (1994) counting method to examine the location of the surface area and volume lessons in the textbook and the number of pages and lessons devoted to these concepts. This method employs a quantitative approach of collecting and analyzing data. I selected Flanders’ (1994) counting method because it is a well-established method of examining the physical characteristics of textbooks that has been previously used in many studies (Flanders, 1994; Jones, 2004; Dogbey, 2010; Alajmi 2012).
Location of the Topic

Based on Flanders’ (1994) counting method, I determined the location of the surface area lessons in the textbook by calculating the percentage of lessons that come before the first lesson containing instruction on surface area. For instance, if a textbook includes 100 lessons and instruction on surface area starts at lesson number 75, then 75% of the lessons in this textbook precede this surface area lesson (75 ÷ 100 = .75). I also calculated the percentage of instructional pages that come before the first instructional page devoted to the concept of surface area. For example, if a textbook contains 500 instructional pages and the first instructional page containing instruction on surface area is page 450, then 90% of the instructional pages in this textbook precede this instructional page on surface area (450 ÷ 500 = .90). I repeated the same process to determine the location of the volume topic in the textbook. I conducted all calculations twice. I then used a table to present and compare the location of the surface area and volume topics in each textbook.

Number of Pages

Further drawing on Flanders’ (1994) counting method, I calculated the number of instructional pages devoted to the concept of surface area by first counting the total number of instructional pages within the textbook by excluding supplemental exercise at the end of the textbook, glossaries, appendices, answer pages, and indices. I then counted the number of instructional pages devoted to the concept of surface area using linear measurement of the pages. Instructional pages that contain other topics I rounded them to the nearest quarter of a page. I then divided the number of instructional pages devoted to surface area by the total number of instructional pages. For example, if a textbook includes 50 instructional pages on surface area and has a total of 500 instructional pages, then 10% of the instructional pages in this textbook are
devoted to the concept of surface area \((50 \div 500 = .10)\). It should be noted that some instructional pages address both concepts. Therefore, I repeated the same process to determine the number of instructional pages devoted to the volume concept and to both concepts in the textbook. I conducted all calculations twice. I then used a table and a visual display to report and compare the number of instructional pages devoted to these concepts within each textbook.

**Number of Lessons**

Finally, I utilized Flanders’ (1994) counting method to quantify the number of lessons devoted to the concept of surface area. I first counted the total number of lessons within the textbook. I then counted the number of lessons devoted to the concept of surface area. Next, I divided the total number of lessons devoted to surface area by the total number of lessons in the textbook. For instance, if a textbook includes 5 lessons on surface area and has a total of 100 lessons, then 5% of the lessons in this textbook are devoted to the concept of surface area \((5 \div 100 = .05)\). It is imperative to note that some lessons address both concepts. Therefore, I repeated the same process to determine the number of lessons devoted to the volume concept and to both concepts within the textbook. I conducted all calculations twice. I also used a table and a visual display to present and compare the number of surface area, volume, and surface area and volume lessons included in the sample of textbooks.

**Structure of Lessons**

During this study, I examined the sequence of the instructional blocks of surface area, volume, and surface area and volume lessons. As previously mentioned, I labeled lessons that address both concepts as surface area and volume lessons. Mathematics textbooks are made up of lessons (Valverde et al., 2002). Each lesson is divided into instructional blocks (e.g., worked
examples, exercises, or question sets) (Valverde et al., 2002). I analyzed the sequence of the instructional blocks of these lessons by using content analysis.

**Sequence of Instructional Blocks**

The unit of analysis was the lessons devoted to the concepts of surface area and volume. I analyzed the sequence of the instructional blocks of these lessons by conducting the following steps. First, I read through the lesson and I used a highlighter to highlight the main parts also called the instructional blocks of the lesson. I then read through the lesson again to make sure that I have marked all of the instructional blocks of the lesson. Next, I used a table to record the instructional blocks of the lesson. I conducted the same process of documenting the sequence of the instructional blocks of each lesson devoted to the concepts of surface area and volume within the textbook. I determined the sequence of the instructional blocks within these lessons by looking for similar patterns. I used tables to display the sequence of the instructional blocks of these lessons in each textbook series.

**Table 3. Sequence of Instructional Blocks of Lessons within Textbooks**

<table>
<thead>
<tr>
<th>Textbook</th>
<th>GM7</th>
<th>GMC7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional Blocks</td>
<td>Essential Question</td>
<td>Inquire Lab</td>
</tr>
<tr>
<td></td>
<td>Activity</td>
<td>Essential Question</td>
</tr>
<tr>
<td></td>
<td>Reflection Questions</td>
<td>Introductory Task</td>
</tr>
<tr>
<td></td>
<td>Description of Concept + Vocabulary</td>
<td>Description of Concept + Vocabulary</td>
</tr>
<tr>
<td></td>
<td>+ Formula</td>
<td>+ Formula</td>
</tr>
<tr>
<td></td>
<td>Worked Examples + Solutions</td>
<td>Worked Examples + Solutions</td>
</tr>
<tr>
<td></td>
<td>Practice Problems</td>
<td>Practice Problems</td>
</tr>
<tr>
<td></td>
<td>Independent Practice</td>
<td>Independent Practice</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Test Practice</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Review Problems</td>
</tr>
</tbody>
</table>

In the table above (Table 3), I provide an example of the sequence of the instructional blocks of two surface area lessons: “Solving Surface Area Problems” from the *Go Math!* (Grade
7) textbook (Burger et al., 2014, pp. 283-288) and “Surface Area of Prisms” from the *Glencoe Math Course (Course 2)* textbook (Carter et al., 2015, pp. 661-672). I have also included a copy of each lesson (see Appendix A).

Both surface area lessons contain several similar instructional blocks such as essential question, activity/inquire lab, and description of concept with vocabulary terms and formula. Both lessons also include worked examples with solutions, practice problems, and independent practice. However, the “Solving Surface Area Problems” lesson from the *Go Math! (Grade 7)* textbook only offers reflection questions and the “Surface Area of Prisms” lesson from the *Glencoe Math Course (Course 2)* textbook only has an introductory task, test practice, and review problems. Some differences can also be observed in the sequence of the instructional blocks within these lessons. The “Solving Surface Area Problems” lesson from the *Go Math! (Grade 7)* textbook introduces the concept of surface area by stating the essential question and then offering an activity to explore this concept. In contrast, the “Surface Area of Prisms” lesson from the *Glencoe Math Course (Course 2)* textbook introduces the concept of surface area by including an inquiry lab and then stating the essential question.

**Pedagogical Features of Tasks**

I examined the pedagogical features of the tasks within the surface area, volume, and surface area and volume lessons in terms of their performance expectations, types of visual representations of 3D objects, and level of mathematical complexity. I used the Performance Expectations: Codes and Definitions framework to analyze the performance expectations of students within these tasks. I developed and used the Visual Representations of 3D Objects framework to examine the types of visual representations of 3D objects and the Mathematics Framework for NAEP 2007 to analyze the mathematical complexity of these tasks. I divided the
tasks within these lessons into three categories: surface area (SA), volume (V), and surface area and volume (SA&V). Tasks that address both concepts were labeled as surface area and volume tasks.

Performance Expectations of Students

The unit of analysis was the surface area, volume, and surface area and volume tasks found within the lessons. I drew from the 2002 Performance Expectations for Mathematics Framework to analyze these tasks in terms of performance expectation of students (Valverde et al., 2002). Performance expectations are defined as the kinds of “performances” students are expected to carry out while engaged with the content (Valverde et al., 2002).

Table 4. TIMSS 2002 Performance Expectations for Mathematics (Valverde et al., 2002)

<table>
<thead>
<tr>
<th>Mathematics Category</th>
<th>TIMSS Framework Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowing &amp; Using Vocabulary</td>
<td>Representing</td>
</tr>
<tr>
<td></td>
<td>Recognizing equivalents</td>
</tr>
<tr>
<td></td>
<td>Recalling mathematical objects &amp; properties</td>
</tr>
<tr>
<td></td>
<td>Using vocabulary &amp; notation</td>
</tr>
<tr>
<td>Using Equipment/Performing Routine Procedures</td>
<td>Using equipment</td>
</tr>
<tr>
<td></td>
<td>Performing routine procedures</td>
</tr>
<tr>
<td>Using Complex Procedures</td>
<td>Using more complex procedures</td>
</tr>
<tr>
<td>Investigating &amp; Problem Solving</td>
<td>Formulating &amp; clarifying problems &amp; situations</td>
</tr>
<tr>
<td></td>
<td>Developing strategy</td>
</tr>
<tr>
<td></td>
<td>Solving</td>
</tr>
<tr>
<td></td>
<td>Predicting</td>
</tr>
<tr>
<td></td>
<td>Verifying</td>
</tr>
<tr>
<td>Mathematical Reasoning</td>
<td>Developing notation &amp; vocabulary</td>
</tr>
<tr>
<td></td>
<td>Developing algorithms</td>
</tr>
<tr>
<td></td>
<td>Generalizing</td>
</tr>
<tr>
<td></td>
<td>Conjecturing</td>
</tr>
<tr>
<td></td>
<td>Justifying &amp; Proving</td>
</tr>
<tr>
<td></td>
<td>Axiomatizing</td>
</tr>
<tr>
<td>Complex Communication</td>
<td>Relating representation</td>
</tr>
<tr>
<td></td>
<td>Describing/Discussing</td>
</tr>
<tr>
<td></td>
<td>Critiquing</td>
</tr>
</tbody>
</table>
This framework consists of six categories of performance expectations for mathematics:

1) knowing and using vocabulary, 2) using equipment and performing routine procedures, 3) using complex procedures, 4) investigating and problem solving, 5) mathematical reasoning, and 6) complex communication. Each category includes several codes (See Table 4). I selected this framework because several well-known researchers used it to analyze the content of lessons in 418 mathematics textbooks from 48 educational systems (Valverde et al., 2002). Table 4 represents the 2002 Performance Expectations for Mathematics Framework.

Table 5. Performance Expectations: Codes and Definitions Framework

<table>
<thead>
<tr>
<th>Mathematics Category</th>
<th>Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Knowing &amp; Using Vocabulary</strong></td>
<td></td>
</tr>
<tr>
<td>Representing (R)- Students are expected to represent</td>
<td></td>
</tr>
<tr>
<td>mathematical information or data using models or</td>
<td></td>
</tr>
<tr>
<td>symbolic, verbal or graphical representations.</td>
<td></td>
</tr>
<tr>
<td>Recognizing Equivalents (RE)- Students are expected to</td>
<td></td>
</tr>
<tr>
<td>recognize equivalent symbolic, verbal, numerical, and</td>
<td></td>
</tr>
<tr>
<td>graphical mathematically entities.</td>
<td></td>
</tr>
<tr>
<td>Recalling Mathematical Objects &amp; Properties (RMOP)-</td>
<td></td>
</tr>
<tr>
<td>Students are expected to recall definitions, vocabulary</td>
<td></td>
</tr>
<tr>
<td>terms, mathematical objects, formulas, properties, and</td>
<td></td>
</tr>
<tr>
<td>concepts.</td>
<td></td>
</tr>
<tr>
<td>Using Vocabulary &amp; Notation (UVN)- Students are</td>
<td></td>
</tr>
<tr>
<td>expected to record a vocabulary term or interpret/create</td>
<td></td>
</tr>
<tr>
<td>a representation of vocabulary.</td>
<td></td>
</tr>
<tr>
<td><strong>Using Equipment/Performing Routine Procedures</strong></td>
<td></td>
</tr>
<tr>
<td>Using Equipment (UE)- Students are expected to use</td>
<td></td>
</tr>
<tr>
<td>physical or technological tools to complete a task.</td>
<td></td>
</tr>
<tr>
<td><strong>Using Procedures</strong></td>
<td></td>
</tr>
<tr>
<td>Performing Routine Procedures (PRP)- Students are</td>
<td></td>
</tr>
<tr>
<td>expected to apply a routine procedure to complete a</td>
<td></td>
</tr>
<tr>
<td>task.</td>
<td></td>
</tr>
<tr>
<td>Using More Complex Procedures (MCP)- Students are</td>
<td></td>
</tr>
<tr>
<td>expected to apply and/or connect facts, concepts, and</td>
<td></td>
</tr>
<tr>
<td>procedures to complete a task.</td>
<td></td>
</tr>
<tr>
<td><strong>Investing &amp; Problem Solving</strong></td>
<td></td>
</tr>
<tr>
<td>Formulating &amp; Clarifying Problems &amp; Situations (FCPS)-</td>
<td></td>
</tr>
<tr>
<td>Students are expected to formulate and/or clarify</td>
<td></td>
</tr>
<tr>
<td>problems and/or situations.</td>
<td></td>
</tr>
<tr>
<td>Developing Strategy (DS)- Students are expected to</td>
<td></td>
</tr>
<tr>
<td>develop problem-solving strategies to solve a problem.</td>
<td></td>
</tr>
<tr>
<td>Solving (S)- Students are expected to solve a problem</td>
<td></td>
</tr>
<tr>
<td>set in a mathematical or real-life context by applying</td>
<td></td>
</tr>
<tr>
<td>mathematical procedures.</td>
<td></td>
</tr>
<tr>
<td>Predicting (P)- Students are expected to predict the</td>
<td></td>
</tr>
<tr>
<td>solution of a problem.</td>
<td></td>
</tr>
<tr>
<td>Verifying (V)- Students are expected to verify the</td>
<td></td>
</tr>
<tr>
<td>accuracy and validity of the solution of a problem.</td>
<td></td>
</tr>
</tbody>
</table>
Table 5. (Continued) Performance Expectations: Codes and Definitions Framework

Mathematics Category

**Mathematical Reasoning**

*Developing Notation & Vocabulary (DNV)- Students are expected to develop mathematical formulas, symbols, and vocabulary terms.*

*Developing Algorithms (DA)- Students are expected to develop algorithmic procedures.*

*Generalizing (G)- Students are expected to extend the mathematical thinking and problem solving by restating results in more general and widely applicable way.*

*Conjecturing (CON)- Students are expected to produce a deductive argument about a problem or situation.*

*Justifying & Proving (JP)- Students are expected to provide a statement to justify or prove the conclusions of a problem or situation.*

*Axiomatizing (A)- Students are expected to formulate and express theories for a mathematical concept.*

**Complex Communication**

*Relating Representations (RR)- Students are expected to relate mathematical models and representations to mathematical concepts and to each other.*

*Describing/Discussing (DD)- Students are expected to describe, discuss, and/or explain mathematical concepts, representations, relationships, and situations.*

*Critiquing (C)- Students are expected to critique and/or compare and contrast mathematical concepts, representations, relationships, and situations.*

I also developed the definition of codes included in the TIMSS 2002 Performance Expectations for Mathematics to examine the performance expectations of students within tasks. I developed the definition of codes under each mathematics category to establish the features of cognitive behavior for each code. I also assigned a label to each code. Table 5 contains the Performance Expectations: Codes and Definitions framework that I used to analyze the performance expectations of students within tasks.

For this analysis, I read and coded the tasks within the lessons in terms of students’ performance expectations using the mathematics codes listed in the Performance Expectations for Mathematics: Codes and Definitions.
### Table 6. Sample Tasks to Illustrate Performance Expectations: Codes and Definitions

<table>
<thead>
<tr>
<th>Code</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recalling Mathematical Objects &amp; Properties (RMOP)</td>
<td>Suppose you observed the camping tent shown from directly above. What geometric figure would you use?</td>
</tr>
<tr>
<td>Developing Notation &amp; Vocabulary (DNV)</td>
<td>The base of a rectangular prism has an area of 19.4 square meters and the prism has a volume of 306.52 cubic meters. Write an equation that can be used to find the height h of the prism. Then find the height of the prism.</td>
</tr>
<tr>
<td>Recalling Mathematical Objects &amp; Properties (RMOP)</td>
<td>The volume of paperclip box is 1.5 cubic inches. Which of the following are possible dimensions of the box? Select all that apply.</td>
</tr>
</tbody>
</table>
| Recognizing Equivalents (RE)         | ▪ 2 in. by 1.5in. by 0.5in.  
▪ 2in. by 1in. by 1in.  
▪ 3in. by 0.5in. by 1.5in.  
▪ 3in. by 1in. by 0.5in.                                                   |
| Performing Routine Procedures (PRP)  | Find the volume of the triangular prism.                                                                                           |
| Solving (S)                          |                                                                                                                                 |
| Recalling Mathematical Objects & Properties (RMOP) | Find the volume of the prism.                                                                                                        |
| Performing Routine Procedures (PRP)  |                                                                                                                                      |
| Solving (S)                          |                                                                                                                                 |

---

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Table 6. (Continued) Sample Tasks to Illustrate Performance Expectations: Codes and Definitions

<table>
<thead>
<tr>
<th>Code</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recalling Mathematical Objects &amp; Properties (RMOP)</td>
<td>How do you find the volume of a composite solid formed by two or more prisms?</td>
</tr>
<tr>
<td>Using More Complex Procedures (MCP)</td>
<td>Josie has 260 cubic centimeters of candle wax. She wants to make a hexagonal prism candle with a base area of 21 square centimeters and a height of 8 centimeters. She also wants to make a triangular prism candle with a height of 14 centimeters. Can the base area of the triangular prism candle be 7 square centimeters? Explain.</td>
</tr>
<tr>
<td>Developing Strategy (DS)</td>
<td></td>
</tr>
<tr>
<td>Describing/Discussing (DD)</td>
<td></td>
</tr>
</tbody>
</table>

Specifically, I read and assigned the appropriate code(s) for each task. I read and coded each task twice. After coding the tasks in each lesson, I used statistical measures and graphical displays to document and compare the performance expectations of students within surface area, volume, and surface area and volume tasks in the selected middle-grades textbooks. Table 6 displays several tasks to illustrate the use of the Performance Expectations: Codes and Definitions framework.

I selected the first three tasks from the *Glencoe Math* (Course 2) textbook (Carter et al., 2015, pp. 639-646) and the last four tasks from the *Go Math!* (Grade 7) textbook (Burger et al., 2014, pp. 289-294). Based on the Performance Expectations: Codes and Definitions framework, I coded the first task as recalling mathematical objects & properties (RMOP). Students are expected to recall a mathematical object. I coded the second task as developing notation & vocabulary (DNV), recalling mathematical objects & properties (RMOP), using more complex procedures (MCP), and solving (S). Students are asked to first write an equation and then find
the height of the prism by connecting concepts and procedures. I coded the third task as recalling mathematical objects & properties (RMOP) and recognizing equivalents (RE). Students are expected to recognize equivalent numerical mathematical entities by recalling mathematical concepts.

I coded the fourth task as performing routine procedures (PRP) and solving (S). For this task, students are asked to find the volume of the triangular prism by filling in the blanks and performing simple calculations. I coded the fifth task as recalling mathematical objects & properties (RMOP), performing routine procedures (PRP), and solving (S). Students are expected to find the volume of the prism by recalling formulas and performing routine procedures. I coded the sixth task as recalling mathematical objects & properties (RMOP), using more complex procedures (MCP), developing strategy (DS), and describing/discussing (DD). For this task, students are asked to explain how to find the volume of composite figures using their existing knowledge to connect mathematical concepts and develop procedures. I coded the last task as recalling mathematical objects & properties (RMOP), using more complex procedures (MCP), developing strategy (DS), verifying (V), and describing/discussing (DD). Students are expected to connect mathematical concepts and develop procedures in order to verify the measurement of the base. Students also are asked to explain their work.

**Visual Representations of 3D Objects**

For the types of visual representations of 3D objects, the unit of analysis was the 2D representations of 3D objects (e.g., nets, pictures, & drawings) included in the surface area, volume, and surface area and volume tasks. I developed a framework based on the importance of exposing students to different types of visual representations of 3D objects (CCSSI, 2010;
NCTM, 2000). Table 7 illustrates the framework that I used to analyze the visual representations of 3D objects.

Table 5. Visual Representations of 3D Objects Framework

<table>
<thead>
<tr>
<th>Component of Analysis</th>
<th>Category</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Format</td>
<td>Net</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>Picture</td>
<td>P</td>
</tr>
<tr>
<td></td>
<td>Drawing</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>Student Generated</td>
<td>SG</td>
</tr>
<tr>
<td></td>
<td>Other</td>
<td>O</td>
</tr>
<tr>
<td>Representation Form</td>
<td>Real-world representation</td>
<td>RW</td>
</tr>
<tr>
<td></td>
<td>Non real-world representation</td>
<td>NRW</td>
</tr>
<tr>
<td>Location</td>
<td>In-text</td>
<td>IT</td>
</tr>
<tr>
<td></td>
<td>Not In-text</td>
<td>NIT</td>
</tr>
</tbody>
</table>

The five formats of the visual representations of 3D objects are nets, pictures, and drawings, student generated, and other. The representation forms of the visual representations of 3D objects are divided into two categories: real-world representations and non real-world representations. A real-world representation means that the visual representation of 3D object is a representation of a real-world object. A non real-world representation means that the visual representation of 3D object is a representation of a mathematical object. The location of the visual representations of 3D objects is divided into two categories: in-text and not in-text. In-text means that the visual representation of 3D object is present in the task. Not in-text means that the visual representation of 3D object is not present in the task.

During this analysis, I used the Visual Representations of 3D Objects framework to code the visual representations of 3D objects found within the surface area, volume, and surface area and volume tasks. To be more precise, I used the Visual Representations of 3D Objects framework to code the visual representations of 3D objects in terms of format, representation form, and location. I used the following labels to code the format, representation form, and
location of the visual representations of 3D objects: net (N), picture (P), drawing (D), student generated (SG), other (O), real-world representation (RW), non real-world representation (NRW), in-text (IT), and not in-text (NIT). First, I evaluated the visual representations of 3D objects to determine if it is a net, picture, drawing, student generated, or other. Next, I examined the visual representations of 3D objects to determine if it is a real-world or non real-world representation. Finally, I determined if the visual representations of 3D objects was in-text or not in-text. I coded all visual representations of 3D objects twice.

I employed simple descriptive statistical measures to compute the proportion of types of visual representations of 3D objects included in surface area, volume, and surface area and volume tasks within each textbook and textbook series. I then presented and compared the proportion of types of visual representations of 3D objects in the different mathematics curricula using graphical displays. I coded the visual representations of 3D objects as demonstrated in Table 8.

I selected the first five examples from the “Solving Surface Area Problems” and “Solving the Volume of Prisms” lessons within the Go Math! (Grade 7) textbook (Burger et al., 2014, pp. 283-294). I also chose the last two examples from the “Surface Area of Prisms” lesson within the Glencoe Math (Course 2) textbook (Carter et al., 2015, pp. 661-672).

Based on the Visual Representations of 3D Objects framework, I coded the first item as net, non real-world, and in-text; the second item as drawing, real-world, and in-text; the third item as drawing, non real-world, and in-text; the fourth item as drawing, real-world, and in-text; the fifth item as picture, real-world, and in-text; the sixth item as net, real-world, and in-text; and the last item as student-generated, non real-world, and not in-text.
<table>
<thead>
<tr>
<th>Code</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net (N)</td>
<td>The surface area of a three-dimensional figure is the sum of the areas of all its surfaces. You know how to use the net of a figure to find its surface area. Now you will discover a formula that you can use.</td>
</tr>
<tr>
<td>Non real-world (NRW)</td>
<td></td>
</tr>
<tr>
<td>In-text (IT)</td>
<td></td>
</tr>
<tr>
<td>Drawing (D)</td>
<td>The oatmeal box shown is shaped like a cylinder. Use a net to find the surface area S of the oatmeal to the nearest tenth. Then find the number of square feet of cardboard needed for 1,500 oatmeal boxes. Round your answer to the nearest whole number.</td>
</tr>
<tr>
<td>Real-world (RW)</td>
<td></td>
</tr>
<tr>
<td>In-text (IT)</td>
<td></td>
</tr>
<tr>
<td>Drawing (D)</td>
<td></td>
</tr>
<tr>
<td>Non real-world (NRW)</td>
<td>Find the volume of the prism.</td>
</tr>
<tr>
<td>In-text (IT)</td>
<td></td>
</tr>
<tr>
<td>Drawing (D)</td>
<td>A movie theater offers popcorn in two different containers for the same price. One container is a trapezoidal prism with a base area of 36 square inches and a height of 5 inches. The other container is a triangular prism with a base area of 32 square inches and a height of 6 inches. Which container is the better deal? Explain.</td>
</tr>
<tr>
<td>Real-world (RW)</td>
<td></td>
</tr>
<tr>
<td>In-text (IT)</td>
<td></td>
</tr>
</tbody>
</table>
Table 8. (Continued) Examples of Visual Representations of 3D Objects within Tasks

<table>
<thead>
<tr>
<th>Code</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Picture (P)</td>
<td>Alex made a sketch for a homemade soccer goal he plans to build. The goal will be in the shape of a tringular prism. The legs of the right triangles at the sides of his goal measure 4ft and 8ft, and the opening along the front is 24ft. How much space is contained within this goals?</td>
</tr>
<tr>
<td>Real-world (RW)</td>
<td></td>
</tr>
<tr>
<td>In-text (IT)</td>
<td></td>
</tr>
<tr>
<td>Net (N)</td>
<td>The net of a cereal box is made up of a total of _____ rectangles. What do you notice about the top and bottom faces, the left and right faces, and the front and back faces?</td>
</tr>
<tr>
<td>Real-world (RW)</td>
<td></td>
</tr>
<tr>
<td>In-text (IT)</td>
<td></td>
</tr>
<tr>
<td>Student-generated (SG)</td>
<td>Draw and label a rectangular prisms that has a total surface area between 100 and 200 square units. Then find the surface area of your prism.</td>
</tr>
<tr>
<td>Non real-world (NRW)</td>
<td></td>
</tr>
<tr>
<td>Not In-text (NIT)</td>
<td></td>
</tr>
</tbody>
</table>

Level of Mathematical Complexity

I used the Mathematics Framework for the 2007 NAEP to examine the level of mathematical complexity of tasks found within the surface area, volume, and surface area and volume lessons. Mathematical complexity is defined as the demands of thinking that a task makes on students (NAEP, 2007). The unit of analysis was the surface area, volume, and surface area and volume tasks included in the lessons. For this study, I defined a task as the mathematics activities and exercises which students are expected to complete cooperatively or independently.
According to the Mathematics Framework for the 2007 NAEP tasks are classified as being of low, moderate, or high complexity. A low-level complexity task’s main focus is to help students to remember previously learned concepts, thus students are not expected to create their own method to solve the problem. At this level, students are also expected to solve problems by computing a sum, difference, product, or quotient. A moderate-level complexity task requires more flexibility of thinking when compared to the low-complexity category. At this level, students are encouraged to solve multi-step tasks by “using informal methods of reasoning and different problem-solving strategies” (NAEP, 2007, p. 40). A high-level complexity task requires students to think critically and analytically, be creative and argumentative in mathematics, use their reasoning, and be able to justify or explain their work.

**Table 9. Mathematics Framework for the 2007 NAEP**

<table>
<thead>
<tr>
<th>HIGH COMPLEXITY</th>
<th>MODERATE COMPLEXITY</th>
<th>LOW COMPLEXITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>High complexity items make heavy demand on students who must engage in more abstract reasoning, planning, analysis, judgment and creative thought. The students are expected to think in abstract and sophisticated ways.</td>
<td>Items in this category involve more flexibility of thinking and choice among alternatives. They require a response that has more than a single step. The students are expected to decide what to do, using informal methods of reasoning and problem solving strategies.</td>
<td>This category consists of the recall and recognition of previously learned concepts and principles. Students carry out some procedure that can be performed mechanically. Students are not expected to produce an original method or solution.</td>
</tr>
</tbody>
</table>
Table 9. (Continued) Mathematics Framework for the 2007 NAEP

<table>
<thead>
<tr>
<th>HIGH COMPLEXITY</th>
<th>MODERATE COMPLEXITY</th>
<th>LOW COMPLEXITY</th>
</tr>
</thead>
</table>

➢ Describe how different representations can be used for different purposes
➢ Perform a procedure having multiple steps
➢ Analyze similarities and differences between procedures and concepts
➢ Generalize a pattern
➢ Formulate an original problem given data
➢ Solve a novel problem
➢ Solve a problem in more than one way
➢ Explain and justify a solution to a problem
➢ Describe, compare and contrast solution methods
➢ Formulate a mathematical model for a complex situation
➢ Analyze the assumptions made in a mathematical model
➢ Analyze or produce a deductive argument
➢ Provide a mathematical justification.

➢ Represent a situation mathematically in more than one way
➢ Select and use different representations, depending on situation and purpose
➢ Solve a problem requiring multiple steps
➢ Compare figures or statements
➢ Provide a justification for steps in a solution process
➢ Interpret a visual representation
➢ Extend a pattern
➢ Retrieve information from a graph, table or figure and use it to solve a problem requiring multiple steps
➢ Formulate a routine problem given data and conditions
➢ Interpret a simple argument
➢ Recall or recognize a fact, term or property
➢ Recognize an example of a concept
➢ Compute a sum, difference, product or quotient
➢ Recognize an equivalent representation
➢ Perform a specified procedure
➢ Evaluate an expression in an equation or formula for a given variable
➢ Solve a one-step word problem
➢ Draw or measure simple geometric figures
➢ Retrieve information from a drawing, table or graph

Table 9 presents the Mathematics Framework for the 2007 NAEP that I utilized to evaluate the level of mathematical complexity of surface area, volume, and surface area and volume tasks. During this analysis, I read and coded the surface area, volume, and surface area...
and volume tasks as low, medium, or high complexity using the Mathematics Framework for the 2007 NAEP. Precisely, I used the criterion under each level of mathematical complexity to code the tasks. I used the following labels to code the level of mathematical complexity of tasks: Low (L), Medium (M), or High (H). A task had to meet at least one criterion to be coded as low, medium, or high complexity. A task cannot be in between levels. Furthermore, a task containing multiple parts was analyzed as a whole. Therefore, I coded each task as requiring a single level of mathematical complexity. I repeated this process twice for each set of tasks.

I used simple descriptive statistical measures to calculate the proportion of the level of mathematical complexity of surface area, volume, and surface area and volume tasks contain in each textbook and textbook series. I then used graphical displays to report and compare the proportion of the level of mathematical complexity of surface area, volume, and surface area and volume tasks within each textbook and textbook series.

In the table below (Table 10), I provide examples for each level of mathematical complexity tasks in order to demonstrate the Mathematics Framework for the 2007 NAEP coding process. I chose all sample tasks from the Glencoe Math (Course 2) textbook (Carter et al., 2015, pp. 643-645).

The first example presents a low-complexity task. The students have to find the volume of a rectangular prism by computing the product. The second example illustrates a moderate-complexity task. The students have to find the cost to air condition the office for one month by performing multiple-step calculations. The third example shows is a high-complexity task. The students have to solve, explain, and justify their solution.
Table 6. Examples of Low, Moderate, and High Complexity Levels of Volume Tasks

<table>
<thead>
<tr>
<th>Code</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-Complexity</td>
<td>Find the volume of the prism. Round to the nearest tenth if necessary.</td>
</tr>
<tr>
<td>Medium-Complexity</td>
<td>The diagram shows the dimensions of an office. It costs about $0.11 per year to air condition one cubic foot of space. On average, how much does it cost to air condition the office for one month?</td>
</tr>
<tr>
<td>High-Complexity</td>
<td>A toy company makes rectangular sandboxes that measure 6 feet by 5 feet by 1.2 feet. A customer buys a sandbox and 40 cubic feet of sand. Did the customer by too much or too little sand? Justify your answer.</td>
</tr>
</tbody>
</table>

The Mathematics Framework for the 2007 NAEP was selected for three reasons. First, the NAEP is a congregational project of the Department of Education’s National Center for Education Statistics that has been gathering information about U.S. students’ performance on different subject area such as reading, writing, science, and mathematics since the early 1970s (NAEP, 2007). Therefore, the Mathematics Frameworks for the NAEP are well-established frameworks that allow the researcher to accurately and consistently assess the mathematical complexity of a task (Thompson, 2011). Second, the Mathematics Framework for the 2007 NAEP has been used to assess the mathematical complexity of task in previous studies (Schneider et al., 2013). In addition, I am very familiar with this framework, as I have used it to assess the mathematical complexity of tasks for two different studies. One study was part of a
final project for a doctoral level course (Hatziminadakis & Ercan, 2016). The other study was part of a research project. Both studies were submitted and accepted for presentation at two different international mathematics textbook conferences but only the first study was presented (Hatziminadakis & Ercan, 2016).

**Content Features of Lessons**

Based on the focus of this study, I also examined the surface area, volume, and surface area and volume lessons to determine if these lessons address the CCCS for 6-8 geometry that are aligned with these topics. I first utilized the Common Core Mathematics Companion: The Standards Decoded (Miles & Williams, 2016) book to break down the components for each standard. I then created and used the CCCS for 6-8 geometry components guideline (see table 11) and the geometric measurement standards for grade 5 (see table 12) to examine the extent to which the content of these lessons address the appropriate CCCS.

**Lesson Content and CCCS**

The unit of analysis was the surface area, volume, and surface area and volume lessons within each textbook. I used the CCCS for 6-8 geometry components guideline and the geometric measurement standards for grade 5 to examine if these lessons address the appropriate geometry content standards. Table 11 contains the CCCS for 6-8 geometry components that I used to evaluate these lessons. The geometric measurement standards for grade 5 are illustrated in Table 12.
I first read each lesson to determine to what extent it addresses the CCCS for 6-8 geometry components and/or the geometric measurement standards for grade 5. I then assigned to each lesson the CCCS for 6-8 geometry components and/or the geometric measurement standards for grade 5 it address. I repeated this process twice. After assigning to each lesson the appropriate CCCS for 6-8 geometry components and/or the geometric measurement standards for grade 5, I used tables to document and compare the extent to which these lessons address the CCCS for 6-8 geometry components and/or the geometric measurement standards for grade 5.

<table>
<thead>
<tr>
<th>CCCS MATH. CONTENT</th>
<th>Component 1</th>
<th>Component 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.G.A.2</td>
<td>Students will determine the volume of a right rectangular prism with fractional side lengths by using unit cubes</td>
<td>Students will apply the formulas ( V = l w h ) and ( V = b h ) to solve real-world and mathematical problems involving volume of right rectangular prisms with fractional edge lengths</td>
</tr>
<tr>
<td>6.G.A.4</td>
<td>Students will represent three-dimensional figures by using nets made up of rectangles and triangles</td>
<td>Students will use nets to solve real-world and mathematical problems involving surface area</td>
</tr>
<tr>
<td>7.G.B.6</td>
<td>Students will work with two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms</td>
<td>Students will solve real-world and mathematical problems involving area, volume, and surface area</td>
</tr>
<tr>
<td>8.G.C.9</td>
<td>Students will learn the volume formulas for cones, cylinders, and spheres</td>
<td>Students will apply the volume formulas to solve real-world and mathematical problems involving volume</td>
</tr>
</tbody>
</table>

Table 7. CCCS for 6-8 Geometry Components Guideline
Table 8. CCCS for Geometric Measurement (Grade 5)

<table>
<thead>
<tr>
<th>CCSS</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.MATH.CONTENT 5.MD.C.3</td>
<td>Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 5.MD.C.3.A</td>
<td>A cube with side length 1 unit, called a &quot;unit cube,&quot; is said to have &quot;one cubic unit&quot; of volume, and can be used to measure volume.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 5.MD.C.3.B</td>
<td>A solid figure which can be packed without gaps or overlaps using ( n ) unit cubes is said to have a volume of ( n ) cubic units.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 5.MD.C.4</td>
<td>Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 5.MD.C.5</td>
<td>Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 5.MD.C.5.A</td>
<td>Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 5.MD.C.5.B</td>
<td>Apply the formulas ( V = l \times w \times h ) and ( V = b \times h ) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT 5.MD.C.5.C</td>
<td>Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.</td>
</tr>
</tbody>
</table>

In the table below (Table 13), I provide an example to demonstrate to what extent two lessons address the CCCS for 6-8 geometry components and the geometric measurement standards for grade 5. Both lessons were drawn from the CM (Grade 6) textbook (Lappan et al., 2014, pp. 80-84)

Table 9. Examples of Lessons and CCCS

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
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<td>6</td>
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<td></td>
<td>X</td>
<td>X</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

84
The first lesson is devoted to the concept of surface area. This lesson addresses both components of the 6.G.A.4 content standard. The second lesson is devoted to the concept of volume. This lesson covers both components of the 6.G.A.2 content standard and two geometric measurement standards (5.MD.5a and 5.MD.5b) for grade 5. Based on the topic and grade level, both lessons address the appropriate CCSS for 6-8 geometry. However, the second lesson also covers two geometric measurement standards from the previous grade level.

**Reliability Measures**

Reliability in content analysis is concerned with stability and reproducibility (Kondracki et al., 2002). Stability also called intra-rater reliability measurement the consistency to which the same coder categorizes characteristics of texts (Stemler, 2001). To ensure stability, I read and coded the data twice. I then check for consistency in coding the data.

In content analysis, inter-rater reliability is another important component of reliability (Krippendorff, 1980). Inter-rater reliability is “often perceived as the standard measure of research quality” (Kolbe & Burnett, 1991, p. 248). Reproducibility also referred to as inter-rater reliability, is concerned with the degree of agreement between coders when coding a text (Stemler, 2001). As noted by Weber (1990), “to make valid inferences from the text, it is important that the classification procedure be reliable in the sense of being consistent: Different people should code the same text in the same way” (p. 12). However, high levels of disagreement among judges suggest weaknesses in research methods (Kolbe & Burnett, 1991, p. 248). To ensure coding reliability, it is recommended that at least two coders should code the sets of data (Kondracki et al., 2002).
For the first research question, I carefully read and coded the data. I also conducted all procedures twice. For the second research question, two coders coded the quantitative data. In particular, the tasks were coded in terms of their performance expectations, types of visual representations of 3D objects, and level of mathematical complexity. The first coder was the author and the second coder was a doctoral level mathematics education student. The author and the second coder have coded tasks using the Mathematics Framework for the 2007 NAEP for a previous study. The study was part of a final project for a doctoral level course and was presented at an international mathematics textbook conference. For the previous study, to ensure coding reliability, the author and the second coder randomly selected one set of tasks from each mathematics textbook. The coders first discussed the coding and reached consensus on the application of the codes. Each coder then coded the tasks independently and reached approximately 90% agreement.

For this study, the coders followed a similar coding procedure for coding the quantitative data. The coders meet three times to discuss the categories, characteristics, and symbols of each framework. During the first meeting, the coders read and discussed the codes. The coders then randomly selected and coded a sample set of tasks together. After they reached consensus on the application of the codes, the coders randomly selected another set of task. Each coder independently coded the set of tasks. During the second and third meeting, the coders discussed and compare their codes. After reaching 100% agreement on the application of codes, the coders randomly selected 10 sets of tasks to code in terms of their performance expectations, types of visual representations of 3D objects, and level of mathematical complexity. Again, each coder independently coded the set of tasks. The coders coded a total of 195 tasks. Approximately 15% of the tasks in the sample of textbooks were coded.
For the third research question, two coders coded the lessons. The first coder was the author and the second coder was her major professor. The author and her major professor have coded tasks together for a previous study. This study was submitted for presentation to an international mathematics textbook conference. The coders first met to discuss the application of the CCCS for 6-8 Geometry Components guideline and the CCCS for Geometric Measurement (Grade 5). After reaching an agreement on the coding process, the coders randomly selected four lessons. Each coder independently coded the lessons. Nearly 10% of the lessons in the sample of textbooks were coded.

To measure the percent of agreement between the two coders, I added the number of tasks coded the same way by both coders and divided it by the total number of tasks. I followed the same process to calculate the percent of agreement between the two coders for all coding types. A 1.00 signified total agreement and .00 indicated no agreement. According to Neuendorf (2002), 90% or greater agreement would be acceptable to all and 80% or greater agreement would be acceptable in most cases. Table 14 displays the reliability measures.

**Table 10. Reliability Measures**

<table>
<thead>
<tr>
<th>Coding Type</th>
<th>Agreement with Second Coder</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Performance Expectations</strong></td>
<td></td>
</tr>
<tr>
<td>Percent of tasks with agreement</td>
<td>83%</td>
</tr>
<tr>
<td>Percent of codes with agreement</td>
<td>93%</td>
</tr>
<tr>
<td><strong>Visual Representations of 3D Objects</strong></td>
<td></td>
</tr>
<tr>
<td>Percent of tasks with agreement</td>
<td>100%</td>
</tr>
<tr>
<td>Percent of codes with agreement</td>
<td>100%</td>
</tr>
<tr>
<td><strong>Level of Mathematical Complexity</strong></td>
<td></td>
</tr>
<tr>
<td>Percent of tasks with agreement</td>
<td>95%</td>
</tr>
<tr>
<td><strong>Lessons and CCCS</strong></td>
<td></td>
</tr>
<tr>
<td>Percent of lessons with agreement</td>
<td>100%</td>
</tr>
</tbody>
</table>

As illustrated in Table 13, the reliability for performance expectations was 83%; the reliability for visual representations of 3D objects was 100%; the reliability for level of
mathematical complexity was 95%; and the reliability for lessons and CCCS was 100%. It is important to point out that coding tasks in regards to performance expectations and visual representations of 3D objects involved more than one code. Therefore, the percent of codes with agreement for both performance expectations and visual representations of 3D objects was 93% and 100%, respectively.

Validity

In content analysis, it is also vital to establish validity (Krippendorf, 2004; White & Marsh, 2006). Holsti (1969) defined validity as “the defensibility of the inferences make from the data collected through the use of an instrument” (p. 90). Along the same line, Holsti and colleagues (1990) described validity as “the appropriateness, meaningfulness, and usefulness of the specific inferences researchers make based on the data they collect” (p. 127). Therefore, validity depends on the amount and type of evidence used to support inferences made from the data collected (Fraenkel & Wallen, 1990).

Krippendorf (2004) identified seven forms of validity: face validity, social validity, empirical validity, content validity, construct validity, concurrent validity, and criterion-related validity. The most common form of validity used in content analysis studies is face validity (Krippendorf, 2004; Weber, 1990; White & Marsh, 2006). Face validity has been defined as “the extent to which a measure “gets at” the essential aspects of the concept being measured” (White & Marsh, 2006, p. 31). To determine face validity, the researcher needs to provide evidence regarding the appropriateness and quality of the content and format of the instrument used in his/her study (Fraenkel & Wallen, 1990).

For this study, I established validity for research question 1 (part b) by implementing Neuendorf’s (2002) validity approach, “what you see is what you get” (WYSIWYG). I used the
textbooks as the source to generate the categories of the instructional blocks of lessons. More specifically, I developed the categories of the instructional blocks of lessons by reading through the data and recognizing patterns. For research questions 1 (part a) and 2 (parts a and c), I established validity by using pre-established frameworks that include categories, codes, and definitions that are valid and relevant to the purpose of this study. That is, these categories, codes, and definitions adequate the purpose of this study. According to Fraenkel and Wallen (1990), “the quality of instruments used in research is very important, for the conclusions researchers draw based on the information they obtain using these instruments” (p. 126).

Therefore, I employed well-known frameworks used in previous content analysis studies to code the data needed to draw appropriate, meaningful, and useful inferences regarding students’ opportunity to learn the concepts of surface area and volume.

For research questions 2 (part b) and 3, I developed the Visual Representations of 3D Objects Framework based on the national recommendations and standards and the CCCS for 6-8 Geometry Components Guideline based on the geometry content standards. As noted by Krippendorf (2004), “a measuring instrument is considered valid if it measures what it user it claims it measures” (p. 313). Thus, the validity of the categories, codes, and definitions developed and included in both frameworks were supported by national recommendations and standards. I also established validity of these categories, codes, and definitions by having the second coder review these measures.

Summary of Research Design and Methodology

In this chapter, I described the research design and methodology for this study. In particular, I presented and discussed the three research questions, the sample of textbooks, the
research design method, the coding scheme, the data collection and procedures, the reliability measures, and validity of this study. In the next chapter, I report the findings of this study.
CHAPTER 4: RESULTS

The purpose of this study was to analyze the treatment of surface area and volume concepts in order to determine students’ opportunities to learn these concepts. I selected four series of middle-grades student edition mathematics textbooks from 2008 to present. More specifically, I chose two popular and two alternative mathematics textbook series. Each series includes textbooks for grades 6, 7, and 8. I examined a total of 12 textbooks during this study.

Research Questions

The following three research questions were addressed in this study:

1) Within published mathematics textbook series and across different publishers, what are the structural features devoted to the concepts of surface area and volume? In particular,
   a. Where are the surface area and volume lessons located and how many pages and lessons are devoted to surface area and volume?
   b. How are the instructional blocks of surface area and volume lessons sequenced?

2) What are the pedagogical features of the tasks included in the surface area and volume lessons within a published mathematics textbook series, and across different publishers? Specifically,
   a. What are the performance expectations of students within these tasks?
   b. What types of visual representations of 3D objects are included in these tasks?
   c. What is the level of the mathematical complexity of these tasks?

3) To what extent do the content of surface area and volume lessons address the Common Core Content Standards for 6-8 geometry that are aligned with these topics?
This chapter is divided into four sections to address the research questions. In the first section, I present the findings regarding the treatment of surface area and volume concepts in terms of the location of surface area and volume lessons in the textbook and the number of pages and lessons devoted to these concepts. In the second section, I report the results related to the sequence of the instructional blocks of surface area and volume lessons. In the third section, I present the findings related to the performance expectations of students within tasks, the types of visual representations of 3D objects included in tasks, and the level of mathematical complexity of tasks. In the final section, I report the results in regards to the extent to which the content of surface area and volume lessons address the CCCS for 6-8 geometry that are aligned with these topics. I conclude this chapter with a brief summary of the results.

**Physical Characteristics of Textbook Series**

In this section, I report the results related to the location of surface area and volume lessons in the textbooks and the number of pages and lessons devoted to these concepts within published mathematics textbook series and across different publishers. I used Flanders’ (1994) counting method to determine the location of the surface area and volume lessons and the number of pages and lessons devoted to these concepts. I also labeled lessons that address both concepts as surface area and volume lessons. I examined a total of 12 middle-grades mathematics textbooks.

**Location of the Topic**

The location of the surface area and volume concepts in each textbook are presented in Table 15. This table displays the total number of instructional pages and lessons devoted to the concepts of surface area and volume in each textbook.
Table 11. Location of Surface Area (SA) and Volume (V) Concepts in Each Textbook

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Number Total Lessons</th>
<th>Number Total Instr. Pages</th>
<th>% Pages Prior to 1st SA Page</th>
<th>% Pages Prior to 1st V Page</th>
<th>% Lessons Prior to 1st SA Lesson</th>
<th>% Lessons Prior to 1st V Lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>GM6</td>
<td>55</td>
<td>488</td>
<td>86</td>
<td>87</td>
<td>87</td>
<td>89</td>
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<tr>
<td>GM7</td>
<td>49</td>
<td>430</td>
<td>66</td>
<td>67</td>
<td>69</td>
<td>71</td>
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<tr>
<td>GM8</td>
<td>51</td>
<td>472</td>
<td>N/A</td>
<td>85</td>
<td>N/A</td>
<td>88</td>
</tr>
<tr>
<td>GMC</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>GMC6</td>
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<td>926</td>
<td>82</td>
<td>79</td>
<td>84</td>
<td>81</td>
</tr>
<tr>
<td>GMC7</td>
<td>72</td>
<td>854</td>
<td>77</td>
<td>75</td>
<td>81</td>
<td>78</td>
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<tr>
<td>GMC8</td>
<td>62</td>
<td>732</td>
<td>84</td>
<td>80</td>
<td>87</td>
<td>82</td>
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<td>CM</td>
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<td>CM6</td>
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<td>51</td>
<td>55</td>
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<td>78</td>
<td>77</td>
</tr>
<tr>
<td>CM8</td>
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<td>841</td>
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<tr>
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<td>89</td>
</tr>
<tr>
<td>U8</td>
<td>108</td>
<td>832</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 15 also shows the percent of pages and lessons prior to the introduction of these concepts. I rounded the data in the table to the nearest whole percent. In most textbooks, the concepts of surface area and volume are introduced after the middle or towards the end of the textbook. In all textbooks, approximately 70% or more of the instructional pages precede the first surface area and volume instructional page. Exception to this is the CM6 textbook that introduces both concepts in the middle of the textbook. Furthermore, 70% or more of the lessons precede the first surface area and/or volume lesson across all textbooks. Again, exception to this is the CM6 textbook; both concepts are introduced in the middle of the textbook. Three textbooks (GM8, CM8, and U8) do not include any surface area lessons. The U8 textbook also does not contain any volume lessons. It is compelling to note that lessons follow one another over the four textbook series. That is, all surface area and volume lessons are grouped together.
within the same unit or chapter. Both the GM and CM textbook series introduce first the concept of surface area and then the concept of volume. In contrast, the GMC textbook series introduces these concepts in reverse order. Finally, the UCSMP textbook series introduces both concepts at the same time.

**Number of Pages**

The total number of instructional pages in each textbook is displayed in Table 16. This table also shows the total number and percent of instructional pages devoted to the concept of surface area and volume in each textbook. I calculated the total number of instructional pages devoted to these concepts by implementing linear measurement of the pages. I rounded instructional pages that included other topics to the nearest quarter of a page. I also used a separate column to report the number and percent of instructional pages that address both concepts.

**Table 12.** Number and Percent of Surface Area (SA), Volume (V), and Surface Area and Volume (SA&V) Pages in Each Textbook

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Number Total Instr. Pages</th>
<th>Number SA Pages</th>
<th>Number V Pages</th>
<th>Number SA&amp;V Pages</th>
<th>Percent SA Pages</th>
<th>Percent V Pages</th>
<th>Percent SA&amp;V Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>2.00</td>
<td>1.8</td>
<td>2.6</td>
<td>0.4</td>
</tr>
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<td>GM7</td>
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<td>6.25</td>
<td>8.25</td>
<td>3.25</td>
<td>1.5</td>
<td>1.9</td>
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<tr>
<td>GM8</td>
<td>472</td>
<td>0.00</td>
<td>25.75</td>
<td>0.25</td>
<td>0.0</td>
<td>5.5</td>
<td>&lt;0.01</td>
</tr>
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<td><strong>GMC</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GMC6</td>
<td>926</td>
<td>31.75</td>
<td>21.00</td>
<td>7.50</td>
<td>3.4</td>
<td>2.3</td>
<td>0.8</td>
</tr>
<tr>
<td>GMC7</td>
<td>854</td>
<td>21.50</td>
<td>22.00</td>
<td>14.75</td>
<td>2.5</td>
<td>2.6</td>
<td>1.7</td>
</tr>
<tr>
<td>GMC8</td>
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<td>24.50</td>
<td>28.75</td>
<td>12.00</td>
<td>3.3</td>
<td>3.9</td>
<td>1.6</td>
</tr>
<tr>
<td><strong>CM</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM6</td>
<td>730</td>
<td>9.0</td>
<td>3.50</td>
<td>10.25</td>
<td>1.2</td>
<td>4.8</td>
<td>1.4</td>
</tr>
<tr>
<td>CM7</td>
<td>812</td>
<td>8.5</td>
<td>25.25</td>
<td>26.00</td>
<td>1.0</td>
<td>3.1</td>
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<tr>
<td>CM8</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>U6</td>
<td>765</td>
<td>6.75</td>
<td>6.00</td>
<td>8.0</td>
<td>0.9</td>
<td>0.8</td>
<td>1.0</td>
</tr>
<tr>
<td>U7</td>
<td>791</td>
<td>11.5</td>
<td>12.75</td>
<td>8.0</td>
<td>1.5</td>
<td>1.6</td>
<td>1.0</td>
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<tr>
<td>U8</td>
<td>835</td>
<td>1.25</td>
<td>6.50</td>
<td>0.0</td>
<td>0.2</td>
<td>0.8</td>
<td>0.0</td>
</tr>
</tbody>
</table>
As indicated in Table 16, the number of total instructional pages in twelve textbooks ranges from 430 to 926. The GM textbook series contains the least amount of instructional pages. The GMC, CM, and UCSMP textbook series have almost double the amount of instructional pages than the GM textbook series. Among all textbooks, less than 6% of the instructional pages are devoted to the concepts of surface area and volume. More specifically, the percent of instructional pages devoted to the concepts of surface area and volume across all textbooks, range from 0.2% to 5.5%. The majority of textbooks also contain instructional pages that address both concepts simultaneously ranging from 0.01% to 3.2%.

![Figure 1. Percent of Instructional Pages in Each Textbook](image)

Figure 1. Percent of Instructional Pages in Each Textbook
A closer examination of Table 16 and Figure 1 also reveals a variation in the percentage of instructional pages devoted to the concepts of surface area and volume across textbooks. The GMC6 textbook has the highest percentage of instructional pages devoted to the concept of surface area followed by the GMC8 textbook. Contrary, less than 1% of the instructional pages in the CM8, U6, and U8 textbooks address the concept of surface area. The GM8 textbook is the only textbook that does not contain any instructional pages devoted to the concept of surface area.

The GM8, GMC8, and CM6 textbooks place a greater emphasis on volume indicated by the higher proportion of instructional pages devoted to this concept. By the way of contrast, the U6 and U8 textbooks contain the least amount of volume pages. Both the U6 and U8 textbooks have less than 1% of volume pages. Almost all textbooks include instructional pages that address both concepts simultaneously. Exception to this is the U8 textbook; it does not contain any instructional pages that address both concepts. The CM7 textbook has the highest percentage of instructional pages that address both concepts followed by the GMC7 and GMC8 textbooks.

**Number of Lessons**

The total number of lessons in each textbook and textbook series is reported in Table 17. This table also presents the total number and percent of surface area and volume lessons in each textbook and textbook series. I used a separate column to report lessons that address both concepts. I also rounded the data to the tenths place.
Table 13. Number and Percent of Surface Area (SA), Volume (V), and Surface Area and Volume (SA&V) Lessons in Each Textbook and Textbook Series

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Number Total Lessons</th>
<th>Number SA Lessons</th>
<th>Number V Lessons</th>
<th>Number SA&amp;V Lessons</th>
<th>Percent SA Lessons</th>
<th>Percent V Lessons</th>
<th>Percent SA&amp;V Lessons</th>
</tr>
</thead>
<tbody>
<tr>
<td>GM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM6</td>
<td>55</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>2.0</td>
<td>3.9</td>
<td>0.0</td>
</tr>
<tr>
<td>GM7</td>
<td>49</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2.0</td>
<td>2.0</td>
<td>0.0</td>
</tr>
<tr>
<td>GM8</td>
<td>51</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>0.0</td>
<td>5.9</td>
<td>0.0</td>
</tr>
<tr>
<td>GM678</td>
<td>155</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>1.3</td>
<td>3.9</td>
<td>0.0</td>
</tr>
<tr>
<td>GMC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GMC6</td>
<td>79</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>3.8</td>
<td>2.5</td>
<td>0.0</td>
</tr>
<tr>
<td>GMC7</td>
<td>72</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2.8</td>
<td>2.8</td>
<td>1.3</td>
</tr>
<tr>
<td>GMC8</td>
<td>62</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3.2</td>
<td>4.8</td>
<td>1.6</td>
</tr>
<tr>
<td>GMC678</td>
<td>213</td>
<td>7</td>
<td>7</td>
<td>2</td>
<td>3.3</td>
<td>3.3</td>
<td>0.9</td>
</tr>
<tr>
<td>CM</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM6</td>
<td>104</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1.9</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>CM7</td>
<td>116</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>3.4</td>
<td>5.2</td>
<td>0.9</td>
</tr>
<tr>
<td>CM8</td>
<td>136</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0.0</td>
<td>1.5</td>
<td>0.0</td>
</tr>
<tr>
<td>CM678</td>
<td>356</td>
<td>6</td>
<td>9</td>
<td>1</td>
<td>1.7</td>
<td>2.5</td>
<td>0.3</td>
</tr>
<tr>
<td>UCSMP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U6</td>
<td>106</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td>2.8</td>
</tr>
<tr>
<td>U7</td>
<td>105</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1.9</td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td>U8</td>
<td>108</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>U678</td>
<td>319</td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>0.6</td>
<td>0.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

An examination of Table 17 shows that the total number of lessons range from 155 to 316. The GM textbook series appears to have the least amount of total lessons followed by the GMC textbook series. The other two textbook series (CM and UCSMP) contain a similar amount of total lessons. Notice that the CM and UCSMP textbook series have double the amount of total lessons than the GM textbook series. Another important observation is that the number of surface area and volume lessons in all textbook series is low. Less than 4% of lessons in all textbook series are devoted to the surface area and volume concepts.

Across all textbooks the total number of lessons range from 49 to 116. The number of surface area, volume, and surface area and volume lessons range from 1 to 5. The analysis also
indicated that the percent of surface area, volume, and surface area and volume lessons range from 1% to 5.9%. That is, most of the textbooks contain a low number and percentage of lessons devoted to the concepts of surface area and volume (see Figure 2).

**Figure 2.** Percent of Lessons in Each Textbook

The GMC6 textbook has the highest amount of surface area lessons followed by the CM7 textbook. Two of the GM textbooks, GM6 and GM7, contain only one surface area lesson. None of the textbooks offer more than 4 surface area lessons. It is also worth noting that three textbooks (GM8, CM8, and U8) do not offer any surface area lessons. The GM8 and GMC8 textbooks include 4 and 5 lessons on volume respectively. However, two textbooks (CM7 and CM6) offer only one volume lesson. The U6 and U7 textbooks have the most lessons that address both concepts simultaneously. The U8 textbook does not include any lessons that address
the concepts of surface area and volume. At last, the GM textbooks do not contain any lessons that address both concepts simultaneously.

**Summary of Physical Characteristics**

On the whole, approximately three-fourths of the instructional pages in all textbooks precede the first instructional page devoted to the concepts of surface area and volume with the exception of the CM6 textbook that introduces these concepts in the middle of the textbook. Likewise, about three-fourths of the lessons in the sample textbooks precede the first lesson devoted to the concepts of surface area and volume. Again, only the CM6 textbook includes lessons devoted to the concepts of surface area volume in the middle of the textbook.

All textbooks contain significantly small amounts of instructional pages devoted to the concepts of surface area and volume. The GMC6 and GMC8 textbooks have the highest percentage of instructional pages devoted to the concept of surface area, 3.4% and 3.3%. The GM8 textbook does not contain any surface area pages. The GM8, GMC8, and CM6 textbooks have the largest percentage of instructional pages devoted to the concept of volume ranging from 3.9% to 5.5%. Nearly 3% or less of the instructional pages in all textbooks address both concepts.

The majority of textbooks contain 1 to 6 lessons devoted to the concepts of surface area and volume. Three 8th grade textbooks (GM8, CM8, and U8) do not offer any surface area lessons. The U8 textbook also does not offer any volume lessons. Five out of twelve textbooks only have lessons that introduce both concepts simultaneously.
Structure of Lessons in Textbook Series

In this section, I present the findings in regards to the sequence of the instructional blocks of lessons devoted to the concepts of surface area and volume within published mathematics textbook series and across different publishers.

Sequence of Instructional Blocks

I determined the sequence of the instructional blocks of surface area, volume, and surface area and volume lessons by using content analysis. As earlier stated, some lessons address both concepts. I labeled these lessons as surface area and volume lessons. I analyzed a total of 49 lessons over the four textbook series. Precisely, I examined 17 surface area, 24 volume, and 8 surface area and volume lessons during this study. The U8 textbook was not part of this analysis because it does not contain any lessons that address the concepts of surface area and volume. In the following paragraphs, I describe and provide a display of the sequence of the instructional blocks of these lessons for each textbook series.

Go Math Textbook Series

There are 2 surface area and 6 volume lessons within the GM textbook series. Analysis indicated that the lessons within the GM textbook series contain eight instructional blocks. Table 18 presents the sequence of the instructional blocks of the lessons within the GM textbook series. For the GM textbook series, lesson sequence begins with the essential question, activity, and description of concept that includes vocabulary terms and formula. In the middle of the lesson, worked examples with solutions, reflection questions, and practice problems are provided. All lessons in this textbook series conclude with independent practice.
Table 14. Sequence of the Instructional Blocks of Lessons within GM Textbook Series

<table>
<thead>
<tr>
<th>Instructional Blocks</th>
<th>Essential Question</th>
<th>Activity</th>
<th>Description of Concept + Vocabulary + Formula</th>
<th>Worked Examples + Solutions</th>
<th>Reflection Questions</th>
<th>Practice Problems</th>
<th>Independent Practice</th>
</tr>
</thead>
</table>

Glencoe Math Textbook Series

The GMC textbook series offers 7 surface area, 7 volume, and 2 surface area and volume lessons. Analysis revealed that most lessons in the GMC textbook series include two parts: inquiry lab and lesson.

Table 15. Sequence of the Instructional Blocks of Lessons within GMC Textbook Series

<table>
<thead>
<tr>
<th>Instructional Blocks within Inquiry Lab</th>
<th>Inquiry Question</th>
<th>Hands-on Activity</th>
<th>Practice Problems</th>
<th>Reflection Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructional Blocks within Lesson</td>
<td>Essential Question</td>
<td>Introductory Task</td>
<td>Description of Concept + Vocabulary + Formula</td>
<td>Worked Examples + Solutions</td>
</tr>
</tbody>
</table>

The sequence of the instructional blocks of the lessons within the GMC textbook series is displayed in Table 19. Within the GMC textbook series, a typical inquiry lab contains four instructional blocks: inquiry question, hands-on activity, practice problems, and reflection questions. The lessons in the GMC textbook series have nine instructional blocks. In particular, the lessons start with the essential question followed by an introductory task, description of concept that includes vocabulary terms and formula, worked examples with solutions, and
practice problems. Lastly, each lesson in the GMC textbook series ends with independent practice, test practice, and review problems.

*Connected Mathematics Textbook Series*

There are 6 surface area, 9 volume, and 1 surface area and volume lessons within the CM textbook series. Analysis showed that lessons within the CM textbook series follow a similar sequence. Table 20 demonstrates the sequence of the instructional blocks of the lessons within the CM textbook series.

**Table 16. Sequence of the Instructional Blocks of Lessons within CM Textbook Series**

<table>
<thead>
<tr>
<th>Instructional Blocks</th>
<th>Introductory Task</th>
<th>Reflection Question</th>
<th>Multi-Step Problems</th>
<th>Independent Practice</th>
</tr>
</thead>
</table>

The lessons in the CM textbook series contain four instructional blocks. A typical lesson in the CM textbook series begins with an introductory task followed by reflection questions. In addition, each lesson in the CM textbook series concludes with a set of multi-step problems and independent practice. A multi-step problem contains several parts. It is also imperative to note that the independent practice section is located at the end of the unit.

*University of Chicago School Mathematics Project Textbook Series*

The UCSMP textbook series contain 2 surface area, 2 volume, and 5 surface area and volume lessons. Analysis indicated that the lessons in the USCMP textbook series follow a similar structure. The lessons in the UCSMP textbook series include eight instructional blocks as shown in Table 21. In the USCMP textbook series, typical lessons start with the big idea, description of concept that includes vocabulary terms and formula, worked examples with solutions; followed by guided practice, practice problems, and activity. All lessons in the UCSMP textbook series conclude with an independent practice and review problems.
Table 17. Sequence of the Instructional Blocks of Lessons within UCSMP Textbook Series

<table>
<thead>
<tr>
<th>Instructional Blocks</th>
<th>Big Idea</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Description of Concept+ Vocabulary + Formula</td>
</tr>
<tr>
<td></td>
<td>Worked Examples + Solutions</td>
</tr>
<tr>
<td></td>
<td>Guided Practice</td>
</tr>
<tr>
<td></td>
<td>Practice Problems</td>
</tr>
<tr>
<td></td>
<td>Activity</td>
</tr>
<tr>
<td></td>
<td>Independent Practice</td>
</tr>
<tr>
<td></td>
<td>Review Problems</td>
</tr>
</tbody>
</table>

Summary of Sequence of Instructional Blocks

Among the four textbook series there are some similarities and differences of the types and sequence of instructional blocks of lessons. Three out of four textbook series (GM, GMC, and UCSMP) include lessons that begin with a question or statement called essential question or big idea. All three textbook series also contain lessons with description of the concept that includes vocabulary terms and formula, worked examples with solutions, and activity. However, only the GMC textbook series offers a separate section for the activity called the inquiry lab. In addition, the activities in the UCSMP textbook series are mostly located towards the middle or end of the lesson. In contrast, all activities in both the GM and GMC textbook series are located in the beginning of the lesson.

In three textbook series (GM, GMC, and CM), the lessons contain reflection questions. The lessons in both the GMC and CM textbook series offer an introductory task. The lessons in both the GMC and USCMP textbook series also have review problems. All lessons over the four textbook series include practice problems and independent practice.

For the most part, the types of instructional blocks of the lessons within the GM and GMC textbook series are similar, with both series containing an essential question, activity, description of concept that includes vocabulary terms and formula, worked examples with
solutions, reflection questions, practice problem, and independent practice. The instructional blocks of the lessons within the UCSMP textbook series are also similar to the GM and GMC textbook series but have a slightly different distribution in sequence. For example, most of the activities are not located in the beginning of the lesson. Another important observation is that the CM textbook series includes lessons with fewer instructional blocks and the practice problems are located at the end of the unit.

**Pedagogical Features of Tasks in Textbook Series**

In this section, I report the results related to the performance expectations of students within tasks, the types of visual representation of 3D objects included in tasks, and the level of mathematical complexity of tasks found within the surface area, volume, and surface area and volume lessons in published mathematics textbooks series and across different publishers.

**Number of Surface Area, Volume, and Surface Area and Volume Tasks**

I evaluated all tasks located within the surface area, volume, and surface area and volume lessons in terms of their performance expectations, types of visual representations of 3D objects, and level of mathematical complexity. Exception to this is the CM textbook series that most of the tasks are located at the end of the unit. The tasks within the CM textbook series contain multiple parts. Each part was coded as one task. The U8 textbook was not part of this analysis because it does not contain any lessons devoted to the concepts of surface area and volume.

I examined a total of 1,186 tasks within the four textbook series. To be more precise, I evaluated a total of 186 tasks in the GM textbook series; 637 tasks in the GMC textbook series; 208 tasks in the CM textbook series; and 155 tasks in the UCSMP textbook series (see Table 22). In the table below, I rounded the data to the nearest whole percent. I also divided the tasks into
three types: surface area, volume, and surface area and volume. I labeled tasks that address both concepts as surface area and volume tasks.

Table 18. Number and Percent of Surface Area (SA), Volume (V), and Surface Area and Volume (SA&V) Tasks in Each Textbook and Textbook Series

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Total Tasks</th>
<th>SA Tasks</th>
<th>V Tasks</th>
<th>SA&amp;V Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>GM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM6</td>
<td>53</td>
<td>13</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>GM7</td>
<td>46</td>
<td>21</td>
<td>46</td>
<td>25</td>
</tr>
<tr>
<td>GM8</td>
<td>87</td>
<td>0</td>
<td>0</td>
<td>87</td>
</tr>
<tr>
<td>GM678</td>
<td>186</td>
<td>34</td>
<td>18</td>
<td>152</td>
</tr>
<tr>
<td><strong>GMC</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GMC6</td>
<td>194</td>
<td>106</td>
<td>55</td>
<td>84</td>
</tr>
<tr>
<td>GMC7</td>
<td>187</td>
<td>81</td>
<td>43</td>
<td>83</td>
</tr>
<tr>
<td>GMC8</td>
<td>256</td>
<td>110</td>
<td>43</td>
<td>133</td>
</tr>
<tr>
<td>GMC678</td>
<td>637</td>
<td>297</td>
<td>47</td>
<td>300</td>
</tr>
<tr>
<td><strong>CM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CM6</td>
<td>51</td>
<td>25</td>
<td>49</td>
<td>11</td>
</tr>
<tr>
<td>CM7</td>
<td>135</td>
<td>27</td>
<td>20</td>
<td>85</td>
</tr>
<tr>
<td>CM8</td>
<td>22</td>
<td>0</td>
<td>0</td>
<td>22</td>
</tr>
<tr>
<td>CM678</td>
<td>208</td>
<td>52</td>
<td>25</td>
<td>118</td>
</tr>
<tr>
<td><strong>UCSMP</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>U6</td>
<td>58</td>
<td>20</td>
<td>35</td>
<td>26</td>
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<td>U7</td>
<td>97</td>
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<td>43</td>
<td>38</td>
</tr>
<tr>
<td>U8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>U678</td>
<td>155</td>
<td>62</td>
<td>40</td>
<td>64</td>
</tr>
</tbody>
</table>

As observed in Table 22, the GMC6 and GMC8 textbooks have the greatest number of surface area tasks. The GM6 textbook includes the least amount of surface area tasks followed by the GM7 and U6 textbooks. Note that two textbooks (GM8 and CM8) do not offer any surface area tasks. In contrast, all textbooks contain volume tasks. The GMC8 textbook has the largest amount of volume tasks. The CM6 textbook has the fewest number of volume tasks followed by the CM8 textbook. In terms of tasks that address both concepts, the GMC7 and CM7 textbooks have the greatest amount. Notice that four textbooks (GM6, GM7, GM8, and CM) do not offer any tasks that address both concepts. The GM8 and CM8 textbooks contain only
volume tasks. That is, all tasks in both the GM8 and CM8 textbooks are devoted to the concept of volume. Figure 3 provides a graphical representation of the total number of surface area, volume, and surface area and volume tasks within each textbook. This representation allows for a visual comparative analysis from one textbook to another.

![Bar chart showing total number of tasks in each textbook](image)

**Figure 3.** Total Number of Tasks in Each Textbook

A further examination of Table 22 indicates that the percentage of surface area, volume, and surface area and volume tasks varies at each grade level. At the sixth grade level, approximately 50% of tasks in the GMC6 and CM6 textbooks are devoted to the concept of surface area. The GM6 textbook has the smallest percentage of surface area tasks. For the concept of volume, the CM6 textbook has the least percentage of volume tasks. However, three-fourths of the tasks in the GM6 textbook are devoted to the concept of volume. The CM6 textbook also provides the largest proportion of tasks that address both concepts.
At the seventh grade level, nearly half of the tasks in three textbooks (GM7, GMC7, and U7) address the concept of surface area. The CM7 textbook contains a larger percent of volume tasks. Both the CM7 and U7 textbooks offer an equal proportion of tasks that address both concepts. At the eighth grade level, only the GMC8 textbook contain surface area tasks and tasks that address both concepts. This textbook also has the smallest proportion of volume tasks. Figure 4 contains a visual representation of the percent of surface area, volume, and surface area and volume tasks within each textbook.

![Figure 4. Percent of Tasks in Each Textbook](image)

Table 22 and Figure 5 also suggest substantial differences in the distribution of the number of tasks over the four textbook series. The GMC textbook series has a significantly larger number of both surface area and volume tasks than the GM, CM, and UCSMP textbook series. Approximately two-thirds of the surface area tasks and half of the volume tasks from the entire sample are located within the GMC textbook series. There are approximately an equivalent
number of tasks that address both concepts in the GMC and CM textbook series followed by the UCSMP textbook series. The GM textbook series does not include any tasks that address both concepts.

![Bar chart showing the total number of tasks in textbook series.](image)

**Figure 5. Total Number Tasks in Textbook Series**

As illustrated in Figure 6, the GM and CM textbook series have a different composition; both textbook series contain a smaller percentage of surface area tasks than the GMC and UCSMP textbook series. The GM textbook series also includes a significantly larger proportion of volume tasks. Nearly four-fifths of the tasks in the GM textbook address the concept of volume. Tasks that address both concepts are less represented across all textbook series. Both the UCSMP and CM textbook series contain a similar proportion of tasks that address both concepts followed by the GMC textbook series.
Figure 6. Percent of Tasks in Textbook Series

In the following paragraphs, I present and describe the results of performance expectations of students within tasks, the types of visual representations of 3D objects included in tasks, and the level of mathematical complexity of tasks using numerical and graphical representations.

Performance Expectations of Students within Tasks

I adopted the TIMSS 2002 Performance Expectations for Mathematics (Valverde et al., 2002) to evaluate the performance expectations of students within tasks. Based on the TIMSS 2002 Performance Expectations for Mathematics framework, I used twenty-one performance expectations codes during this analysis (see Table 23).
<table>
<thead>
<tr>
<th>Code</th>
<th>Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representing</td>
<td>R</td>
</tr>
<tr>
<td>Recognizing Equivalents</td>
<td>RE</td>
</tr>
<tr>
<td>Recalling Mathematical Objects &amp; Properties</td>
<td>RMOP</td>
</tr>
<tr>
<td>Using Vocabulary &amp; Notation</td>
<td>UVN</td>
</tr>
<tr>
<td>Using Equipment</td>
<td>UE</td>
</tr>
<tr>
<td>Performing Routine Procedures</td>
<td>PRP</td>
</tr>
<tr>
<td>Using More Complex Procedures</td>
<td>MCP</td>
</tr>
<tr>
<td>Formulating &amp; Clarifying Problems &amp; Situations</td>
<td>FCPS</td>
</tr>
<tr>
<td>Developing Strategy</td>
<td>DS</td>
</tr>
<tr>
<td>Solving</td>
<td>S</td>
</tr>
<tr>
<td>Predicting</td>
<td>P</td>
</tr>
<tr>
<td>Verifying</td>
<td>V</td>
</tr>
<tr>
<td>Developing Notation &amp; Vocabulary</td>
<td>DNV</td>
</tr>
<tr>
<td>Developing Algorithms</td>
<td>DA</td>
</tr>
<tr>
<td>Generalizing</td>
<td>G</td>
</tr>
<tr>
<td>Conjecturing</td>
<td>CON</td>
</tr>
<tr>
<td>Justifying &amp; Proving</td>
<td>JP</td>
</tr>
<tr>
<td>Axiomatizing</td>
<td>A</td>
</tr>
<tr>
<td>Relating Representation</td>
<td>RR</td>
</tr>
<tr>
<td>Describing/Discussing</td>
<td>DD</td>
</tr>
<tr>
<td>Critiquing</td>
<td>C</td>
</tr>
</tbody>
</table>
For a detailed description of the performance expectations codes refer to Chapter 3. For tasks that contained more than one type of performance expectation, I recorded each type during this analysis. In the following paragraphs, I report and describe the findings related to the performance expectations of students within surface area, volume, and surface area and volume tasks.

**Performance Expectations of Students within Surface Area Tasks**

The findings of the performance expectations found within surface area tasks in each textbook and textbook series are featured in Table 24. Nearly half of the surface area tasks in the CM6 textbook require representing. In the GMC7 and GMC8 textbooks, almost all surface area tasks address the performance expectation of recalling mathematical objects & properties. By the way of contrast, approximately half of the surface area tasks in CM6 and CM7 textbooks involve recalling mathematical objects & properties. No surface area tasks that require the use of equipment were found in the GM6 and GM7 textbooks. In both the GMC8 and U6 textbooks, a large percentage of surface area tasks involve performing routine procedures, 83% and 70%, respectively. Furthermore, more than three-fourths of surface area tasks in the CM6 and CM7 textbooks require the use of more complex mathematical procedures followed by the GM6 textbook.

The performance expectation of developing strategies is present in almost 50% of surface area tasks in the CM6 and CM7 textbooks. In the U6 textbook, almost all surface area tasks require solving. The performance expectations of justifying & proving and relating representation are mostly present within surface area tasks in the CM6 and CM7 textbooks. Around half of the surface area tasks in the CM6 and CM7 textbooks require justifying and proving. The GMC8, U6, and U7 textbooks have the smallest proportion of surface area tasks
that involve describing/discussing than the other eight textbooks. The inclusion of several
performance expectations such as recognizing equivalents, formulating & clarifying problems &
situations, developing notation & vocabulary, predicting, generalizing, conjecturing, and
critiquing are underrepresented across all nine textbooks. The GM8 and CM8 textbooks were not
part of this analysis because these textbooks do not contain any surface area tasks.
Table 20. Percent of Each Type of Performance Expectations to the Total Number of Surface Area Tasks in Each Textbook and Textbook Series

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Note: The data in the table were rounded to the nearest whole percent.
Figure 7 contains comparable findings for the types of performance expectations within surface area tasks across the four textbook series. Among the four textbook series, a small proportion of surface area tasks require representing. Exception to this is the CM textbook series; approximately one-fourth of the surface area tasks involve representing. It is also compelling to note that the GMC textbook series contains a substantial proportion of surface area tasks that involve recalling mathematical objects & properties. Indeed, ninety percent of surface tasks in the GMC textbook series require recalling mathematical objects & properties followed by the UCSMP and GM textbook series, 77% and 68% respectively. Three textbook series (GM, GMC, and UCSMP) have a large proportion of surface area tasks that require performing routine procedures, ranging from 38% to 68%. The CM textbook series includes the least amount of surface area tasks that involve recalling mathematical objects & properties.

The proportion of surface area tasks that require the use of more complex mathematical procedures is significantly higher in the CM textbook series than the other three textbook series. Eighty percent of surface area tasks in the CM textbook series involve the use of more complex mathematical procedures. Nearly half of the surface area tasks in the CM textbook series require developing strategies. In all textbook series, solving is present in at least three-fourths of the surface area tasks. Analysis also showed that the GM textbook series does not offer any surface area tasks that require justification and proving. None of the four textbooks series contain surface area tasks that involve using vocabulary & notation, predicting, developing algorithms, and axiomatizing.
**Figure 7.** Percent of Performance Expectations Required by Students within Surface Area Tasks in Textbook Series
Performance Expectations of Students within Volume Tasks

The results of the performance expectations found within volume tasks in each textbook and textbook series are shown in Table 25. Both the GMC6 and CM7 textbooks contain almost an equal percent of volume tasks that require representing, 12% and 14% respectively. Most textbooks have volume tasks that require recalling mathematical objects & properties, ranging from 59% to 96%. With the exception of the CM8 textbook, only 32% of volume tasks require recalling mathematical objects & properties. The majority of textbooks have a large proportion of volume tasks that require performing routine procedures with the exception of the CM6 and CM8 textbooks. All volume tasks in the CM8 textbook require using more complex procedures.

The U6 textbook, as well as the U7 textbook, contain the most volume tasks that require solving followed by the GMC8 textbook. The inclusion of developing strategies within volume tasks is less evident in the GMC6 and GMC8 textbooks. Less than 5% of volume tasks in the GMC6 and GMC8 textbooks require developing strategies. Only the CM8 textbook contains a significant proportion of volume tasks that involve developing notation & vocabulary. The CM6 textbook places the greatest emphasis on verifying. More than one-fourth of the volume tasks within the CM6 textbook require verifying. The performance expectation of conjecturing is only evident in two textbooks (GM8 and U7). Three textbooks (GM7, GM8, and U6) do not contain any volume tasks that address the performance expectation of justifying and proving. Note that the U7 textbook does not offer any volume tasks that involve describing/discussing. Whereas 55% of volume tasks in the CM8 textbook involve describing/discussing.
Table 21. Percent of Each Type of Performance Expectations to the Total Number of Volume Tasks in Each Textbook and Textbook Series

| Textbook | To | R | R | R | U | U | P | M | F | D | S | P | V | D | D | G | C | J | A | R | D | C |
| #         | O  | N | P | P | P |   |   |   |   |   |   |   | V | N |   |   |   |   |   |   |   |   |
| **GM**    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| GM6       | 40 | 0 | 3 | 90 | 0 | 0 | 60 | 33 | 0 | 10 | 85 | 0 | 5 | 3 | 0 | 0 | 0 | 3 | 0 | 0 | 20 | 3 |
| GM7       | 25 | 0 | 0 | 92 | 0 | 0 | 56 | 40 | 0 | 24 | 72 | 0 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 32 | 4 |
| GM8       | 87 | 0 | 0 | 84 | 0 | 1 | 61 | 33 | 0 | 20 | 77 | 0 | 3 | 9 | 0 | 0 | 2 | 0 | 0 | 1 | 24 | 6 |
| GM678     | 152| 0 | 1 | 87 | 0 | 1 | 60 | 34 | 0 | 18 | 78 | 0 | 5 | 6 | 0 | 0 | 1 | 1 | 0 | 1 | 24 | 5 |
| **GMC**   |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| GMC6      | 84 | 12| 1 | 92 | 1 | 4 | 74 | 16 | 5 | 4  | 87 | 0 | 0 | 1 | 0 | 0 | 0 | 4 | 0 | 1 | 10 | 4 |
| GMC7      | 83 | 5 | 1 | 95 | 0 | 1 | 55 | 39 | 0 | 8  | 80 | 0 | 2 | 4 | 0 | 0 | 0 | 4 | 0 | 0 | 11 | 10 |
| GMC8      | 133| 2 | 0 | 96 | 0 | 1 | 59 | 41 | 1 | 3  | 91 | 0 | 5 | 2 | 0 | 0 | 0 | 5 | 0 | 1 | 5  | 2 |
| GMC678    | 300| 6 | 1 | 95 | 0 | 2 | 62 | 33 | 2 | 5  | 87 | 0 | 3 | 2 | 0 | 0 | 0 | 4 | 0 | 1 | 8  | 4 |
| **CM**    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| CM6       | 11 | 9 | 0 | 82 | 0 | 0 | 36 | 55 | 0 | 46 | 73 | 0 | 27| 9 | 0 | 0 | 0 | 0 | 27| 0 | 0 | 36 | 18 |
| CM7       | 85 | 14| 0 | 59 | 0 | 4 | 13 | 84 | 0 | 41 | 77 | 0 | 2 | 5 | 0 | 1 | 0 | 4 | 0 | 8 | 39 | 5 |
| CM8       | 22 | 5 | 0 | 32 | 0 | 0 | 100| 0  | 5  | 23 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 14| 55| 0  |
| CM678     | 118| 12| 0 | 56 | 0 | 3 | 13 | 84 | 0 | 42 | 74 | 0 | 5 | 9 | 0 | 1 | 0 | 6 | 0 | 9 | 42 | 5 |
| **UC**    |    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| U6        | 26 | 4 | 0 | 81 | 0 | 0 | 46 | 54 | 0 | 8  | 96 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 4 |
| U7        | 38 | 3 | 0 | 92 | 3 | 0 | 61 | 34 | 0 | 11 | 92 | 0 | 3 | 0 | 3 | 3 | 3 | 5 | 0 | 0 | 0 | 3 | 3 |
| U8        | 0  | 0 | 0 | 0  | 0 | 0 | 0  | 0  | 0 | 0  | 0  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| U678      | 64 | 3 | 0 | 88 | 2 | 0 | 55 | 42 | 0 | 9  | 94 | 0 | 3 | 0 | 2 | 2 | 3 | 0 | 0 | 2 | 3 | 0 | 0 |

Note: The data in the table were rounded to the nearest whole percent.
The data in Figure 8 reflect substantial differences in the percent of each type of performance expectations within volume tasks across the four textbook series. In the GM textbook series, none of the volume tasks require representing. The performance expectation of recalling mathematical objects & properties is heavily present within volume tasks in three textbook series (GMC, GM and UCSMP), ranging from 87% to 95%. For performing routine procedures, approximately three-fifths of the volume tasks in the GM, GMC, and UCSMP textbook series have volume tasks that address this performance expectation. Whereas more than four-fifths of the volume tasks in the CM textbook series involve using more complex procedures. Additionally, nearly two-fifths of the volume tasks in the CM textbook series require developing strategies.

Over the four textbook series, the inclusion of solving is present in more than three-fourths of the volume tasks. All four textbook series have nearly an equal amount of volume tasks that involve verifying or critiquing. However, the amount of volume tasks that require justifying and proving in all textbook series was low. The UCSMP textbook series does not contain any volume tasks that involve relating representation. For describing/discussing, two textbook series (CM and GM) contain the largest amount of volume tasks that address this performance expectation, 42% and 24% respectively. Some of the least represented performance expectations within volume tasks are recognizing equivalents, using vocabulary & notation, using equipment, formulating & clarifying problems & situations, conjecturing and generalizing. In addition, several performance expectations such as predicting, developing algorithms, and axiomatizing are not present in volume tasks over the four textbook series.
Figure 8. Percent of Performance Expectations Required by Students within Volume Tasks in Textbook Series
Performance Expectations of Students within Surface Area and Volume Tasks

The findings of the performance expectations found within surface area and volume tasks in each textbook and textbook series are summarized in Table 26. Both textbooks (GMC8 and U6) do not offer any surface area and volume tasks that address representing. The GMC8 textbook also places greater emphasis on recalling mathematical objects & properties. All surface area and volume tasks within the GMC8 textbook require recalling mathematical objects & properties. Only the U7 textbook contains surface area and volume tasks that involve using vocabulary & notation. The performance expectation of using more complex procedures is present in all surface area and volume tasks within the GMC6 textbook.

All textbooks have a high percentage of surface area and volume tasks that require solving with the exception of the GMC6 textbook. None of the surface area and volume tasks in the GMC6 textbook require solving. For developing strategies, the CM7 textbook offers the greatest amount of surface area and volume tasks that include this type of performance expectation. More than half of the surface area and volume tasks within the CM7 textbook address this type of performance expectation. It is also worth noting that the GMC8 and U7 textbooks do not contain surface area and volume tasks that address developing strategies. The findings also showed that the performance expectation of verifying within surface area and volume tasks only appears in three textbooks (CM6, GMC8, and U7), ranging from 6% to 13%. Two textbooks (CM6 and CM7) only offer surface area and volume tasks that require justifying and proving, 13% and 22% respectively. The CM8 textbook was not part of this analysis because it does not contain any tasks that address both concepts.
| Textbook | Total | R | R | R | U | U | P | M | F | D | S | P | V | D | D | G | C | J | A | R | D | C |
|          | #     | R | E | M | V | E | R | C | C | S | N | A | O | P | R | D | O | N | P | P | P | V |
|          |       | P |   | S |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| **GM**   |       |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| GM6      | 0     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| GM7      | 0     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| GM8      | 0     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| GM678    | 0     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **GMC**  |       |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| GMC6     | 4     | 50| 0 | 75| 0 | 0 | 0 | 100| 0 | 50| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 50|
| GMC7     | 23    | 48| 0 | 65| 0 | 17| 4 | 87| 9 | 26| 57| 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 35| 4|
| GMC8     | 13    | 0 | 0 | 100| 0 | 0 | 62| 31| 0 | 0 | 69| 0 | 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15| 0|
| GMC678   | 40    | 33| 0 | 78| 0 | 10| 23| 70| 5 | 20| 55| 0 | 3 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 30| 3|
| **CM**   |       |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| CM6      | 15    | 20| 0 | 60| 0 | 0 | 33| 67| 0 | 33| 80| 0 | 13| 0 | 0 | 0 | 13| 0 | 0 | 0 | 0 | 13| 0|
| CM7      | 23    | 9 | 0 | 35| 0 | 17| 17| 83| 0 | 57| 83| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 22| 0|
| CM8      | 0     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0|
| CM678    | 38    | 13| 0 | 45| 0 | 11| 24| 76| 0 | 47| 82| 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 18| 0|
| **UC**   |       |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| U6       | 12    | 0 | 0 | 92| 0 | 0 | 58| 42| 0 | 25| 100| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 8|
| U7       | 17    | 12| 0 | 94| 6 | 0 | 41| 35| 0 | 0 | 77| 0 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6|
| U8       | 0     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0|
| U678     | 29    | 7 | 0 | 93| 3 | 0 | 48| 38| 0 | 10| 86| 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 3 | 3|

Note: The data in the table were rounded to the nearest whole percent.
Figure 9 includes the percent of each type of performance expectations within surface area and volume tasks to illustrate the differences over the four textbook series. The performance expectation of representing is primarily limited in the UCSMP textbook series. The reverse is true about recalling mathematical objects & properties that is prominent within surface area and volume tasks in the UCSMP textbook series. For using more complex procedures, roughly three-fourths of the surface area and volume tasks in the GMC and CM textbook series address this performance expectation. Only the GMC textbook series have surface area and volume tasks that involve formulating & clarifying problems & situations and conjecturing.

All three textbook series contain surface area and volume tasks that require solving; the UCSMP textbook series has the highest percentage (86%), followed by the CM textbook series (82%). For developing strategies, the results are mixed among the three textbooks series. Approximately half of the surface area and volume tasks in the CM textbook series involve developing strategies followed by the GMC and UCSMP textbook series, 20% and 10% respectively. For verifying, there are small differences across the three textbook series. With the exception of the CM textbook series, justifying and proving is absent from the surface area and volume tasks. Less than 5% of surface area and volume tasks in the UCSMP textbook series address the performance expectation of describing/discussing. The performance expectations of recognizing equivalent, predicting, developing notation & vocabulary, developing algorithms, generalizing, and axiomatizing are absent from the surface area and volume tasks in the sample. The GM textbook series was not part of this analysis because this textbook series does not contain any tasks that address both concepts.
Figure 9. Percent of Performance Expectations Required by Students within Surface Area and Volume Tasks in Textbook Series
Summary of Performance Expectations

In all, the most common performance expectations within surface area and volume tasks in three textbook series (GM, GMC, and UCSMP) are recalling mathematical objects & properties, performing routine procedures, and solving. The CM textbook series contain a greater proportion of surface area and volume tasks that require representing, developing strategies, using more complex procedures, justifying and proving, and describing/discussing than the other three textbook series. The findings for tasks that address both concepts are slightly different. For example, the GMC textbook series contain a large percentage of surface area and volume tasks that involve representing, using more complex procedures, and describing/discussing. Other performance expectations such as recognizing equivalents, using vocabulary and notation, using equipment, formulating & clarifying problems & situations, developing notation and vocabulary, conjecturing, and critiquing are less address in surface area, volume, and surface area and volume tasks across all textbook series.

Visual Representations of 3D Objects within Tasks

I developed and used the Visual Representations of 3D Objects framework to examine the types of visual representations of 3D objects within tasks. According to the Visual Representations of 3D Objects framework, I used ten different types of visual representations of 3D objects to code the data during this study (see table 27). For a detailed description of the codes refer to Chapter 3. For tasks that included more than one type of visual representation of 3D objects, I documented each type during this analysis. In the following paragraphs, I present and describe the findings related to the types of visual representations of 3D objects found within surface area, volume, and surface area and volume tasks.
Table 23. Visual Representations of 3D Objects: Types and Labels

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Visual Representations of 3D Objects within Surface Area Tasks

Table 28 presents the findings of the types of visual representations of 3D objects found within surface area tasks. I rounded the data in the table to the nearest whole percent. As displayed in the table, the U7 textbook includes the highest percentage of surface area tasks with non real-world nets followed by the CM7 textbook, 19% and 15% respectively. More than half of the surface area tasks in three textbooks (GMC6, GMC7, and GM7) contain non real-world drawings, ranging from 58% to 52%. For real-world drawings, about 25% of surface area tasks in the GM6 and CM7 textbooks contain this type of visual representation of 3D objects. However, these two textbooks (GM6 and CM7) do not offer any surface area tasks with real-world pictures. The U7 textbook has the largest percentage of surface area tasks with real-world pictures. In terms of student opportunity to generate visual representations of 3D objects, 52% of
surface area tasks in the CM6 textbook offer this type of opportunity. In a way of contrast, only 5% of surface area tasks in the GM7 and GMC8 textbooks provide students with opportunities to generate visual representations of 3D objects. The GM8 and CM8 textbooks were not part of this analysis because these textbooks do not contain any surface area tasks.

**Table 24.** Percent of Each Type of Visual Representations of 3D Objects to the Total Number of Surface Area Tasks in Each Textbook and Textbook Series

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Figure 10 reports comparable results of the types of visual representations of 3D objects within surface area tasks across the four textbook series. The most common type of visual representation of 3D objects within surface area tasks in all textbook series is non real-world drawing. The least common type of visual representation of 3D objects within surface area tasks
over the four textbook series is real-world net. Among all textbook series, no surface area tasks were observed that contain non real-world pictures.

![Figure 10. Percent of Types of Visual Representations of 3D Objects within Surface Area Tasks in Textbook Series](image)

For non real-world nets, both the CM and UCSMP textbook series have the largest proportion of surface area tasks with this type of visual representations of 3D objects, 14% and 15% respectively. The UCSMP textbook series only has surface area tasks that contain real-world nets. Nearly half of the surface area tasks in the GM and GMC textbook series include real-world drawings. Whereas approximately 20% of surface area tasks in the CM and UCSMP textbook series include real-world drawings. The amount of surface area tasks with real-world drawings is low among the four textbook series, ranging from 15% to 3%. Note that more than one-third of the surface area tasks in the CM textbook series incorporate opportunities for student
to generate visual representations of 3D objects. In contrast, less than 10% of surface area tasks in the GMC textbook series offer this type of opportunity.

*Visual Representations of 3D Objects within Volume Tasks*

Table 29 displays the results of types of visual representations of 3D objects found within volume tasks in each textbook and textbook series. I rounded the data in the table to the nearest whole percent.

As revealed in the table, the CM6 textbook has a significantly larger amount of volume tasks with non real-world nets than the other ten textbooks. However, none of the textbooks offer volume tasks with real-world nets. All textbooks contain volume tasks with non real-world drawings, ranging from 14% to 56%. For real-world drawings, both the CM6 and U6 textbooks do not offer any volume tasks with this type of visual representation of 3D objects. In addition, approximately half of the textbooks do not contain any volume tasks with real-world pictures. The U7 textbook has the largest percentage of volume tasks with real-world pictures. The CM7 textbook offers the greatest percentage of volume tasks with opportunities for students to generate visual representations of 3D objects. Contrary, the GM textbook series does not provide any opportunities for students to generate visual representations of 3D objects.
Figure 11 provides a visual display of the percent of types of visual representations of 3D objects within volume tasks in each textbook series. The majority of volume tasks over the four textbook series include non real-world drawings. In all textbook series, the least represented type of visual representation of 3D objects is real-world net. Additionally, none of the volume tasks in all textbook series contain real-world nets or non real-world pictures. Three out of four textbook series (GMC, CM, UCSMP) have volume tasks that offer opportunities for student to generate visual representations of 3D objects.
Figure 11. Percent of Types of Visual Representations of 3D Objects within Volume Tasks in Textbook Series

In the GMC textbook series, almost half of the volume tasks include non real-world drawings. Whereas only 16% of volume tasks in the CM textbook series contain this type of visual representation of 3D objects. In other words, the GMC textbook series has three times more volume tasks with non real-world drawings than the CM textbook series. For real-world drawings, less than 10% of volume tasks in three textbook series (GM, GMC, and UCSMP) contain this type of visual representation of 3D objects. The UCSMP textbook series has the most real-world pictures within volume tasks than all the other textbook series. Findings also indicated that the CM textbook series has the largest amount of volume tasks that offer opportunities for students to generate visual representations of 3D objects.
Visual Representations of 3D Objects within Surface Area and Volume Tasks

Table 30 shows the findings of the types of visual representations of 3D objects found within surface area and volume tasks in each textbook and textbook series. I rounded the data in the table to the nearest whole percent. As featured in the table, non real-world nets within surface area and volume tasks are negligible across all textbooks. Exception to this is the CM7 textbook that approximately 20% of surface area and volume tasks include this type of visual representation of 3D objects. Nearly two-thirds of the surface area and volume tasks in the CM6 textbook contain non real-world drawings followed by the U6 textbook (42%).

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<tr>
<td>UC SMP</td>
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<td>8</td>
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<tr>
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<td>24</td>
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<td>12</td>
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<tr>
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<td>7</td>
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</table>
For real-world drawings, an equal amount of surface area and volume tasks in two textbooks (GMC7 and CM7) offer this type of visual representation of 3D objects. The GMC6 textbook does not contain any surface area and volume tasks with non real-world or real-world drawings. Low amounts of surface area and volume tasks in all textbooks contain real-world pictures with the exception of the GMC8 textbook. With respect to student opportunity to generate visual representations of 3D objects, three textbooks (GMC6, GMC7 and CM7) offer the highest percentage of this type of opportunity, ranging from 52% to 47%. The U6 textbook does not provide any opportunities for students to generate visual representations of 3D objects. The CM8 textbook was not part of this analysis because it does not contain any tasks that address both concepts.

Figure 12 illustrates the differences in the percent of the types of visual representations of 3D objects in each textbook series. The most common type of visual representations of 3D objects within surface area and volume tasks in all textbook series is non real-world drawings. The least represented types of visual representations of 3D objects within surface area and volume tasks are non real-world nets, real-world drawings, and real-world pictures. Findings were mixed in regards to student opportunity to generate visual representations of 3D objects. Another important observation is that none of the textbook series have tasks that contain real-world nets or non-real world pictures.
In particular, the UCSMP textbook series does not contain any surface area and volume tasks with non real-world nets. All three textbook series (CM, UCSMP and GMC) typically have surface area and volume tasks that include non real-world drawings, ranging from 28% to 40%. For real-world drawings, only two textbook series (GMC and CM) offer surface area and volume tasks with this type of visual representation of 3D objects, 11% and 10% respectively. More than one-third of the surface area and volume tasks in the GMC and CM textbook series offer opportunities for students to generate visual representations of 3D objects. However, less than 10% of surface area and volume tasks in the UCSMP textbook series offer this type of opportunity. The GM textbook was not part of this analysis because it does not contain any tasks that address both concepts.
Summary of Visual Representations of 3D Objects

In sum, some types of visual representations of 3D objects are more represented than others among the four textbook series. All textbook series have a large amount of surface area, volume, and surface area and volume tasks that contain non real-world drawings. However, smaller amounts of surface area, volume, and surface area and volume tasks in all textbook series contain non real-world nets, real-world drawings, and real-world pictures. For student opportunity to generate visual representations of 3D objects, the CM textbook series offer a significant greater amount of surface area, volume, and surface area and volume tasks that contain this type of opportunity than the other textbook series.

Level of Mathematical Complexity of Tasks

I used the Mathematics Framework for the 2007 NAEP (NAEP, 2007) to determine the level of mathematical complexity of tasks. According to this framework, I divided the levels of mathematical complexity of tasks into three levels: Low, Moderate, and High. For a detailed description of the levels of mathematical complexity of tasks refer to Chapter 3. In the following paragraphs, I report and describe the findings related to the level of mathematical complexity of surface area, volume, and surface area and volume tasks.

Level of Mathematical Complexity of Surface Area Tasks

Table 31 reports the total number of surface area tasks as well as the number and percent of the level of mathematical complexity of surface area tasks within each textbook and textbook series. I rounded the data in the table to the nearest whole percent.
Table 27. Number and Percent of Level of Mathematical Complexity of Surface Area (SA) Tasks within Each Textbook and Textbook Series

<table>
<thead>
<tr>
<th>Textbook</th>
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<th>Moderate</th>
<th>High</th>
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<td>#</td>
<td>%</td>
<td>#</td>
<td>%</td>
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<tr>
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<tr>
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<td>GM7</td>
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<td>GM678</td>
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<td>GMC</td>
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<tr>
<td>GMC6</td>
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<tr>
<td>U678</td>
<td>62</td>
<td>14</td>
<td>23</td>
<td>42</td>
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</tbody>
</table>

As seen in Table 31, the most common level of mathematical complexity required by surface area tasks in all textbooks is moderate, ranging from 48% to 77%. For low complexity, the GMC8 textbook has the greatest number of surface tasks followed by the U7 and U6 textbooks, 30%, 24% and 20%. The CM6 and CM7 textbooks do not offer any surface area tasks of low complexity. The GMC8, U6, and U7 textbooks also have the least amount of surface area of high complexity. Whereas the CM6 textbook, as well as the CM7 textbook have the largest amount of surface area tasks of high complexity, 52% and 44% respectively. The GM8 and CM8 textbooks do not contain any surface area tasks. Figure 13 provides a graphical representation of the percent of level of mathematical complexity of surface area tasks in each textbook.
Figure 13. Percent of Level of Mathematical Complexity of Surface Area Tasks in Each Textbook

Figure 14 illustrates comparable findings of the percent of level of mathematical complexity of surface area tasks in each textbook series. The distribution of levels of mathematical complexity of surface area tasks varies among the four textbook series. Both the GMC and UCSMP textbook series have a similar composition of the levels of complexity of surface area tasks. The other two textbook series (GM and CM) contain a significantly different distribution of the levels of complexity of surface area tasks. Analysis also revealed that the CM textbook series has a higher proportion of surface area tasks of high complexity than the other three textbook series and the UCSMP textbook series has the greatest amount of surface area tasks of low complexity.
The UCSMP textbook series offers the greatest percentage of surface area tasks of low complexity followed by the GMC and GM textbook series, 23%, 16%, and 9% respectively. The CM textbook series also does not contain any surface area tasks of low complexity. Nearly half of the surface area tasks in the GM and CM textbook series are of moderate complexity. Both the GMC and UCSMP textbook series contain a higher percentage of surface area tasks of moderate complexity than the other textbook series. The CM textbook series has the largest percentage of surface area tasks of high complexity followed by the GM, GMC, and UCSMP textbook series.

**Level of Mathematical Complexity of Volume Tasks**

Table 32 displays the total number of volume tasks in each textbook and textbook series. This table also contains the total number and percent of the level of mathematical complexity of volume tasks within each textbook and textbook series. I rounded the data in the table to the nearest whole percent.
As presented in Table 32, approximately two-thirds of the volume tasks in the GMC6 textbook are of low complexity followed by the GM6 and GM7 textbooks, 58% and 50% respectively. In contrast, both the CM6 and CM7 textbooks have the lowest percentage of volume tasks of low complexity. It should also be noted that the CM8 textbook does not contain any volume tasks of low complexity. In regards to moderate complexity, the GM7 textbook offer the highest proportion of volume tasks. The majority of volume tasks in the CM6, CM7, and CM8 textbooks are of high complexity, ranging from 52% to 82%. At last, both the U6 and U7 textbooks offer similar amounts of volume tasks of low and moderate complexity. Figure 15 displays a visual analysis of the percent of the level of mathematical complexity of volume tasks in each textbook.
The extent to which the composition of the level of complexity of volume tasks varies over the four textbook series is reflected in Figure 16. Both the GM and GMC textbook series appear to have a similar composition of the level of complexity of volume tasks. Contrary, the other two textbook series (CM and UCSMP) appear to be different in the distribution of the levels of complexity of volume tasks.
Figure 16. Percent of Level of Mathematical Complexity of Volume Tasks in Textbook Series

Approximately half of the volume tasks in three textbook series (GM, GMC, and UCSMP) are of low complexity, whereas less than 10% of volume tasks in the CM textbook series are of low complexity. In terms of moderate complexity, the UCSMP textbook series contain the greatest percentage of volume tasks (48.4%) followed by the CM textbook series (33.9%) and GMC textbook series (32%). The CM textbook series has the highest percentage of volume tasks of high complexity (58.5%) followed by the GM textbook series (27%). In fact, the CM textbook series offers almost six times more volume tasks of high complexity than the UCSMP textbook series.

Level of Mathematical Complexity of Surface Area and Volume Tasks

Table 33 provides an overview of the total number of tasks as well as the number and percent of the level of mathematical complexity of tasks that address both concepts within each textbook and textbook series. I rounded the data in the table to the nearest whole percent.
As evidenced in Table 33, four textbooks (GMC7, GMC7, U6, and U8) only contain surface area and volume tasks of low complexity, ranging from 13% to 24%. Approximately half of the textbooks have a large proportion of surface area and volume tasks of moderate complexity. For instance, more than two-thirds of the surface area and volume tasks in the U7 and GMC7 textbooks are of low complexity. It is also important to note that all surface area and volume tasks in the GMC6 textbook are of high complexity followed by the CM7 textbook. The CM8 textbook was not part of this analysis because it does not contain any tasks that address both concepts. Figure 17 provides a graphical analysis of the percent of the level of mathematical complexity of surface area and volume tasks in each textbook.

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Total SA&amp;V Tasks</th>
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<th>Low %</th>
<th>Moderate #</th>
<th>Moderate %</th>
<th>High #</th>
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<td>21</td>
<td>20</td>
<td>69</td>
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<td>10</td>
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</tbody>
</table>

As evidenced in Table 33, four textbooks (GMC7, GMC7, U6, and U8) only contain surface area and volume tasks of low complexity, ranging from 13% to 24%. Approximately half of the textbooks have a large proportion of surface area and volume tasks of moderate complexity. For instance, more than two-thirds of the surface area and volume tasks in the U7 and GMC7 textbooks are of low complexity. It is also important to note that all surface area and volume tasks in the GMC6 textbook are of high complexity followed by the CM7 textbook. The CM8 textbook was not part of this analysis because it does not contain any tasks that address both concepts. Figure 17 provides a graphical analysis of the percent of the level of mathematical complexity of surface area and volume tasks in each textbook.
Figure 17. Percent of Level of Mathematical Complexity of Surface Area and Volume Tasks in Each Textbook

Figure 18 reports the variations of the level of complexity of surface area and volume tasks across the three textbook series. All three textbook series (GMC, CM, and UCSM) have a quite different distribution of the level of complexity of surface area and volume tasks. The CM textbook series does not contain any surface area and volume tasks of low complexity. This textbook series offers almost five times more surface area and volume tasks of high complexity than the UCSMP textbook series. The GMC textbook series also has half of the amount of surface area and volume tasks of low complexity than the UCSMP textbook series. Indeed, the UCSMP textbook series has the highest percentage of surface area and volume tasks of low complexity than the other two textbook series.
Figure 18. Percent of Level of Mathematical Complexity of Surface Area and Volume Tasks in Textbook Series

The UCSMP textbook series has the highest percentage (69%) of surface area and volume tasks of moderate complexity. The CM textbook series has the greatest percentage (53%) of surface area and volume tasks of high complexity followed by the GMC textbook series (45%). Analysis also indicated that the GMC textbook series has approximately the same percentage of surface area and volume tasks of moderate and high complexity, 43% and 45% respectively. The GM textbook series does not contain any tasks that address both concepts. Thus, the GM textbook series was not part of this analysis.

Summary of Level of Mathematical Complexity

In general, three textbooks series (GM, GMC, and UCSMP) only offer surface area tasks of low complexity, ranging from 9% to 23%. These three textbook series (GM, GMC, and UCSMP) also contain a large proportion of volume tasks of low complexity. In all textbook
series, 50% or more of the surface area tasks are of moderate complexity. However, 50% or less of the volume tasks in all textbook series are of moderate complexity. The CM textbook series does not contain any surface area or surface area and volume tasks of low complexity. The CM textbook series also has the largest proportion of surface area, volume, and surface area and volume tasks of high complexity.

**Content Features of Lessons in Textbook Series**

In this section, I present the findings in regards to the extent to which the content of surface area and volume lessons address the CCCS for 6-8 geometry that are aligned with these topics in published mathematics textbook series and across different publishers.

**Lesson Content and CCCS**

I used the CCCS for 6-8 geometry components guideline and the geometric measurement standards for grade 5 (CCSSI, 2010) to examine the extent to which the surface area, volume, and surface area and volume lessons within the sample textbooks address these content standards. I evaluated a total of 49 lessons (17 surface area, 24 volume, and 8 surface area and volume) over the four textbook series during this analysis. As previously noted, I labeled lessons that address both concepts as surface area and volume lessons. I used three codes to label the lesson topic: surface area (SA), volume (V), and surface area and volume (SA&V). I also labeled the content standards the same way they are labeled in the Common Core. For a detailed description of the CCCS for 6-8 geometry components and the geometric measurement standards for grade 5 refer to Chapter 3. The U8 textbook was not part of this analysis because it does not contain any lessons that are devoted to the concepts of surface area and volume. In the following paragraphs, I describe and provide a display of the findings.
Go Math Textbook Series

I examined 2 surface area and 6 volume lessons within the GM textbook series. Table 34 illustrates the findings by grade level, lesson number, and topic. At the sixth grade level, the surface area lesson addresses both components of the 6.G.A.4 content standard. The first volume lesson covers both components of the 6.G.A.2 content standard while the second lesson only covers the 2nd component of this content standard. Both volume lessons also address the 5.MD.5b content standard.

Table 30. Results of CCCS for 6-8 Geometry & Geometric Measurement Standards for Grade 5 Address in Lessons within GM Textbook Series

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</tbody>
</table>

At the seventh grade level, both surface area and volume lessons address both components of the 7.G.B.6 content standard. At the eighth grade level, all volume lessons cover both components of the 8.G.C.9 content standard. Each volume lesson is dedicated to one of the three 3D objects (cylinder, cone, and sphere) as listed in the CCCS for grade 8 geometry. The GM textbook series does not contain any lessons that address both concepts.

Glencoe Math Textbook Series

I evaluated 7 surface area, 7 volume, and 2 surface area and volume lessons within the GMC textbook series. The results of this analysis are presented in Table 35. Both components of
the 6.G.A.4 content standard are addressed in all 6th grade surface area lessons. One 6th grade volume lesson covers both components of the 6.G.A.2 content standard and all five geometric measurement standards for grade 5. While the other 6th grade volume lesson addresses both components of the 7.G.B.6 content standard.

All 7th grade surface area, volume, and surface area and volume lessons cover both components of the appropriate grade level content standard. The 7th grade surface area lessons also address both components of the 6.G.A.4 content standard. One 7th grade volume lesson also covers the 5.MD.5a and 5.MD.5b content standards and the 7th grade surface area and volume lesson covers the 5.MD.4 content standard.

Table 31. Results of CCCS for 6-8 Geometry & Geometric Measurement Standards for Grade 5 Address in Lessons within GMC Textbook Series

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Two out of three 8th grade lessons address both components of the 8.G.C.9 content standard. Each volume lesson is dedicated to one of the three 3D objects (cylinder, cone, and
sphere) as listed in the CCCS for grade 8 geometry. The 8th grade volume lesson on spheres does not address the 1st component of the 8.G.C.9 content standard. This textbook series also contains two surface area and one surface area and volume lesson at the 8th grade level. These lessons cover both components of the 7.G.B.6 content standard. The 8th grade surface area and volume lesson also addresses the 2nd component of the 8.G.C.9 content standard.

Connected Mathematics Textbook Series

I examined 6 surface area, 9 volume, and 1 surface area and volume lessons within the CM textbook series. The data in Table 36 provide a summary of the findings. All 6th grade surface area lessons address both components of the 6.G.A.4 content standard. The 6th grade volume lesson covers both components of the 6.G.A.2 content standard and two (5.MD.5a and 5.MD.5b) geometric measurement standards for grade 5.

Table 32. Results of CCCS For 6-8 Geometry & Geometric Measurement Standards for Grade 5 Address in Lessons within CM Textbook Series

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Both components of the 7.G.B.6 content standard are addressed in all 7th grade surface area, half of the volume, and one surface area and volume lesson. The rest of the 7th grade volume lessons address both components of the 8.G.C.9 content standard, with the exception of one lesson that only covers the 2nd component of the 8.G.C.9 content standard. Two 7th grade volume lessons also cover the 5.MD.5a and 5.MD.5b content standards. Both 8th grade volume lessons cover the 2nd component of the 8.G.C.9 content standard but only one lesson addresses the 1st component of the 8.G.C.9 content standard.

*University of Chicago School Mathematics Project Textbook Series*

I evaluated 2 surface area, 2 volume, and 5 surface area and volume lessons within the UCSMP textbook series. The findings of this analysis are shown in Table 37. There is a variation in the coverage of content standards across the 6th grade lessons. It is also imperative to note that all 6th grade lessons address both surface area and volume concepts.

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The first lesson only covers the two (5.MD.3a and 5.MD.4) geometric measurement standards for grade 5. This lesson is about finding the surface area and volume of cubes. The

There are also significantly differences in the coverage of content standards among 7th grade lessons that address both concepts. One lesson covers both components of 7.G.B.6 content standard and three (5.MD.4, 5.MD.5a, and 5.MD.5b) geometric measurement standards for grade 5, while the other lesson only addresses the 2nd component of the 8.G.C.9 content standard. This lesson is about finding the surface area and volume of spheres. Both 7th grade surface area lessons cover both components of the 7.G.B.6 content standard. With the expectation of one lesson that addresses both components of the 6.G.A.4 content standard. Both components of the 7.G.B.6 are covered in the 7th grade volume lessons. However, one volume lesson also addresses both components of the 8.G.C.9 content standard, whereas the other lesson covers only the 2nd component of the 8.G.C.9 content standard.

**Summary of Lesson Content and CCCS**

Taken together, the lessons within the GM textbook series address both components of the corresponding content standard for each grade level and topic. Exception to this is one 6th grade volume lesson that covers only the 2nd component of the corresponding content standard. Both 6th grade volume lessons also cover one geometric measurement standards for grade 5. This geometric measurement standard for grade 5 addresses the concept of finding volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. The lessons in the GM textbook series do not cover content standards for the next grade level.
There are also some variations in regards to addressing the corresponding content standard for each grade level and topic in both the GMC and CM textbook series. Almost all lessons in the GMC and CM textbook series address both components of the content standard for the appropriate grade level and topic. However, some lessons address only one component of the content standard while other lessons also cover the content standard from the previous or next grade level. I also observed that the GMC textbook series contains lessons that address both concepts at the 8th grade level. Whereas the GM and CM textbook series only offers lessons that address the volume concept at the 8th grade level. This is an important observation because only the concept of volume is addressed in the 8th grade content standard.

Similarly to the GMC and CM textbook series, the UCSMP textbook series contains lessons that cover the content standards from the previous or next grade level. However, there is a greater variation in the UCSMP textbook series regarding the coverage of the corresponding content standard for each grade level and topic. Some lessons in the UCSMP textbook series do not cover the components of the content standard for the appropriate grade level and topic. For example, one 6th grade lesson only addresses the geometric measurement standards for grade 5. This lesson does not cover the 6th grade content standards. In addition, one 7th grade lesson addresses only the 2nd component of the 8th grade content standard. Finally, the UCSMP textbook series does not contain any lessons that address the volume concept at the 8th grade level.

**Summary of Results**

In this chapter, I presented the results of the treatment of surface area and volume concepts within four middle-grades student edition mathematics textbook series. In particular, I examined the physical characteristics of textbooks, the structure of lessons, the pedagogical
features of tasks, and the content features of lessons within published mathematics textbook series and across different publishers.

In terms of the physical characteristics of textbooks, I observed some differences across the four textbook series. All textbooks address the concepts of surface area and volume after the middle or towards the end of the textbook. Only the CM6 textbook introduces these concepts in the middle of the textbook. Most textbooks devote more instructional pages to the concept of volume with the exception of the GMC6 textbook that devotes more instructional pages to the concept of surface area. A relatively small amount of lessons are devoted to the concepts of surface area and volume in all textbook series. The GMC and CM textbook series have a larger number of lessons devoted to the concepts of surface area and volume than the GM and UCSMP textbook series. However, the GMC textbook series has the highest percentage of lessons devoted to the concept of surface area and the GM textbook series has the greatest percentage of lessons devoted to the concept of volume. The GM textbook series also does not contain any lessons that address both concepts.

In regards to the structure of lessons, the majority of the lessons in the three textbook series (GM, GMC, and UCSMP) contain similar instructional blocks. The GM, GMC, and UCSMP textbook series have lessons with multiple instructional blocks. Nearly all lessons within the GM, GMC, and UCSMP textbook series contain an essential question also called big idea, activity, description of concept with vocabulary terms and formula. These three textbook series also have lessons that include worked examples with solutions, practice problems, and independent practice. I also found some variations in the sequence of these instructional blocks. For example, the activities in the GM and GMC textbook series are located in the beginning of the lesson whereas the activities in the UCSMP textbook series are mostly located in the middle
or towards the end of the lesson. The CM textbook series also contains lessons with fewer instructional blocks. The majority of the lessons in the CM textbook series have four instructional blocks. The CM textbook series does not include any worked examples with solutions or formulas. This textbook series also contains tasks with multi-parts and the independent practice problems are located in the end of the unit.

With respect to the pedagogical features of tasks, I found significant differences over the four textbook series. For performance expectations, the majority of tasks within the GM, GMC, and UCSMP textbook series require recalling mathematical operations & properties and performing routine procedures. Contrary, the CM textbook series appear to contain more tasks that require the use of more complex procedures and developing strategies than the other three textbook series. The CM textbook series also has the most tasks that require justifying & proving and describing/discussing. All textbooks contain large proportions of tasks that require solving. Furthermore, all textbooks have low amounts of tasks that involve representing, recognizing equivalents, using vocabulary & notation, using equipment, formulating & clarifying problems & situations, verifying, developing notation & vocabulary, generalizing, conjecturing, relating representation, and critiquing. It is also vital to note that some performance exceptions within tasks such as predicting, developing algorithms, and axiomatizing are negligent among the four textbook series.

For types of visual representations of 3D objects, the most frequent type of visual representation of 3D object within tasks in all textbooks is non real-world drawings. However, the GM and GMC textbook series have the largest amounts of tasks with non real-world drawings. Both the GMC and UCSMP textbook series have the greatest proportion of tasks that contain real-world pictures. The UCSMP textbook series also has the least amount of tasks with
real-world drawings. In terms of student opportunity to generate visual representations of 3D objects, the CM textbook series appears to have the most tasks with this type of opportunity. The CM textbook series also has more tasks with non real-world nets than the other three textbook series.

For level of mathematical complexity, the most common level of complexity of tasks in all textbook series is moderate. The UCSMP textbook series has a larger proportion of tasks of moderate complexity. Results also showed that the CM textbook series contains minuscule amounts of low complexity tasks than the other textbook series. Both the GMC and UCSMP textbook series have a greater proportion of low complexity tasks. The UCSMP textbook series also offers the least amount of high complexity tasks.

Regarding the content features of lessons, I observed some variations among the four textbook series. Almost all lessons in the GM textbook series address both components of the appropriate grade level geometry content standard. Some lessons in the GM textbook series address the content standards from the previous grade level but none of the lessons cover the content standards for the next grade level. In a way of contrast, the GMC, CM, and UCSMP textbook series contain lessons that cover the content standards from the previous and/or next grade level. It is also important to note that the GM, GMC, and CM textbook series include lessons that address the geometry content standards for grades 6-8. However, the UCSMP textbook series has one lesson that covers only the geometric measurement standards for grade 5. This textbook series also does not contain any lessons that address the 8th grade geometry content standard at the 8th grade level.

In the next chapter, I discuss the results, limitations, and significance of this study. I also present the implications for mathematics education and future research.
CHAPTER 5: SUMMARY, DISCUSSION, AND CONCLUSIONS

The ultimate goal of this study was to investigate the treatment of surface area and volume concepts in four middle-grades student edition mathematics textbooks in order to document students’ opportunity to learn these concepts. Specifically, I examined the structural, pedagogical, and content features devoted to the concepts of surface area and volume within published mathematics textbook series and across different publishers.

In this chapter, I first provide a summary of the study. I then report and discuss the findings and limitations of this study. Next, I present the significance of this study, implications for mathematics education, and recommendations for future research. I conclude this chapter with some final remarks.

**Summary of the Study**

Geometry is an important branch of mathematics (Choi & Park, 2013). As noted by NCTM (2000), “geometry offers an aspect of mathematical thinking that is different from, but connected to, the world of numbers” (p. 97). Battista (2007) also stated that geometry can be used to conceptualize and interpret physical and spatial environments. A significant component of geometry is spatial geometry. In spatial geometry, students are required to use their geometric reasoning and spatial abilities to solve mathematical tasks.

At the middle grade level, two essential concepts in spatial geometry are surface area and volume (CCSSI, 2010; NCTM 2000). These geometric concepts can help develop students’ geometric reasoning and spatial abilities. However, it has been repeatedly reported that the
average achievement of U.S. students is weak in the content area of geometry, especially on geometric tasks involving geometric reasoning and spatial abilities (Mullis et al., 2012; Mullis et al., 2016). These findings demonstrate the need to provide students with increased opportunities to develop their geometric reasoning and spatial abilities required to solve surface area and volume tasks.

Among all curriculum materials, textbooks have the greatest influence in the teaching and learning of mathematics (Reys et al., 2004; Tarr et al., 2008). Both teachers and students use textbooks as their primary resource on a daily basis (Mullis et al., 2012). Textbooks dictate what topics students are exposed to and how students’ learn mathematics (Alajmi, 2012). As noted by Tornroos (2005), textbooks greatly influence students’ opportunity to learn mathematics. Therefore, differences in the structure and content in textbooks signify differences in students’ opportunities to learn mathematics.

In this study, I conducted detailed content analysis of the treatment of the geometric concepts of surface area and volume in popular and alternative middle-grades student edition mathematics textbook series in order to document students’ opportunity to learn these concepts.

**Research Questions**

This study was guided by the following three research questions:

1) Within published mathematics textbook series and across different publishers, what are the structural features devoted to the concepts of surface area and volume? In particular,

   a. Where are the surface area and volume lessons located and how many pages and lessons are devoted to surface area and volume?

   b. How are the instructional blocks of surface area and volume lessons sequenced?
2) What are the pedagogical features of the tasks included in the surface area and volume lessons within a published mathematics textbook series, and across different publishers? Specifically,

   a. What are the performance expectations of students within these tasks?
   b. What types of visual representations of 3D objects are included in these tasks?
   c. What is the level of the mathematical complexity of these tasks?

3) To what extent do the content of surface area and volume lessons address the Common Core Content Standards for 6-8 geometry that are aligned with these topics?

   **Discussion of Findings**

   The purpose of this study was to examine the treatment of the geometric concepts of surface area and volume to determine the extent to which these textbooks offer students the opportunity to learn these concepts. During this study, I examined the treatment of surface area and volume concepts in terms of the location, number of pages and lessons, structure of lessons, pedagogical features of tasks, and content features of lessons devoted to these concepts within four middle-grades student edition mathematics textbook series. In this section, I report and discuss the findings in regards to students’ opportunity to learn the concepts of surface area and volume.

   **Opportunity to Learn and Physical Characteristics of Textbooks**

   Research has indicated that less attention and time is given to topics located at the end of the textbook (Valverde et al., 2002; Weiss et al., 2001). In fact, studies have shown that teachers usually cover 75% of the textbook during a school year (Valverde et al., 2002; Weiss et al., 2001). It has also been stated that topics located in the last half of the textbook might be omitted
or not covered by the teacher (NCTM, 1989; Stein et al., 2007). Hence, if a topic is located in the last part of the textbook it might impact students’ opportunity to learn this topic.

Across all textbooks, the concepts of surface area and volume are located in the third or fourth quartile of pages. Exception to this is the CM6 textbook that introduces these concepts in the second quartile of pages. In particular, the placement of the concept of surface area in six textbooks (GM6, GMC6, GMC7, GMC8, CM7, U6, and U7) is in the fourth quartile of pages. In contrast, the GM7 and CM6 textbooks place the concept of surface area in the third quartile of pages. The GM8, CM8, and U8 textbooks also do not contain any surface area lessons. Thus, students’ opportunity to learn the concept of surface area in (GM6, GMC6, GMC7, GMC8, CM7, U6, and U7) textbooks is extremely low. The placement of the concept of volume in the majority of textbooks (GM6, GM8, GMC6, GMC7, GMC8, CM7, U6, and U7) is in the fourth quartile of pages. Hence, apart from the GM7, CM6, and CM8 textbooks that place the concept of volume in the third quartile of pages, students’ opportunity to learn the concepts of volume is limited in all other textbooks. The U8 textbook also does not include any volume lessons.

These results highlight the limited opportunities for students to learn the concepts of surface area and volume. The placement of the surface area and volume concepts towards the end of the textbooks might further indicate that textbook authors and publishers are not fully implementing the national recommendations that suggest that the majority of instructional time in middle grades should be devoted to algebra and geometry (NCTM, 2000). A possible explanation might be that most of the geometric topics are repeated across the K-8 mathematics curriculum or geometry and measurement are considered to be less important than arithmetic and algebra (Flanders, 1994). Textbook developers need to reconsider the content placement and
coverage of these concepts in order to provide ample opportunities for students to encounter and learn these concepts.

For decades, researchers have also argued that the attention a topic receives in textbooks influence students’ opportunity to learn this topic (Begle 1973; Flanders, 1994; Schmidt et al., 1996; Stein et al., 2007; Thompson et al. 2012; Tornroos, 2005). Stein and colleagues (2007) stated that, “what mathematical topics covered in a given set of curriculum materials is of fundamental importance” (p. 327). When a topic is not present in the textbook, students most likely will not be exposed to this topic (Begle 1973; Stein et al., 2007; Thompson et al., 2012).

I observed important differences in the opportunities for students to learn the concepts of surface area and volume among all textbook series. A small percentage of instructional pages and lessons are devoted to the concepts of surface area and volume among all textbooks. However, the GM and CM textbook series have more instructional pages and lessons that are devoted to the concept of volume. A possible explanation to the variation of total instructional pages and lessons devoted to the concepts of surface area and volume might be due to the fact that the GM8 and CM8 textbooks do not contain any surface area lessons. This might also be due to the fact that the concept of surface area is not addressed in the CCCS for 8th grade geometry.

The GMC and UCSMP textbook series place approximately an equal attention to both concepts. A similar amount of instructional pages and lessons in the GMC and UCSMP textbooks are devoted to the concepts of surface area and volume. However, the UCSMP textbook series offers a significant lower percentage of instructional pages and lessons devoted to the concepts of surface area and volume than the GMC textbook series. This limited emphasis on surface area and volume in the UCSMP textbook series might be due to the fact that the U8 textbook does not contain any lessons devoted to these concepts.
Opportunity to Learn and Structure of Lessons

Textbooks contain lessons that are units of instruction written to guide the teaching and learning of mathematics (Valverde et al., 2002). Therefore, the way content is presented in textbooks influences students’ opportunity to learn mathematics (Valverde et al., 2002). The results of this study revealed similarities in the sequence of the instructional blocks of lessons among the textbook series. Exception to this is the CM textbook series that contain lessons with fewer instructional blocks than the other three textbook series. A standard lesson in the CM textbook series has an introductory task, reflection question, multi-step questions, and independent practice. A typical lesson in the GM, GMC, and UCSMP textbook series includes an essential question or big ideas, activity, description of concept, vocabulary terms, formula, worked examples with solutions, practice problems, and independent practice.

Other instructional blocks such as introductory task, reflection questions, guided practice, test practice, and review problems are present within the lessons in some textbook series. For example, the GMC and UCSMP textbook series only have lessons with review problems. Similarly, Ponte and Marques (2011) found that the lessons in the U.S. popular textbook contained an introductory task, worked example with solution, explanation of concepts, application tasks, and practice problems. Ponte and Marques also stated that the U.S. popular textbook included review problems in the beginning of the chapter.

Further, the instructional blocks of lessons in the GM, GMC, and CM textbook series follow a linear pattern, whereas the instructional blocks of lessons in the UCSMP textbook series do not necessarily follow a linear pattern. For example, the activities in the UCSMP textbooks series are sometimes placed in the beginning, middle, or towards the end of the lesson. Ponte and
Marques (2011) also reported that all lessons in the U.S. popular textbook followed a similar linear pattern in the sequence of the instructional blocks.

Another important observation is that the lessons in the CM textbook series do not contain any formulas or definitions. These findings support Stein and colleagues (2007) statement that “standards-based curricula embody an approach to learning that focused on the students’ active construction of important and ideas” (p. 360). These results might also indicate some possible connections between national recommendations in regards to providing students with opportunities to explore mathematical concepts and the instructional blocks of lessons within alternative textbook series. The observed differences in the instructional blocks of lessons might also be due to the authors’ different philosophical approaches to teaching mathematics.

These differences in the instructional blocks of lessons might signify differences in students’ opportunities to learn the concepts of surface area and volume. Hence, some questions that arise from these findings are, is it better to include more or less instructional blocks? Should students be given the formulas and definition of mathematical terms? Does this lesson structure suit all students’ learning levels, needs, and styles? Do all students read and do all the parts of the lesson? The answers to these questions might vary depending on the group of students, time, and place. Further empirical investigation is also required to answer these types of questions.

Opportunity to Learn and Performance Expectations of Students Within Tasks

Another important variable in students’ opportunity to learn mathematics are the tasks contained in textbooks. As noted by Stein and colleagues (2007), the nature of tasks included in textbooks influence students’ mathematical thinking and learning. Thus, it is crucial to examine the nature of tasks because not all tasks offer the same opportunity to learn mathematics. In this
study, I examined two aspects of the nature of tasks: performance expectations and level of mathematical complexity.

A significant percentage of surface area, volume, and surface area and volume tasks in all textbook series require recalling mathematical objects & properties and solving. In addition, most of the surface area, volume, and surface area and volume tasks in the GM, GMC, and UCSMP textbook series involve performing routine procedures. In contrast, the majority of surface area, volume, and surface area and volume tasks in the CM textbook series require using more complex procedures. The CM textbook series also contain a greater percentage of surface area, volume, and surface area and volume tasks that involve developing strategies. These results coincide with the findings of Li’s (2000, 2007) and Ponte’s and Marquis’s (2011) studies that popular textbooks tend to contain large proportions of tasks that require performing routine procedures. Furthermore, the findings of this study in regards to the large proportion of tasks within popular textbooks that require recalling mathematical objects & properties support previous results from Incikabi’s and Tjoe’s (2013) study.

Being able to predict, verify, and justify or prove solutions are important mathematical skills; yet most textbooks that I examined, students are offered few opportunities to engage and develop these mathematical skills. In particular, analyses of the tasks indicated that, across the textbook series, less than 10% of the surface area, volume, and surface area and volume tasks require verifying and justifying and proving. Exception to this is the CM textbook series that offers more opportunities for students to justify and prove their reasoning than the other three textbook series. Opportunities for students to engage with tasks that involve representing, recognizing equivalents, using equipment, formulating & clarifying problems & situations, and conjecturing are also rare among all textbook series. Other important performance expectations
such as developing notation and vocabulary, predicting, generalizing, axiomatizing, and critiquing are negligent across all textbook series.

These low percentages of tasks that require conjecturing, verifying, generalizing and justifying and proving contradict NCTM’s (1989, 2000) and CCSSI’s (2010) recommendations of providing students with ample opportunities to make conjectures, gather evidence, and build arguments. This inefficient exposure of students to tasks that require constructing arguments, interpreting results, and generalizing solutions might lead to their inability to develop and deepen their understanding of these geometric concepts.

These findings coincide with previous results from Jones’s and Tarr’s (2007), NCR (2004), and Project 2016 (AAAS, 2000) studies that alternative textbook series such as CM textbook series are more closely aligned to the national recommendations. Choi and Park (2013) also reported that the CM textbooks included a compelling amount of tasks that require reasoning, generalizing, and solving complex situations. While Incikabi and Tjoe (2013) found that few tasks in popular textbooks required analyzing, generalizing, and justifying. This might be an indication that some textbook authors are partially implementing national recommendations and standards.

It is also important to note that all textbooks offer limited opportunities for students to describe and discuss their reasoning; with the exception of the CM textbook series that contain a larger percentage of surface area, volume tasks, and surface area and volume tasks that require describing/discussing. These results support the findings of Li (2000, 2007) and Ponte and Marquis (2011) that the majority of tasks within popular textbooks do not require an explanation; while Hang and Choi (2013) reported that a compelling amount of tasks within the CM textbook
series required explanation. Based on these findings, CM textbook series appears to be a wealthy source of tasks for students to develop their problem solving and communication abilities.

**Opportunity to Learn and Level of Mathematical Complexity of Tasks**

The frequency of opportunities for students to engage with tasks of high complexity differs by textbook series. Most of the surface area, volume, and surface area and volume tasks in the GM, GMC, and UCSMP textbook series are of low or moderate complexity. With the exception of tasks that address both concepts in the GMC textbook series that are mostly either of moderate or high complexity. Nearly all surface area, volume, and surface area and volume tasks in the CM textbook series are of moderate or high complexity.

The large percentage of high complexity surface area, volume, and surface area and volume tasks within the CM textbook series appears to coincide with the national recommendations of NCTM (2000) that advocate for the inclusion of high complexity tasks that support the development of conceptual understanding. The CM textbook series also adheres to the recommendations of CCSSI (2010) that students should have opportunities to engage with tasks of sufficient richness. Therefore, these findings might be a reflection of the CM textbook authors attempts to implement national recommendations and standards.

The CM textbook series also received the highest quality rating in Project 2061 study of middle-grades mathematics textbooks (AAAS, 2000). Finally, the results that the CM textbook series contains more opportunities for students to engage with high complexity tasks support the findings of previous studies by Jones and Tarr (2007), Arnold and Son (2011), and Choi and Park (2013). However, these results do not coincide with the findings of Hong and Choi (2013) that the majority of tasks in the CM textbooks are of low complexity. These variations in
findings might be due to the different methodological approaches used in textbook content analysis studies.

Similarly to Jones’s and Tarr’s, Arnold’s and Son’s (2011), Ponte’s and Marquis’s (2011), Huntley’s and Terrell’s (2014), and Polikoff’s (2015) results regarding the lack of high complexity tasks in popular textbooks, I also found limited opportunities for students to engage with high complexity surface area and volume tasks in both popular (GM and GMC) textbook series. The findings of Project 2061 (AAAS, 2000) and NCR (2004) also indicated that the popular textbooks series were of lower quality than the alternative textbooks. Both Project 2061 (AAAS, 2000) and NCR (2004) studies reported that the popular textbooks contained less rigorous than the alternative textbooks. Therefore, the recommendations of NCTM (2000) for curricular materials to include tasks that support and promote the development of conceptual understanding are seldom implemented in the popular textbooks that I examined.

The results of this study further indicated that the UCSMP textbook series also contain limited opportunities for students to engage with surface area, volume, and surface and volume tasks of high complexity. Likewise, Zhu’s and Fan’s (2006) findings also showed that the UCSMP textbooks mostly included traditional tasks of low complexity. Huntley and Terrell (2014) also claimed that the UCSMP textbooks contained miniscule amounts of high complexity tasks. Thus, the differences observed in the mathematical complexity of tasks seemed to indicate that national recommendations and standards influence the design of textbooks to some degree.

**Opportunity to Learn and Visual Representations of 3D Objects within Tasks**

As stated in relevant literature, students should be given the opportunity to explore the concepts of surface area and volume by visually and physically building and manipulating different types of visual representations of 3D objects (Battista, 2007; Obara, 2009; Revina et al.,
The NCTM (1989, 2000) and CCSSI (2010) documents also recommend that students should be exposed to a variety of tasks that involve constructing nets and creating 3D shapes using 2D shapes in order for students to develop their spatial abilities required to solve surface area and volume tasks. These recommendations to provide opportunities for students to generate different types of visual representations of 3D objects are rarely implemented in the majority of the textbooks. All textbook series provide more opportunities for students to view and examine nets, drawings, or pictures of visual representations of 3D objects than to generate visual representations of 3D objects. Exception to this is the CM textbook series that offers the most opportunities for students to generate visual representations of 3D objects.

The most dominant type of visual representation of 3D objects within surface area, volume tasks, and surface area and volume in all textbook series is drawings. Approximately one-third or more of the surface area, volume, and surface area and volume tasks in all textbook series contain drawings. A significant smaller amount of surface area, volume, and surface area and volume tasks in all textbook series include other types of visual representations of 3D objects such as nets and pictures. For examples, less than 20% of surface area, 10% of volume, and 15% of surface area and volume tasks in the four textbook series contain nets.

These results appear not to be consistent with the findings from previous studies on the types of visual representations included in tasks. For instance, Zhu and Fan (2006) reported that less than 10% of tasks in the UCSMP textbooks contained a visual representation. Incikabi and Tjoe (2013) also found that U.S. popular textbooks contained less than 1% of tasks with some type of visual representation. Incikabi and Tjoe further stated that U.S. popular textbooks do not emphasize the use of representations in problems that is required for developing a deeper understanding of mathematics. In contrast, Ponte and Marques (2011) observed that the U.S.
popular textbook contains a large amount of visual representations such as pictures and
drawings. These differences in findings might be due to the fact that these studies examined
different content areas using various frameworks and methods of analysis. However, these
findings are consistent with Yang’s and colleagues’s (2017) results that CM textbooks contain a
significant amount of tasks with visual representations. This consistency in findings might be due
to the fact that Yang and colleagues also examined the types of visual representations within
geometric tasks.

Another important observation is the low percentage of surface area, volume, and surface
area and volume tasks that offer opportunities for students to generate visual representations of
3D objects across all textbook series. These observations might explain the findings by
researchers that, despite the national recommendations to provide opportunities for students to
develop their spatial abilities by drawing and constructing visual representations of 3D objects,
most students still have difficulties with solving surface area and volume tasks.

Exception to this is the CM textbooks series that offers the greatest frequency of
opportunities for students to generate visual representations of 3D objects. Having more
opportunities to generate visual representations of 3D objects is an advantage for CM textbooks
since research has explicitly stated the benefits of providing students with the opportunity to
draw and construct visual representations of 3D objects (Battista, 2007; Obara, 2009; Revina et
al., 2011). These differences might also indicate that CM textbook authors are designing
mathematics textbooks based on the national recommendations and standards.

**Opportunity to Learn and Content Features of Lessons**

The CCSSM as reflected in the intended curriculum, explicitly state what students should
learn (Porter et al., 2011). It has also been argued that the successful implementation of the
standards depends on the quality and alignment of the curriculum materials used by teachers (Polikoff, 2015). All publishers of the four textbook series have stated alignment of their textbooks to the standards on their websites. Nonetheless, the results reported here reveal that not all lessons within the four textbook series fully address the appropriate grade level geometry content standards.

The results demonstrated differences in the coverage of the CCCS for 6-8 geometry across topics, grade levels, and publishers. Most lessons in the GM, GMC, and CM textbook series address the appropriate CCCS for 6-8 geometry. For each grade level, the majority of the lessons also cover both components of the corresponding geometry content standard. However, I observed less variation in the coverage of the CCCS for 6-8 geometry in the GM textbook series than the other three textbook series. The lessons also in the UCSMP textbook series have the greatest variation in addressing the appropriate CCCS for 6-8 geometry. For instance, one 6th grade surface area and volume lesson addresses only the geometric measurement standards for grade 5. The 8.G.C.9 content standard is also partially covered only at the sixth and seventh grade level. At last, some of the lessons in the GMC, CM, and UCSMP textbook series address the content standards from the previous and/or next grade level.

These findings might indicate that textbook authors and publishers are designing lessons that address the CCCS for 6-8 geometry to some extent. Given that teachers rely heavily on the use of textbooks to plan their instruction, differences in the extent of coverage of the CCCS for 6-8 geometry in the four textbook series might signify differences in students’ opportunities to learn the concepts of surface area and volume.

Taken as a whole, the results of this study indicate that the concepts of surface area and volume are located in the last half of the textbooks- the part of the textbook most often skipped
or omitted by teachers. Given these findings, it is reasonable to state that the placement of these concepts in the last half of textbooks might further diminished students’ opportunity to learn these concepts. In addition, all textbook series contain a small percentage of both instructional pages and lessons devoted to the concepts of surface area and volume. Therefore, students who are taught mathematics with these curriculum materials might not be given enough opportunities to encounter and learn these concepts.

The structure of the lessons within the four textbook series also demonstrates a linear pattern in the teaching and learning of these concepts. Besides, the instructional blocks of the lessons in the UCSMP textbook series that does not always follow a linear pattern. The CM textbook series also contains lessons that embody a different approach to teaching mathematics; students are expected to develop the definitions and formulas. Another positive note to these findings is that the majority of lessons within the four textbook series offer potential opportunities for students to explore the concepts of surface area and volume through the implementation of hands-on activities. This state of affair might be due to authors’ attempts to design lessons that adhere to the national recommendations and standards.

The findings of this study also revealed that the CM textbook series generally offers more learning opportunities for students to explore the concepts of surface area and volume by including more rigorous materials. The majority of surface area, volume, and surface area and volume tasks within the CM textbook series are designed to challenge students’ thinking by asking them to develop strategies and explain their reasoning. In contrast, most surface area, volume, and surface area most and volume tasks within the GM, GMC, and UCSMP textbook series appear to be less challenging for students. Despite the efforts of the CM textbook authors to include more rigorous materials, such as a large portion of challenging tasks might overwhelm
students. It is possible that students might give up or refuse to solve tasks that are perceived as too hard.

The surface area, volume, and surface area tasks included in the CM textbook series also offer greater learning opportunities for students to engage with these concepts by requiring them to generate visual representations of 3D objects. This lack of attention on providing students with opportunities to generate visual representation of 3D objects within the GM, GMC, and UCSMP textbook series might explain some of the difficulties students have with solving surface area and volume tasks.

The majority of the lessons across the four textbook series address both components of the appropriate grade level geometry content standards. However, the lessons in the GM textbook series better address both components of the appropriate 6-8 geometry content standards, while the lessons in the UCSMP textbook series exhibit the greatest deviation in the coverage of the 6-8 geometry content standards. These results indicate the need to develop lessons that address both components of the 6-8 geometry content standards at the appropriate grade level.

Finally, analysis indicated that differences exist between the amount of tasks that contain a real-world visual representation of 3D objects and tasks set in real-world context. The majority of tasks contain non real-world visual representation of 3D objects across all four textbook series. Whereas nearly all lessons within the four textbook series address the 2nd component of the geometry content standards that states student should solve real-world problems involving surface area and volume. This discrepancy between the amount of tasks that contain a real-world visual representation of 3D objects and tasks set in real-world context might be an indication that the national recommendations are partially implemented in textbooks.
Opportunity to Learn in Popular and Alternative Textbook Series

For this study, I purposefully selected and examined two popular and two alternative textbook series because these textbooks are authored under different educational philosophies. As previously mentioned, the popular textbooks are commercially and widely used textbooks that usually focus on the development of procedural skills rather than conceptual skills and the alternative textbooks are designed based on the national recommendations and standards to provide greater emphasis on the development of conceptual understanding through problem solving (Stein et al., 2007). However, the findings revealed that significance similarities and differences exist between and within popular and alternative textbook series.

In particular, all popular and alternative textbooks place the concepts of surface area and volume in the 4th quartile of pages. With the exception of one popular (GM7) and two alternative (CM6 and CM8) textbooks that place these concepts in the 3rd quartile of pages. Both popular and alternative textbook series include small percentages of instructional pages devoted to the concepts of surface area and volume. Both popular (GM and GMC) and one alternative (UCSMP) textbooks series have similar instructional blocks of lessons. Only one alternative (CM) textbook series has lessons with fewer instructional blocks.

All popular and alternative textbook series contain miniscule amounts of important performance expectations such as justifying and proving and visual representations of 3D objects such as nets and pictures. Both popular and alternative textbook series also contain large amounts of tasks of moderate complexity. However, one alternative (CM) textbook series offers the greatest amount of tasks that require justifying and proving, generating visual representations of 3D objects, and tasks of high complexity followed by the popular (GMC) textbook series. All popular and alternative textbook series offer lessons that address the appropriate geometry
content standards. Nonetheless, the lessons in the alternative (UCSMP) textbook series exhibit the greatest variation in the coverage of the geometry content standards. Given these findings, it’s important to note that there is no clear distinguish between popular and alternative textbook series in terms of students’ opportunity to learn the concepts of surface area and volume. That is, analysis showed that similarities and differences exist in terms of the structural, pedagogical, and content features between and within popular and alternative textbook series.

**Limitations of the Study**

The main limitation of this study is related to the sample of textbooks. I examined two popular and two alternative middle-grades student edition mathematics textbook series from three main publishing companies. I selected a small sample size of textbooks because I wanted to conduct an in-depth analysis of the textbooks content. I also chose these four textbook series based on their market share and different pedagogical approaches to teaching mathematics. As previously stated, the popular and alternative textbooks are authored under different educational philosophies. However, this limited sample size does not represent all textbooks and educational philosophies used in the U.S. classrooms. This sample also does not represent all of the textbooks published by these three publishing companies. Therefore, I acknowledge that the results might have been different, if I have chosen other textbooks. Furthermore, I only examined the student edition textbooks. Have I examined the teacher edition textbooks, I might have obtained different results. I also only examined certain sections of the textbooks. It is possible that the findings might have been different if I have examined other parts of the textbooks.

Lastly, I used content analysis and simple descriptive statistical measures to collect and analyze the data. It is possible that if I have used other simple descriptive statistical measures such as frequency or conducted analysis of statistics tests, the results might have been slightly different.
Significance of the Study

The U.S. students are underperforming in the content area of geometry and especially on geometric tasks that require the use of their geometric reasoning and spatial abilities (Mullis et al., 2012; Mullis et al., 2016). Many studies have also indicated that teachers and students use textbooks as their main teaching and learning resource on a regular basis (Banilower et al., 2013; Mullis et al., 2012). This dependence on textbooks for the teaching and learning of mathematics affects students’ opportunity to learn mathematics and thus their achievement in mathematics. For this reason, it is important to document the opportunities presented in textbooks to learn mathematics. An examination of the structural, pedagogical, and content features of textbooks can reveal students’ opportunity to learn mathematics.

This study’s objective was to examine middle-grades student edition textbooks in order to determine students’ opportunity to learn the concepts of surface area and volume. Both the strengths and weaknesses of four middle-grades student edition mathematics textbook series are highlighted in this study. Therefore, the findings of this study add to the body of knowledge regarding students’ opportunity to learn mathematics offered by different curriculum materials.

The curricular developers might want to familiarize themselves with the research findings of this study and use the information to improve these curriculum materials. The findings of this study can also provide valuable information to curriculum specialists and teachers about the content of middle-grades student edition textbooks in terms of the location and sequence of the concepts of surface area and volume, as well as the pedagogical features of tasks and content features of lessons devoted to these concepts. For instance, awareness of the pedagogical features of tasks can help teachers make better instructional decisions regarding including or omitting some tasks.
The analysis of the treatment of the concepts of surface area and volume in popular and alternative textbooks provide an opportunity for curriculum specialists and teachers to compare and contrast different types of curriculum materials. For example, the result of this study can help them select curriculum materials that follow the national recommendations and standards. Teachers can also use the findings from this study to select curriculum materials that fit their students’ learning level and needs.

The methodology used in this study can contribute towards the knowledge and use of methods in textbook content analysis studies. I used content analysis to collect and analyze the data during this study. Hence, the methods employed in this study to collect and analyze the data can be used and modified if necessary to conduct future research in regards to students’ opportunity to learn other mathematical concepts presented in textbooks. The frameworks also employed in this study can be used in teacher education and professional development programs to help pre- and in-service teachers learn how to analyze textbooks in order to provide better learning opportunities for their students. For example, teachers can learn how to use these frameworks to identify and select worthwhile tasks.

**Implications for Mathematics Education**

Textbooks are the most common element of the teaching and learning of mathematics (Alajmi, 2012). The textbooks adopted and used in the classroom impact students’ opportunity to learn mathematics and thus their achievement in mathematics (Tornroos, 2005). Based on the usage and influence of textbooks on students’ learning and achievement in mathematics, the wise selection and implementation of well-designed textbooks can reshape the classroom environment (Flanders, 1994) and improve students’ mathematics learning (Reys et al., 2004) and achievement in mathematics (Tornroos, 2005). Therefore, it is imperative that curriculum
specialists and teachers select and employ curriculum materials that provide equal opportunities for all students to learn mathematics.

By examining the structural, pedagogical, and content features of textbooks, curriculum specialists and teachers can determine the strengths and weaknesses of different curriculum materials and make necessary adjustments. The results from this study indicated that differences exist in the location and sequence of the concepts of surface area and volume, as well as the pedagogical features of tasks and content features of lessons devoted to these concepts. These differences in textbooks signify differences in students’ opportunities to learn these concepts.

Research has indicated that teachers usually don’t cover lessons located in the last part of the textbook (Valverde et al., 2002; Weiss et al., 2001). The results of this study showed that the concepts of surface area and volume are introduced in the last part in the majority of textbooks included in this sample. Based on these findings, curriculum and textbook developers might want to consider including the concepts of surface area and volume in the first or middle part of the textbook in order to increase students’ opportunity to encounter and learn these concepts. Textbook developers might want to also consider including more instructional pages and lessons devoted to these concepts in order to offer additional opportunities for students to encounter and learn these concepts.

Teachers also need to be aware that the concepts of surface area and volume are located in the end of the textbook. This awareness can help teachers attend to this issue by purposefully covering these concepts regardless of their location in the textbook.

The results of this study also shed light on the lack of opportunity for students to engage, explore, and learn certain mathematical terms and concepts. For instance, most textbooks do not offer opportunities for students to develop the formula for the surface area and volume of
spheres. Nearly all textbooks include the vocabulary terms in the beginning of the lesson. These findings indicate the need to develop more mathematical lessons that allow students to develop mathematical vocabulary and formulas. As noted by NCTM (2000), lessons should allow students to engage in the process of developing definition and notations. Being cognizant that teachers rely heavily on their textbooks for instruction, greater attention needs to be placed on the mathematical structure of lessons. A deeper examination of the structure of lessons might help us better understand how textbooks influence students’ opportunity to learn mathematics.

Curriculum and textbook developers and teachers should also be aware of the gap that exists between the national recommendations and standards and the intended curriculum as represented in the textbooks. As the results of this study indicate a significant low percentage of tasks contain performance expectations such as conjecturing, verifying, justifying and proving, and critiquing. Furthermore, a large percentage of the tasks required low levels of mathematical thinking. These findings contradict NCTM (1989, 2000) and CCSSI (2010) recommendations that students must be provided with opportunities to make conjectures and construct arguments. It has also been noted that students should be encouraged to interpret, verify, and justify and prove their mathematical ideas and solutions (CCSSI, 2010; NCTM, 1989, 2000). Therefore, textbook developers might want to consider including more tasks of high complexity that require students to make conjectures and construct arguments, explain their mathematical thinking, and verify and justify their solutions. Teachers also need to be aware of the low amounts of tasks that require justifying and proving within textbooks. Teachers can attend to this matter by providing students with additional opportunities to justify and prove their answers. For example, teachers can ask students to justify and prove their answers by adding this component to the task or use other rigorous supplementary materials that include this type of opportunity.
Students also need to construct different types of visual representations of 3D objects (CCSSI, 2010; NCTM, 2000) in order to develop their geometric reasoning and spatial abilities required to solve surface area and volume tasks. The findings of this study revealed that certain types of visual representations of 3D objects are barely present in tasks. This lack of opportunity for students to encounter and construct certain types of visual representations of 3D objects might influence their learning of surface area and volume. Given these findings, textbook developers might want to consider including larger amounts of all types of visual representations of 3D objects in tasks in order to increase students’ opportunity to learn the concepts of surface area and volume. Textbook developers should also include tasks that offer more opportunities for students to generate visual representations of 3D objects. Based on this lack of visual representation of 3D objects within textbooks, teachers might want to use additional visual representations of 3D objects during instruction in order to provide students with ample opportunities to encounter and construct visual representations of 3D objects.

All textbook series analyzed in this study are developed by major publishers and marketed as Common Core aligned. Unfortunately, the results of this study indicated that some of the lessons within the four textbook series address only one component of the appropriate grade level geometry content standards. Furthermore, some lessons within the four textbook series only address the geometry content standards for the previous or next grade level. Being aware that teachers heavily rely on textbooks to help them implement the Common Core Mathematics Standards (Polikoff, 2015), it’s important to select textbooks that address all components of the appropriate grade level geometry content standards. For example, teachers might overemphasize, underemphasize, or neglect some standards topics, if the lesson does not address all components of the appropriate grade level geometry content standards. Therefore,
teachers need to be aware of the differences in the extent of coverage of the CCCS for 6-8 geometry in textbooks. This awareness can help teachers examine and adjust the content of their lessons in order to address all components of the appropriate grade level geometry content standards, if necessary. However, unless textbook developers include lessons in the textbooks that fully address all components of the appropriate grade level geometry content standards, implementation of the standards might be less effective than is desired.

**Recommendations for Future Research**

In this study, I analyzed the intended curriculum to investigate students’ opportunity to learn the concepts of surface area and volume in middle-grades mathematics textbooks. Based on the purpose of this study, the results revealed the potential opportunities students have to learn the concepts of surface area and volume in middle-grades mathematics textbook. Therefore, a natural extension to this study is to observe and understand when and how students learn these concepts. An examination of the enacted curriculum can provide valuable information about the differences in students’ opportunities to learn mathematics that exist between the intended and enacted curriculum. It can help the mathematics education research community learn more about the transformations from the intended to the enacted curriculum. For example, a study on the enacted curriculum can help us examine the differences between the tasks in the intended and enacted curriculum. It can also shed light on teachers’ instructional decisions and practices such as which tasks teachers tend to assign, why teachers choose to implement or omit certain tasks, and how these tasks are implemented.

Another potential direction for future research is the examining of the assessed curriculum. In the present study, I examined certain features and sections of the textbooks. However, assessments are another important component of the textbooks. For this reason future
research might need to examine if the assessment tasks are aligned to the tasks included in the lessons. For instance, it is possible that the assessment tasks might require higher or lower levels of mathematical complexity than the tasks in the lessons. By examining the assessment tasks included in the textbooks, researchers can also gather important information about the alignment of the assessment tasks with the national recommendations and standards.

Future research might also investigate the attained curriculum. Differences in curricular materials might reflect potential differences in students’ learning and thus achievement in mathematics. A study on the attained curriculum can help us investigate the actual impact of the curricular materials on students’ achievement in mathematics. This kind of study might yield interesting results regarding the relationship between the quality of curricular materials and students’ achievement in mathematics. It might also explain some of the differences in students’ achievement. Research on the interaction among the intended, enacted, and attained curriculum is limited. A study on the relationship among the intended, enacted, and attained curriculum might provide important information on how each level of curriculum influence the other levels.

Another area to be considered is the sample size of textbooks. The sample size for this study was small and limited to specific educational philosophies and approaches to teaching mathematics. Therefore, I recommend conducting a study that contains a larger sample of various middle-grades mathematics textbooks used in the U.S. classrooms. A larger sample of textbooks that contains various educational philosophies and approaches to teaching mathematics might generate interesting results about students’ opportunity to learn the concepts of surface area and volume across the country. Researchers might also want to include the teacher edition textbooks in the sample. It is possible that the teacher edition textbooks might include different
strategies and activities. Analysis of both the teacher and student edition textbooks might provide us with a more holistic picture of students’ learning opportunities.

The sample was also limited to only middle-grades mathematics textbooks used in the U.S. Given that U.S. students are underperforming in the area of geometry on international comparative studies, further research might want to examine and compare the treatment of the concepts of surface area and volume in the U.S. textbooks with the textbooks of other countries. A cross-national comparison textbook content analysis study can provide new perspectives and findings about the treatment of these concepts in other countries. By examining various international perspectives, it can help us understand how differences in curriculum materials are related to variations of students’ learning and performance in mathematics.

During this study, I also used well-established frameworks and methods of analysis to investigate the treatment of the surface area and volume concepts in middle-grades mathematics textbooks. Hence, the frameworks and methods of analysis used in this study can be modified and used in future textbook content analysis studies. For example, researchers might want to use the frameworks used in this study to examine other geometrical concepts such as area and perimeter that are related to the concepts of surface area and volume. This type of study can provide insights on how other geometrical concepts are introduced and developed in mathematics textbooks. It can also provide valuable information on how various geometrical concepts are interrelated in mathematics textbooks.

**Final Remarks**

Many variables influence the teaching and learning of mathematics. One important variable is the intended curriculum as presented in textbooks. In mathematics classrooms, textbooks play an essential role as instruction is geared around them. Thus, students’ opportunity
to learn mathematics is based on the breadth and depth of the textbook content. Given the importance of textbooks in the teaching and learning of mathematics, content analysis can be used to assess the quality of textbooks in order to determine students’ opportunity to learn mathematics.

In this study, I evaluated students’ opportunity to learn the geometric concepts of surface area and volume by examining the treatment of these concepts in four middle-grades mathematics textbook series. The findings of this study are a valuable addition to the body of knowledge regarding students’ opportunity to learn mathematics offered by different curriculum materials. However, as students are continuing to underperform in the content area of geometry, I believe that additional content analysis of mathematics textbooks needs to be conducted to further investigate this matter.
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APPENDIX A: EXAMPLES OF LESSONS

Lesson From Go Math! Grade 7 Textbook (Burger et al., 2014, pp. 283-288)

Finding the Surface Area of a Prism
Given a prism's dimensions, you can use a formula to find the surface area.

Surface Area of a Prism
The surface area $S$ of a prism with base perimeter $P$, height $h$, and base area $B$ is $S = Ph + 2B$.

EXAMPLE 1
Erin is making a jewelry box of wood in the shape of a rectangular prism. The jewelry box will have the dimensions shown. She plans to spray paint the exterior of the box. How many square inches will she have to paint?

**STEP 1**
Make a sketch of the box. Drawing a diagram helps you understand and solve the problem.

**STEP 2**
Identify a base, and find its area and perimeter.
Any pair of opposite faces can be the bases. For example, you can choose the bottom and top of the box as the bases.

- $B = \ell \times w$  
  - $= 12 \times 15$  
  - $= 180$ square inches

- $P = 2(12) + 2(15)$  
  - $= 24 + 30$  
  - $= 54$ inches

**STEP 3**
Identify the height, and find the surface area.
The height $h$ of the prism is 6 inches. Use the formula to find the surface area.

$S = Ph + 2B$  
$S = 54(6) + 2(180) = 684$ square inches

Erin will have to spray paint 684 square inches of wood.

3. A brand of uncooked spaghetti comes in a box that is a rectangular prism with a length of 9 inches, a width of 2 inches, and a height of $1\frac{1}{2}$ inches.

What is the surface area of the box? ____________
Finding the Surface Area of a Composite Solid

A composite solid is made up of two or more solid figures. To find the surface area of a composite solid, find the surface area of each figure. Subtract any area not on the surface.

EXAMPLE 2

Daniel built the birdhouse shown. What was the surface area of the birdhouse before the hole was drilled?

1. **Analyze Information**
   - The top is a triangular prism with \( h = 24 \text{ cm} \).
   - The base is a triangle with height 8 cm and base 30 cm.
   - The bottom is a rectangular prism with \( h = 18 \text{ cm} \). The base is a 30 cm by 24 cm rectangle.
   - One face of each prism is not on the surface of the figure.

2. **Formulate a Plan**
   - Find the surface area of each prism.
   - Add the areas. Subtract the areas of the parts not on the surface.

3. **Solve**
   - Find the area of the triangular prism.
     \[
     \text{Perimeter} = 17 + 17 + 30 = 64 \text{ cm}; \quad \text{Base area} = \frac{1}{2} (30)(8) = 120 \text{ cm}^2
     \]
     \[
     \text{Surface area} = Ph + 2B \]
     \[
     = 64(24) + 2(120) = 1,776 \text{ cm}^2
     \]
   - Find the area of the rectangular prism.
     \[
     \text{Perimeter} = 2(30) + 2(24) = 108 \text{ cm}; \quad \text{Base area} = 30(24) = 720 \text{ cm}^2
     \]
     \[
     \text{Surface area} = Ph + 2B \]
     \[
     = 108(18) + 2(720) = 3,384 \text{ cm}^2
     \]
   - Add. Then subtract twice the areas of the parts not on the surface.
     \[
     \text{Surface area} = 1,776 + 3,384 - 2(720) = 3,720 \text{ cm}^2
     \]
   - The surface area before the hole was drilled was 3,720 cm².

4. **Justify and Evaluate**
   - You can check your work by using a net to find the surface areas.
YOUR TURN

4. Dara is building a plant stand. She wants to stain the plant stand, except for the bottom of the larger prism. Find the surface area of the part of the plant stand she will stain.

Guided Practice

Find the surface area of each solid figure. (Examples 1 and 2)

1. 
   \[ \text{Perimeter of base = } \]
   \[ \text{Height = } \]
   \[ \text{Base area = } \]
   \[ \text{Surface area: } S = (\text{____})(\text{____}) + 2(\text{____}) \]

2. 
   \[ \text{Surface area of cube: } S = \]
   \[ \text{Surface area of rectangular prism: } S = \]
   \[ \text{Overlapping area: } A = \]
   \[ \text{Surface area of composite figure: } = \text{____} + \text{____} - 2(\text{____}) = \text{____} \text{ m}^2 \]

ESSENTIAL QUESTION CHECK-IN

3. How can you find the surface area of a composite solid made up of prisms?

286 Unit 4
4. Carla is wrapping a present in the box shown. How much wrapping paper does she need, not including overlap?

   4 in.  
   10 in.  
   3 in.

5. Dmitri wants to cover the top and sides of the box shown with glass tiles that are 5 mm square. How many tiles does he need?

   20 cm  
   15 cm  
   9 cm

6. Shera is building a cabinet. She is making wooden braces for the corners of the cabinet. Find the surface area of each brace.

   3 in.  
   1 in.  
   1 in.  
   3 in.

7. The doghouse shown has a floor, but no windows. Find the total surface area of the doghouse, including the door.

   2 ft  
   2.5 ft  
   2.5 ft  
   2 ft

   3 ft  
   4 ft

8. **Analyze Relationships** Describe two ways to find the surface area of the ramp.

   12 in.  
   20 in.  
   24 in.  
   16 in.  
   16 in.  
   16 in.

9. What is the surface area of the ramp?

10. Marco and Elaine are building a stand like the one shown to display trophies. Use the figure for 10–11.

   1 ft  
   3 ft  
   3 ft

   2 ft  
   3 ft  
   1 ft

   7 ft

11. **Critique Reasoning** Marco and Elaine want to paint the entire stand silver. A can of paint covers 25 square feet and costs $6.79. They set aside $15 for paint. Is that enough? Explain.
12. Henry wants to cover the box shown with paper without any overlap. How many square centimeters will be covered with paper?

______________

13. **What If?** Suppose the length and width of the box in Exercise 12 double. Does the surface area double? Explain.

14. **H.O.T. Focus on Higher Order Thinking** Enya is building a storage cupboard in the shape of a rectangular prism. The rectangular prism has a square base with side lengths of 2.5 feet and a height of 3.5 feet. Compare the amount of paint she would use to paint all but the bottom surface of the prism to the amount she would use to paint the entire prism.

15. **Interpret the Answer** The oatmeal box shown is shaped like a cylinder. Use a net to find the surface area S of the oatmeal box to the nearest tenth. Then find the number of square feet of cardboard needed for 1,500 oatmeal boxes. Round your answer to the nearest whole number.

16. **Analyze Relationships** A prism is made of centimeter cubes. How can you find the surface area of the prism in Figure 1 without using a net or a formula? How does the surface area change in Figures 2, 3, and 4? Explain.

288 Unit 4
August 22, 2018

Sofia Hatzinikadakis
University of South Florida

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Inquiry Lab
Nets of Three-Dimensional Figures

Inquiry
HOW can models and nets help you find the surface area of prisms?

Nets are used to design and manufacture items such as boxes and labels. Find the shapes that make up the net of a cereal box.

Hands-On Activity 1
Make a net from a rectangular prism.

**Step 1**
Use an empty cereal box. Cut off one of the two top flaps. The remaining top flap is the top face.

**Step 2**
Label the top and bottom faces using a green marker.
Label the front and back faces using a blue marker.
Label the left and right faces using a red marker.

**Step 3**
Carefully cut along the three edges of the top face.
Then cut down each vertical edge.

The net of a cereal box is made up of a total of rectangles.

What do you notice about the top and bottom faces, the left and right faces, and the front and back faces?
Hands-On Activity 2

Make a triangular prism from a net.

**Step 1** Draw a net on a piece of card stock with the dimensions shown below.

```
      5 cm
      
   6 cm      10 cm
    left      right
      5 cm
      5 cm

  side
```

**Step 2** Fold the net into a triangular prism. Tape together adjacent edges.

```
   5 cm
   
  6 cm      10 cm

  5 cm
```

The triangular prism is made up of □ triangles and □ rectangles.

What is true about the triangular bases?

How is the side of one of the rectangles related to the base of one of the triangles?

Explain one way to find the total surface area of a triangular prism.
Work with a partner to solve each problem.

1. A net of a rectangular prism that is 24 inches by 18 inches by 4 inches is shown. The net of the prism is labeled with top, bottom, side, and end. Fill in the boxes to find the total area of the rectangular prism.

2. Describe in words how you could find the total surface area of a rectangular prism.

3. A net of a triangular prism is shown. Fill in the boxes to find the total area of the triangular prism.

4. Describe in words how you could find the total surface area of a triangular prism.
Work with a partner.

5. **Reason Inductively** Suppose Ladell wants to wrap a present in a container that is a rectangular prism. How can he determine the amount of wrapping paper that he will need?

Circle each correct surface area. Draw and label the net for each figure if needed. The first one is done for you.

<table>
<thead>
<tr>
<th>Prism</th>
<th>Measures</th>
<th>Surface Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>Length: 10 cm, Width: 8 cm, Height: 5 cm</td>
<td>170 cm²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>340 cm²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>400 cm²</td>
</tr>
<tr>
<td>Rectangular</td>
<td>Length: 3 ft, Width: 2 ft, Height: 5 ft</td>
<td>30 ft²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>31 ft²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>62 ft²</td>
</tr>
<tr>
<td>Rectangular</td>
<td>Length: 2 m, Width: 1 m, Height: 1.5 m</td>
<td>3 m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.5 m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13 m²</td>
</tr>
<tr>
<td>Triangular</td>
<td>Area of Top and Bottom Triangles: 3 mm²</td>
<td>25 mm²</td>
</tr>
<tr>
<td></td>
<td>Area of Center Rectangle: 12 mm²</td>
<td>28 mm²</td>
</tr>
<tr>
<td></td>
<td>Area of Left and Right Rectangles: 10 mm²</td>
<td>38 mm²</td>
</tr>
<tr>
<td>Triangular</td>
<td>Area of Top and Bottom Triangles: 6 in²</td>
<td>174.4 in²</td>
</tr>
<tr>
<td></td>
<td>Area of Center Rectangle: 50.4 in²</td>
<td>118.4 in²</td>
</tr>
<tr>
<td></td>
<td>Area of Left and Right Rectangles: 56 in²</td>
<td>112.4 in²</td>
</tr>
</tbody>
</table>

10. **Be Precise** Surface area is the sum of the areas of all the surfaces of a three-dimensional figure. Write the formula for the total surface area of a rectangular prism.

11. **Inquiry** How can models and nets help you find the surface area of prisms?
Surface Area of Prisms

Real-World Link
Message Board  Members of a local recreation center are permitted to post messages on 8.5-inch by 11-inch paper on the board. Assume the signs are posted vertically and do not overlap, as shown below.

Lost Dog
Post Kitten's
To a Good
Home
Tutoring

1. Suppose 6 messages fit across the board widthwise. What is the width of the board in inches? ______ inches

2. Suppose 3 messages fit down the board lengthwise. What is the length of the board in inches? ______ inches

3. What is the area in square inches of the message board?

4. Messages can also be posted on the other side of the board. What is the total area of the front and back of the board in square inches?

Which Mathematical Practices did you use? Shade the circle(s) that applies.

1. Persevere with Problems
2. Reason Abstractly
3. Construct an Argument
4. Model with Mathematics
5. Use Math Tools
6. Attend to Precision
7. Make Use of Structure
8. Use Repeated Reasoning

Lesson 6  Surface Area of Prisms 665
The sum of the areas of all the surfaces, or faces, of a three-dimensional figure is the **surface area**. In the previous Inquiry Lab, you used a net to find the surface area of a rectangular prism. You can also use a formula to find surface area.

When you find the surface area of a three-dimensional figure, the units are square units, not cubic units.

**Example**

1. Find the surface area of the rectangular prism shown at the right.

   Replace \( \ell \) with 9, \( w \) with 7, and \( h \) with 13.

   Surface area = \( 2\ell h + 2\ell w + 2hw \)
   
   \[
   = 2 \cdot 9 \cdot 13 + 2 \cdot 9 \cdot 7 + 2 \cdot 13 \cdot 7
   \]
   
   \[
   = 234 + 126 + 182
   \]
   
   \[
   = 542
   \]

   The surface area of the prism is 542 square inches.

**Got it? Do these problems to find out.**

Find the surface area of each rectangular prism.

a. \[ \begin{array}{c}
6 \text{ m} \\
10 \text{ m}
\end{array} \]

b. \[ \begin{array}{c}
11 \text{ mm} \\
11 \text{ mm}
\end{array} \]
Example

2. Domingo built a toy box 60 inches long, 24 inches wide, and 36 inches high. He has 1 quart of paint that covers about 87 square feet of surface. Does he have enough to paint outside of the toy box? Justify your answer.

Step 1
Find the surface area of the toy box.
Replace \( l \) with 60, \( w \) with 24, and \( h \) with 36.

Surface area \( = 2lh + 2lw + 2hw \)
\[ = 2 \cdot 60 \cdot 36 + 2 \cdot 60 \cdot 24 + 2 \cdot 36 \cdot 24 \]
\[ = 8,928 \text{ in}^2 \]

Step 2
Find the number of square inches the paint will cover.

\( 1 \text{ ft}^2 = 1 \text{ ft} \times 1 \text{ ft} \)
\[ = 12 \text{ in.} \times 12 \text{ in.} \]
\[ = 144 \text{ in}^2 \]

So, 87 square feet is equal to \( 87 \times 144 \) or 12,528 square inches.

Since 12,528 > 8,928, Domingo has enough paint.

Get it? Do this problem to find out.

c. The largest corrugated cardboard box ever constructed measured about 23 feet long, 9 feet high, and 8 feet wide. Would 950 square feet of paper be enough to cover the box? Justify your answer.

Surface Area of Triangular Prisms

To find the surface area of a triangular prism, it is more efficient to find the area of each face and calculate the sum of all of the faces rather than using a formula.
3. Marty is mailing his aunt the package shown. How much cardboard is used to create the shipping container?

Find the area of each face and add.
The area of each triangle is $\frac{1}{2} \cdot 4 \cdot 3$ or 6.
The area of two of the rectangles is $14 \cdot 3.6$ or 50.4. The area of the third rectangle is $14 \cdot 4$ or 56.
The sum of the areas of the faces is $6 + 6 + 50.4 + 50.4 + 56$ or 168.8 square inches.

Got it? Do this problem to find out.

d. Find the surface area of the triangular prism.

Guided Practice

Find the surface area of each prism. (Examples 1–3)

1.  

2.  

3. Building on the Essential Question Why is the surface area of a three-dimensional figure measured in square units rather than in cubic units?
Independent Practice

Find the surface area of each rectangular prism. Round to the nearest tenth if necessary. (Example 1)

1. 8 cm
   9 cm
   5 cm

2. 12 ft
   17 ft
   6.4 ft

When making a book cover, Anwar adds an additional 20 square inches to the surface area to allow for overlap. How many square inches of paper will Anwar use to make a book cover for a book 11 inches long, 8 inches wide, and 1 inch high? (Example 2)

Find the surface area of each triangular prism. (Example 2)

4. 10 cm
   8 cm
   3 cm

5. 13 in
   5 in
   4 in

6. **Model with Mathematics** Refer to the graphic novel frame below. What whole number dimensions would allow the students to maximize the volume while keeping the surface area at most 160 square feet? Explain.

We are designing a dunk tank. Remember, we want to maximize the volume and minimize the surface area.
7. Write a formula for the surface area S.A. of a cube in which each side measures x units.

8. A company will make a cereal box with whole number dimensions and a volume of 100 cubic centimeters. If cardboard costs $0.05 per 100 square centimeters, what is the least cost to make 100 boxes?

**H.O.T. Problems** Higher Order Thinking

9. **Reason Inductively** Determine if the following statement is true or false. Explain your reasoning.
   
   *If you double one of the dimensions of a rectangular prism, the surface area will double.*

10. **Reason Inductively** A prism with a base that is a regular hexagon is shown. How would you find the surface area of the hexagonal prism if the area of the base of the prism is x square centimeters?

11. **Persevere with Problems** The figure at the right is made by placing a cube with 12-centimeter sides on top of another cube with 15-centimeter sides. Find the surface area of the figure.

12. **Model with Mathematics** Draw and label a rectangular prism that has a total surface area between 100 and 200 square units. Then find the surface area of your prism.
Extra Practice

Find the surface area of each prism. Round to the nearest tenth if necessary.

13.  
\[ S.A. = 2lh + 2lw + 2hw \]
\[ = 2 \times 12.3 \times 15 + 2 \times 12.3 \times 8.5 + 2 \times 15 \times 8.5 \]
\[ = 369 + 207.1 + 255 \]
\[ = 833.1 \text{ mm}^2 \]

14.  
\[ S.A. = 2lh + 2lw + 2hw \]
\[ = 2 \times 3 \times 6.25 + 2 \times 3 \times 8.5 + 2 \times 6.25 \times 8.5 \]
\[ = 37.5 + 51 + 103.75 \]
\[ = 192.25 \text{ in}^2 \]

15.  
\[ S.A. = 2lh + 2lw + 2hw \]
\[ = 2 \times 3 \times 4 + 2 \times 3 \times 7 + 2 \times 4 \times 7 \]
\[ = 24 + 42 + 56 \]
\[ = 122 \text{ ft}^2 \]

16.  
\[ S.A. = 2lh + 2lw + 2hw \]
\[ = 2 \times 24 \times 14 + 2 \times 24 \times 10 + 2 \times 14 \times 10 \]
\[ = 672 + 480 + 280 \]
\[ = 1432 \text{ m}^2 \]

17. If one gallon of paint covers 350 square feet, will 8 gallons of paint be enough to paint the inside and outside of the fence shown once? Explain.

18. The attic shown is a triangular prism. Insulation will be placed inside all walls, not including the floor. Find the surface area that will be covered with insulation.

19. **Be Precise** To the nearest tenth, find the approximate amount of plastic covering the outside of the CD case.
20. A cardboard box has the dimensions shown. Select the correct values to complete the formula to find the surface area of the box.

\[ SA = \square \cdot \square \cdot \square + \square \cdot \square \cdot \square + \square \cdot \square \cdot \square \]

How much cardboard is needed to make the box?

21. A triangular prism has the dimensions shown. Fill in each box to complete each statement.
   a. The area of each triangular base is \square \square \square \square \square square inches.
   b. The area of each of the two congruent rectangular faces is \square \square \square \square \square square inches.
   c. The area of the third rectangular face is \square \square \square \square \square square inches.
   d. The total surface area of the prism is \square \square \square \square \square square inches.

Common Core Spiral Review

Describe the shape resulting from a vertical, horizontal, and angled cross section for each figure. 7.6.3

22. [Diagram of a rectangular prism]
   - vertical: 
   - horizontal: 
   - angled: 

23. [Diagram of a square pyramid]
   - vertical: 
   - horizontal: 
   - angled: 

24. [Diagram of a cone]
   - vertical: 
   - horizontal: 
   - angled: 

25. [Diagram of a cylinder]
   - vertical: 
   - horizontal: 
   - angled: 

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