Multistable Shape-Shifting Surfaces (MSSSs)

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Multistable Shape-Shifting Surfaces (MSSSs)

by

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A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering
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Bistable, Compliant Mechanisms, Pseudo-Rigid-Body Model, Virtual Work, Kinematics

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This paper presents designs for Multistable Shape-Shifting Surfaces (MSSS) by introducing bistability into the Shape-Shifting Surface (SSS). SSSs are defined as surfaces that retain their effectiveness as a physical barrier while undergoing changes in shape. The addition of bistability to the SSS gives the surface multiple distinct positions in which it remains when shifted to, i.e. by designing bistability into a single SSS link, the SSS unit cell can change into multiple shapes, and stabilize within the resulting shape, while maintaining integrity against various forms of external assaults normal to its surface. Planar stable configurations of the unit cell include, expanded, compressed, sheared, half-compressed, and partially-compressed, resulting in the planar shapes of a large square, small square, rhombus, rectangle, and trapezoid respectively. Tiling methods were introduced which gave the ability to produce out-of-plane assemblies using planar MSSS unit cells. A five-walled rigid storage container prototype was produced that allowed for numerous stable positions and volumes. Applications for MSSSs can include size-changing vehicle beds, expandable laptop screens, deformable walls, and volume-changing rigid-storage containers. Analysis of the MSSS was done using pseudo-rigid-Body Models (PRBMs) and Finite Element Analysis (FEA) which ensured bistable characteristics before prototypes were fabricated.
CHAPTER 1: INTRODUCTION

1.1 Objective

The objective of this paper is to describe designs of Multistable Shape-Shifting Surfaces (MSSSs) [1]. A Shape-Shifting Surface (SSS) is defined as a surface that can change its shape while retaining its integrity as a physical barrier. Shape changes include expanding, shrinking, shearing, twisting, and encircling. The addition of bistability to the individual SSS links gives the surface multiple distinct positions in which it remains when shifted to, i.e. by designing bistability into a single SSS link, the SSS unit cell can change into multiple shapes, and stabilize within the resulting shape, while maintaining integrity against various forms of external assaults normal to its surface.

1.2 Motivation

A surface is defined as a material layer constituting a boundary. Examples of surfaces are walls, ceilings, doors, tables, armor, and vehicle bodies [1]. In some cases, it may be valuable for these surfaces to change shape while maintaining rigidity in the direction normal to the surface. In addition, having surfaces able to change between multiple distinct shapes and sizes on demand may add value. One valuable application of size-changing surfaces may apply to rigid storage containers, such as:

- milk crates
- trash barrels
- dumpsters
- laundry baskets
- suit cases
Such containers are designed for large, heavy volumes, however, when not in use, they may become cumbersome. Thus, containers capable of transitioning between large volumes when in use and small volumes when empty are of value. For example, truck beds could start off at a normal size, and have the capability to expand to twice the volume by pulling on the bed. Other applications of a deformable surface with multiple stable positions may appeal to the communication business. Take for instance, a cell phone that initially has a pocket-sized screen. With the push of a link, the screen could expand to a laptop sized display, giving better visibility when space permits. Another advantage of having a Shape-Shifting Surface with multiple stable positions is the ability to absorb energy. Because work is done to change the shape of the surface from one position to the other, external kinetic energy can be absorbed, and stored internally as potential energy within compliant links. Usually, impact absorbers require permanent deformation to absorb energy or shock. Allowing surfaces to absorb impacts by transitioning to a second stable position could save time and money in repair as they can be ‘fixed’ or returned to their initial state by transitioning the unharmed surface back to its first stable shape. The surface is essentially unharmed due to the mechanisms ability to re-direct the energy through elastic body movement.

1.3 Scope

The scope of this paper is to describe design iterations on how the compliant section of the Shape-Shifting Surface link may be bistable, i.e. each SSS link is designed to have two stable positions; this results in an SSS unit cell that contains multiple stable positions due to the SSSs unique tiling system and degrees of freedom. Prototypes,
pseudo-rigid-body models, and finite element analysis (FEA) provide proof-of-concept. Prototypes were developed further in order to suggest commercial applications. Tiling of unit cells is considered within the design, in chapters 5 and 6, to allow three-dimensional assemblies of the MSSS.

1.4 Background

The Shape-Shifting Surface is an innovative concept that uses compliant mechanisms to achieve its motion. Research has been done on compliant mechanisms and on bistability, and is described in the following section.

1.4.1 Compliant Mechanisms

A compliant mechanism is defined as a flexible member that transfers an input force or displacement from one point to another through elastic body deformation [2]. Examples of compliant mechanisms are nail clippers, paper clips, diving boards and bows. The advantages of compliant mechanisms are considered in two categories [2]:

1) “Cost reduction:
   • Part-count reduction
   • Reduced assembly time
   • Simplified manufacturing process

2) Increase performance
   • Increased precision
   • Increased reliability
   • Reduced wear
   • Reduced weight
   • Reduced maintenance” [2]

There are two types of compliant mechanisms. The first is called fully compliant mechanisms. These mechanisms have no joints, and therefore no links. They use only the deflection of compliant members to obtain their motion. The second type is called
partially compliant mechanisms. These may contain one or more kinematic pairs, such as pins or sliders within the compliant portions.

Links can be compliant; a link is defined as a continuum of matter connecting one or more kinematic pairs [2]. A two-pin link may either be a binary link, which is a rigid link with no movement between two pins, or a structurally binary link, which is a compliant, or flexible link with two pin joints. When a structurally binary link is loaded only at its joints, it is said to be functionally binary. The SSS is composed of partially compliant links that are functionally binary.

1.4.2 Compliant Mechanisms in MEMS

Compliant mechanisms are well known in Microelectromechanical systems (MEMS). MEMS devices use mechanical and electrical components on the micrometer and millimeter scale. MEMS are fabricated using planar layers of material. Assembly of MEMS at the micro scale tends to be difficult. Compliant mechanisms offer solutions to this problem as they are easy to assemble and do not require many parts. Shape shifting surfaces are similar to the production of compliant MEMS in that they can be easily built using planar fabrication. Planar layers overlap and are attached with pins. Three-dimensional configurations of the SSS are possible using similar techniques.

1.4.3 Pseudo-Rigid-Body Replacement Method (PRBM)

Methods for predicting the behavior of compliant mechanisms can be complex. Because compliant mechanisms experience large, non-linear deformations, small-deflection force-deflection equations cannot be used. Elliptic integrals and topology optimization are sometimes used. However, these require intense calculation and time.
Therefore, a popular method of compliant mechanism analysis (used on the SSS) is using pseudo-rigid-body models [2]. Pseudo-Rigid-Body models offer a simplified technique for determining the motion of mechanisms undergoing large, nonlinear deflections. The compliant mechanism is analyzed as a rigid-body mechanism with equivalent force-deflection characteristics. To achieve this, joints, whose locations are determined by the pseudo-rigid-body model, are placed within a skeletal model to represent the kinematics of the compliant mechanism. Torsional springs are then added to these joints to mimic the stiffness of the flexible members in the compliant mechanism. The spring constant at each joint is determined by the geometry and material property of the compliant segment [3]. Flexible segments can include, but are not limited to [2]:

- Small-length flexural pivots (living hinges)
- Cantilever beam with force at free end
- Fixed guided
- End-moment loaded cantilever
- Initially curved cantilever
- Pinned-pinned segment [2]

The location of the pin’s torsional spring is different for each geometry type and loading case, and is determined using the pseudo-rigid-body models. Each compliant segment can be modeled as a portion of a rigid-link mechanism and analyzed using rigid-link mechanism theory. “In this way, the pseudo-rigid-body model is a bridge that connects rigid-body mechanism theory and compliant mechanism theory” [2].

A flexible segment that was extensively used in this research is the small-length flexural pivot, or living hinge. This flexible segment is a thin, short-length compliant section that can be modeled as a kinematic pin. Because this segment is thin, it can be assumed to have minimal resistance, therefore, minimal stress [2].
1.4.4 Special-Purpose Mechanisms

Because compliant mechanisms have excellent energy absorption capabilities from their deflections, they are useful in a wide range of applications. Three specific special-purpose mechanisms that are used in compliant mechanisms are constant-force mechanisms, parallel-guiding mechanisms, and bistable mechanisms. There are many more types of special-purpose mechanisms, however, these three are discussed because of their possible applications to the SSS. Constant-force mechanisms and parallel-guiding mechanisms are briefly discussed in this section while bistable mechanisms are more extensively discussed in the following section.

1.4.4.1 Constant-Force Mechanisms

A constant-force mechanism maintains a constant reaction force to an applied load throughout its entire motion. Designs for rigid-link constant-force mechanisms [4-6] have been developed. Constant-force springs [7] have also been studied, and produce a constant force as they are extended. Recently, compliance has been incorporated into constant-force mechanisms [8-10]. Future work on SSS design can incorporate a constant-force mechanism within the compliant section to allow for a flat, linear reaction force as the SSS is deformed into its many possible shapes. Applications can range from statically balanced to gravity compensating SSSs.

1.4.4.2 Parallel Mechanisms

Parallel-guided mechanisms contain two opposing links that stay parallel through their entire motion. Examples of parallel-guided mechanisms are tackle boxes, desktop lamps, and playground swings. Compliant parallel-guiding mechanisms are designed to retain all the advantages associated with compliant mechanisms, including the
elimination of joint friction, backlash, and the need for lubrication, in addition to a reduction in part count, weight, and assembly time [2]. Another key feature of parallel mechanisms is their ability to produce pure planar translation. Compliant parallel mechanisms could be used within the SSS to reduce the effect of planar rotation of nodes.

1.4.5 Bistable Mechanisms

A bistable mechanism is defined as a mechanism with two stable equilibrium positions separated by a peak in energy. Examples of bistable mechanisms include light switches, self-closing gates, cabinet hinges, and three-ring binders.

1.4.5.1 Definition of Stability

A system is in a state of equilibrium when it is experiencing no acceleration [2]. This can either be a stable equilibrium, or unstable equilibrium. When in stable equilibrium, any small external disturbances will cause the system to oscillate about its equilibrium. When in unstable equilibrium, any small external disturbance will cause it to diverge from its initial equilibrium state to another. Figure 1.1 illustrates the “ball-on-the-

![Figure 1.1: Ball on a hill analogy.](image)
hill” analogy of bistability, where the Y-axis represents potential energy (gravity), and the X-axis represents position. Position 1 and 4 are stable equilibrium positions. This means that even if the ball is shifted from these positions by a small amount, it will return to the same position, or oscillate around it. Position 2 is an unstable equilibrium position. Though the ball can rest in this position, it will return to one of its stable equilibrium positions (1 or 4) when shifted by any small amount. Position 3 is classified as a “hard-stop”. This is also a stable equilibrium position, however, is only so, due to an external reaction force holding it in place. These key bistable mechanism characteristics may be summarized with the following statements [11]:

- “A mechanism will have a stable equilibrium position when the first derivative of the potential energy curve is zero and the second derivative of the potential energy curve is positive.”
- “A mechanism will have an unstable equilibrium position when the first derivative of the potential energy curve is zero and the second derivative of the potential energy curve is negative.”
- “A mechanism will have a neutrally stable equilibrium position when the first derivative of the potential energy curve is zero and the second derivative of the potential energy curve is also zero.”
- “Because two local minima must always contain one local maximum between them, an unstable or neutrally stable position will always occur between any two stable states.”
- “The critical moment (the maximum load required for the mechanism to change stable states) may be found by evaluating the moment curve when the second derivative of potential energy is zero.
- “The stiffness of a stable equilibrium position is equal to the value of the second derivative of potential energy at that position.” [11]

1.4.5.2 Multistable Mechanisms

Extensive studies have been done on bistable mechanisms with two stable configurations; see citations [3, 11, 12]. Studies have also been done on multistable compliant mechanisms which gives three stable equilibrium positions by using a combination of bistable mechanisms [13]. The research presented in this paper is similar
in that it uses a combination of bistable links to achieve a Multistable-Shape-Shifting Surface (MSSS). The unique tiling scheme of the SSS is what provides the possibility of multistability using bistable mechanisms.

1.4.5.3 Bistability in Compliant Mechanisms

Compliant mechanisms are specifically practical in bistability because they have the ability to store energy within their members [2]. This is an efficient method to achieve bistability because the mechanism can be made from one piece without the need to add springs for energy storage.

Using the pseudo-rigid-body model, the potential energy equation for a compliant mechanism is found. For a small-length flexural pivot or a fixed-pinned segment, the potential energy $V$ stored in the segment is [2]

$$V = \frac{1}{2} K \Theta^2$$

where $V$ is the potential energy, $K$ is the torsional spring constant, and $\Theta$ is the pseudo-rigid-body angle. The torsional spring constant is found using the pseudo-rigid-body model.

1.4.5.4 Bistability in Four-Bar Mechanisms

Figure 1.2 shows a general four-bar mechanism. This four-bar may either be classified as a Grashof or non-Grashof mechanism. A Grashof four-bar allows the shortest link to rotate through a full revolution. A non-Grashof four-bar does not allow any link to rotate a full revolution. Labeling the four-bar correctly is important in
determining the location of the springs that cause bistable behavior. The Grashof's criterion is stated mathematically as

\[ s + l \leq p + q \quad \text{Grashof} \]  \hspace{1cm} (2)

where \(s\) is the length of the shortest link, \(l\) is the length of the longest link, and \(p\) and \(q\) are the lengths of the intermediate links. Examples of Grashof mechanisms are crank rockers, double cranks, and double rockers. The mathematical model of a non-Grashof is as follows [2]

\[ s + l > p + q \quad \text{non-Grashof} \]  \hspace{1cm} (3)

Figure 1.2: Four-Bar pseudo-rigid-body model with torsional springs [2].
A change-point mechanism occurs when the sum of the lengths of the longest and shortest links is equal to the sum of the lengths of the other two links, or [2]

\[ s + l = p + q \]  \hspace{1cm} \text{change-point} \tag{4}

Knowing the four-bar’s classification is important in determining how to make the mechanism bistable. The following theorems apply for a pseudo-rigid-body four-bar mechanism to determine bistability:

- **Theorem 1.** A compliant mechanism whose pseudo-rigid-body model behaves like a Grashof four-link mechanism with a torsional spring placed at one joint will be bistable if and only if the torsional spring is located opposite the shortest link and the spring’s undeflected state does not correspond to a mechanism position in which the shortest link and the other link opposite the spring are collinear.
- **Theorem 2.** A compliant mechanism whose pseudo-rigid-body model behaves like a non-Grashof four-link mechanism with a torsional spring at any one joint will be bistable if and only if the spring’s undeflected state does not correspond to a mechanism position in which the two links opposite the spring are collinear.
- **Theorem 3.** A compliant mechanism whose pseudo-rigid-body model behaves like a change-point four-link mechanism with a torsional spring placed at any one joint will be bistable if and only if the spring’s undeflected state does not correspond to a mechanism position in which the two links opposite the spring are collinear. [14]

Once the four-bar is properly classified and a bistable design is chosen, virtual work can be used to predict the force characteristic of the mechanism.

1.4.6 Bistability Using Virtual Work

The principle of virtual work says that the net virtual work of all active forces is zero if the system is in equilibrium i.e. due to conservation of energy, the total net virtual work on a system is zero [15]. The principle of virtual work can be used to find the values of reaction forces or moments caused by a given displacement [16]. Considering
the four-bar linkage system shown in Figure 1.2, the total virtual work of the system can be expressed as [2]

$$\delta W = \sum_{i=2}^{4} \vec{F}_i \cdot \delta \vec{z}_i + \sum_{i=2}^{4} \vec{M}_i \cdot \delta \vec{\theta}_i + \sum_{i=1}^{4} \vec{T}_i \cdot \delta \vec{\Psi}_i = 0 \quad (5)$$

where $\vec{F}_i$ is the force applied to link $i$, which is expressed as

$$\vec{F}_i = X_i \hat{i} + Y_i \hat{j} \quad (6)$$

where $X_i$ is the force in the X-direction and $Y_i$ is the force in the Y-direction.

$\vec{M}_i$, in equation (5) is the moment applied to link $i$, and $\vec{T}_i$ is the moment at characteristic pivot $i$. Take for instance the $Z_2$ vector shown in Figure 1.2

$$\vec{Z}_2 = (a_2 \cos \theta_2 - b_2 \sin \theta_2) \hat{i} + (a_2 \sin \theta_2 + b_2 \cos \theta_2) \hat{j} \quad (7)$$

The differential or virtual displacement from equation (5) is then expressed as

$$\delta \vec{Z}_2 = (-a_2 \sin \theta_2 - b_2 \cos \theta_2) \delta \theta_2 \hat{i} + (a_2 \cos \theta_2 - b_2 \sin \theta_2) \delta \theta_2 \hat{j} \quad (8)$$

The potential energy, due to the torsional springs for the four-bar shown in Figure 1.2 is

$$V = \frac{1}{2} (K_1 \psi_1^2 + K_2 \psi_2^2 + K_3 \psi_3^2 + K_4 \psi_4^2) \quad (9)$$

where

$$\psi_1 = \theta_2 - \theta_{2o} \quad (10)$$

$$\psi_2 = \theta_2 - \theta_{2o} - (\theta_3 - \theta_{3o}) \quad (11)$$

$$\psi_3 = \theta_4 - \theta_{4o} - (\theta_3 - \theta_{3o}) \quad (12)$$
where the “o” subscript indicates the initial un-deflected value of the angle.

In addition, the torque from the torsional springs, $T_i$, is found using

$$T_i = -K_i \Psi_i$$

(14)

The final total virtual work for the four-bar is found using the following equations [2]:

$$\delta W = A \delta \theta_2 + B \delta \theta_3 + C \delta \theta_4 = 0$$

(15)

where

$$A = \left(-X_2 a_2 - Y_2 b_2 - r_2 X_3 \right) \sin \theta_2$$

$$+ \left(-X_2 b_2 + Y_2 a_2 + r_2 Y_3 \right) \cos \theta_2$$

$$+ M_2 + T_1 + T_2$$

(16)

$$B = \left(-X_3 a_3 - Y_3 b_3 \right) \sin \theta_3$$

$$+ \left(-X_3 b_3 + Y_3 a_3 \right) \cos \theta_3 + M_3 - T_2$$

$$- T_3$$

(17)

$$C = \left(-X_4 a_4 - Y_4 b_4 \right) \sin \theta_4$$

$$+ \left(-X_4 b_4 + Y_4 a_4 \right) \cos \theta_4 + M_4 + T_3$$

$$+ T_4$$

(18)

where, $X_i$ and $Y_i$ are the forces acting in the x and y direction respectively. $M_i$ is the moment acting on the coupler links and $T_i$ is the torque due to the deflection of the torsional spring for each joint.
1.4.7 The SSS

The Shape-Shifting Surface (SSS) is an innovative concept in which tiled arrays of overlapping surfaces undergo relative displacement, changing the overall shape of the mechanism. This is done by layering eight compliant, pin-pinned links, connected at their pins, in a way to form a closed square unit cell. These links consists of two parts, a compliant portion and a shield portion. The compliant portion gives the link mobility, while the shield portion gives the unit cell its effectiveness as a barrier.

1.4.7.1 Kinematic Structure of the SSS

Because the SSS is designed to cover a plane, its shape is chosen to tile a plane, e.g. squares on a checkerboard. Therefore, each square has a particular structure that enables it to move and maintain its integrity.

Each square unit cell has four, straight sides with four nodes, represented by Figure 1.3a). In order to achieve the SSSs relative displacement, each side link of the unit cell is modeled with a kinematic slider, as shown in Figure 1.3b). Two overlapping links form each side and produce this kinematic slider; this gives the unit cell a total of

![Figure 1.3](image)

Figure 1.3: Part a) shows the SSS unit cell with four nodes while part b) shows the eight-link kinematic skeleton with the same number of nodes. [1].

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eight links. In order to maintain the square shape of the unit cell, these overlapping sliders must undergo linear deflections along the line of action connecting nodes. Therefore, four sets of two coupled sliders are formed together to produce the sides of unit cell. This provides the SSS unit cell with a total of five degrees of freedom, i.e. each of the four sides of the square can move independently, plus an additional shear movement from the four pinned links.

1.4.7.2 SSS Shield Portion

“The SSS has a unique tiling system that maintains surface integrity under numerous conditions including: uniform and non-uniform compression, expansion, and shear” [1]. This means that the unit cell serves as a line-of-sight barrier to assaults normal to the surface as it experiences these movements. This is done by overlapping sliding and rotating layers so that the unit cell can experience changes in area coverage in all three modes of deformation (compression, expansion, and shear) without compromising the line-of-sight coverage. In order to have this un-broken area coverage in both compression and expansion, the unit cell is divided into thirds so that the initial position is halfway between complete overlap and no overlap. This provides equal stress in tension and compression when expanded and compressed. Figure 1.4 demonstrates this theory using

![Figure 1.4](image-url)

Figure 1.4: Part a) shows minimum area coverage; part b) shows maximum area coverage; part c) shows its initial position with one-third overlapping area coverage [1].
square coverage. Figure 1.4a) shows the minimum area coverage when the squares are coincident. Figure 1.4b) shows the maximum area coverage when the layers are adjacent. Figure 1.4c) shows the initial half-way position with a 1/3 overlapping area coverage allowing a 1/3rd movement in each direction. Similarly for angles, the 1/3rd overlap is the mean between complete overlap and no overlap. Thus, the right angle of the square shown in Figure 1.5 is divided into thirds where the initial angular coverage is 30 degrees. This allows 30-degree relative rotation in both directions (counterclockwise and clockwise) while still maintaining the line-of-sight coverage.

Figure 1.5: Part a) shows the minimum area coverage with the smallest angle; part b) shows the maximum area coverage with the largest angle; part c) shows the initial angular position.

1.4.7.3 SSS Compliant Portion

The original SSS implements eight identical compliant mechanisms (shown in Figure 1.6) to achieve its relative linear displacement between nodes. The pin on the left follows the path of the solid black line allowing a straight-line deflection with respect to the unit cell’s nodes. It can be seen that the rigid area-covering portion constitutes two-thirds of the entire length of the link; a one-third movement is possible in compression and expansion before gaps form. In addition, the angle at the node is two-thirds of a right angle. This allows a one-third movement in either rotational direction.
A total of eight of these compliant links overlap and form the unit cell shown in Figure 1.7.

Figure 1.7a) shows the unit cell in its initial position.

Figure 1.7b) shows the unit cell shearing. Figure 1.7c) shows the unit cell in its expanded position. Figure 1.7d) shows the unit cell in its compressed position. These motions maintain line-of-sight surface area coverage throughout their entire motions.

Figure 1.6: One of eight identical compliant links that makes a square unit cell [1].

Figure 1.7: Square SSS polypropylene prototype where part a) is unstressed, part b) is sheared, part c) is expanded, and part d) is compressed.
1.5 Thesis Overview

This section outlines the design process used for the bistable SSS. In chapter 2, a bistable SSS link (BSSSL) was designed with a compliant portion which allows motion, and a shield portion which provides for surface integrity similar to the SSS link. The design presented in this chapter demonstrated the feasibility of the BSSSL.

In chapter 3, a BSSSL was designed to allow two links to be coupled. The design objective was to ensure that the links undergo horizontal motion with respect to each other as required for each side of the kinematic model, i.e. each pair of overlapping links maintain straight-line deformation with respect to nodes, as shown in Figure 1.8 with links 1 and 2 as the coupled overlapping layers.

![Figure 1.8: One side of kinematic structure of SSS.](image)

In chapter 4, eight BSSSLs, designed in chapter 3, were put together to form the entire MSSS unit cell. Polypropylene prototypes were fabricated to test the behavior of the unit cell. Node-link interferences that became apparent from the prototypes were analyzed and resolved. New prototypes were fabricated, showing functional MSSS unit cells.

In chapter 5, a BSSSL was designed so that there were no protrusions that could interfere with other tiled unit cells. A tiling method of attaching unit cells was achieved and shown using prototypes.
In chapter 6, an additional BSSSL and tiling method was designed which permitted rods or electrical cables to pass through the center of a MSSS unit cell without interfering the necessary motions for multistability. In chapter 7, future work and applications are elaborated. In chapter 8, thesis conclusion summarizes results.
CHAPTER 2: DETERMINING BISTABLE BEHAVIOR IN A SINGLE SSS LINK

This chapter describes the first proof-of-concept design for a BSSSL; the design of this link closely follows the Bistable Light Switch shown in Figure 2.1, which was the basis for the addition of bistability to the SSS link. Subsequent chapters describe design iterations that allowed for improved performance as BSSSLs were joined together to form the sides of the unit cells, complete unit cells, and arrays of unit cells.

2.1 Bistable Four-Bar Analysis

Howell outlines numerous bistable mechanisms [2]. One of these, which was incorporated into the SSS link, is the compliant bistable switch [2]. The bistable compliant mechanism’s two stable positions are shown in Figure 2.1a) and Figure 2.1b). Its pseudo-rigid-body model has two stable positions shown in Figure 2.1c) and Figure 2.1d) and can be categorized as a non-Grashof four-bar mechanism. The small-length flexural pivots shown as ‘Living hinges’ (joints A, B, and C), can be modeled as a characteristic pivot, or revolute joint at the center of the short-length flexural pivot, i.e. these joints do not contribute significant torsional stiffness to the mechanism [2]. Joint D contains a stiff joint, shown with a torsional spring on the pseudo-rigid-body model; the combination of the types of joints and links are what gives the four-bar its bistable behavior.

According to Theorem 1 in Requirements for Bistable Behavior [2] from section 1.4.5.4, “A Compliant mechanism whose pseudo-rigid-body model behaves like a non-
Grashof four-link mechanism with a torsional spring at any one joint will be bistable if and only if the spring’s undeflected state does not correspond to a mechanism position in which the two links opposite the spring are collinear” [14]. Thus, because joint D contains a torsional spring, the two opposite links are links 2 and 3. It can be seen that these links are not collinear in either of their stable positions. To explain this further, consider the triangle formed by links 1, 2 and 3 as shown in Figure 2.2. Figure 2.2a) shows the triangle in its first stable equilibrium position, with the spring unstretched, and links 2 and 3 separated by an angle $\alpha$. Figure 2.2b) shows the system as $\alpha$ increases, forcing link 1 (the spring) to expand allowing links 2 and 3 to pass through a collinear stage. Once links 2 and 3 pass through this collinear stage, the potential energy stored in the spring forces links 2 and 3 to a second stable position with an angle of $-\alpha$ between

Figure 2.1: Bistable light switch [2] showing first stable position in part a), second stable position in part b), first stable kinematic structure in part c) and second stable kinematic structure in part d).
links, shown in Figure 2.2c); the spring is back to its unstretched, zero energy state in this position, however, links 2 and 3 are inverted. This is similar to the behavior of the bistable light switch. Link 4 (Figure 2.1) is placed with a torsional spring on joint D to oppose the change in length of hypotenuse connecting links 2 and 3, just like the linear spring did in Figure 2.2. This gives the mechanism two stable configurations symmetric about its unstable equilibrium position, or hypotenuse. The hypotenuse, or linear spring, in this case, represents the collinear stage of links 2 and 3. If links 2 and 3 were initially collinear, the spring would only contain one position in which it was undeflected, and therefore, would only contain one stable equilibrium position.

![Figure 2.2: Representation of bistable behavior in a four-bar where part a) is first stable position, part b) is unstable equilibrium position, and part c) is second stable position.](image)

In addition, the light switch design allows for a large displacement of the light switch handle (link 3) which can be re-purposed as a straight-line movement of the SSS pin, in accordance with the objective of straight-line movement between nodes, i.e. the
bistable link must have straight-line deflection in order to keep the linear edges on the final unit cell.

2.2 BSSSL Design

Once a bistable design was determined from the light switch, it was then integrated into the compliant portion of a single SSS link. Pin distances, shield design, and range of motion on the SSS were unchanged. The bistable light switch was simply superimposed onto the shield portion of the SSS. However, the bistable light-switch was rotated 90 degrees counter-clockwise before the integration, in order to allow the shield portion of the SSS to align with link 1 of the light-switch, or its ground link. Figure 2.3 shows a solid-model of the integration of the bistable light-switch into the SSS link.

Having rotated the bistable light-switch counter-clockwise, link 1 of the light switch became the shield portion of the SSS. Links 2, 3, and 4 of the light switch became the extruded segments that connect to the movable pin (bottom left node) using the connector link. In comparing the bistable light-switch shown in Figure 2.1 and the integrated design in Figure 2.3, it can be seen that the connector link was moved to link 4 in the integrated design, as compared to where the original switch handle was attached to link 3 in the bistable light-switch. This was done to allow horizontal displacement of the movable pin. The initial bistable light-switch handle tip has a horizontal displacement. However, because it was rotated 90 degrees counter-clockwise, it would then have a vertical displacement when attached to the shield portion of the SSS. Therefore, the connector link was moved to link 4 to allow a near horizontal movement of the movable pin. No exact method was followed in this modification; numerous iterative ‘trial and error’ steps were performed to find a desired behavior. Each iterative step was assessed with
geometric constraint programming [17] until a design was found with feasible bistable behavior.

![Diagram of BSSS design](image)

Figure 2.3: BSSSL design.

2.2.1 Motion Prediction

Motion prediction of this bistable mechanism was done using geometric constraint programming [17] and algebraic position analysis [18]. The first step was to draw the initial position of the kinematic skeleton of the compliant portion of the BSSS in its first stable position, shown in Figure 2.3 as links 1, 2, 3 and 4 (solid black lines). Relative to the shield portion of the BSSS link, joints D and B are virtually immobile. Therefore, a line of symmetry can be drawn between these two joints, and is shown with a black dotted line. This dotted line is analogous to the hypotenuse spring in Figure 2.2. It represents the mechanisms unstable equilibrium position (when links 3 and 4 are collinear passing through to the second stable position). In this unstable position, the
links store high compressive forces within their members and release this energy as the mechanism moves through to its second stable equilibrium position. To achieve this behavior, link 2 must be stiff enough to oppose the separation of joints B and D so that the potential energy due to compression has a more significant effect than the potential energy due to the bending of the flexural pivots, i.e. the strain energy must be a maximum near the symmetry line between joints B and D. The second stable position was then found by mirroring the initial kinematic skeleton across this line of symmetry and is shown with the solid red line in Figure 2.3.

As links 3 and 4 pass through this unstable position, the movable pin follows a small curved path from its initial position from the left to the final position to the right. The path of the movable pin is shown in Figure 2.3 using motion prediction [17] and was also found using algebraic position analysis [18] in Figure 2.4. The origin of Figure 2.4 is shown in Figure 2.3 at the movable pin (bottom left node). The x-axis in Figure 2.4 represents the desired line of action of the BSSSL. It can be seen that the movable pin

![Path of movable pin](image)

*Figure 2.4: Path of movable pin found using algebraic position analysis.*
does not return back to the x-axis (line of action) in its final position. It reaches a maximum displacement of -0.31 inches in the y-direction at 1.2 inches of x-deflection, and -0.3 inches of y-deflection at its final position of 1.53 inches of x-deflection. Figure 2.5 shows the final compressed position using finite element analysis (FEA). It is also shown here that the final position of the movable pin does not align with the line of action between nodes in its compressed position.

Also, it should also be noted that the curvature of link 4 of this bistable design was such that when compressed, it would not interfere with links 2 and 3. This can be seen from the final FEA position shown in Figure 2.5, where link 4 curves around links 2 and 3 in the compressed position.

![Figure 2.5: FEA model of compressed position.](image)

### 2.3 FEA vs. Pseudo-Rigid-Body Model Analysis

This design demonstrates bistability, allowing the SSS to maintain two stable configurations. This is proven by Figure 2.6, which shows the force vs. deflection results from the FEA model and pseudo-rigid-body model (PRBM) results. The force in this
model represents the horizontal reaction force exerted by the movable pin vs. the corresponding x-deflection of the movable pin. Each location the graph crosses the x-axis (zero force) indicates an equilibrium position (whether stable or unstable). However, the mechanism never fully reaches its second stable equilibrium position because of a hard-stop, i.e. the mechanism was physically stopped by an external force (shield) before reaching its second stable state; nevertheless, this is still classified as a stable equilibrium position. The hard-stop occurs at 1.4 inches of deflection and is located at the right end of the graph in Figure 2.6. Therefore, the stable equilibrium positions occur at 0 and 1.4 inches of deflection. The unstable equilibrium position occurs between the two stable equilibrium positions, and at the intersection of the x-axis, i.e. at 0.85 inches of deflection. At this position, high compressive forces are internally stored and strain energy is a maximum.

![Force vs. Deflection](image)

Figure 2.6: Force vs. deflection of design one.
Another way to view the bistable behavior is using the potential energy vs. deflection graph which is shown in Figure 2.7. The graph shows the potential energy vs. deflection results from FEA and PRBM approach analyses. It can be seen, as stated in Section 1.4.5.1, that the unstable equilibrium positions occur where the first derivative of the potential energy curve is zero, and its second derivative is positive. This can be seen to occur at 0, and 1.4 inches of deflection, which is consistent with the force-deflection results in Figure 2.6. Also, the unstable equilibrium position occurs where the first derivative of the potential energy curve is zero and its second derivative is negative. This occurs at 0.85 inches of deflection and is again consistent with the results found from the force vs. deflection results.

![Potential Energy Graph](image)

Figure 2.7: Potential energy vs. deflection of design one.

### 2.4 Discussion of Initial Design

The bistable shape-shifting-surface design in this chapter successfully demonstrated bistability. This is proven by the geometric representation of the stable
equilibrium positions, and by the FEA and PRBM results of force vs. deflection and potential energy curve. However, it can be seen that the movable pin does not follow a straight path along the line of action, nor does its second stable position occur on this line of action. There is significant rotation throughout the movable pin’s translation. This is seen in the graphical motion prediction (Figure 2.3), the algebraic position analysis (Figure 2.4) and the FEA model (Figure 2.5). The pin drops significantly below the line of action. This is due to the symmetry about joint D. Because the connector link is attached to link 4, which in turn pivots about joint D, the movable pin will inherently rotate about joint D. Furthermore, because joint D is not symmetric about the intended starting and ending position of the movable pin, it will not begin and end along the horizontal line of action. This can be foreseen to cause problems within the final unit cell of the MSSS, as each side will have two small intersecting angles, breaking the integrity of the square unit cell.

Nevertheless, this design helped demonstrate the feasibility of the BSSSL as well as provide possible future work. Future work based on this chapter can include bistable designs in which translation and rotation is desired within the MSSS unit cell. In the next chapter, a new design is produced that accommodates the coupling of two BSSSLs together.
CHAPTER 3: DESIGN FOR COUPLED BSSSLs

In this chapter, knowledge gained from design experience described in chapter 2 is used to re-design a BSSSL that can be coupled, to produce one side of the kinematic structure of a unit cell. Three specific design issues were addressed in order to achieve better BSSSL performance.

3.1 Performance Improvements for the BSSSL

First, the trade-off between the maximum shield coverage and maximum displacement of the BSSSL was formalized in order to determine the distance between the two bistable positions. Second, this distance was modified to account for the radius of the pins connecting the BSSSLs. Lastly, the four-bar joint locations were defined in order to achieve the proper kinematics resulting in the bistable positions located at the correct distance apart.

3.1.1 Defining Terminology

To facilitate discussion, considering the BSSSL, the left-hand movable pin that connects to the connector link, is termed the flexure pin, while the right-hand (fixed) pin is termed the shield pin. The flexural pin is the movable pin which defines the mechanisms position (compressed or expanded). In addition, the circular cut-out at the lower left corner of the shield is called the pin slot, as labeled in Figure 3.1a). This is the slot that the flexure pin tucks within to allow smooth, non-extruding edges in the four-bar’s compressed position, as seen in Figure 3.1b). Therefore, the size of the pin slot
matches the size of the flexure pin. In addition, the pin slot is located along the horizontal line of action between nodes. This constrains the movable pin’s second stable position along this line of action.

3.1.2 Trade-Off: Shield Coverage vs. Displacement

The SSS is composed of two portions, a shield portion, and a compliant (flexure) portion. The larger the shield, the more shield integrity, i.e. coverage without gaps, while the larger the compliant portion, the larger the displacement the link permits. The goal was to obtain the maximum displacement of the flexural pin while still maintaining shield integrity. Therefore, the deformation of the BSSSL cannot exceed the horizontal length (width) of the shield; this prevents the compliant portion from displacing past its adjacent layer’s shield and opening gaps or extrusions in either direction of movement (expansion or contraction). This can be seen in Figure 3.1, which shows two shields, coupled as one of the four sides of the unit cell. Figure 3.1a) shows the two layers at its maximum expanded position before gaps open at the ‘edge of shield coverage’ from the pin slot.

![Figure 3.1](image)

Figure 3.1: a) shows two overlapping layers in their fully expanded position, while b) shows the layers in their fully compressed position.

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Figure 3.1b) shows how the two layers align when compressed. In both cases, the length from the ‘edge of shield coverage’ to the ‘flexure pin’ cannot exceed the width of the shield, and is denoted by \( l \). Therefore, link displacement and shield width are inversely proportional. In order to maximize both displacement and shield width, a simple relation was chosen; the lengths of each section (compliant portion and shield) were half the width of the unit cell. Therefore, considering the line of action between pins (or nodes), the length from the shield pin to the ‘edge of shield coverage’ is equal the length of the flexural pin to the ‘edge of the shield coverage’. This is shown in Figure 3.1 and Figure 3.2 where each length is equal to \( l \).

![Figure 3.2: BSSSL length consideration.](image-url)
3.1.3 Accommodating the Flexure Pin

The diameter of the pins was selected as 0.4 inches. This is slightly smaller than the size of the SSS pins but permitted more space for the compliant portion design. However, the one-half rule relation was developed assuming the displacement of each pin behaved as a point; thus pin diameter was not accounted for in this relation. Consequently then, the displacement of the flexural pin is the length of the shield, minus the radius of the pin, or:

\[ D_f = l_s - r_{pin} \]  \hspace{1cm} (19)

Therefore, having a diameter equal to 0.4 inches subtracts 0.2 inches (radius of the pin) from the displacement of the flexural pin.

Therefore, because \( l \) is equal in both the compliant portion and shield portion, the value of it is a design choice. This can be changed depending on the application and how large a unit cell is desired. For this design, the shield length was given a value of 2.3 inches, giving an overall length of 4.6 inches to the BSSSL from pin to pin; this was slightly larger than the length of the original SSS. The larger length allowed the pin size to not significantly affect the shield coverage vs. displacement relation, because the pin diameter was not accounted for in the one-half rule.

Once the flexural pin and shield pin were defined, rigid-body replacement concepts were used to design the kinematics of the compliant portion [2]. This entailed determining proper joint locations for a four-bar mechanism like the one created in chapter 2. The bistable compliant mechanism was to remain within the given design area shown in Figure 3.2, which is labeled “Compliant Portion to be Designed”.

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3.1.4 Designing for a Specific Displacement

For ease of reference, let joints (1,2), (2,3), (3,4) and (1,4) of a typical four-bar mechanism be designated as joints A, B, C, and D respectively, or as shown Figure 3.3. Before the joints were placed, a general concept of the bistable four-bar was conceived. The structure of the four-bar design shown in Figure 3.3 was used as the basis for the design presented in this chapter. According to Theorem 1 in section 1.4.5.4, the joint opposite of the shortest link must contain a strong torsional spring, thus, joint D was pre-planned to contain a strong torsional spring. As a result, joint C can be thought to be virtually immobile, as joint D deflects only slightly and resists the movement of joint C. The following sections constrain the ground points and joint C to achieve the proper kinematics of the bistable four-bar.

3.1.4.1 Ground Joints A and D

First, the two ground points of the four-bar (Joints A and D) were constrained. This ensured that the kinematic linkages of the four-bar remained within the given design space shown in Figure 3.2 as “Compliant Portion to be Designed”. Therefore, joint A was placed near the bottom left corner of the shield while joint D was placed near the top left corner of the shield; see Figure 3.4. This bounded the design area within the height of the shield so that linkages did not extend past the top or bottom of the shield portion, i.e. the
shield portion design was left unchanged from the original SSS and it provided size constraints for the design area.

3.1.4.2 Joint C Location

The design presented in chapter 2 extended the connector link from link 4, which rotated about the stationary joint D. This produced a circular curvature that the flexure pin followed. In addition, because joint D was not symmetric about the intended initial and final positions (along the horizontal line of action), the two stable positions were offset from the horizontal line. Therefore, two design decisions in this chapter were made to adjust for this offset. First, the design decision was made to use a coupler link (attached to link 3). A coupler link is capable of having an instant center that produces a large radius of curvature that rotates about joint C. This large radius of curvature enables a near linear displacement of the flexure pin. The second design decision was to place

Figure 3.4: Positioning of joints A, D and C.
joint C at the perpendicular bisector of the intended initial and final positions of the flexure pin shown in Figure 3.4. Based on the structure of this bistable four-bar, joint D must contain a strong torsional spring in order to make joint C virtually immobile. As a result, link 3, or the coupler link, moves symmetrically about this virtually immobile joint C. Having this joint symmetric about the intended initial and final positions allows the flexure pin to have near straight line deflection with its two stable positions being on the horizontal line of action.

The procedure for specifying joint B is described in the next section. Its location affects the displacement, actuation force, and stress experienced by the mechanism.

3.2 Designing Kinematic Links

With joint A, C and D defined, a graphical synthesis method was used to define the initial position of joint B, and hence, the kinematics of the four-bar mechanism. For the first stage of the graphical synthesis, links were arbitrarily placed, giving an initial guess, before optimization was implemented to find a more specific solution.

3.2.1 Generic Placement of Joint B to be Optimized

The first step in the graphical synthesis was to define an estimation of the location of joint B using the surrounding joints (A and C). This was done by connecting joints A and C. First, they were connected by a straight dotted line; defining the mechanism’s unstable equilibrium position (when joints A, B and C are collinear). Then, joints A and C were connected again, this time with two, arbitrarily placed, non-collinear links, where the intersection of these two non-collinear links defined the first stable position of joint B. The lengths and angles of these links are later optimized to give a more specific solution.
The second stable position is then defined by mirroring the two non-collinear links previously drawn, across the unstable equilibrium line connecting joints A and C. The intersection of these mirrored, non-collinear lines, defines the second stable position of joint B, which will also be further refined when the lengths and angles are optimized. Figure 3.5 shows these steps, where links 2 and 3 are the two initial non-collinear links of the four bar mechanism (solid black lines); links 2’ and 3’ (solid red lines) are the links final (mirrored) position, i.e., solid black lines are the mechanisms first stable expanded position, while solid red lines are the mechanisms second stable compressed position.

The next step was to connect the coupler link from each stable position, to its counter-part flexural pin position, i.e. the black coupler link connected to the initial flexure pin position, while the red coupler link connected to the final flexure pin position. Joint C

![Figure 3.5: Determining stable positions.](image-url)
was then connected to joint D (ground point), giving two symmetric four-bars, giving the position of its two stable positions shown in Figure 3.6.

![Diagram](image)

**Figure 3.6: Two stable positions of kinematic structure.**

3.2.2 Graphical Optimization

Parametric constraints were applied in the previous section which ensured that initial and final position linkages remained equal, i.e. any manipulation of a link, will automatically update the four-bar’s second stable position location. Therefore, iterative manipulation was done to allow both stable positions to not only fit within the given design area, but to also produce enough strain energy at the mechanisms unstable equilibrium position to overcome any flexural strain produced by the other joints. This was done by prototype iterations where the force behavior of the bistability was quickly
analyzed. Thus, the angles between links 3 and 3’, and links 2 and 2’ were optimized to allow adequate force characteristics. This angle is one of the factors that determine the force of the mechanism’s bistability. The greater the initial angle between links 3 and 3’ and links 2 and 2’, the farther joint C will displace to allow for links 2 and 3 to pass through their collinear stage (unstable equilibrium position). The farther joint C has to displace, the farther link 4 will deflect. Because link 4 is modeled with a linear torsional spring (shown in Figure 3.7), it will resist this movement proportionally to its displacement, and force joint C back to its initial position, giving two preferred stable positions for links 2 and 3, i.e. the greater the initial angle between links 2 and 2’ and 3 and 3’, the greater the bistable force. This angle can change depending on the desired bistable behavior, but is not the only factor that affects the force of bistability. The length

![Diagram of four-bar model](image)

Figure 3.7: Pseudo-Rigid-Body model of four-bar and its two stable positions.
of link 4 and the torsional spring constant both have significant effects on the bistable behavior as they determine the force exerted on joint C.

This motion prediction (Figure 3.6) shows the initial and final stable positions aligned with the line of action and was the skeletal model used to make the pseudo-rigid-body model of the four-bar mechanism shown in Figure 3.7.

3.2.3 Geometric Motion Prediction

The path of the flexural pin follows a shallow parabolic trend as it moves from the first stable equilibrium position to its second and was founding using algebraic position analysis [18]. This shows that the coupler link’s instant center allowed for a large radius of curvature with a near straight line displacement. The flexural pin’s path is shown in Figure 3.8. In this graph, the x-axis represents the line of action that the flexural pin is forced to follow. The final compressed position of the flexural pin is located at 2.05 inches along the x-axis, and zero in the y-direction (on the line of action). However, the flexure pin does not remain on the x-axis throughout its entire path. It reaches a

![Path of Movable Pin](image)

Figure 3.8: Path of flexure pin of kinematic four-bar.
maximum y-value of 0.06 inches at 1.2 inches of x-deflection. However, compared to the x-displacement traveled, it is negligibly small and said to follow a near linear path.

Throughout the flexure pin’s movement, link 4 rotates slightly and was found using the same kinematic equations used to find the flexural pin path [18]. Link 4 rotates through a total of 14.7 degrees from its initial preferred position to its unstable equilibrium position. Knowing the spring constant and the rotational angle provides the ability to calculate stresses within the compliant member. Figure 3.9 shows a graph of the rotational angle of link 4 where the angular deflection of link 4 is plotted with respect to the x-displacement of the flexural pin. Link 4 nearly returns back to its initial angle once links 2 and 3 pass through their co-linear stage, or their unstable equilibrium position; however, there is a 0.1 degree offset from the initial angle of link 4 to its final angle.

![Theta 4 vs. x-disp.](image)

Figure 3.9: Theta 4 vs. movable pin x-displacement.

### 3.3 Kinematics to Compliance

The kinematic model of the previous four-bar mechanism was converted to a compliant model using pseudo-rigid-body replacement [2]. Each joint in the kinematic
model was replaced with a short-length flexural pivot, excluding joint D. Joint D is crucial in making the mechanism bistable in that it requires a stiff torsional spring in order to produce compressive forces upon links 2 and 3. Therefore, link 4 was modeled as a tapered fixed-pinned cantilever beam. This allowed for high stiffness to oppose the separation of joints A and C, while allowing the torsional spring constant of joint D to be easily manipulated by changing only the thickness of its base. However, there is no supporting pseudo-rigid-body model to approximate cantilevered beams. Therefore, a simplistic pseudo-rigid-body model of a cantilever beam was used instead, allowing for an efficient method to predict motions. Consequently, large errors in stress approximation became evident. Because of these errors and the complexity of links and joints, FEA will be the primary source for stress analysis. Small deflection equations could not be used in link 4 because it experiences large deflection. Figure 3.10 illustrates the final

![Figure 3.10: Kinematics to compliance.](image-url)
transformation of the pseudo-rigid-body model to a compliant mechanism using pseudo-rigid-body replacement techniques [2].

The short-length flexural pivot locations were placed directly centered over the pseudo-rigid joints, or kinematic joints. Small-length flexural pivots, in general, can be modeled with a kinematic joint at its center of its flexural section. However, because link 4 is a tapered cantilever beam, the exact location of its pseudo-rigid joint is unknown. There is, however, data supporting cantilever beam conversion, and therefore were assumed using the pseudo-rigid-body replacement technique [2]; the characteristic pivot was placed at approximately 85% of the link length, to the joint, i.e. 15% of the link length away from the base of the cantilever. However, the exact location of this joint is relatively unimportant as link 4 primarily remains within the two stable positions and its path between is not crucial, i.e. regardless of the accuracy in the model representing link 4, joint C is forced back to its original location once links 2 and 3 pass through their collinear stage, so the intermediate path it takes is irrelevant.

The following section uses FEA and the pseudo-rigid-body model estimation to ensure the kinematics was preserved in the conversion between the kinematic model to a compliant mechanism.

3.4 FEA vs. Pseudo-Rigid-Body Model Approach

Figure 3.11 shows the final compressed position of the design presented in this chapter. It can be seen that the flexure pin ends its path within the pin slot, beginning and ending its path on the line of action between nodes. In addition, the curvature of link 3 is such that prevents interferences with joint A and link 2 when in its compressed position.
The two stable positions shown in this figure are consistent with the geometric motion rendering shown from previous figures within this chapter. Figure 3.12 shows the comparative results of the force vs. deflection curve using FEA and PRBM. It can be seen that there is a slight difference in scale between the two methods of approximation.

Figure 3.11: FEA deflection results.

Figure 3.12: Force vs. deflection of design two.
This is the result of modeling link 4 as a cantilevered beam. However, because the objective of this paper is to design bistability into the SSS link, and both methods show bistable characteristics, they both can be used to show bistability within this design.

The unstable equilibrium position occurs around 1.2 inches of deflection where the graph crosses the x-axis between peaks. This is labeled as the unstable equilibrium position because it occurs at a zero force intersection in between two maxima. The other two stable equilibrium positions lie on the x-axis at the outer ends of the two local maxima. The potential energy and stress results are shown in chapter 4 as the design presented in this chapter is only slightly modified in chapter 4, i.e. the resulting design in this chapter is not complete in that it has not been considered tiled as a unit cell. Nevertheless, the FEA and PRBM results in this chapter show bistability within the design, ensuring the kinematics are correct.

3.5 Discussion of BSSSL Design

The BSSSL designed in this chapter successfully demonstrated bistability. This is proven by the geometric constraint programming, FEA results, and the force vs. deflection graph using FEA and the PRBM approach. The coupled BSSSLs model a kinematic slider, which represents one side of a unit cell. They properly ensure line-of-sight shield coverage while producing near linear displacement with two stable positions. However, consideration of the unit cell as a whole has not been considered yet, and will be analyzed in the next chapters, i.e. eight BSSSLs will be assembled to ensure proper behavior as a unit cell.

Future work based on this chapter may involve designs for triangular, hexagonal and other tiling patterns, which will have different pin interferences and geometries than
the one discussed in chapter 4. Other future work may consider the use of the coupled BSSSLs as a bistable kinematic slider, as modeled in Figure 1.8.
CHAPTER 4: DESIGN INTEGRATION FOR MSSS UNIT CELL WITH PROTOTYPES

Chapter 3 described a design that demonstrated bistability within the BSSSL. It maximized the link’s displacement, as well as ensured no gaps or extrusions formed within the shield coverage when coupled together. However, upon assembly of a unit cell prototype, it became evident that there was pin interference between adjacent layers. This chapter addresses this issue, and describes additional prototypes and stress analysis that improve on the design produced in chapter 3.

4.1 Pin Interference

The BSSSL designed in chapter 3 is 4.5 inches from pin to pin in its fully expanded position. It then shrinks down to 2.5 inches, from pin to pin, in its compressed position. Upon assembly of a unit cell in a CAD program, it became apparent that there existed significant interferences with adjacent pins. The interference came from the height of the shield portion being 2.9 inches. Because the pin distance compresses down to 2.5 inches, a modification in the shield design was essential. Figure 4.1 shows the compressed size of the unit cell on top of the BSSSL. The top left of the shield portion contains an interfering pin from the adjacent layers. Therefore, the shield portion of the BSSSL was manipulated slightly to account for this pin interference (specifically joints B, C and D).

The structure of the four-bar was changed slightly to accommodate for this shift in position of joint B, C and D. Using previously applied geometric constraints in a
parametric CAD program [17], pins and linkages were re-positioned so that the
kinematics were unchanged. As a result, joint A remained unmoved, while joints B, C
and D shifted downward. This provided a pocket for the adjacent pins to fall within the
shield portion.

Figure 4.2 shows the adjusted SSS link with the shield portion modified to avoid
interferences when the unit cell was fully compressed. In this figure, \(l'\) is the length of the
compressed unit cell and \(L\) is the length of the expanded unit cell from pin to pin. Recall
from Figure 3.2 that \(l\) was the length from either major pin to the edge of shield coverage.
A new equation is formed relating the displacement vs. shield coverage with respect to \(L\):

\[
L = \text{Displacement} + l'
\]  
(20)

where

\[
l' = l + r_{\text{pin}}
\]  
(21)

where \(r_{\text{pin}}\) is the radius of the pin.
4.2 FEA vs. Virtual Work Predictions

This section analyses results found using FEA and PRBM analysis to show the BSSSL’s behavior after the slight linkage modification. Figure 4.3 shows the final position of the BSSSL found using FEA. It can be seen that the flexure pin ends its path

Figure 4.2: BSSSL with tiling consideration.

Figure 4.3: FEA displacement model of BSSSL.
on the line of action within the pin-cutout slot. Figure 4.4 shows comparative results of force vs. deflection of the BSSSL using FEA and PRMB. These results are similar to the results found in chapter 3; the unstable equilibrium position occurs around 1.0 inch and occurs between the two maxima. The two stable equilibrium positions are shown at the graph’s outer bounds where it touches the x-axis. This zero-force indicates a stable position. Because the stable positions and the unstable position are separated by a peak in force, it was concluded that this design exhibits bistability. To provide further details, Figure 4.5 shows the graph of the potential energy of the four-bar with respect to the x-deflection of the flexure pin. It can be seen that this potential energy curve exhibits two stable equilibrium positions where the first derivative is zero, and its second derivative is positive. These occur at zero inches of deflection and around 2.1 inches of deflection. These two positions are separated by the unstable equilibrium position in which the first derivative of the curve is zero, and the second derivative is negative (around 1 inch of
deflection). The two methods (RPBM and FEA) give results with differing scale, however, both show strong bistable characteristics and thus can be used to verify bistability within this design. However, it can be foreseen that these errors of scale will produce significant errors when calculating stress. Because of the complicated joints and linkages used, pseudo-rigid-body models with hand calculations will not be used to calculate stress. Instead, FEA will be used.

![Potential Energy Graph](image)

Figure 4.5: Potential energy of BSSSL in design three.

### 4.3 Prototype

Figure 4.6, a prototype of this design in its expanded, sheared, semi-compressed and compressed positions, shows furthermore, the concept of bistability within this MSSS design. It was seen from the prototype that the MSSS unit cell maintained the five degrees of freedom of the original SSS. Each side can collapse independently, giving four modes of deformation, as well as an additional mode of shear. Figure 4.6a) shows the unit cell in its first stable un-deflected equilibrium position as an expanded SSS. Figure 4.6b) shows the unit cell in its sheared mode. Figure 4.6c) shows the unit cell compressed in
one direction, forming a rectangle. In this figure, two sides are compressed. It should be noted that another position is possible with only one of the four sides of the unit cell compressed, forming a trapezoid. Figure 4.6d) shows the complete compressed position of the multistable unit cell.

It can be seen that in all of the deformed positions (b, c and d), that the unit cell contains linkage protrusion outside the shield portions of the unit cell. If this unit cell in
Figure 4.6 was tiled to another unit cell, the linkage protrusion would interfere with adjacent unit cells and angular shapes would not be possible.

4.4 Stress

Stress was tested within compliant links and joints. Because link 4 of this design is complex and cannot be accurately modeled with the pseudo-rigid-body model, hand calculations cannot be used as they will have significant errors from the inaccurate representation of link 4. Small deflection methods also cannot be used as link 4 has significant deflection. Motion prediction was not affected by this inaccuracy because only the starting and ending points of the tip of link 4 was crucial, not its path. Therefore, FEA was extensively used to analyze stress.

Stress was found to be a maximum when link 4 was at its maximum deflection. It can be seen from Figure 3.9 that when the flexure pin is displaced 0.96 inches, link 4 is displaced to its maximum amount. Therefore, FEA was analyzed in this position. Figure 4.7 shows the von Mises stress distribution of the four-bar mechanism. Link 4 has a stress of 6,000 psi on top of the beam and 6,400 psi at its bottom. Joint 1 experiences 10,000 psi, joint 2 experiences 12,000 psi, and joint 3 experiences 12,600 psi of stress. The reason the small-length flexural pivots contain significant stress is due to the compressive force being exerted by link 4, causing axial stress within the small-length flexural segments.

The yield strength of polypropylene is around 5000 psi. This means that the stress experienced by the BSSSL is over twice its yield strength. However, because polypropylene has a high percent elongation, it is capable of handling higher stresses.
without experiencing significant or noticeable permanent deformation. Future work will entail minimizing stresses within these given joints.

![FEA model of von Mises stress distribution](image)

Figure 4.7: FEA model of von Mises stress distribution.

4.5 Discussion of MSSS

This chapter modified the design presented from chapter 3 to allow for effective assembly of a unit cell. Pin interferences were resolved by manipulating pin and link locations while the kinematics were preserved. This was proven using motion prediction with FEA and the PRBM approach. Prototypes were developed to further prove multistability within the design. Prototypes also ensured no pin interferences throughout all modes of deformation.

However, linkage protrusion became evident from prototype assemblies. This introduces problems when tiling multiple unit cells together, as linkage protrusion will
interfere with other unit cell’s linkage protrusion. The next chapter will resolve the linkage protrusion of the MSSS unit cell.

Future work on the design presented in this chapter could include applications in which one unit cell is desired, or when linkage protrusion does not pose to be an issue. In addition, methods to achieve lower stress all joints of the four-bar presented in this chapter could be investigated. Link 4 must maintain a strong resistive force for bistability, while still allowing moderate deflection in order to allow links 2 and 3 to pass through their co-linear stage. Manipulation of material properties and geometry may help improve this stress while maintaining these constraints.
CHAPTER 5: BSSSL DESIGN FOR TILED UNIT CELLS

This chapter focuses on creating a new design for the MSSS that does not contain linkage protrusions. In order to do so, the compliant portion of the BSSSL must lie within the shield portion when in its compressed position, i.e. the compliant portion must serve as shield coverage when compressed.

5.1 Graphical Design

A four-bar mechanism was chosen again for this new design due to its well-known characteristics and ease of design. All previously applied constraints were applied from the design presented in chapter 3 and 4, i.e. shield coverage vs. pin displacement relationship, flexure pin accommodation, as well as the structure of the bistable four-bar. However, to ensure that the compressed position of the four-bar remained entirely within the shield portion, the BSSSL was initially designed in its compressed configuration; this gave more control in designing the final shape to ensure that the bistable four-bar remained within the shield portion when compressed.

5.1.1 Graphical Iterations for Joint Locations

Few design decisions were made for the generation of this new four-bar. Many decisions were experimental, however, focused on containing the four-bar within the shield portion in its compressed position. Multiple iterative steps were done in order to achieve the final configuration of the four-bar shown in Figure 5.1. The majority of the iterations were not only directed at containing the four-bar within the shield, but also
minimizing the angle of rotation of link 4; this would minimize its stress, as the torsional spring is located on joint D.

Therefore, one method used to minimize the rotation of Link 4 is to maximize the length of the attached link. This gives a larger tip displacement for a small given angular deflection. Thus, joint D was placed near the bottom right of the shield while joint A was placed at the top left of the shield. This oblique configuration of link 4 allowed the longest possible link length of link 4, thus inherently reducing the stress joint D would contain. Joint A was placed near the bottom left portion of the shield. It was placed past the compressed position of the flexure pin so that it would not interfere with the movable pin or the linkages connecting to it. Joint A and C were then connected with two non-collinear links buckling toward the center of the shield portion, allowing joint B to remain deep within the shield. A coupler link was then drawn from link 3 and attached at

Figure 5.1: Containing linkages within shield.
the compressed position of the flexural pin. At this point in the design, all joint locations and link lengths were subjective, and were to be further constrained using geometric constraint programming in the next section [17].

5.1.2 Geometric Motion Design

The geometric constraint programming presented in this chapter slightly differs than the one presented in chapter 3. Because joint C is constrained within the shield portion in the four-bar’s compressed position, it cannot contain the symmetry between stable positions that Joint C had in previous designs. Therefore, graphical synthesis [17] was used to determine the link lengths and joint locations of the four-bar to achieve the two stable positions set in chapter 3 (shield coverage vs. pin displacement relationship) without the need for a symmetric placement of joint C.

The expanded position four-bar was constructed on top of the compressed position drawn from Figure 5.1 with the same ground points (Joints A and D). It was drawn in a similar way, instead, connecting to the expanded pin position, or as shown in Figure 5.2. Only in the compressed position is when the compliant links must serve as the shield portion, so this second configuration of the four bar being past the shield does not pose an issue. Figure 5.2 shows the two stable positions where the compressed position is shown as the solid black lines with links 2, 3, and 4, and the expanded position is shown as the solid red lines with links 2’, 3’, and 4’. Using parametric constraints in CAD [17], the links were then made equal to their counter-parts (2 to 2’ etc.). Iterative manipulation steps were done to simultaneously allow the four-bar’s compressed position to remain within the shield portion, while allowing both stable positions to reach their respective flexure pin (set by shield coverage vs. displacement relationship). Thus, it can be seen...
from Figure 5.2 that link 4 will not return to its original position when shifted from its first stable position to its next, like it did in previous designs. It has an angle of Φ between its first stable position to its second. This change in angle of link 4 is necessary to allow the proper displacement of the flexural pin with the given constraints set forth in chapter 3, along with the constraint set in this chapter of no protrusions.

Figure 5.2: CAD optimization giving two stable positions.

Thus, after numerous manipulation steps with CAD, a final kinematic structure was made giving two stable positions, where the compressed position lies entirely within the shield portion.

5.2 Pseudo-Rigid-Body Model Synthesis

Figure 5.3 shows the kinematic structure of the bistable design. Figure 5.3a shows the models expanded position and Figure 5.3b shows the models compressed position. Φ
is the rotational angle of link 4. It should be noted that fabricating the mechanism in each of its two stable positions will give entirely different results.

5.2.1 Fabrication Procedures

First, consider the four-bar mechanism fabricated in the expanded position, (links 2', 3', and 4' or Figure 5.3a)); this will give bistable behavior. This is due to the behavior of joint D. When the flexural pin is moved from the expanded position to the compressed position, Links 2 and 3 must pass through their collinear stage, pushing joint C, which is resisted by joint D, i.e. there exists compressive forces within links 2 and 3 as they pass

![Figure 5.3: Kinematic structure of four-bar and its two stable positions.](image-url)
through their collinear stage. These compressive forces within links 2 and 3 gives them bistable characteristics by giving two sides in which the links can *buckle*, providing the lowest energy state for joint D.

Next, consider the case when the mechanism is fabricated in the compressed position (links 2, 3 and 4, or Figure 5.3b)); this will not be bistable. When the flexural pin is moved from the compressed position to the expanded position, links 2 and 3 move away from their collinear stage, *pulling* joint C, which is resisted by joint D, i.e., there exists tensile forces within links 2 and 3 as they rotate away from their collinear stage. In order for the mechanism to be bistable, these two links must have compressive forces in order to force them into two non-collinear positions symmetric about their collinear stage. These tensile forces acting on links 2 and 3 effectively straighten them like a taught rope, resulting in one preferred position where joint D has the lowest energy state.

5.2.2 Geometric Motion Prediction

The origins of the next two graphs are represented by the origin shown in Figure 5.3 (at the shield pin). Figure 5.4 shows the kinematic path of the flexural pin [18]. Note that the flexure pin does not follow an exact linear path; it displaces 0.18 inches in the y-direction. However, the final bistable positions are located on the line-of-action and this small y-displacement is negligible. The first stable position (expanded) is shown to occur at -4.34 inches from the origin, while the second stable position (compressed) occurs at -2.48 inches from the origin. This gives an overall deflection of 1.86 inches of the movable pin and a percent reduction in length of 43%.
The rotational angle $\phi$ is shown in Figure 5.5 [18]. Link 4 rotates from the links expanded position where $\phi$ is equal to 144.9°, to the links compressed position where $\phi$ is equal to 136.5°. Link 4 rotates through a total of 9.9°, but has a difference in angle of 8.4° from its initial stable position to its second. The maximum deflection of Link 4 occurs at the mechanisms unstable equilibrium position at -3.08 inches of deflection from
the origin. It can be seen that Link 4 does not return to its initial angular position when the bistable link is compressed; it stops at 136.5° instead of rotating to the initial angle of 144.9°. This is because the four-bar does not return back to a zero-force stable state. A hard-stop inhibits the mechanism from continuing to its second stable equilibrium position. However, as long as the hard-stop is located past a local maximum of energy or force, the mechanism will still exhibit bistability. This is further discussed in section 5.4.

5.3 Pseudo-Rigid-Body Model Replacement

Now that the geometric design is complete and the kinematics is known, the next step taken was to transform the pseudo-rigid-body model into a compliant mechanism. Small-length flexural pivots were used to replace the joints which do not require torsional springs (joints A, B, and C). A novel torsional spring was placed at joint D in order to give the link its bistable behavior. Because this joint lies near the shield pin, it uses the pin hole as a joint; a thin cut was made up to the shield pin hole. The cut stops at approximately 0.1 inches before the hole. If the cut continued into the hole, making a non-continuous hole, the joint would not contain adequate stiffness to achieve its bistability. This, however, creates a novel joint with unknown characteristics.

Figure 5.6 shows the compliant bistable four-bar. The shape of each link was precisely chosen so that when compressed, formed the shape of the SSS shield portion. This can be seen in Figure 5.7, showing an FEA model of the compressed position of the BSSSL. If this design was fabricated in its compressed position, it would behave similar to the original SSS, i.e. it would not be bistable. The major difference would be that the non-bistable version of the design presented in this chapter would only resist movement as it expanded from its compressed to expanded position. The original SSS resists
movement in both expansion and contraction. Nevertheless, if only resistive expansion was desired, this design would offer a solution without linkage protrusion, unlike the original SSS that has linkage protrusion in its compressed position.

5.4 FEA vs. Virtual Work Prediction with Prototype

Figure 5.7 shows the compressed position of the BSSSL using FEA. It can be seen that in the compressed position, the linkages provide shield coverage. However, it
can also be seen that there exists a small slit along the shield portion. Design iterations were performed to optimize the location of this slit over redundant area coverage in the following section.

Figure 5.8 shows the force vs. deflection curve of the BSSSL. It can be seen that the PRBM and FEA results are identical. Because there was no approximate pseudo-rigid-body model for joint D, an arbitrary value for its stiffness was used. This arbitrary value was then optimized to match the FEA results, as was found to be 24.7 in-lbs. of torque.

![Comparative Results of Force vs. Deflection](image)

**Figure 5.8**: Force vs. Deflection of design four.

It can be seen that the force required to displace from its expanded position (zero deflection) to its compressed position (1.75 inch deflection) requires over 0.7 lbs. of force. Once 0.7 lbs. of force is reached and held, the BSSSL will shift into its second stable position at 1.75 inches, giving approximately 0.12 lbs. of reaction force in the opposite direction of the initial input force. The force vs. deflection graph never returns to
the zero x-axis because of its hard-stop which halts the mechanism before reaching its second stable equilibrium position. In addition, the hard-stop occurs just at the peak of the force. Because of this, the force required to move the BSSSL from its second stable equilibrium position to its first stable equilibrium position is merely the static force keeping the mechanism at its hard-stop. Therefore, only 0.12 lbs. of force will shift the BSSSL from its second stable equilibrium position (compressed) to its first stable equilibrium position (expanded), giving 0.7 lbs. of a reaction force in the opposite direction of the initial 0.12 lbs. of input force. This means that to expand the mechanism, only 14% of the force to compress the mechanism is required, i.e. the force required to compress the BSSSL is seven times greater than the force required to expand it.

Furthermore, when expanding the BSSSL, the applied force is required for only 14% of its entire deflection, i.e. the mechanism will travel six times farther than the initial push, with seven times the force, when expanding. Thus, going from the compressed to expanded position gives an unusual mechanical advantage. 0.12 lbs. of input force over the length of 14% of its movement, results in a 0.7 lb. output force that travels seven times the distance of the input force. This is opposite of the traditional intuition of mechanical advantage; usually a longer, weaker input force results in a shorter, stronger output force or vice versa.

This can be further elaborated using the potential energy curve shown in Figure 5.9. It can be seen that the stable equilibrium positions occur around 0 and 1.8 inches of deflection, or where the first derivative of the energy curve is zero and its second derivative is positive. Also, the unstable equilibrium position occurs at 1.5 inches of deflection, or where the first derivative of the energy curve is zero and its second
derivative is negative. The energy curve does not return back to its original zero-energy state. This is due to the hard-stop. This hard stop is located so close to the unstable equilibrium position, that the mechanism stores nearly all of its kinetic energy while in

![Potential Energy Graph]

Figure 5.9: Potential energy of BSSSL in design four.

the second stable equilibrium position. This is the reason for the unusual mechanical advantage of this bistable four-bar. Most of the energy is stored in its second state, and is released as it shifts from its second stable state to its first (from 1.8 inches to zero inches).

5.4.1 Link 4 Path Manipulation

Because the four-bar does not return to its zero-energy state, joint D remains slightly deflected ($8.4^\circ$, found in section 5.2) while the BSSSL is in its compressed position. As a result, a large visible slit propagates diagonally across the shield and can be seen from Figure 5.7. This large slit slightly affects the line of sight coverage of the unit cell during its deformation. Therefore, using iterative steps in a parametric CAD program [17], link 4 was created with a unique curve that ensured complete shield
coverage throughout the unit cells deformations. The new slit, created by link 4, shown in Figure 5.10, is positioned over redundant shield coverage from adjacent layers in the unit cell. Because links are independent of path, this manipulation of link 4 did not

![Figure 5.10: Modified BSSSL.](image)

affect the kinematics of the four-bar. This modified BSSSL was then fabricated into a prototype and assembled into a functioning unit cell. Figure 5.11 show the configurations of the MSSS. Figure 5.11a) shows the unit cell in its initial, un-deformed expanded position. The MSSS maintains the SSS’s five degrees of freedom, and therefore, will exhibit five modes of deflection. The first is shown in Figure 5.11b), where the unit cell exhibits shear. It can be seen that no slits or gaps form through any of these movements. Another deflection mode is shown in Figure 5.11c), where the unit cell is half-compressed, and deformed in one direction. It should be noted that another form of deflection not shown entails partially compressed, or when only one of the four sides of the unit cell is compressed, forming a trapezoid. Another mode, shown in Figure 5.11d), contains the unit cell in its final compressed position.
It can be seen that in all modes of deflection, there exists no linkage protrusion, unlike the designs in previous chapters. All linkages remain within the shield portion when compressed. Therefore, tiling is now conceivable and will be exploited.

Figure 5.11: Polypropylene prototype of MSSS where part a) is the first stable position, part b) is its sheared position, part c) is a second stable position, and part d) is a third stable position.

5.5 **Tiling Bistable Unit Cells**

A tiling system was developed to achieve out-of-plane shapes with planar MSSS unit cells. In order to attach faces of adjacent unit cells, a compliant pin connector was
made to attach two bisecting pins. This compliant pin connector is shown in Figure 5.12. These compliant pin connectors are easily deformed into any angle, as the segment between pins is a small-length compliant segment.

Bistable connectors were made and five planar MSSS’ s were attached at the pins using them; see Figure 5.13. Figure 5.13a) shows the container in its initial, un-deflected, expanded state. This configuration consists of five unit cells fully expanded (like Figure 5.11a). Figure 5.13b) shows the container compressed downward; this configuration consists of the four side-wall unit cells half-compressed, like the one shown in Figure 5.11c). The bottom face unit cell is still fully expanded. In Figure 5.13c), the original container is compressed horizontally. In this configuration, the bottom unit cell, as well as two parallel side-wall unit cells is half-compressed. The other two parallel side-wall unit cells are in their fully expanded position. In Figure 5.13d), two parallel side-wall unit cells, as well as the bottom face unit cell is half-compressed in one direction. The other two side-wall unit cells are completely compressed (like Figure 5.11d). Figure 5.13e), shows the unit cell composed of four side-wall unit cells half-compressed with the bottom face unit cell completely compressed. In Figure 5.13f), all six faces of the unit cell are completely compressed. These six configurations are the container’s basic shapes. In addition, the container has the ability to shear (like Figure 5.11b) in any direction, in any configuration shown, except when fully compressed (Figure 5.13f). In the unit cells smallest
Figure 5.13: Polypropylene prototype of multistable container made with five planar unit cells where part a) is fully expanded, part b) is vertically compressed, part c) is horizontally compressed in one direction, part d) is horizontally compressed in one direction and vertically compressed, part e) is horizontally compressed in both directions, and part f) is fully compressed.
configuration, no shear mode is possible due to the boundary conditions set by pins, i.e. the pins prevent lateral movement when fully bound due to interference issues.

Because these polypropylene prototypes successfully demonstrated bistability, stress was analyzed to verify if the resilient polypropylene was concealing large stresses within this design.

5.6 Stress

Stress was analyzed similar to the first design using FEA. The maximum rotational angle experienced by link 4 was found from Figure 5.5 and stress was analyzed in this position. Figure 5.14 shows the von Mises stress in each of the small-length flexural pivots. It can be seen that the stress is up to four times greater than the yield strength of polypropylene (5000 psi). This stress is caused by the compressive force

Figure 5.14: Von Mises stress in small-length flexural pivots.
exerted by link 4. Typically small-length flexural pivots do not contain significant bending stresses, thus, the majority of this stress is due to axial compressive stress.

Figure 5.15: FEA model of von Mises stress distribution in joint D.

Stress was analyzed at joint D as well and is shown in Figure 5.15. The maximum von Mises stress found was 19,528.7 psi. Even though most links in a compliant mechanism are assumed to be rigid, they are still capable of absorbing stresses through elastic body deformations. It can be seen from Figure 5.15 that the stress is distributed to the surrounding joint in sections that were assumed to be rigid; the stress resides far along link 4. Because significant stresses reside within the material, it can be assumed that there is elastic body deformation within the material that may not be noticeable. This elastic body deformation may absorb, and reduce the total stress within a joint. Nevertheless, the stress is still significantly higher than polypropylenes yielding point. This is proven by small permanent deformation that can be seen within the polypropylene prototype at joint
4. Link 4 of this mechanism does not rotate back completely to its starting position (fully expanded) after numerous repeated cycles through its stable positions. Overall, link 4 experiences approximately two degrees of permanent rotational deformation. This is not significant enough for a re-design, instead, more consideration on stress of these joints are held for future work.

5.7 Discussion of BSSSL Design

This chapter presented a BSSSL design which contained no linkage protrusion in any of the deformed positions. As a result, a tiling scheme was produced in order to assemble the MSSS unit cells into a three-dimensional storage container.

Future work based on this chapter could involve producing additional tiling schemes. As a result, more three-dimensional geometries could be produced. Also, stress and fatigue consideration could be addressed, while developing a more accurate representation of joint D. To reduce the stress of this joint is challenging because it must maintain a stiff torsional resistance, however, must also displace a moderate amount to allow for the large displacement of the flexural pin.
CHAPTER 6: ADDITIONAL TILING DESIGN

In chapter 5, a tiling design was presented which attached unit cells by their outer pins, or nodes. In this chapter, a new tiling design was created by utilizing an additional node at the center of the unit cell. Unit cells were then stacked and attached by this center-node. It should be noted that the compliant portion of this design was left unchanged from chapter 5. Therefore, results found in chapter 5 apply to the design presented in this chapter, i.e. links, kinematic motion, path of flexural pin, \( \varphi \) vs. \( X \) position of flexural pin, force vs. deflection, potential energy, stress, etc.

6.1 Kinematic Structure of Center-Node Design

The original kinematic structure for the MSSS is shown in Figure 1.3. It has four nodes connected by kinematic sliders. For the design presented in this chapter, an additional node was placed at the center of the unit cell and attached to each of the outer four nodes by kinematic sliders. Figure 6.1a) shows the 5 node unit cell. Figure 6.1b)

![Diagram](image)

Figure 6.1: Part a) shows a five node structure while part b) shows the kinematic model of the five node unit cell.
shows the skeletal model of the 5 node unit cell. The addition of this center-node allows for layering of the MSSS. Deformable objects have been developed with a similar kinematic structure to the one shown in Figure 6.1a) by using a rheological object with interconnected mass-spring -dampers [19].

6.2 Design of Center-Node Linkage

The center-node must be connected to each of the four outer nodes by a kinematic slider. Therefore, a design was produced that gave this desired behavior. It uses a centered, large-radius circle plate that serves as shield coverage, with four initially-curved compliant linkages that connect to the outer four nodes of the unit cell. These linkages serve as kinematic sliders that are capable of rotation as well as lateral motion. Figure 6.2 shows the design produced for the center-plate of the MSSS.

Because all four compliant links on the center-plate are initially curved in the same direction, they will have a tendency to give a biased torsional resistance when compressing or expanding the unit cell. Therefore, two center-plates are used on each side of the unit cell, each being in opposite direction. Having the two center-plates in opposite direction cancel any biased torsional resistance, giving a more accurate representation of the model presented in Figure 6.1. Figure 6.3 shows the
two center-plates on top of the other in reverse order. One center-plate is shown highlighted in blue, while the other is highlighted in dark grey.

### 6.3 Design Integrations of the BSSSL

Because an additional node was added to the MSSS unit cell, design integrations were performed on the BSSSL to allow for adequate clearance of the center-node as the unit cell deformed, while still preserving the shield coverage of the unit cell.

Numerous prototypes were made as an iterative process to find the most efficient shield portion shape for the BSSSL. Prototypes were assembled which brought out the design decision to attempt a four-layer MSSS unit cell, instead of the eight-layers used in previous designs. It was seen from these prototypes that the center-plates contributed significant shield coverage, and the use of eight layers of the BSSSL was redundant. Therefore, the shield portion was re-designed to allow for complete shield coverage using four layers, while allowing adequate clearance for the center-node.

#### 6.3.1 Center-Pin Path

Using initial prototypes, the paths of the center-pin were traced and then geometrically rendered onto the BSSSL in CAD. The shield portion was then developed around these multiple paths that the center-pin was seen to follow. The center-pin paths are shown in Figure 6.4.

#### 6.3.2 Angle of Shield Corner

In addition to the rendered path cut-out, the BSSSL also required a larger shield portion to account for the lack of shield coverage from the fewer layers used. Consequently, due to the larger shield portion, shear was not possible without significant
shield protrusion. The $2/3^{rd}$ initial angular area coverage of the shield corners (60 degrees), discussed in section 1.4.7.2 and Figure 1.5, allowed the unit cell’s to have 30 degrees of shearing motion in either direction. The shield angle for this design was designed to 83 degrees, as shown in Figure 6.4. This angle was found by optimizing a trade-off. If the angle was smaller than 83 degrees, it did not provide enough shield coverage in the expanded position, however, if the angle was larger than 83 degrees, shield portions protruded in the unit cell’s compressed position. To explain this, recall from Figure 5.5, in section 5.2.2, that the angular change of link 4 from the first stable position to the second is 8.4 degrees. Because the BSSSL in this chapter maintained the compliant design from chapter 5, the angle of link 4 ($\phi$) will increase by 8.4 degrees, i.e. the shield corner angle will increase by 8.4 degrees from its first stable position to its second.

However, because the shield corner angle is less than a right angle, when assembled into a unit cell, it creates non-linear edges when expanded. Figure 6.5, a solid-

Figure 6.4: BSSS link showing different paths that the center-pin can follow.
model of a unit cell with four BSSSLs and two center-plates, shows the broken edges created by the 83 degree angle shield corner. The front facing center-plate is solid blue, while the back facing center-plate is outlined in a light blue.

Because the purpose of this chapter is tiling, a compromise was made to have non continuous edges in the unit cells expanded position, or as shown in Figure 6.5. However, if collinear edges (four right angles) are desired in both stable positions, using an eight-layer MSSS configuration with the two center-plates is a viable solution (ten layers total). The only change in design would be to reduce the shield corner angle to 60 degrees. This would also allow for 30 degrees of shear in each direction.

6.4 Prototype Results

6.4.1 Center-Node Unit Cell Prototype

A polypropylene prototype of a unit cell was made. It contains four BSSSLs and two center-plates. The center-plates sandwich the BSSSLs, where one lies on the top face, and the other lies on the bottom face in opposite direction. The two center-plates are then pinned together, clasping all four BSSSLs together. Figure 6.6a) shows the unit cell in its initial expanded position. The top face center-plate can be seen in these figures. The bottom face is mirrored beneath the unit cell. Figure 6.6b) shows a side angle of the expanded unit cell. This figure shows how the center-pin clasps the center-plates together. Figure 6.6c) shows the unit cell in its half- compressed position. Only two BSSSLs are compressed in this configuration. Figure 6.6d) shows the unit cell in its fully
compressed position. All four BSSSLs are compressed in this configuration. It can be seen that the final compressed shape forms a square with continuous edges. This is due to the shield angle forming a right angle in its compressed position allowing a complete square. Note that no shear mode was presented in these figures. This is because the center-plate resists any shearing within the unit cell. Small shear modes are possible, however, are not stable positions.

Figure 6.6: Polypropylene prototype of unit cell with center-plate where part a) is fully expanded, part b) is a side view, part c) is a second stable position, and part d) is a third stable position.
6.4.2 Intrinsically Curved Surface

The center-node unit cell can be useful in multiple ways. One practical use can be passing items such as electrical wires through the center, providing a safe path that contains no interferences throughout the unit cell’s movement. Another practical use is using the center node as a connection point to stack unit cells, allowing each unit cell the ability to move independently while coupled together. This allows three dimensional configurations from planar deformations. For example, Figure 6.7 shows a prototype of

Figure 6.7: Prototype of intrinsically curved surface where part a) is unstressed and flat, and part b) has two unit cells compressed and is then curved out-of-plane.
an intrinsically curved surface. Four unit cells are used in this prototype; two sets of stacked unit cells (connected by their center node) are connected by their outer nodes together, forming a one by two matrix of unit cells, two layers thick. Figure 6.7a) shows the surface flat, having all four unit cells in their expanded position. However, when the top plane is compressed (the two top layer unit cells), out-of-plane deformations can be seen, and is shown in Figure 6.7b). This is analogous to the way stress behaves in a beam that experiences pure bending. One side of the beam experiences compressive stress, while the other side experiences tensile stress. The top, compressed layer of unit cells provides tensile forces, while the bottom, expanded layer of unit cells provides compressive forces, resulting in an out-of-plane curved surface. Future work can entail creating a flat surface capable of forming into a closed box by using six sets of stacked unit cells connected at their outer nodes.

6.5 Torsionally Induced Deformation

Another practical use of the center-node unit cell is by using its center-node as an input. Since the two center-plates are initially curved, a torsional input to the center-node causes a tensile force exerted on all four outer nodes. This tensile force compresses the unit cell symmetrically. However, since the two center-plates are in opposite direction, the center node can only be rigidly attached to one center-plate. Therefore, a square rod can be used as the center node, where one plate has a circular hole while the other has a square cross hole, allowing torsion to be transferred to only one plate.

6.6 Discussion of Center-Node Unit Cell

This chapter successfully demonstrated a multistable unit cell with five nodes. It was done so using a center-plate with four initially curved compliant links connecting the
center-node to the four outer nodes. A unit cell prototype was developed that ensured the geometry of the shield did not interfere with the center-node. Also, an intrinsically curved surface prototype was developed to show the feasibility of stacking unit cells by its center-node to create out-of-plane shapes with planar deformations. Also, planar deformation caused by a torsional input to the center-node was discussed. The compliant portion in this design was left unchanged from chapter 5, therefore, all analysis done in chapter 5 such as force vs. deflection, potential energy, and geometric analysis applies to the design presented in this chapter. Future work entails an initially flat surface that can fold into a closed box or trap.
CHAPTER 7: FUTURE WORK AND APPLICATIONS

7.1 Future Work

7.1.1 Stress Consideration

The BSSSLs use novel joints to achieve its bistable behavior, specifically joint D in the four-bar designs. This joint requires strong torsional resistance with moderate deformation. The challenge lies in controlling the stress. Because these joints experience fixed displacements, the segment normally would be made thinner to decrease its stress. However, doing so decreases its resistance, which is needed for bistable behavior. Therefore, future work will entail decreasing the stress in these joints while still maintaining their strong torsional resistance and moderate deformations.

7.1.2 Actuation

Other future work on the MSSS could entail actuation. The MSSS requires an input force to move it to and from its stable positions. Using pneumatics or actuators would allow the MSSS to change its shape from one shape to the other with the push of a button. Because the force vs. deflection curve can be manipulated, actuation can be placed on the weak end of the curve, allowing the mechanism to use its mechanical advantage to push with a force greater than that capable of the actuators.

7.2 Applications of MSSS

The MSSSs presented in chapter 5 and 6 have a unique force vs. deflection characteristic. It uses an unusual method of mechanical advantage by storing energy
within its members to be used in a specific part of its deflection. Section 5.4 elaborated on this unusual force characteristic. Many applications can stem from the mechanical advantage and usefulness of this MSSS design.

7.2.1 Collapsible Truck Bed

Because the MSSS is capable of resisting normal forces to its face, it could be useful in applications where large, heavy loads are contained or carried, i.e. vehicle beds in standard pick-up trucks, dump trucks, 18-wheelers, etc. The bed would be composed of large square unit cells forming an open container. The vehicle bed could start off small, as the stock, storing size of the vehicle bed when not in use. However, when more space is permitted, or needed from the bed, a small push of a link from the user would allow the bed to double in size. Multiple shapes of the bed would be possible as shown in the container prototype shown in Figure 5.13. This would give many options to the user such as making the vehicle bed wider, taller, longer, or all of the above. As mentioned, the force to expand the bed would be 14% of the force required to collapse the bed down to its stock size. This is advantageous is the sense that the cargo held within the bed would not collapse the bed down to its original size from its own weight. A mechanism could be implemented to acquire mechanical advantage over the MSSS to allow manual input using average human force to collapse the bed when needed. This would decrease cost and increase reliability without the need of motors or pneumatics that may require repair or replacement over time.

7.2.2 Trash Compactor

The same theory of using an undistributed force vs. deflection curve can be used as a trash compactor. The user could apply a reasonably small force to a trash compactor
made up of MSSSs. The compactor could be capable of exerting seven times the force of the user over a range six times longer than the initial push. This would also acquire impact forces within the compactor depending on the size and fill level.

7.2.3 Energy Absorption

Compliant mechanisms have a large use in energy absorption as they can absorb energy within their links. Bistable Compliant mechanisms use this stored energy to transfer motion to two defined positions. The MSSS includes these qualities, as well as the ability to resist forces normal to its face. Therefore, MSSSs could be implemented into things such as vehicle bodies, impact safety gear, or damper systems for energy absorption, i.e., the MSSS would be useful as a multiple use impact absorber. Implementing the MSSS into vehicle bodies would allow the vehicle the ability to absorb significant impact energy, without experiencing permanent damage. For example, when a vehicle experiences a collision, the frame and body of the vehicle absorb much of the impact through permanent deformation within its members. Implementing the MSSS into the structure of the vehicle would allow its surfaces to absorb high impacts by transitioning to its second stable position and could save time and money in repair as they can be ‘fixed’ or returned to their initial state by transitioning the unharmed surface back to its first stable shape.

7.2.4 General Storage Containers

The MSSS can also be designed using a symmetric force vs. deflection curve. This way, it can be used in every-day storage containers where having different force behaviors within the shape change does not necessarily benefit its function. For instance, trash cans, milk crates and cabinets could all be collapsible, so when not in use, do not
take up valuable space. In addition, the force characteristics can be designed so that no additional mechanical advantage is required to open and close the container; the force of a human input would suffice. A prototype of a simple storage container was made and is shown in Figure 5.13. This shows some of the different shapes possible with a container made of six, five-degrees of freedom MSSS unit cells.

7.2.5 Box Traps

The design presented in chapter 6 resulted in an intrinsically curved surface. This surface started flat, and using planar deformations, resulted into a three dimensional curve. Future work will entail optimizing the out-of-plane motion using these planar deformations, to assemble a six-sided collapsible box that initially starts out flat. This box can then be used as a safe and harmless animal trap by using planar actuation that would not compromise its concealability when lying flat.
CHAPTER 8: CONCLUSIONS

Numerous iterative designs for Multistable-Shape-Shifting-Surfaces were presented in this paper; some designs may be more advantageous than others depending on its application. They used a combination of compliant mechanism theory with the concept of bistability and incorporated them into the Shape-Shifting Surface.

The design presented in chapter 2 established a bistable link which behaved similar to a kinematic slider. Rotation was evident throughout the sliding motion, producing an arc-like motion during its deflection. This link was not designed to be connected to form a unit cell, but performs adequately as a single compliant bistable slider with relative rotation.

The design presented in chapter 3 produced a bistable slider with no relative rotation, allowing for the link to be coupled with another overlapping link. The two links together formed one side of the kinematic structure of the unit cell defined in chapter 1. It also produced effective shield coverage during its relative displacement with the other coupled link, allowing close to a 50% reduction in length from pin to pin. Tiling was not considered within this design.

The design presented in chapter 4 manipulated the design produced in chapter 3 to allow the assembly of links into a complete unit cell with eight overlapping layers. Motions were optimized and interferences were resolved. The unit cell maintained complete surface area coverage in all of its stable positions. Stable positions included expanded, compressed, sheared, semi-compressed and partially compressed. However,
because there was significant linkage protrusion within the unit cell, tiling was not feasible as this linkage protrusion would interfere with other unit cells. However, if tiling is not desired, the design presented in this chapter provides an effective solution for a stand-alone MSSS unit cell.

The design presented in chapter 5 focused on permitting the tiling of unit cells. Therefore, linkage protrusion was eliminated. The design used the compliant portion of the BSSSL as shield coverage, allowing all compliant links to be hidden within the unit cell when compressed. This design allowed for three-dimensional assemblies using planar unit cells. An open container prototype was developed which contained numerous stable positions.

Chapter 6 introduced an additional tiling system that allowed unit cells to be connected by a center-node. This allowed stacking of unit cells which provided a design and prototype to be developed for an intrinsically curved surface. In addition, it provided another way of deforming the unit cell by using an applied input torque to the center node.

Applications for the MSSS can include size-changing vehicle beds, expandable laptop screens, deformable walls, and volume-changing rigid-storage containers. Future work entails stress reduction, producing more accurate models of joints, as well as impact absorption using the bistable characteristics.
REFERENCES


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