

Using the SIR Epidemiology Model with Vector Transmission to Predict the Effectiveness of a Viral Marketing Campaign and the Spread of Product Adoption

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Using the SIR Epidemiology Model with Vector Transmission to Predict the Effectiveness of a Viral Marketing Campaign and the Spread of Product Adoption

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Abstract

Stevelab, LLC is working on an app that will be released within the year. For planning purposes, the company needs to be able to predict how well the app will spread in its initial phase. Because the pilot launch will be limited to Hillsborough County, FL, there is a limited population. The marketing strategy is to sell the application to places where people gather to mingle and socialize, with a particular focus to the bar scene. Customers of the bar can use the app for free. In addition to our direct sales efforts, we can expect the bar owners to talk to other owners, users will talk to other people, and users will talk to other bar employees creating a web of additional growth. These components are very similar to an SIR model used to predict the spread of infection by dividing a population into 3 categories, Susceptible, Infected, and Recovered. Ratios are applied to show how each division interacts with the other two. (Nicho, 2010) Adding a vector as a means of transmission gives a second population to track divided into 3 similar categories. (Wei, Li, & Martcheva, 2007) In addition to reactions among these three, there are also constants to represent how the two populations interact. This study tests the viability of using an SIR vector model to predict sales of our new app. We conclude that there are good matches between the biological data points and the business data points making the use of an SIR model for this purpose plausible and a prediction can be made. Due to pervasive estimations in actual figures, the predictions are not expected to be extremely accurate at this time. However, we have the ability to directly monitor all of these points as they happen, meaning we can use the same model with increasing accuracy as business progresses, even as early as our first week.

Keywords

Viral Marketing, App Marketing, SIR Model

PROBLEM STATEMENT

How can Stevelab predict how quickly the product, the Maka mobile app, will be adopted by customers?

MOTIVATION

While sitting in a bar having a conversation, my business partner at Stevelab and I noticed something about the patrons around us. Many of the people at the bar were using their mobile phones and not interacting with anyone else apart from the occasional words with the bartender. This has become such a common sight that many of us take for granted how strange this is. Why would you leave the house and go to a public place traditionally used for mingling and meeting people and not mingle and meet people? The answer is that most of these people want to have a social interaction, but they feel they can't for a number of reasons. Suffice it to say, we have done quite a bit of homework since then to determine many of the reasons for this. We have developed an app which we are calling Maka to help people use their phones to put down their phones by facilitating interpersonal communication in real life. How this product works and why is omitted to protect the intellectual property and because beyond what has already been put forth, it is not relevant to this specific problem.

Stevelab has the resources to get us through the initial roll out and shakedown phase, but will need external resources to expand nationally. External investment means we need a business plan and one of the most important parts of the business plan is a sales forecast. While we don't necessarily have all the information we need now to make accurate predictions, it is vital to establish a methodology and proof of concept so that we know what information is important to track in our rollout.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

Fermi Estimation

In science, researchers take a great deal of time designing controlled experiments where every variable is knowable and within their power. By contrast, in business we often have to make decisions with little or no information at all, and we often have to do it quickly. If we simply ask ourselves how many users we will have within a year of launching our product, at first glance it may appear impossible to answer or at least a complete shot in the dark. However the physicist Enrico Fermi, one of the geniuses behind the Manhattan project gave us a process by which we can at least get in the ball park which is all we really need for long term strategic planning. This process, called a Fermi estimation or Fermi Equation, has three major steps.

1. Break the question into a large number of smaller questions that can affect the outcome.
2. Determine how those answers are related to each other.

3. Use information that you do have available to you to make a reasonable guess. If such a decision is not evident, determine the lowest and highest reasonable values for each variable and take the geometric mean between them.

This process has two big advantages. By estimating on a large number of related variables, you are just as likely to estimate high on one and low on another cancelling out the difference. You can also make predictions more accurate down the road as you obtain more reliable values for each variable. That means that although we have no hard data before day one and we have to use a lot of estimates, we can plug in real information as the process starts moving forward to fine tune our predictions. (NASA, 2000) (Lamb & McCormick, 2017)

The SIR w/ vector model

We can accomplish the first two steps of the Fermi process by developing a model by which we can calculate our growth. The strategy we intend to use has been known for years as viral marketing, a strategy that relies heavily on customers recruiting other customers. As a low cost product with high utility value, we believe this could work for us. Across the industry, 36% of new downloads are based on word of mouth (Comscore, 2017) so this is an important tactic to consider for any app. Viral marketing is more than just an expression, it is a process of spreading a product person to person like a virus. This made looking for an existing model for the spread of a virus seem like an obvious parallel.

Because we have two different customer classes, venues and end users, a simple SIR approach is not suitable. The vector transmission model described by (Wei, Li, & Martcheva, 2007) is much closer to our needs. Their process is a bilinear mass action process. *“Mass action is a fundamental notion in many situations in Chemistry, Biochemistry, Population Dynamics and Social Systems. In this class of phenomena, one has a large population of individuals partitioned into several types of “species”, whose dynamics is specified by a set of reaction rules. Each reaction indicates the transformation that is likely to take place when individuals of specific types come into contact.”* (Maler, Halasz, Lebeltel, & Maler, 2013)

In this model, two separate populations are tracked, In the original form, this is the human population which is the primary concern and the Mosquito population, the vector. In our version, the venue population is our primary customer, and the user population is our vector. The primary population is divided into three categories, S, I, & R. Everyone is assumed to be born susceptible. Once they are infected, they move from the susceptible group to the Infected group. Here, they can spread the infection to others. Once the infection has run its course, they move to the Recovered group where they can still interact, but cannot be infected. People are moved from one group to another by mass action calculations such as the rate at which people infect each other $\lambda_1 I(t)S(t)$. The second population is categorized in similar fashion. M for uninfected and V for active Vector. In the original model, there was no third group because mosquitos don't recover. In our case, people will eventually tire of using our app and abandon it. For this reason, we added category Z. The subpopulations interact in the same way using mass action terms. The infected portion of each population interacts with the other population as well. In the following sections, you will find the original

equations from the virus model and the modified equations used for our purposes, followed by a table of what each variable was originally used for and the business parallel we adapted it to.

EQUATIONS

Original Reference Equations

- I. Original System of equations for the change in infected people
(Wei, Li, & Martcheva, 2007):

- $\frac{dS(t)}{dt} = b_1 - \lambda_1 S(t)I(t) - \lambda_2 S(t)V(t) - \mu_1 S(t)$
- $\frac{dI(t)}{dt} = \lambda_1 S(t)I(t) + \lambda_2 S(t)V(t) - \gamma I(t) - \mu_1 I(t)$
- $\frac{dR(t)}{dt} = \gamma I(t) - \mu_1 R(t)$

- II. Original System of equations for the change in infected vectors
(Wei, Li, & Martcheva, 2007):

- $\frac{dM(t)}{dt} = b_2 - \lambda_3 M(t)I(t) - \mu_2 M(t)$
- $\frac{dV(t)}{dt} = \lambda_3 M(t)I(t) - \mu_2 V(t)$

Original equation for change in population: $N'_1 = b_1 - \mu_1 N_1$ (Wei, Li, & Martcheva, 2007)

Original equation for change in vector population: $N'_2 = b_2 - \mu_2 N_2$ (Wei, Li, & Martcheva, 2007)

Modified Purpose built Equations

The original model is good, but it lacks a few key details that exist in our scenario. First mosquitos don't infect each other. There is no one actively pushing the disease forward as our account executive will do, and mosquitos don't recover. For these reasons we have to make a few changes. Our equations work like this.

- III. Maka system of equations for participating venues (patients):

- $\frac{dS(t)}{dt} = b_1 N_1 - \lambda_1 S(t)I(t) - \lambda_2 S(t)V(t) - \lambda_4 - \mu_1 S(t)$
- $\frac{dI(t)}{dt} = \lambda_1 S(t)I(t) + \lambda_2 S(t)V(t) + \lambda_4 - \gamma I(t) - \mu_1 I(t)$
- $\frac{dR(t)}{dt} = \gamma I(t) - \mu_1 R(t)$

- IV. Maka system of equations for end users (vectors):

- $\frac{dM(t)}{dt} = b_2 N_2 - \lambda_3 M(t)I(t) - \lambda_5 M(t)V(t) - \mu_2 M(t)$
- $\frac{dV(t)}{dt} = \lambda_3 M(t)I(t) + \lambda_5 M(t)V(t) - \gamma_2 V(t) - \mu_2 V(t)$
- $\frac{dZ(t)}{dt} = \gamma_2 V(t) - \mu_2 Z(t)$

MAKA population changes:

- $N'_1 = N_1(b_1 - \mu_1)$
- $N'_2 = N_2(b_2 - \mu_2)$

Once all variables have been assigned we use a table in excel (attached) to calculate the value of each mass action unit as a sequence, then add or subtract them from the previous week's totals to get that week's totals.

NOMENCLATURE

Table of Variables

(original variables from) (Wei, Li, & Martcheva, 2007)

Variable	Symbol	Marketing Analog	
number of people Susceptible	S	Securable - untapped portion of the market	1530
number of people Infected	I	Involved – current users (venues)	0
number of people Recovered/immune	R	Released, those who have tried the service and stopped using it or those which are not likely to accept, analogous to immunized	0
uninfected vectors	M	potential individual users	146207
infected vectors	V	number of secondary users (people)	0
	Z	those individual users who have used the app ad rejected it.	0
time	t	time	1 week intervals
human birth/recruited rate	b_1	rate of venue growth	.01
vector birth rate	b_2	rate of influx of people	.243%
direct transmission rate	λ_1	rate at which venue owners convince others to adopt the product	.098%
vector transmission rate	λ_2	Percent of bars each user can recruit per week	.00065%
infection rate of vector	λ_3	rate at which venues convince patrons to use the product	.0043%
	λ_4	rate at which account executives recruit new venues	Hard 2 per week, not a ratio.
	λ_5	Person to person	.000144%
human population natural death rate	μ_1	rate of venue closures	.96%

vector death rate	μ_2	rate of people leaving	.238%
per capita recovery rate	γ	rate at which venues will discard the product	1.28%
	γ_2	rate at which individuals abandon the product	1.92%
human population size	N_1	total of all venues in market area	1530
total vector population	N_2	total of people in market area	146207

VARIABLES

Segments

S: Securable - untapped portion of the market

We believe all bars included in N_1 can potentially be secured.

I: Involved – current users (venues)

This will be zero at launch.

R: Released, those who have tried the service and stopped using it

This will be zero at launch.

M: potential individual users

We believe all people in N_2 are potential users.

V: number of end users (people)

This will be zero at launch.

Z: individual users who have used and abandoned the app

This will be zero at launch.

N_1 : total of all venues in market area

We decided to use the number of onsite liquor licenses issued in Hillsborough county to estimate the number of venues. While a significant portion of these will be restaurants where the primary focus is eating, not socializing, most of these establishments still have a bar area that meets our needs. For those that do not, this number can be offset by the number of coffee shops and kava bars in the local area which will also serve our purpose. It will likely get us close to a good count. The total number of on site liquor and beer wine licenses in Hillsborough county was slowly declining between 2004 and 2010 given by the following table, but rose quickly following the great recession. (Moore, Young, & Snelling, 2013)

Year	2004	2006	2008	2010	2011	2012
Licenses	1359	1463	1199	927	1247	1223

We can draw two possible conclusions from this data. The downward trend could continue at the slope of the trend line for all of these years at -33.34 per year giving us a population of 886 bars

in 2019. We could also recognize that 2008 was the beginning of the recession and we have been in recovery since 2010. Looking at only 2010 through 2012 we get a slope of 148 and a projected number of licenses at 2316 in 2019. There is another reasonable correlation we can explore to reach an approximation. Between 2004 and 2012, there was an average of 1 bar for every 998 people in Hillsborough County and 1 for every 213 people in the 20-35 age group. This gives us a projected total for 2019 of 1448 or 1470 respectively and a growth rate of 20.1 and 24.2 respectively. If we take the geometric mean of these four methods we can estimate our total number of venues at 1530 and a growth rate at 37.2 bars per year.

N₂ Total target population

This will be the total number of people who meet our target demo criteria also called the target market segment. These are people who are interacting with the bar scene and likely to try a new app. For this reason, we will be focusing our efforts on the 21 to 35 crowd. This is the segment most likely to try new apps. (Comscore, 2017) The best statistical information available can only give us 20 to 34, though it is shifted by a year, it still makes for a great approximation. While we really only need the 2019 number to start with, the historical data helps give us an approximate rate of change and other key details. (Florida Department of Health, 2018)

range	0-14	15-19	20-24	25-34	35-85+	Total	Target
2004	239811	82399	78340	154965	560302	1115817	233305
2006	248230	82346	84765	165480	597075	1177896	250245
2008	253217	84381	87050	169185	614364	1208197	256235
2010	240567	87365	90853	172535	640233	1231553	263388
2011	244539	88313	93167	176306	640166	1242491	269473
2012	243080	85631	94990	181826	655360	1260887	276816
2013	248317	85132	95223	186395	667444	1282511	281618
2014	253818	86256	94716	191612	681504	1307906	286328
2015	257284	87009	94168	197232	696304	1331997	291400
2016	261071	88096	92381	204378	713924	1359850	296759
2017	265440	89021	91061	210876	731713	1388111	301937
2018	270923	90861	92940	215233	746830	1416787	308173
2019	276413	92701	94825	219598	761972	1445509	314423

The total population is only part of the story. We need to narrow it down to the number of people who actually interact with the local bars. According to (Trocki & Drabble, 2008 Nov), 40% of women and 53% of men were regular bar goers. Assuming that men and women are distributed close to evenly, that gives us 46.5%. A more recent article from CNBC (Breuninger, 2017) states that 42% of all millennials go to a bar at least once a week.” While more recent, the second article poses a few problems. It doesn’t describe how that result was obtained and a more than monthly visit to the bar should be sufficient to consider a person a carrier. Because they are similar answers, we can consider the CNBC article as a bolster to our original estimate. 46.5% of the total 314423 in our target age bracket gives us a market segment of 146207 people with a growth rate of 2407 people per year.

ENVIRONMENTAL RATIOS

B₁: rate of venue growth - how often do new bars pop up?

There are two ways new bars will appear, by buying out an existing license or by obtaining a new one.

The average lifespan of a nightclub in Manhattan is 18 months. (Rose, 2006) The average life of a nightclub or bar is two years. (College Foundation of North Carolina). The CFNC article while not specifically dated contained clues that it was within the last 5 years and gave a more generalized statement of all night clubs and bars vs just Manhattan. The Rose article does bolster the CFNC article, having a similar number. This means that 1/104 of the existing bars will be switched out for a new one every week.

This should be added to the rate of change of N_1 or N'_1 was previously determined to be 37.2 per year which is a ratio of the current population of 4.68×10^{-4} .

The combined ratio of weekly increase is .01

B₂: rate of influx of people in demographic

There are two ways a person can enter our target population. They can age in, .114% or they can migrate in, .129%. Total .243%

- Aging in – The projection for 2019 population is 92701 for the age category 15-19 which precedes our target. Assuming an even distribution, this is 18540 that will age in per year and 357 per week. We multiply this by our .465 bar goes for 166 individuals. Compared to our starting population size the ratio is 0.114% per week
- How many new people immigrate into Tampa Bay per year? I was unable to pin down an estimate for this, but it can be inferred through other data points we have discovered.

Pop growth = births + move ins – deaths – move outs.

$20727 = 18069 + X - 10661 - 62727$, $X = 76,046$ move ins for the entire population

$76046 * .278$ age group to population * .465 bar going portion / 52 weeks = 189.04/wk or .129%

- Pop growth = 20,727 per year
 - slope of population totals for 15 years (Florida Department of Health, 2018) is 20,727 per year.
- Births = 18069 per year
 - 12.5 per 1000 population (Florida's Bureau of Vital Statistics, 2017)
 - 2019 projected population is 1445509 (Florida Department of Health, 2018)
- Deaths = 10661 per year
 - 10 year average for the total population is 737.5 per 100,000. (Florida's Bureau of Vital Statistics, 2017)
 - Current population is 1445509 (Florida Department of Health, 2018)
- Move Outs = 62727
 - As estimated using the “U-Haul method” for μ_2 below

μ_1 : rate of venue closures

This will equal the number of bars obtaining an existing license from the calculation for new bars. .0096

 μ_2 : rate of people leaving the geographic area

- People can leave the demographic in 3 ways. They can age out, 0.13%/wk. They can move, .106% of total/wk, and they can die, .002%/wk. Total .00216/wk.
- There are 219598 in the 25-34 range. (Florida Department of Health, 2018) We are again forced to assume an even distribution. 21959.8 will age out per year, 422 per week. This is a ratio of .13% of the market segment.
- Published vital statistics include growth rates, births, and deaths, but they don't seem to publish migration in and out. So this problem had to be estimated from a more indirect angle. Hillsborough County has an average of 500 one way U-hauls rented each month, but a "large number" are from one side of town to another. (Collins, 2018) "Large number" is imprecise, but it still tells us a lot. We know it is probably less than half or the word 'most' would have been used. This gives us an upper and lower limit for our Fermi estimate between 250 and 500 with an average of 375. U-Haul has over 50% of the market share in moving trucks (Tuozhi Yang, 2013) and 33% of mover rent a truck. (Simply Self-Storage, 2017) The average size of a moving household is 2.3 persons. (Simply Self-Storage, 2017) From these data points we can get a rough calculation of 375 trucks divided by .5 market share divided by .33 ratio over movers renting trucks multiplied by 2.3 household members. The result is 62727 per year. Multiplied by our established ratios for our target segment, we get 156 per week. and 1206 people per week. That's a ratio of .106%
- 10 year average death rate is 108.1 per 100,000 for the subject age group. This is a weekly ratio of .002% for the subject group. (Florida's Bureau of Vital Statistics, 2017)

Usage Ratios

These are the primary ratios determining how our app will actually spread through the environment. These figures are a double edged sword because they are unique to our product. This means that we have complete and immediate control over this information once we start selling our app. However, until we get started we have absolutely no data with which to create our ratios. That doesn't mean we have no insight at all. It just means we have to rely on informed intuition and put our trust Dr. Fermi's methods. As it happens, I and my social group are part of the target market segment. I have made a point of befriending the owner's of the bars I frequent, I have an A.S. in business administration, and have many years of experience in marketing. Having polled the aforementioned associates, I must rely on myself to make a better than random estimate on each of these points. Once we get moving, we can greatly improve our predictions with real feedback.

γ_1 : rate at which venues will discard the product

While there is no similar product that I am aware of, it does provide a similar utility to a venue's rewards program. Once a company adopts a rewards program, they tend to keep it for a couple years before switching to a new one. Therefore, I'd put a conservative estimate a year and a half. A turnover rate of 1.28%.

γ_2 : rate at which individuals abandon the product

Apps such as Facebook seem to make active customers for several years if not for life, while some flash in the pan games only stay interesting for a few weeks. The more useful application is the longer people tend to keep it. Our application does have a significant utility value which can be used continuously. We stand by the belief that improving a person's ability to escape the alone in a crowd feeling can be very impactful on a person's life. Therefore we will place our lifespan on the high end, but not quite arrogant enough to say we'll be the next Facebook. We should be able to expect an average of at least a year 1.92% turnover.

λ_1 : rate at which venue owners convince others to adopt the product

Based on the small sample of bar employees and managers I had it available to me. They had an average of 6 employees. Each with an average of 2 other bars they had direct social ties to. If we assume a 20% adoption rate, each participating bar could spread participation to others in the course of their use. $6emp * 2links * .20conv / 78weeks / 1530pop = .002\%$ of the population per week

λ_2 : rate at which customers who have tried the product convince new venues to adopt it.

This rate is highly dependent on three factors, how often our user visits a bar, how many different bars they frequent, and how likely they are to promote our product at each bar. The average bar goer is going to be exposed to 26 different bars in the year they are likely to use the app. 50% opportunity, 40% transmission, 10% adoption. This equals .52 per year and .01 per week. Divided by total number of venues, we have .00065%

- We have selected our target market to include only those who visit the bar monthly or more. Therefore the lowest number is 1 per month. The highest possible number is 30ish times per month and there will be a significant portion of people who do go every day.

The true geometric mean will be 15 times per month, every other day. It seems likely that this will not be an even distribution however, because drinking for most people, going out is a weekend activity, not a daily activity. My intuition puts the average at 8, so I'll choose the middle 11.5 per month, 2.6 visits per week, 135.2 per year

- I could find no information on how many unique bars a customer visits, so our estimate will have to be based on my own behavior and that of my peer group until we are able to collect a better sample from our own customers. Personally I go to my "home bar" about 60% of the time. I'll go to a familiar, but not necessarily comfortable venue of which I consider 5 places in this category about 30%. I try a completely new place about 10% of the time. Compared to my peers, I'm a little on the habitual side, so I'd place my estimate for typical around 50%, 35%, 15% .
- How likely a person is to talk up our product, assuming they are using it will really depend on two main factors. How busy is the bar and how much they really like us. If the bartender is not terribly busy making drinks, they will chat with customers, guaranteed. It's actually part of the job. This question is yes or no with fairly even odds. So we can estimate 50%. Again we're talking about an app that can really change lives so let's say 40% of our users are willing to talk about us. And we'll take a shot in the dark that if communication is successful, adoption will be successful 10% of the time since owners get all kinds of suggestions from customers all the time.

λ_3 : rate at which venues convince patrons to use the product

Again comparing the app we are using to a rewards program where the staff is actively working to recruit new users, the average conversion rate among the limited 3 bar sample I had to work with was 90% of all of their customers. Our product is a little more abstract and takes more effort on the part of the user so it is unlikely we can achieve such high numbers, but it is encouraging. If we take the middle between zero and 90, 45% is a reasonable expectation. I don't have data on all bars, but my favorite establishment was nice enough to share their figures. They have had 720 people join their rewards program in 14 months and they believe they have 85% participation. (O'Neill, 2018) This means they see 14 new faces per week. Since they're a smallish bar in their initial growth phase, we can probably double that for a normal venue to 28. We'd have to half that again to target the millennial population. So each bar using our app can influence .0043% of the target market.

λ_4 : rate at which account executives recruit new venues

This is the only ratio truly in our control at least indirectly. We can increase or decrease the number of AEs we employ. For an inexpensive, and utilitarian tool, a conversion rate of 2 per week seems more than reasonable.

λ_5 : Recruiting person to person

People don't often rave to everyone they know about a product they love, but they are likely to tell their closest friends and the others where it becomes relevant. Since it is an app that is used in the location where talking to acquaintances happens often, and it is designed to facilitate communication, we can guess this ratio won't be high, but may come up in 1 in 100 interactions. According to Why We Live – Counting The People Your Life Impacts (Vital, 2013), we can

assume we have 3 unique interactions a day. It should be noted that this sourced figure also appears to be an educated guess, but it seems to be a qualified one. Each user can be expected to recruit .21 others per week.

DISCUSSION

We were able to meet our first objective which was to find a model similar to our needs and adapt it for business use. We were able to successfully draw parallels between the epidemiological model and factors specific to the spread of our product. We were able to complete the table of variables and run a simulation. That simulation told us that we would start seeing significant growth between 10 and 22 weeks. At this point we reach market saturation and new customers dry up. After a year, we count 2/3 of the total market within Hillsborough county as customers. But is this accurate? We have no way of knowing because we have nothing to compare it to. We'd have to see where we stand after the first year of pushing the product. Can we expect it to be accurate. Based on the lessons we've learned from Enrico Fermi, we can expect that this is a reasonable approximation. However many of our key variables had to be based on very tenuous logic and could potentially be wildly inaccurate throwing off the end result. That does not mean the results were useless. For starters we can expect to see peak saturation somewhere between 4 and 8 months even if we give the simulation a wide margin of error. The model can also be a vital tool down the road as more accurate information can be obtained for each data point.

CONCLUSION

This exercise showed that as a proof of concept, it is certainly possible to use an SIR approach to model the effectiveness of a viral marketing campaign. But the practicality is questionable as key information required to run the simulation is difficult to obtain. It will be interesting to see how close our estimation lies to the true path when we get theirs and whether we can accurately model our growth as more accurate data is obtained.

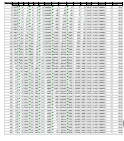
REFERENCES

- Breuninger, K. (2017, June 26). *20-Somethings Spend More than Other Americans on Coffee, Dining Out and Bar Hopping*. Retrieved from CNBC:
<https://www.cnbc.com/2017/06/26/20-somethings-spend-more-than-other-americans-on-coffee-dining-out-and-bar-hopping.html>
- College Foundation of North Carolina. (n.d.). *Nightclub Owner > The Buzz*. Retrieved from CFNC.org:
https://www1.cfnc.org/Plan/For_A_Career/Career_Cluster_Profile/Cluster_Article.aspx?articleId=4XAP2BPAXuq3lgI61sjL0FszDu8jgXAP3DPAXXAP3DPAX&cId=yJF7dgNzUI6xZl0h6IIInegXAP3DPAXXAP3DPAX§ionId=3
- Collins, A. (2018, December 7). in person query on Uhaul statistics. (M. Freed, Interviewer)

- Comscore. (2017). 2017 U.S. Mobile App Webinar. Retrieved from https://www.comscore.com/layout/set/popup/Request/Presentations/2017/The-2017-US-Mobile-App-Report?logo=0&c=12?utm_campaign=CONFIRMED_OPT_IN_AUTO_RESPONDER_ALL&utm_medium=email&utm_source=comscore_elq_OCT2018_OPTIN_CONFIRMATION_CONTENT_ALL_AR
- Dogtiev, A. (2018). *App Statistics Report Guide*. Soko Media; BusinessofApps. Retrieved from www.businessofapps.com
- Florida Department of Health. (2018). *Florida Population Estimates*. Retrieved from FL Health Charts: <http://www.flhealthcharts.com/FLQUERY/Population/PopulationRpt.aspx>
- Florida's Bureau of Vital Statistics. (2017). *Florida Death Rate Query System*. Retrieved from FL Health Charts: <http://www.flhealthcharts.com/FLQUERY/Death/DeathRate.aspx>
- Florida's Bureau of Vital Statistics. (2017). *Florida Brith Rates Query System*. Retrieved from Florida Health Charts: <http://www.flhealthcharts.com/FLQuery/Birth/BirthRateRpt.aspx>
- Gaille, B. (2018, Jun 24). *22 Bar And Nightclub Industry Statistics and Trends*. Retrieved from Brandon Gaille Small Business & Marketing Advice: <https://brandongaille.com/22-bar-and-nightclub-industry-statistics-and-trends/>
- Hillsborough County City-County Planning Commission. (2017). *Hillsborough County Population and Employment Projections and Allocations*. Retrieved from http://www.planhillsborough.org/wp-content/uploads/2017/12/TPC-Dec-11-2017-presentation_KT.pdf
- Lamb, R., & McCormick, J. (2017, March 23). Be a Better Guesser With Fermi Estimation. *Stuff Media, LLC*. Retrieved from <https://www.stufftoblowyourmind.com/podcasts/fermi-estimation.htm>
- Maler, O., Halasz, A. M., Lebeltel, O., & Maler, O. (2013). *Exploring the dynamics of Mass Action Systems*. Retrieved from Cornell University Library: <https://arxiv.org/pdf/1309.0871.pdf>
- Moore, K. A., Young, M. S., & Snelling, E. (2013). Profile of Alcohol and Drug Indicators for Hillsborough County, Florida. *Mental Health Law & Policy Faculty Publications*, 857. Retrieved from https://scholarcommons.usf.edu/mhlp_facpub/857/
- NASA. (2000, 03 27). *Fermi's Piano Tuner Problem*. Retrieved from Glenn Research Center: https://www.grc.nasa.gov/www/k-12/Numbers/Math/Mathematical_Thinking/fermis_piano_tuner.htm
- Nicho, J. (2010). The SIR Epidemiology Model in Predicting Herd Immunity. *Undergraduate Journal of Mathematical Modeling: One+Two*, 2; 2010 Spring(2), Article 8. Retrieved from <https://scholarcommons.usf.edu/ujmm/vol2/iss2/8/>

- Rose, L. (2006, 8 24). Nightclub business: More ego trip than profit? *Forbes*. Retrieved from http://www.nbcnews.com/id/14487574/ns/business-forbes_com/t/nightclub-business-more-ego-trip-profit/#.XAsoEmhKiUk
- Sadun, L. (2014, July 29). *Running the SIR Model*. Retrieved from YouTube: https://www.youtube.com/watch?v=-e_igCrevl0
- Simply Self-Storage. (2017, October 31). *35 Interesting Moving Industry Statistics for 2018 and Beyond*. Retrieved from Simply Self-Storage: <https://www.simplyss.com/blog/moving-statistics/>
- Trocki, K., & Drabble, L. (2008 Nov). Bar patronage and motivational predictors of drinking in the San Francisco Bay Area: Gender and Sexual Identity differences. *J Psychoactive Drugs*, Suppl 5: 345-356. Retrieved from <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC2801063/>
- Tuozhi Yang, T. (2013, December 2). *Why Amerco Still Has More Upside*. Retrieved from Seeking Alpha: <https://seekingalpha.com/article/1871121-why-amerco-still-has-more-upside>
- Vital, A. (2013, April 29). *Why We Live – Counting The People Your Life Impacts*. Retrieved from fundersandfounders.com: <https://blog.adioma.com/counting-the-people-you-impact-infographic/>
- Wei, H.-M., Li, X.-Z., & Martcheva, M. (2007, June 21). An Epidemic Model of a Vector-Borne Disease with Direct Transmission and Time Delay. *Journal of Mathematical Analysis and Applications*. doi:10.1016/j.jmaa.2007.12.058

APPENDIX



Click the image to open the calculations workbook.

