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Design of Nozzle for High-Powered Solid Rocket Propellant

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Problem Suggested By: Society of Aeronautics and Rocketry

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Design of Nozzle for High-Powered Solid Rocket Propellant

Abstract

This paper presents a preliminary nozzle design for a high-altitude rocket to be built by SOAR, the Society of Aeronautics and Rocketry. The equations and methods used for analysis came from professional sources via text or informational videos. The equations in focus will look at the temperature/pressure/area-Mach relationships for isentropic flow, pressure expansions in selected materials, and finding other characteristic properties using safe chemical property assumptions using CEARUN. The equations were used under the assumption of an isentropic flow under sea level conditions. If the design and the projected results look promising, they will be implemented soon for machining and eventually a test fire.

Keywords

nozzle, pressure, propellants, Young's modulus of elasticity, Poisson's ratio.

PROBLEM STATEMENT

The Society of Aeronautics and Rocketry (SOAR) at the University of South Florida is a multidisciplinary engineering organization, with a strong interest and focus in aerospace projects and research. One of the SOAR projects under development is a high-powered rocket designed to break the collegiate apogee record. A rocket of this caliber will require stronger build materials, more advanced and powerful means of propulsion and thorough analysis. The first major consideration for a rocket of this magnitude is the size and power of the motor, to which SOAR will use a solid propellant due to available resources and mentor experience. This paper will look specifically to the design and analysis of a solid propellant and optimized rocket nozzle to deliver a student-built launch vehicle to record heights.

MOTIVATION

The research and effort into this project is to present analytic methods and approaches to effectively selecting solid propellant propulsion systems for collegiate and amateur rocketeers. This is a major step towards not only facilitating learning of more advanced engineering projects, but also presenting research opportunities and collaborative efforts with professionals and other students. The goal of this project is to advance research for the high-altitude program in SOAR and aid in producing an important subsystem for the rocket.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

To manufacture and successfully launch a high-altitude rocket, a mission goal and an altitude goal must be set to aid preliminary designs. Payloads and scientific experiments must be designed properly to avoid potential system and launch vehicle failures, and to optimize both physical space and weight for the rocket. Before we discuss how to design an effective solid propellant propulsion system, it is important to know the general operational and chemical procedures of traditional rocket propulsion methods. The main "ingredients" are heat, oxygen and fuel. The oxygen is presented through oxidizing agents like powdered chemicals, or liquid oxygen. The fuel is presented through combustible and flammable material like metals or special synthetic materials and catalysts. The initial source heat is necessary to start a combustion reaction between all three of these main "ingredients". Using air breathing ventilation systems to provide the oxidizing agent is not a reliable source for rocket propulsion. It does not create a large enough combustible reaction to produce a significant and needed change in acceleration and velocity, and a rocket itself does not produce lift like an aircraft thus needing more reliance from a powerful propulsive force versus a sustained lifting force. The thrust, or the ejected propulsive force, comes from the exhaust gases produced by the combustion reaction which are accelerated through a nozzle. Nozzle design and casing considerations will be discussed later. The basic principles of converging-diverging nozzle theory are as follows;

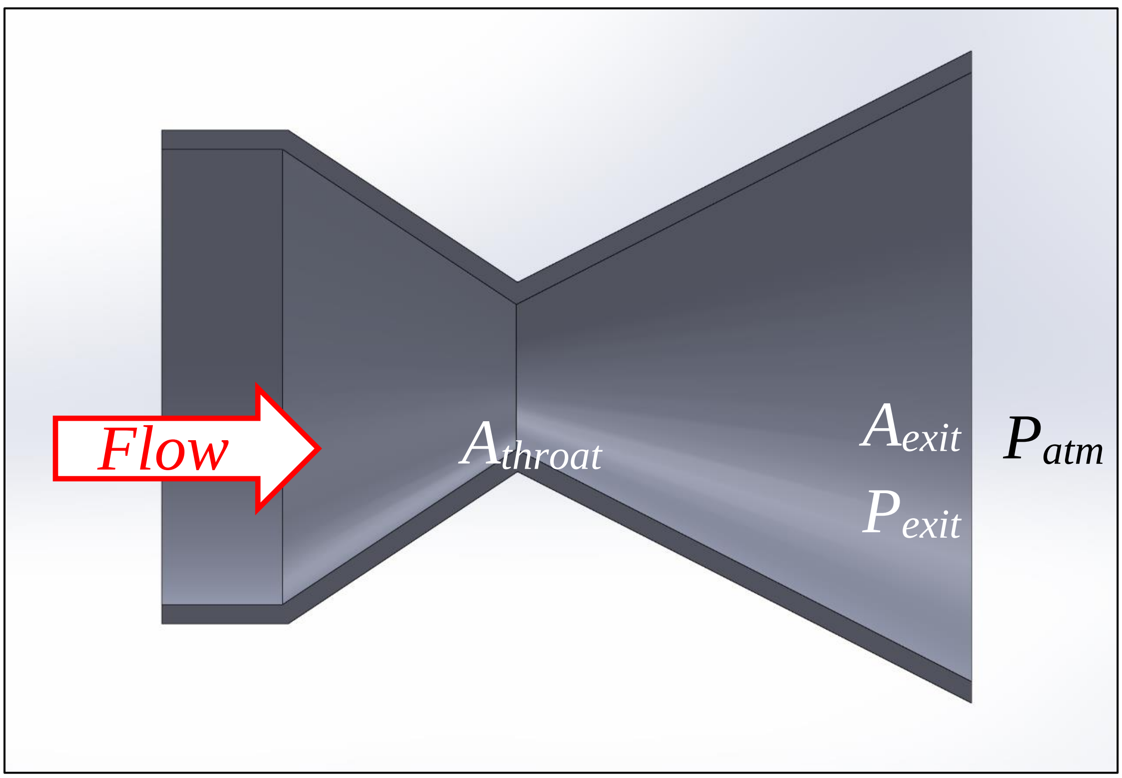

Figure 1: A simple diagram of a converging-diverging nozzle

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1. The converging section lies between the throat of the nozzle, A_{throat}, and the combustion chamber. This area is characterized by a mass flow of subsonic speed. For subsonic speeds, as area decreases then velocity of the fluid increases.
2. The throat of the nozzle is intended to create a sonic state. Here, the velocity of the fluid is equal to the speed of sound and thus represents an effectively “choked” nozzle.
3. The diverging section of the nozzle lies between the throat and the exit, A_{exit}. In this section, the fluid undergoes supersonic flow. For a supersonic flow, as the area increases so does the velocity of the fluid. This results in a Mach number greater than 1 at the nozzle exit.
4. For an effective nozzle, the exit pressure of the nozzle, P_{exit}, must equal the atmospheric pressure, P_{atm}, adjacent to the nozzle exit. In the case where $P_{exit} > P_{atm}$, the flow experiences over-expanded flow and under-expanded flow for when $P_{exit} < P_{atm}$. These irregular flows decrease the overall efficiency of the propulsion system but are unavoidable. Mechanisms and subsystems such as extending nozzles are designed to ensure for the exit pressure of the exhaust gases to be equivalent to the atmospheric pressure.

Solid propellants, or motors, are preferable for high-altitude flights due to their ease of design and manufacturing methods. Typical motors use a mixture of powdered chemicals and liquid agents to bond and cure these combinations into reactive fuels. Ammonium Perchlorate, NH_4O_4, is the most common and among the most reliable and readily available oxidizers on the market. There are varied sizes for purchase, ranging from 90 microns in diameter to 400 and greater. Metal powders and additives such as magnesium, aluminum and beryllium act as both a catalyst and a fuel. Other elements of motors such as binders and plasticizers bound all these ingredients together. Motors are commonly stored in metal tubular casing divided into a desired number of segments called grains, where they will burn upon ignition and expel their gases out of an open end fixed with a nozzle. There are many possible combinations to formulate and test, but there are restrictions given from the launch vehicle system based on materials selection. Two equations can be used to approximate the expansion of a motor casing, for both length and diameter, during a propellant burn due to chamber pressure:

$$\Delta L = \frac{pLD}{4Ed}(1 - 2v) \tag{1}$$

$$\Delta D = \frac{pD^2}{4Ed}\left(1 - \frac{v}{2}\right) \tag{2}$$

where ΔL is the expansion longitudinally in inches, ΔD is the expansion diametrically, p is the chamber pressure in pounds/in^2, L is the length of the case in inches, D is the inner diameter of the case in inches, E is Young's Modulus of Elasticity measured in kilopound/in^2 (or 1000 lb/in^2), d is the casing thickness in inches, and v is Poisson's ratio specific to the casing material.

The Ares team and the Society of Aeronautics and Rocketry have purchased 10-foot-long 6 ID x 6.5" OD tubing composed of 6061-T6 Aluminum. This type of aluminum is incredibly strong, relatively lightweight and is commonly used as motor casings for commercially produced amateur rocket motors. A minimum diameter rocket is a type of rocket which uses the walls of the rocket airframe as the casing for the motor, whereas traditional collegiate and amateur building methods use a motor mount to fit a motor which has a smaller diameter than the airframe. High-altitude rockets are incredibly efficient when they are built minimum diameter, therefore maximizing the potential amount of solid propellant.

There are several material characteristics of metals to pay attention to: Young's Modulus of Elasticity, thickness, and Elongation at Break. Elongation at Break is a characteristic in metals that defines the total longitudinal stretch allowed before the material breaks, and is given as a percentage of the original length. For 6061-T6 Aluminum, the elongation at break is 12%, the Young's Modulus is 10,000 ksi, Poisson's Ratio is 0.33 and the thickness is ¼" in this particular case.

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With these material characteristics known, we can find the maximum chamber pressure permissible the solid propellant can produce before fracturing the rocket airframe. By rewriting equation (1) with respect to *p* we obtain:

$$p = \frac{4\Delta LEd}{(1-2v)LD} \tag{3}$$

Preliminary designs for Ares 1-S have the airframe at 107 inches long, but this number must not be the *L* in the following equations. The area of the airframe exposed to the chamber pressure and acted as the casing will be your *L*. For example, the length *L* is 72 inches and thus its elongation at break value is 8.64. Using the elongation at break of *L* is a good initial safety measure for ensuring the casing does not fracture or break from the chamber pressure. Now, by adding the corresponding values we obtain from (3):

$$\frac{4*8.64*10{,}000*.25}{(1-2*.33)*72*6} = 588.235 \tag{4}$$

With this 6061-T6 casing and the motor dimensions given above, the maximum chamber pressure allowed is 588.235 lb/in^2. Using 588.235 as *p*, diametrical expansion of the casing can be approximated as:

$$\frac{588.235*6^2}{10{,}000}\left(1-\frac{.33}{2}\right) = .72 \tag{5}$$

With this 6061-T6 casing and the max pressure at 588.235 lb/in^2, the diametrical expansion is .72 inches, making the maximum diameter 7.22 inches during burn. The values for maximum pressure allowed will change depending on the length of the motor casing, and so will the elongation of break

(ΔL). Examine the chart below to note the differences between thicknesses, casing length and elongation;

6061-T6 Aluminum, ¼” thickness

Length	ΔL	Max psi
12”	1.44”	588.235
24”	2.88”	588.235
36”	4.32”	588.235
48”	5.76”	588.235
60”	7.2”	588.235
66”	7.92”	588.235
72”	8.64”	588.235

6061-T6 Aluminum, ½” thickness

Length	ΔL	Max psi
12”	1.44”	1176.471
24”	2.88”	1176.471
36”	4.32”	1176.471
48”	5.76”	1176.471
60”	7.2”	1176.471
66”	7.92”	1176.471
72”	8.64”	1176.471

Figure 2: A visual for the relationship between elongation at break and max internal pressure

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These changes in length and diameter are only applicable if the pressure is applied outwardly and directly to the internal of the case walls. If a motor is cast, molded and bonded directly to the airframe walls, then these material expansions are expected. Liners are a possible solution for dispersing this internal pressure. However, inducing a chamber pressure that is greater than the strength of the casing, would present a danger to safety. A liner is a sleeve or an outer casing bonded directly to the grain. Materials and thicknesses for liners vary, depending on the needs and thus stresses of the launch vehicle, and aid in loading solid propellants into the casing.

An interesting trend develops based on these numbers. The maximum internal psi before fracture or breakage is the same for a certain uniform thickness despite the length of the motor casing. To safely ensure a higher chamber pressure is achieved without failure, increase the thicknesses of the case walls and or finding a material with a higher Young's Modulus or lower Poisson's ratio.

NASA employee Chris Snyder created an interactive program called CEARUN, Chemical Equilibrium with Applications, where users can calculate thermodynamic and chemical properties of propellants. On CEARUN's main page (https://cearun.grc.nasa.gov/), select the "rocket" option and input a 4-letter word for the name of the file. Next, add the max psi found from the equations above, and convert 588.235 to 4.056 for one of the slots under Option 2. Ares 1-S will be a solid propellant rocket, so selecting the solid propellant option will progress the user to the fuels selection page. The following paragraph will go into further depth on fuel selection and how to implement the values produced by CEARUN.

The preliminary design for Ares 1-S has the dry mass of the launch vehicle at 95 pounds. A general rule of thumb for high-performance and efficient rockets suggest the propellant account for half of the total pad weight of the rocket. If the dry mass is 95 pounds, then aiming for a propellant weight of 100 pounds is acceptable. Continuing from the last paragraph on fuels selection using CEARUN, although the exact propellant mixture is not yet known, NH_4ClO_4 will serve as the main oxidizing compound. There are general recipes found online for high-performance applications including military missiles and the SRBs used for the Space Shuttle. Solid propellants, especially at the collegiate and amateur level, rarely burn for more than half a minute due to limited resources and obvious size restrictions.

A common fuel and binder are available for selection on CEARUN, Al(cr) and PBAN. PBAN, polybutadiene acrylonitrile, is a copolymer used as a binding agent that is slower to cure but less toxic and lower cost than HTPB, hydroxyl-terminated polybutadiene. Using the following composition of Al(cr), PBAN and NH_4ClO_4, and assuming these three compounds take up the full 100lb estimate (this would not be the case for the actual motor composition), convert it into kilograms for entry in CEARUN;

Compound	Abbreviation	Pounds	Kilograms
Ammonium Perchlorate	NH_4ClO_4	69.6	31.571
Polybutadiene Acrylonitrile	PBAN	12.04	5.61
Aluminum (powder)	Al(cr)	16	7.258

Figure 3: The CEARUN composition of the solid propellant

Viewing the output file below, take note of the circled values in red:

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          THEORETICAL ROCKET PERFORMANCE ASSUMING EQUILIBRIUM

       COMPOSITION DURING EXPANSION FROM INFINITE AREA COMBUSTOR

Pin =    588.2 PSIA
CASE = Ares4184

            REACTANT                    WT FRACTION      ENERGY      TEMP
                                         (SEE NOTE)     KJ/KG-MOL      K
            PBAN                         0.1233000     -63220.000   298.150
            NH4CLO4(I)                   0.7128000    -295767.000   298.150
            AL(cr)                       0.1639000          0.000   298.150

O/F=    0.00000  %FUEL=100.000000  R,EQ.RATIO= 1.571894  PHI,EQ.RATIO= 0.000000

                CHAMBER   THROAT     EXIT
Pinf/P            1.0000   1.7285   40.016
P, BAR            40.557   23.464   1.0135
T, K             3490.65  3310.39  2327.00
RHO, KG/CU M    4.0934 0 2.5202 0 1.6037-1
H, KJ/KG        -1872.37 -2398.09 -4836.60
U, KJ/KG        -2863.18 -3329.13 -5468.60
G, KJ/KG        -35063.0 -33874.7 -26962.7
S, KJ/(KG)(K)     9.5084   9.5084   9.5084

M, (1/n)          29.292   29.563   30.614
MW, MOL WT        27.071   27.238   28.011
(dLV/dLP)t      -1.02568 -1.02057 -1.00236
(dLV/dLT)p        1.4496   1.3744   0.0000
Cp, KJ/(KG)(K)    4.3020   3.9333   0.0000
GAMMAs            1.1273   1.1293   0.9976
SON VEL,M/SEC     1056.9   1025.4    794.0
MACH NUMBER        0.000    1.000    3.066

PERFORMANCE PARAMETERS

Ae/At                      1.0000   6.6182
CSTAR, M/SEC               1569.4   1569.4
CF                         0.6534   1.5514
Ivac, M/SEC                1933.4   2694.4
Isp, M/SEC                 1025.4   2434.8
```

Figure 4: The CEARUN chart, with the red circled values being the ones focused on in this paper

Without inputting sub or supersonic area ratios or chamber temperature, CEARUN will give a reliable estimate for these values. The circled values will be needed for computing the mass flow rate later.

BurnSim3 is a very helpful program for examining grain geometries for general propellant compositions, and approximating key characteristics of such propellants. There are two propellant types available for the user, but in order to simulate as accurately as possible, the propellant of choice listed above (Al, PBAN and NH_4ClO) will be created within the program. Chapters 12 and 13 of Rocket

Propulsion Elements, by George Sutton and Oscar Biblarz, list basic properties of many high-grade solid propellants including the propellant of choice. The specific impulse, I_{sp}, temperature in Kelvin, *K*, density in lb/in^3, and the *n*, burning rate exponent, are given. These properties can be entered directly to create a new propellant labeled "PBAN / AP / Al". However, the burning rate coefficient, *a*, is not given directly and must be found to use BurnSim3 effectively and get usable results. The equation for burning rate, in/sec or mm/sec is given by:

$$r = ap^n \tag{6}$$

Burning rate is *r*, *p* is the chamber pressure, and *n* is the pressure exponent. In the text, Rocket Propulsion Elements, there is a range given for the burning rate from 0.25 to 1 at 1000 psia. A linear or proportional relationship cannot be made to compare 1000 psia to the max chamber pressure of 588.235 psi, for the behavior of solid propellants and their reactions represents more exponential forms. The graph is a log-log plot and shows the number 0.28, which roughly represents the burning rate at 588.235 psia for an ambient temperature around 70 degrees Fahrenheit:

$$0.28 = a * 588.235^{0.28} \tag{7}$$

From (7) we get: a = 0.04.

Now, use *a* as the Burning Rate Coefficient for the "PBAN / AP / Al" custom propellant in BurnSim3 along with the other characteristics to approximate thrust and chamber pressure values for the propellant of choice.

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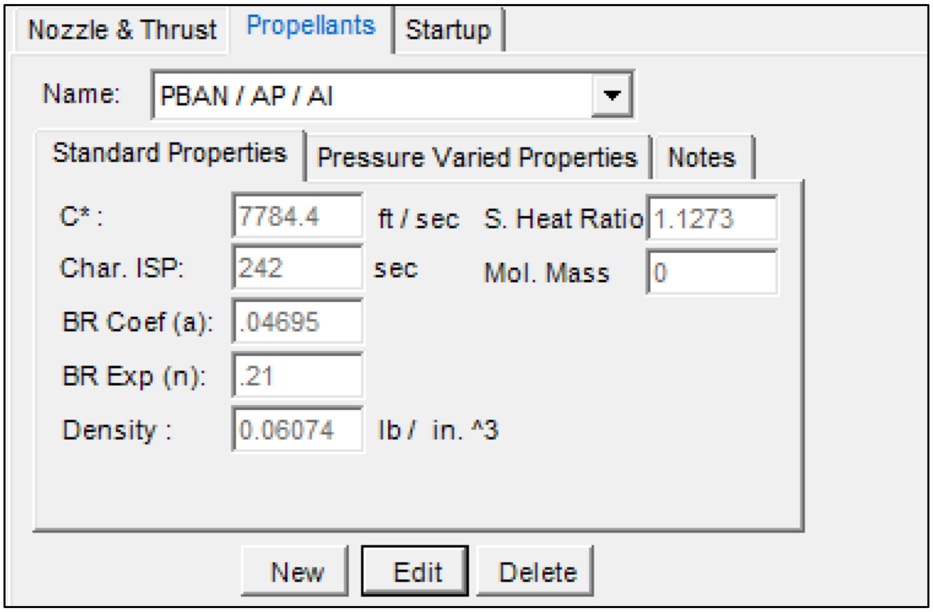

Figure 5: The custom propellant properties on BurnSim3. Note that molar mass and C are negligible. "Char. ISP" is the characteristic impulse of this propellant compound.*

BurnSim3 is now used to create and simulate burns with various grain geometries, lengths and diameters to find an arrangement whose max chamber pressure does not exceed 588.235 psi (4055737.556 Pa). After multiple trials and simulation runs, this was found to be the best propellant arrangement.

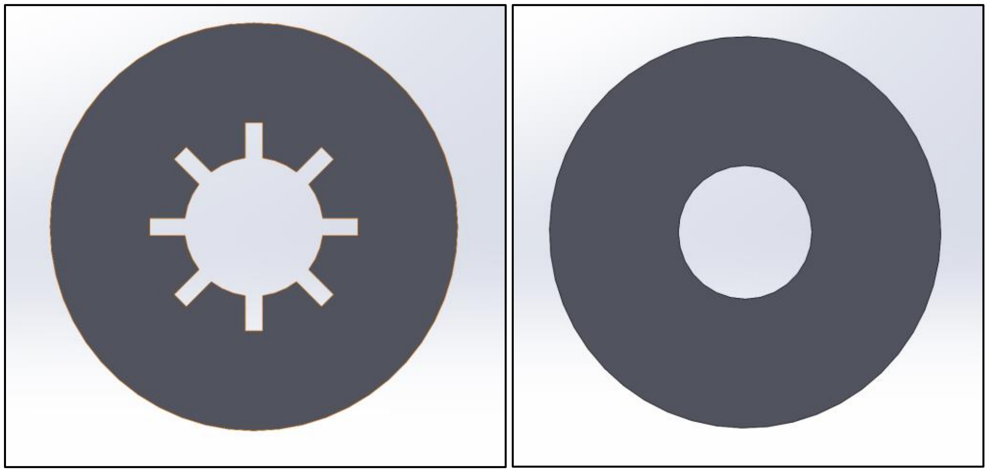

Figure 6: The left grain geometry is an 8-point star "finocyl". The star pattern extensions are .25" thick and .5" long. The right grain geometry is BATES, the most common among commercially manufactured propellants for amateur use. The cores for both geometries are 2".

There is a total of six grains; five 6"-long BATES and one 3.125"-long finocyl. The liner for the grains is carbon fiber or carbon phenolic tube. The aft grain is pressed against the engine block plate, therefore has a surface inhibited from burning. The next four grains are BATES, and the grain adjacent to the nozzle is the finocyl. The grains are spaced out with 1/8" rubber O-rings. The purpose for these O-rings is to expose the base surfaces during the burn, which adds to the pressure and propellant flow. The finocyl is placed next to the nozzle to increase the initial *Kn*, and ensure a satisfactory burn.

The area of the nozzle throat contributes to the chamber pressure and *Kn* of the propulsion system. Many diameters were inputted into BurnSim3 and the most efficient throat diameter was 1.68 inches, which equates to a max chamber pressure of 587.8 psi, under the 588.235 psi limit of the casing.

Kn is a term describing the ratio of the burning surface area of the propellant, all uninhibited surfaces, to the area of the throat, and it correlates to the chamber pressure. The max *Kn* of a motor occurs with the

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maximum chamber pressure and so forth. However, just like the burning rate equation, this relationship is not proportionally linear. The equation for *Kn* is as follows:

$$Kn = \frac{A_p}{A_t}\,. \tag{8}$$

Using the dimensions from the grain geometry in Figure 6 we obtain:

$$\frac{494.6215\ in^2}{2.217\ in^2} = 223$$

It is possible to calculate the Mach exit speed based upon chamber and exit conditions of temperature and pressure. When prompted for specific heat ratio, γ, we use the value found within the chamber state. This value is 1.1273, given from Figure 4.

As mentioned above, the Mach speed of the exhaust gases from the flow of propellant can be evaluated from knowing the chamber pressure and the ambient pressure, or desirable nozzle exit pressure;

$$\frac{p_e}{p_c} = (1 + \frac{\gamma - 1}{2} M^2)^{-\frac{\gamma}{\gamma - 1}} \tag{9}$$

The left side of the equation contains p_e, exit or ambient pressure at sea level which is 14.7 psi, and p_c, chamber pressure which is 588.235 psi. Any unit for pressure is acceptable here since it represents only a ratio and Mach number, *M*, and specific heat ratio, γ, is dimensionless. Recalling from the circled values produced by CEARUN, the Mach can be estimated based on the parameters for system pressure. The roots of the equation can be used to find *M*, although values greater than 10 would be discarded, for rockets traveling faster than Mach 10 at the collegiate level are highly unlikely. The only root given from the rewritten equation is therefore:

$$M= 2.853 \tag{10}$$

Using temperature, the Mach exit speed can be calculated as well. The circled values from Figure 4 give the chamber, throat and exit temperature in Kelvin but only the chamber and exit are needed in the assumption that isentropic flow is occurring with a choked nozzle. We have:

$$\frac{T_e}{T_c} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma - 1}} . \tag{11}$$

Solving for the roots of the equation will lead to a Mach exit value like the equivalent equation under pressure:

$$M=2.803 \tag{12}$$

It is not surprising that there are two different Mach exit numbers. Indeed the pressure numbers used are given from the ideal conditions, and they are not the result of the actual propellants. When CEARUN calculates, it is much easier to fill out as an infinite area combustor versus a finite combustor since nozzle measurements are not inputted.

The area expansion ratio is another relation used to find the Mach exit number. This equation can be rewritten to find the exit area of the nozzle. This exit area can be used to verify the nozzle exit diameter result in BurnSim3, and if they are remotely equal, then that Mach exit value is realistic and verifies the data for the custom propellant “PBAN / AP / Al”. We use the higher value for Mach exit between the temperature and pressure Mach relation equations given from (10):

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$$\left(\frac{A_e}{A_t}\right)^2 = \frac{1}{M^2} * \left(\frac{2+(\gamma-1)M^2}{(\gamma+1)}\right)^{\frac{\gamma+1}{\gamma-1}} \tag{13}$$

We rewrite the equation to find the A_e, area of the nozzle exit:

$$A_e = A_t * \sqrt{\frac{1}{M^2} * \left(\frac{2+(\gamma-1)M^2}{(\gamma+1)}\right)^{\frac{\gamma+1}{\gamma-1}}} \tag{14}$$

Substituting 2.853 for M, 2.217 in^2 for A_t, and 1.1273 for γ:

$$A_e = 15.1\ in^2 \tag{15}$$

Now we convert this area into a diameter, d, in inches;

$$d_e = 2\sqrt{\frac{A_e}{\pi}} \tag{16}$$

Substituting 15.1 for A_e:

$$d_e = 2\sqrt{\frac{15.1}{\pi}} \quad , \qquad d_e = 4.39 \text{ in.} \tag{17}$$

Now, the nozzle throat and exit diameter are found, 1.68 and 4.39 inches respectively. The next step, after the previous solutions are found, would be to find the mass flow rate of the nozzle at the throat with the propellant of choice. Due to the assumed isentropic flow of the converging-diverging nozzle, the use of chamber values is permitted. The equation for mass flow rate is given below:

$$\dot{m} = \frac{A_t p_c}{\sqrt{T_c}} * \sqrt{\frac{\gamma}{R}} * \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \tag{18}$$

where A_t is the throat area in meters, 0.00143 m^2 from 2.217 in^2, p_c is the chamber pressure in Pascals not psi, 4054117.288 Pa from 588.235 psi, T_c is the chamber pressure in Kelvin, 3490.5K from the Chamber column in CEARUN, R is the gas constant for this propellant. The universal gas constant is 8314 J/kmol*K, and the molecular weight in the chamber, displayed as M (1/n) from CEARUN is 29.292 kg/kmol. We divide 8314 by 29.292 and the gas constant for the propellant, R, is 283.832 J/kg*K. The specific heat ratio, γ, is again 1.1273:

$$\dot{m} = \frac{0.00143 * 4054117.288}{\sqrt{3490.5}} * \sqrt{\frac{1.1273}{283.832}} * \left(\frac{1.1273+1}{2}\right)^{-\frac{1.1273+1}{2(1.1273-1)}} \tag{19}$$

$$\dot{m} = 3.693 \text{ kg/s} = 8.142 \text{ lb/s} \tag{20}$$

With the mass flow rate above, the velocity of the exhaust gases can be found:

$$V_e = M * \sqrt{\gamma * R * T_e}\ , \tag{21}$$

where M is the Mach exit number (should always be a supersonic value), γ is the specific heat ratio at the exit which is 0.9976 given by CEARUN, R is the propellant gas constant from earlier, and T_e is the exit temperature in Kelvin;

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$$V_e = 2.853 * \sqrt{0.9976 * 283.832 * 2327} \tag{22}$$

$$V_e = 2315.84 \text{ m/s} = 7597.9 \text{ ft/s}. \tag{23}$$

The exit velocity and mass flow rate can be used to find peak thrust with the given equation:

$$F = (\dot{m} * V_e) + (p_e - p_a) * A_e \tag{24}$$

$$F = (3.693 * 2315.84) + (101352.9 - 101352) * 0.00977 \tag{25}$$

$$F = |-13381.71| = 8552.43 \text{ Newtons} = 1922.66 \text{ lbs-f of peak thrust} \tag{26}$$

The last critical measurement for the nozzle design is the chamber volume in the converging section. The Rocket Propulsion page of the Rocket & Space Technology website lists common converging-throat half angles, *Θ*, from 20 to 45 degrees as well as diverging-throat half angles, *α*, from 12 to 18 degrees. One of the most common diverging-throat half angles is 15 degrees, which will be used for preliminary nozzle design. The nozzle "block", a single converging-diverging nozzle machined out of a single block of material, itself will be made of 6061-T6 Aluminum. Ablative resins will be researched, created and tested further to protect the nozzle. There will be a small lip machined into the nozzle block to prevent the grains from falling into the chamber while in the upright position, but this will be neglected when computing chamber volume and can easily be considered and factored into future designs. The chamber volume can be found using an integration method.

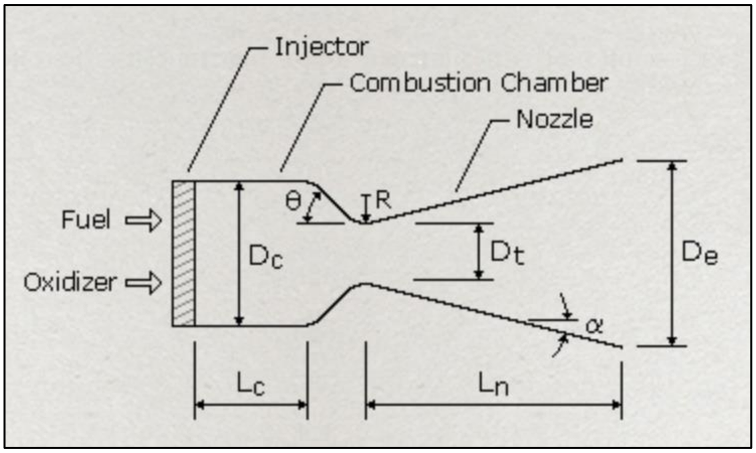

Figure 7: A diagram of a converging-diverging nozzle with considerations to half angles near the throat. Image courtesy of http://www.braeunig.us/space/propuls.htm

The initial width of the chamber will be the same as the width of the propellant grain, in this case 5.86" diameter. The throat diameter is the same before at 1.68". Two shapes can be made to evaluate the chamber volume, a rectangle (A) beginning along the longitudinal axis and length of the radius of the throat, and a triangle (B) beginning from the throat radius to the chamber radius. The throat has a radial fillet equivalent to its own radius, but can be neglected for the following mathematical methods to show application.

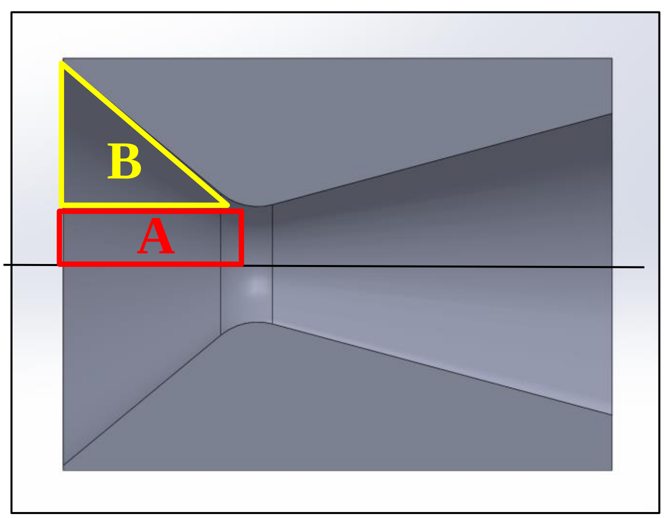

Figure 8: A visual representation of integrating two separate areas to find the chamber volume

The initial radius of the chamber is 2.93”, and the radius of the throat is 0.84”. The longitudinal length of the triangle, x_L, is not known since there is a range of angles that can be used. To find the volume from integration, the equation of the line (hypotenuse) of triangle B needs to be found. The *y* axis is the parallel face to the nozzle exit and chamber entrance. The *x* axis is the longitudinal axis:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{.84 - 2.93}{x_L - 0} \tag{27}$$

$$\frac{\Delta y}{\Delta x} = \frac{-2.09}{x_L} \tag{28}$$

Above, the radii were used to evaluate the slope of the line. The value of x_L is obtained by using equation (29) below:

$$y - y_1 = m * (x - x_1)\ . \tag{29}$$

We can substitute to obtain:

$$y - 2.09 = \frac{-2.09}{x_L} * (x - 0)\ . \tag{30}$$

Evaluating the length x_L for each degree of inclination:

$$\tan\theta = \frac{2.09}{x_L}\ , \tag{31}$$

we obtain:

$$x_L = \frac{2.09}{\tan\theta}\Big|_{20}^{45} \ . \tag{32}$$

Degree of Inclination (Θ)	**Resulting Length (x_L) in inches**	**Degree of Inclination (Θ)**	**Resulting Length (x_L) in inches**
20	5.74	33	3.22
21	5.44	34	3.1
22	5.17	35	2.98
23	4.92	36	2.88
24	4.69	37	2.77
25	4.48	38	2.68
26	4.29	39	2.58
27	4.1	40	2.49
28	3.93	41	2.4
29	3.77	42	2.32
30	3.62	43	2.24
31	3.48	44	2.16
32	3.34	45	2.09

Figure 9: Degree of inclination in the converging section versus the longitudinal length

The following integral, using the washer method, is for the volume of triangle B around the longitudinal axis of the nozzle only and not for rectangle A. The volume for rectangle A can be calculated separately and added in the sum of the following integral:

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$$V_B = \pi \int_0^{x_L} \left(\left(\frac{-2.09 * x}{x_L} + 2.09 \right)^2 - 0.84^2 \right) dx \tag{33}$$

The first squared term of the integral is the equation of the line in triangle B minus the square of the height of the rectangle. Integrating the above expression we have:

$$V_B = \pi * \frac{x(43681x^2 - 1310431x_L x + 109875{x_L}^2)}{30000{x_L}^2} \bigg|_0^{x_L} . \tag{34}$$

Evaluate the sum of the integral for each length x_L found in the tables above.

Length, x_L (inches)	**$\pi * \int_0^{x_L} V_B$**	**Volume of triangle B (inches^3)**
5.74	π * 4.31	13.54
5.44	π * 4.08	12.82
5.17	π * 3.88	12.19
4.92	π * 3.69	11.59
4.69	π * 3.52	11.06
4.48	π * 3.36	10.56
4.29	π * 3.22	10.12
4.1	π * 3.08	9.68
3.93	π * 2.95	9.27
3.77	π * 2.83	8.89
3.62	π * 2.72	8.55
3.48	π * 2.61	8.2
3.34	π * 2.51	7.89
Length, x_L (inches)	**$\pi * \int_0^{x_L} V_B$**	**Volume of triangle B (inches^3)**
3.22	π * 2.42	7.6
3.1	π * 2.33	7.32
2.98	π * 2.24	7.04
2.88	π * 2.16	6.79
2.77	π * 2.08	6.53
2.68	π * 2.01	6.31
2.58	π * 1.94	6.09

2.49	π * 1.87	5.87
2.4	π * 1.8	5.65
2.32	π * 1.74	5.46
2.24	π * 1.68	5.28
2.16	π * 1.62	5.09
2.09	π * 1.57	4.93

Figure 10: The volume of triangle B given from the integration of its line equation

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To find the total volume, calculate the area of rectangle A (the cylinder of the nozzle along the longitudinal axis) using the simple integral:

$$V_A = \pi \int_0^{x_L} 0.84^2 dx \tag{35}$$

Length, x_L (inches)	**Volume of triangle A (inches^3)**	**Total Volume (A+B) (inches^3)**
5.74	12.72	26.26
5.44	12.06	24.88
5.17	11.46	23.65
4.92	10.91	22.5
4.69	10.4	21.46
4.48	9.93	20.49
4.29	9.51	19.63
4.1	9.01	18.69
3.93	8.71	17.98
3.77	8.36	17.25
3.62	8.02	16.57
3.48	7.71	15.91
3.34	7.4	15.29
3.22	7.14	14.74
3.1	6.87	14.19
2.98	6.61	13.65

2.88	6.38	13.17
2.77	6.14	12.67
2.68	5.94	12.25
2.58	5.72	11.82
2.49	5.52	11.39
2.4	5.32	10.97
2.32	5.14	10.6
2.24	4.97	10.25
2.16	4.79	9.88
2.09	4.63	9.56

Figure 11: A table depicting the total chamber volume given its length

The above tables provide good insight into the chamber volumes given specific half-angles and their corresponding lengths. Currently there is not enough concrete information in this paper to find the most efficient volume. That will come with motor manufacturing and testing to collect data, specifically for nominal chamber and throat conditions.

However, there are some similarities and surprising ratios that can be helpful. There is an equation that yields desirable chamber volume given chamber length and the converging half angle, Θ:

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$$V_c = \frac{\pi}{24}\left[\left(6 * L_c * {D_c}^2\right) + \frac{D_c^3 - D_t^3}{\tan\theta}\right] \tag{36}$$

Where L_c is the chamber length, and Θ is the converging half angle. Both values are not known for certain, since they are dependent on each and the chamber volume, V_c. But the chamber diameter, D_c, has been selected to be 5.86”. By looking at the table and performing quick calculations, the following ratios occur for every length, angle and volume:

$$L_c = 0.2185V_c \tag{37}$$

$$\theta = 4.571V_c \tag{38}$$

Using the ratios to fill in L_c and Θ with V_c:

$$V_c = \frac{\pi}{24}\left[(6 * 0.2185V_c * 5.86^2) + \frac{5.86^3 - 1.68^3}{\tan(4.571V_c)}\right] \tag{39}$$

There are two plausible roots to this expression. However, one lies outside of the total chamber volume greater than 30 on the x-axis and the steep is so critical that it registers as a singularity on a TI-83. The other root is at 21.843, and by comparing with the tables above, the converging half-angle of 25 degrees has a chamber volume of 21.46 in^3. The nozzle dimensions are on the next page and are subject to change.

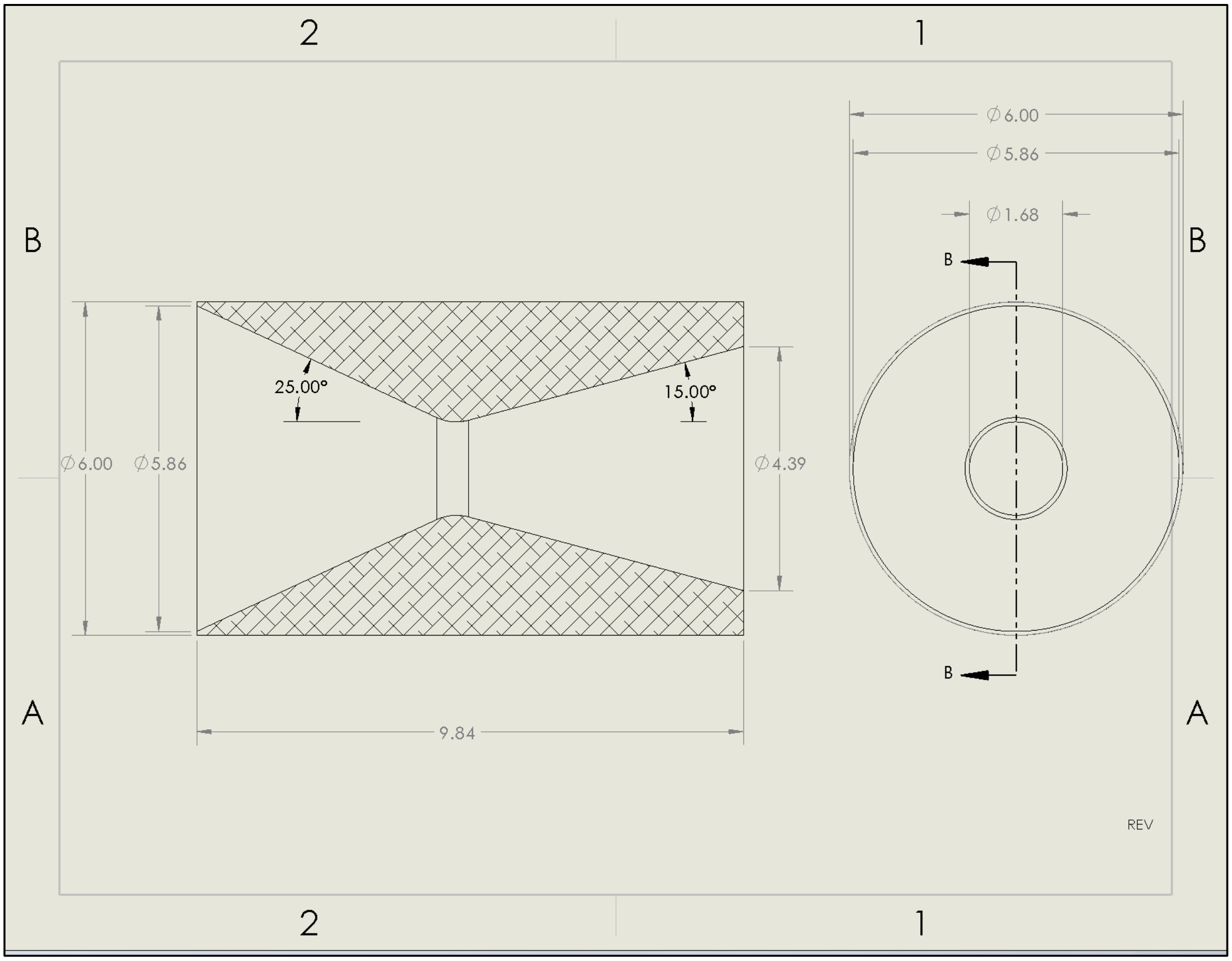

Figure 12: The first complete design of the Ares 1-S nozzle.

https://scholarcommons.usf.edu/ujmm/vol9/iss1/2
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Conclusion and Recommendations

If the calculations and mathematical methods have been applied correctly, then with a solid propellant composed of roughly 72% Ammonium Perchlorate, 16% Aluminum and 12% PBAN reaching a max contained combustion pressure of 588.235 psi, there will be an isentropic and resulting supersonic flow in the nozzle with the dimensions specified in Figure 12. There is a suspected exit Mach number of 2.85, and a flow rate of 8.1 lbs/s but these were dependent upon the thermal and chemical properties. The real propellant will have several more ingredients and its properties cannot be known until an effective test and data are received and interpreted. The area of the throat and exit, and their subsequent expansion ratio are subject to change and likely will reflect the differences in chamber pressure, Temperature and propellant flow rate. A recommendation to anyone else performing this analysis is to have thermal properties data on hand, and to compare between types and compositions of propellants. Propellants with lower combustibility or slower combustion rates can contribute to a lower internal pressure and thus change every dimension of the nozzle. Another recommendation would be to invest in BurnSim3 wo use, for it is hard to formulate the throat diameter and area using only pressure and temperature. The area expansion ratio can be found, but many equations are dependent on the throat, and exit, of the nozzle. BurnSim3 also takes into consideration the propellant properties and can estimate the average thrust, total impulse, burn time and many other important motor features that were not covered in this paper.

NOMENCLATURE

Symbol	Meaning	Unit(s)
a	Burn Coefficient	N/A
$A_{t,e}$	Area of the nozzle throat and exit	Inches^2
d	Material thickness	Inches
d_e	Exit Diameter	Inches
E	Young's Modulus of Elasticity	Kilo-pounds-per-square-inch (ksi)
F	Peak Thrust	Pounds-force (lb(s)-f)
L_c	Length of the chamber, along the nozzle (longitudinal) axis	Inches
M	Exit Mach number	N/A
$\dot{m}$	Mass Flow Rate	Lb/s
n	Burn Exponent	N/A
p	Max chamber pressure	Pounds-per-square-inch (psi)
$p_{c,t,e}$	Pressure at the chamber, throat and exit of the nozzle	Pounds-per-square-inch (psi)
r	Burn Rate	Inches/sec
$T_{c,t,e}$	Temperature at the chamber, throat and exit of the nozzle	Kelvin (K)

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v	Poisons Ratio	N/A
$V_{A,B,C}$	Volume of the chamber for parts A, B & total (C)	Inches^3
V_e	Exhaust exit velocity	Feet/sec
x_L	Nozzle axis (longitudinal) length	Inches

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Programs

- BurnSim3
- SolidWorks 2016
- MatLab R2017b
- OpenRocket
- Integral Calculator https://www.integral-calculator.com/
- Graphing calculator https://www.desmos.com/calculator
- CEARUN https://cearun.grc.nasa.gov/

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