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Calculating the Solar Energy of a Flat Plate Collector

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Calculating the Solar Energy of a Flat Plate Collector

Abstract

The amount of solar energy that could be obtained by a flat plate solar collector of one square meter dimension is calculated in three different locations: Tampa FL, Fairbanks AL, and Pontianak Indonesia, considering the varying sunset time for each day of the year. The results show that if the collectors are placed near the equator, more total energy could be obtained. In fact, by placing a solar collector in Pontianak, Indonesia 12.42% more solar energy can be obtained than by placing it in Tampa and 96.9% more solar energy than Alaska.

Keywords

solar energy, renewable energy, solar energy collector

PROBLEM STATEMENT

The objective of this paper is to calculate the amount of solar energy that could be achieved in Tampa for one year and compare it to two different locations. This has been achieved by considering a flat plate solar collector placed in three different locations and calculating and comparing their absorbed energy through a year.

MOTIVATION

The demand for energy has been growing exponentially in the recent years along with the increase in global population. Based on a report by the US Energy Information Administration, 90% percent of the United States and 85% percent of global energy needs are satisfied by non-renewable resources that are quickly diminishing (U.S. Energy Information Administration; 2013). Although experts predict that coal and natural gas resources are expected to last another two centuries, petroleum, which accounts for over 50% of energy needs in the U.S. will be depleted within 30-40 years in the U.S. (Greiner and Semmler; 2008). Figure 1compares the production and consumption of various energy sources in the United States in 2012. The world is facing an energy crisis that can only be resolved by finding and utilizing clean, renewable energy sources (Combs; 2008).

This paper considers the possibility of solar power as a future energy resource. The use of solar energy has been increasing steadily, becoming the fastest growing power technology in the world. Photovoltaic (PV) cell solar collectors are able to absorb energy with almost no carbon emissions by converting solar energy into hydrogen fuels (Reddy; 2012). However, solar panels

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Keywords: Solar energy, Renewable energy, Solar energy collector

are not effective everywhere. In areas with more sunlight, more solar energy can be obtained. Due to the tilt and rotation of the earth, the amount of energy from the sun reaching a solar collector decreases as the latitude from the equator increases. This also causes changes in length of day and the temperature of the location. The objective of this paper is to calculate the efficiency of solar energy collectors in three different locations throughout the world. Ideal places to set up solar collectors can be determined by comparing the energy collected by each panel throughout a year in these varying locations. This information is useful for governments, businesses, and home-owners to establish renewable and more efficient energy resources for the future.

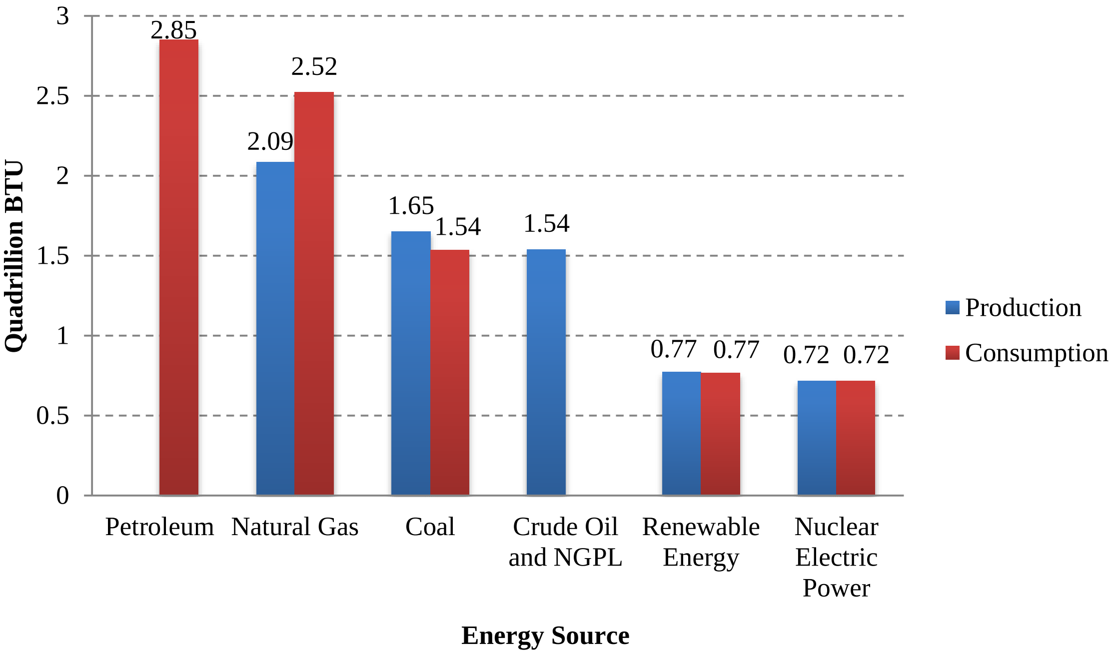

Figure 1: US Energy Consumption and Production in 2012

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

To determine the amount of solar energy absorbed by a solar collector, several key pieces of information are needed. The days are assumed to be perfectly clear for this research. The amount of solar energy reaching earth from the sun on a daily basis is given by the following formula:

$$S = IE_o \tag{1}$$

where I equals 1000 W/m^2 on the earth's surface and E_o is a correcting factor accounting for Earth's elliptical orbit. The value of E_o which depend on the day of the year, n ($n = 1$ for January 1st), is given by:

$$E_o = 1 + 0.033\cos\left(\frac{2\pi n}{365}\right) \tag{2}$$

where the cosine term is in radians. The total energy per unit of area falling onto a solar collector in one day can be computed by:

$$E = 2\int_0^{\tau} S\cos(\theta_z)\,dt \tag{3}$$

where τ is the sunset time and the two doubles this value to account for the entire period of the day ($t = 0$ corresponds to solar noon), θ_z is the zenith angle given by

$$\cos\theta_z = \cos(\varphi)\cos(\omega)\cos(\delta) + \sin(\varphi)\sin(\delta) \tag{4}$$

where φ is the latitude of the collector's location, ω is the hour-angle, which corresponds to t by

$$\omega = 0.2618\,t \tag{5}$$

and δ is the angle of the Earth's declination, in radians, given by

$$\delta = \frac{\pi}{7.6759}\sin\left(\frac{2\pi(n + 284)}{365}\right) \tag{6}$$

Combining equations (1), (3) and (4) and simplifying, the total absorbed energy can be written as

$$E = 2IE_o \left[\cos(\varphi)\cos(\delta)\int_0^{\tau} \cos\,(\omega)\ dt + \sin(\varphi)\sin(\delta)\int_0^{\tau} dt\right] \tag{7}$$

The sunset time can be solved for by substituting equation (5) into equation (4) after setting $\cos\theta_z = 0$. Therefore, an equation to find τ can be formulated as the following

$$\tau = cos^{-1}\left(\frac{-\sin(\varphi)\sin(\delta)}{\cos(\varphi)\cos(\delta)}\right) \tag{8}$$

Finally, by solving the two integrals in (7), the solution can be derived as

$$E = 2IE_o \left[\cos(\varphi)\cos(\delta)\frac{\sin(0.2618\,t)}{0.2618} + \sin(\varphi)\sin(\delta)\,(\tau)\right] \tag{9}$$

This equation represents the solar energy obtained each day by changing values of n. An excel spreadsheet calculator was set up to calculate E for all values of n ranging from 1 to 365, and for three locations with different latitudes as presented in Table 4 of the appendix.

DISCUSSION

The objective of this project was to calculate the amount of solar energy that could be obtained by a square meter solar collector in three locations throughout a year. The results are presented in Figure 2.

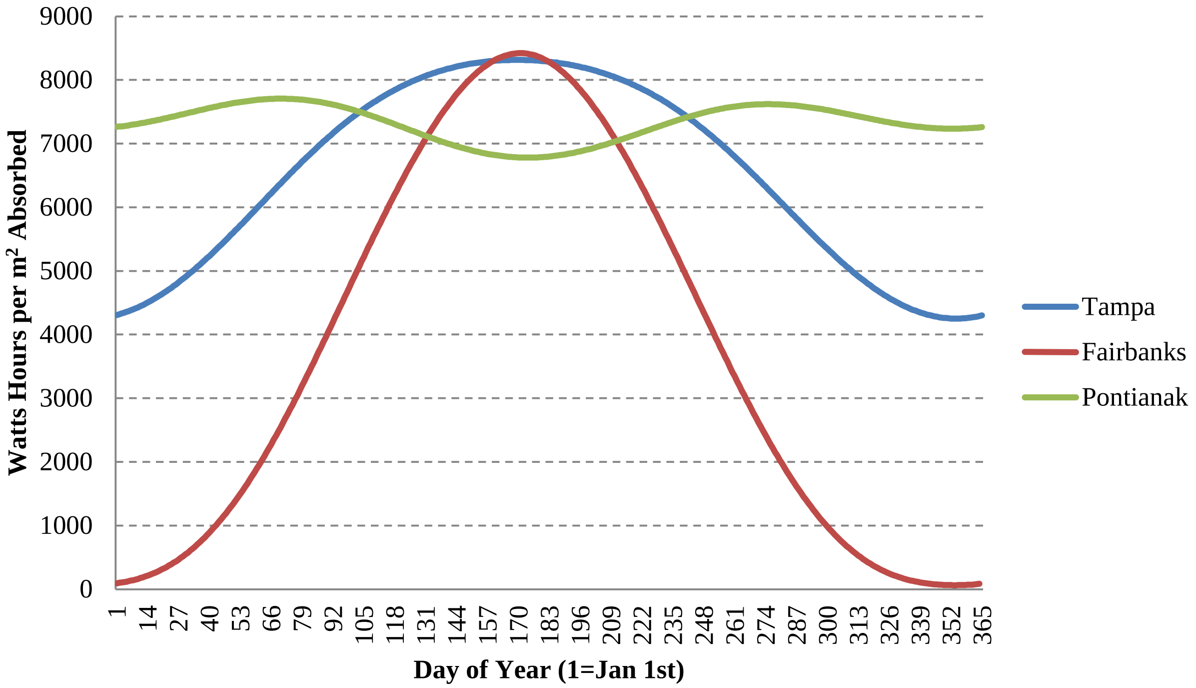

Figure 2: Possible Energy obtained each day throughout one year for the three locations

As expected, the results verify that the panels on the equator can absorb the most sunlight over the course of one year, while the panels near the poles absorb the least total energy. This is due to the tilt of the earth and the decrease in temperature that accompanies this tilt. As the incidence angle increases, the light has to travel more through the atmosphere over a greater distance and therefore the panel will absorb less total energy. The possible maximum value for the Tampa collector, with a latitude location of 27.947, is around the Summer Solstice, on June 18, with a total energy of 8.315 kW-hrs/m^2 (Solstice; 2013). The possible minimum value is on December 20, a day from the Winter Solstice, with a value of 4.25 kW-hrs/m^2. The difference between these possible accumulated energies is 4.065 kW-hrs/m^2, showing a 95% difference in efficiency. The total solar energy that could be obtained is 2,378 kW-hrs/m^2. This information is represented in Table 1, Table 2, and Table 3.

In Fairbanks, Alaska, the possible obtainable energies are spread throughout a greater range: 8.42 kW-hrs/m^2 on June 21 and 0.063 kW-hrs/m2 on December 21. The difference between these values is 8.357 kW-hrs/m^2, which reflects a 13365% difference. These possible values make sense because Alaska is closer to the North Pole and it will undergo a more dramatic shift in the amount of daylight due to the tilt of the Earth. As it tilts towards the sun, Fairbanks experiences almost completely sunny days and extremely short days as it tilts away. That is why Fairbanks, Alaska has the largest energy absorption range, but the lowest total energy of the three, with 1,357 kW-hrs/m^2. Figure 3 compares the sunset times of the three locations throughout a year. As the latitude increases, the length of the day experiences greater change, leading to less possible sunlight absorption on average.

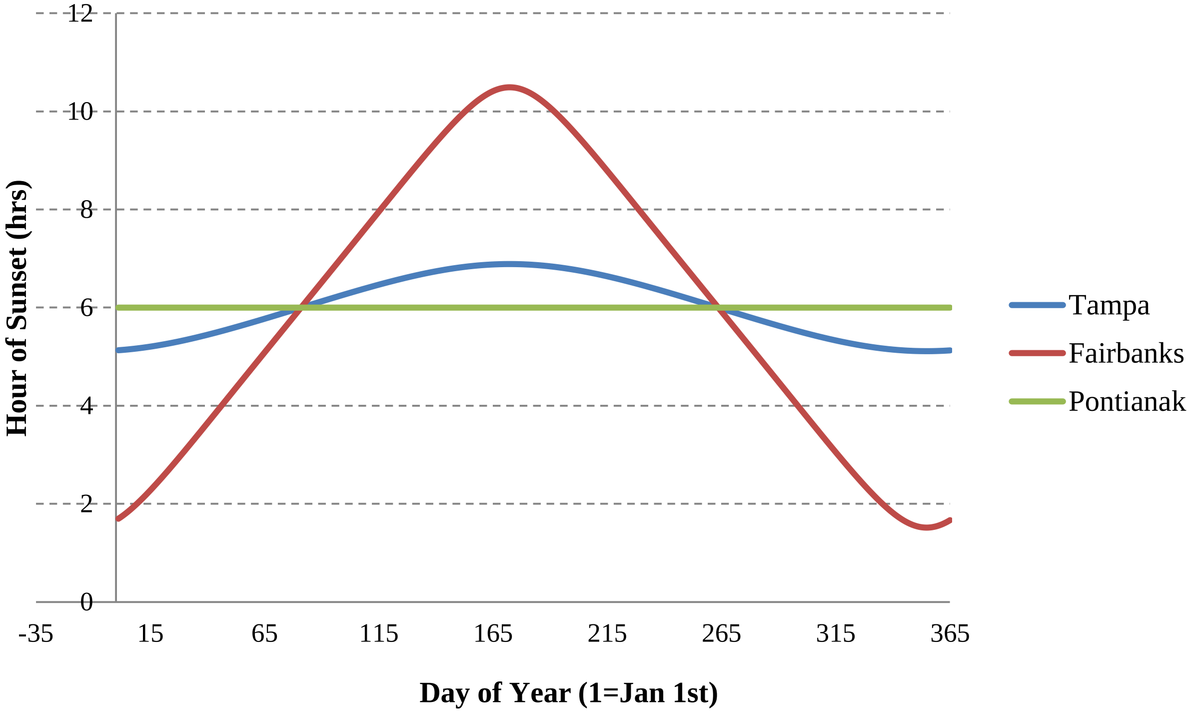

Figure 3: Sunset time of the three locations throughout a Year

In Pontianak, Indonesia, the energy levels remain relatively constant through a year, with two possible maximum attainable values in the curve and minimum possible during the Summer Solstice. The energy flux reaches its possible maximum of 7.71 kW-hrs/m^2 on March 10th, a possible local maximum of 7.62 kW-hrs/m^2 on October 1st, and its possible minimum of 6.78 kW-hrs/m^2 on June 22nd. The difference between these values is a mere 0.93 kW-hrs/m^2, or 13.7%. These values make sense since the equator is least prone to the effects of the Earth's tilt. During the two possible attainable maximum values, the spring equinox and the fall equinox, the Earth's axis is neither leaning towards the sun nor away, so the equator is exposed directly to the sun's light: it always experiences the same sunset hour (March Equinox: Time and Date; 2013). The elliptical orbit of the Earth also accounts for the slight variations in energy, as the Earth's proximity to the sun varies throughout its orbit.

In 2011, the average nuclear power plant in the United States generated approximately 12.2 billion kWh of energy (U.S. Energy Information Administration). In order to recreate this sort of intake with a solar panel placed on the equator, one would need 4,564,450 square meters of solar collectors, assuming there are only clear days. In Tampa that number grows to 5,132,519 square meters and in Fairbanks, 8,990,420 square meters of solar collectors are needed to generate the equivalent amount of energy as the average nuclear power plant. This requires about 3.471 square miles, or 2,221 acres, verses 1.762 square miles and 1,128 acres in Pontianak and 1.982 square miles and 1,268 acres in Tampa. These values, however, do not take into account the space needed between the panels for maintenance and cleaning, or the generators required to store the final energy to distribute to the population (U.S. Energy Administration; 2012).

An average American household uses about 940 kW-hrs of energy per month (U.S. Energy Administration). If a family in Tampa installed a one square meter solar energy collector onto to their roof, they could provide an additional 142 kW-hrs of energy in January and 249 kW-hrs in June. With several of these panels, that household's energy costs could easily drop by as much as 25% each month to the clean, renewable energy source provided by solar panels. With several of these panels, an average household would have little to no energy costs.

Solar energy is a feasible alternative for the future. Although the efficiency of the solar panels is not yet ideal and the prices are still relatively high compared to the electric grid, advances in technology show a promising outlook for this resource. The price of solar collectors is expected to drop to a quarter of the current value by 2015 and because of the abundance of silicon in the crust, the primary material for creating most solar collectors, panels can be produced at relatively low costs (Reddy; 2012). As the fastest growing power technology in the world, solar collectors can easily become a staple source of energy. By placing these solar collectors in the ideal locations, the renewable energy of the sun can be harnessed and distributed throughout the world.

CONCLUSION AND RECOMMENDATIONS

The results show that solar panels on or near the equator are most efficient in their collection of energy. Overall, the solar collector in Pontianak, Indonesia was the most efficient, generating 12.42% more energy than Tampa, Florida and 96.9% more energy than Fairbanks, Alaska. This result suggests that both Pontianak and Tampa would be ideal locations for solar energy collectors, while panels in Alaska are less likely to be as effective.

Several aspects of this research could have been done differently to obtain a greater accuracy in the results. Throughout the calculations, the sky and panels were assumed to be free of obstructions such as weather, debris, or dust that may have decreased the efficiency of the solar energy collector, something that could very likely happen in a real-life setting. This can reduce performance of the collector by as much as 7% per year (Dowd; 2008). Additionally, solar panels are sensitive to damage from weather fluctuations or electricity that may cause some discrepancies in the data, especially in locations with more extreme weather patterns.

The calculations could also be improved by increasing the number of time intervals throughout the year. Rather than using units consisting of days, perhaps calculating the energy by hours, or even minutes, would give a more precise estimation of the solar energy absorbed during that time.

NOMENCLATURE

W	Watts	J/s
E	Energy	kW-hrs/m^2
I	Solar Constant	W/m^2
S	Solar Energy	W/ m^2
t	Sunset Time	Hours
θ_z	Zenith Angle	Radians
ω	Hour Angle	Radians
T	Time	hours
φ	Latitude	°
D	Angle of Declination	Radians
Btu	British Thermal Units	J

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APPENDICES

Table 1: Locations of Solar collectors and their Latitude

City	Latitude (°N)
Tampa, Florida	27.947
Fairbanks, Alaska	64.82
Pontianak, Indonesia	0.00

Table 2: Total Energy Absorbed by Solar Collectors throughout Year

Location	kW-h/m^2
Tampa, Florida	2377.554396
Fairbanks, Alaska	1357.3644
Pontianak, Indonesia	2672.834101

Table 3: Maximum and Minimum values of Solar Flux throughout year with Day

Pontianak Indonesia	Value (KW-hrs/m^2)	Month and Day
Max 1	7.7067	10-Mar
Max 2	7.6200	1-Oct
Min	6.7802	22-Jun
Tampa, FL		
Max	8.3158	18-Jun
Min	4.2515	20-Dec
Fairbanks, AL		
Max	8.4213	21-Jun
Min	0.0627	21-Dec

Table 4: Excerpt of Excel Spreadsheet Calculator

Day	Constants	Delta	Cos	Sin	cos(ω)	Sunset Time	First Integral	Final Value
1	2065.9902	-0.401629	0.8130867	-0.183206	0.2253212	5.13187	3.7214841	4309.03872
2	2065.9609	-0.400214	0.8135747	-0.182595	0.2244354	5.135342	3.7222649	4319.20654
3	2065.912	-0.39868	0.8141017	-0.181933	0.2234766	5.1391	3.7231065	4330.18624
4	2065.8436	-0.397028	0.8146671	-0.181219	0.2224454	5.143141	3.7240075	4341.9732
5	2065.7557	-0.395258	0.8152703	-0.180454	0.2213422	5.147463	3.7249664	4354.56242
6	2065.6483	-0.393371	0.8159107	-0.179637	0.2201679	5.152062	3.7259816	4367.94853
7	2065.5214	-0.391368	0.8165874	-0.17877	0.218923	5.156936	3.7270517	4382.12579
8	2065.3751	-0.389248	0.8172997	-0.177851	0.2176083	5.162081	3.7281748	4397.08805
9	2065.2095	-0.387014	0.8180468	-0.176882	0.2162246	5.167496	3.7293493	4412.82884
10	2065.0245	-0.384664	0.8188279	-0.175862	0.2147725	5.173176	3.7305734	4429.34126
11	2064.8203	-0.382201	0.819642	-0.174791	0.2132529	5.179118	3.731845	4446.61807
12	2064.5968	-0.379624	0.8204882	-0.17367	0.2116667	5.185318	3.7331623	4464.65165
13	2064.3543	-0.376935	0.8213655	-0.172499	0.2100148	5.191773	3.7345234	4483.43399
14	2064.0926	-0.374134	0.822273	-0.171278	0.2082979	5.19848	3.735926	4502.95672
15	2063.812	-0.371223	0.8232095	-0.170007	0.2065171	5.205433	3.7373682	4523.21111
16	2063.5124	-0.368201	0.8241741	-0.168686	0.2046733	5.21263	3.7388478	4544.18804
17	2063.194	-0.36507	0.8251655	-0.167317	0.2027674	5.220065	3.7403626	4565.87804
18	2062.8569	-0.361831	0.8261827	-0.165898	0.2008004	5.227736	3.7419105	4588.27126
19	2062.5012	-0.358485	0.8272244	-0.16443	0.1987734	5.235638	3.7434892	4611.35751
20	2062.1269	-0.355033	0.8282895	-0.162914	0.1966874	5.243767	3.7450964	4635.1262
21	2061.7343	-0.351475	0.8293768	-0.16135	0.1945434	5.252118	3.74673	4659.56643
22	2061.3233	-0.347813	0.8304849	-0.159738	0.1923425	5.260686	3.7483876	4684.66691
23	2060.8942	-0.344048	0.8316126	-0.158078	0.1900857	5.269469	3.7500669	4710.41604
24	2060.447	-0.340182	0.8327585	-0.15637	0.1877741	5.27846	3.7517656	4736.80184
25	2059.982	-0.336214	0.8339214	-0.154616	0.1854088	5.287656	3.7534816	4763.812
26	2059.4991	-0.332147	0.8350999	-0.152816	0.1829909	5.297053	3.7552124	4791.43389
27	2058.9986	-0.327981	0.8362925	-0.150969	0.1805216	5.306645	3.7569558	4819.65453
28	2058.4807	-0.323718	0.837498	-0.149076	0.1780018	5.316428	3.7587095	4848.46064
29	2057.9454	-0.31936	0.8387148	-0.147138	0.1754328	5.326398	3.7604714	4877.8386
30	2057.3929	-0.314906	0.8399416	-0.145155	0.1728157	5.33655	3.762239	4907.77451
31	2056.8234	-0.310359	0.8411769	-0.143128	0.1701515	5.346879	3.7640103	4938.25413
32	2056.2371	-0.305721	0.8424193	-0.141056	0.1674415	5.357381	3.7657831	4969.26296
33	2055.6341	-0.300992	0.8436673	-0.138941	0.1646867	5.368052	3.7675551	5000.7862
34	2055.0147	-0.296173	0.8449194	-0.136782	0.1618882	5.378887	3.7693243	5032.80876
35	2054.3789	-0.291267	0.8461742	-0.134582	0.1590472	5.389881	3.7710885	5065.31531
36	2053.727	-0.286274	0.8474301	-0.132339	0.1561648	5.40103	3.7728457	5098.29023
37	2053.0592	-0.281197	0.8486858	-0.130054	0.1532421	5.41233	3.7745939	5131.71769
38	2052.3757	-0.276036	0.8499396	-0.127729	0.1502802	5.423776	3.776331	5165.58158
39	2051.6767	-0.270794	0.8511901	-0.125363	0.1472802	5.435364	3.778055	5199.86559
40	2050.9623	-0.265471	0.8524358	-0.122958	0.1442432	5.44709	3.7797642	5234.55319
41	2050.2329	-0.26007	0.8536752	-0.120514	0.1411703	5.458949	3.7814566	5269.62763
42	2049.4885	-0.254591	0.8549068	-0.118031	0.1380626	5.470937	3.7831303	5305.07199