Undergraduate Journal of Mathematical Modeling: One + Two

Volume 5 | 2012 Fall

Article 4

2013

High Jump Analysis

Paige Cooke
University of South Florida, pcooke@mail.usf.edu

Advisors:
Arcadii Grinshpan, Mathematics and Statistics
Lisa Olson, USF Track and Field

Problem Suggested By: Paige Cooke

Recommended Citation

Cooke, Paige (2013) "High Jump Analysis," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 5: Iss. 1, Article 4.
DOI: http://dx.doi.org/10.5038/2326-3652.5.1.4
Available at: https://scholarcommons.usf.edu/ujmm/vol5/iss1/4

High Jump Analysis

Abstract

This project presents a mathematical analysis of the high jump, a popular track and field event. The first and second stages of the high jump correspond to the athlete's run along two distinct trajectories. The third stage is the actual jump. We propose an individual model for each of these stages and show how to combine these models to study the dynamics of the entire high jump.

Keywords

High Jump, Trajectory Analysis, Projectile Motion

PROBLEM STATEMENT

Develop a mathematical model of the high jump.

MOTIVATION

According to Plunkett Research, 422 billion dollars were spent on the sports industry in the United States in the year 2011 (Plunkett Research Ltd.). This statistic shows the tremendous importance of sports in our lives. Studying the dynamics of sporting events is important because it may help to improve athletes' performance and reduce the risk of injury (Pettibone). This project aims to develop a mathematical model of the high jump, a popular track and field event.

Figure 1: The three stages of the high jump.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

We observe that the high jump consist of three individual stages. During the first stage, the athlete runs along a linear trajectory. The trajectory changes during the second stage in preparation for the jump. The third stage is the actual jump. These steps are schematically depicted in Figure 1.

First stage: a run along a straight line. During the first stage, the athlete runs along a straight line (or linear trajectory). It's important for the athlete to control their horizontal velocity and other factors: "During running and jumping… forces must be applied to and from a stable base to produce efficient movement and maximal displacements and minimize the internal distortion of the system" (Schexnayder). We somewhat simplify our analysis by assuming that the athlete's motion is uniform during this stage. Note that average length of this segment of the high jump is about 11 meters. That is,

$$v = \frac{d}{t} \quad \text{or} \quad v = \frac{11}{t}$$

where v is the athlete's speed, d is the distance (taken here to be 11 meters as explained above), and t is the time.

Second stage: a run along an arc of a circle. The athlete changes his or her trajectory during the second stage. Instead of running along a straight line, the athlete runs along a trajectory that we approximate by an arc of a circle. Here, the centrifugal force plays a critical role. This force allows the jumper to clear the bar horizontally. The centrifugal force is calculated using the following equation

$$F = \frac{m\,v^2}{r},$$

where m is the athlete's mass, v is his/her speed, and r is the radius of the trajectory (see Figure 2).

The athlete may jump high enough, but if the angular momentum is off during the second stage of the jump, then the athlete will either (a) hit the bar off during the upward ascent or (b) hit the bar off during the descent.

DOI: http://dx.doi.org/10.5038/2326-3652.5.1.4

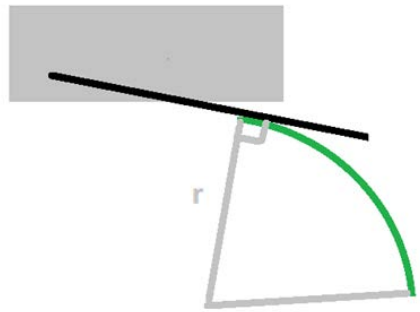

Figure 2: Schematic representation of the second stage of the long jump.

Third stage: projectile motion of a jump. The third stage of the high jump is the actual jump that we model as a projectile motion. During this stage, the jumper ascends over the bar, reaches his or hers maximum height, and then begins the descent onto the high jump mat. The trajectory of this jump is a parabola.

The equation we use to model the athlete's trajectory is

$$y = y_0 + v_i t + \frac{g\,t^2}{2},$$

where y_0 is the initial distance of the athlete's center of mass from the ground, v_i is the initial velocity, g is the standard gravity, and t is time. The initial velocity in the above formula can be determined using the equation

$$v_i = \sqrt{v_h^2 + v_v^2}$$

where v_h are the horizontal velocity and v_v is the vertical velocity of the athlete before the jump. The horizontal velocity can be determined from the centrifugal force equation given during discussion of step 2.

Multistage model. Assume that the athlete reaches each of the three stages at times t_0, t_1, and t_2 respectively. Then the distance at a time t is given by

$$d(t) = \begin{cases} \frac{11}{t_0} t \text{ for } t < t_0 \\ 11 + \frac{\pi r}{2t_1}(\mathrm{t} - t_0) \text{ for } t_0 \leq t < t_1 \\ 11 + \frac{\pi r}{2} + \frac{F}{m}(t - t_1) \text{ for } t_1 \leq t \end{cases}$$

were all of the variables are as before.

DISCUSSION

We have presented our model of the high jump. We showed that high jump can be separated into three stages, all of which describe the motion of the athlete along different trajectories.

During the first stage, the athlete builds up a consistent speed which plays a factor during the second stage. It seems that the arc trajectory that the athlete chooses during the second stage is extremely important: a tighter arc, corresponding to a smaller radius, generates greater centrifugal force; whereas, a wider curve generates less force.

During the third stage, consisting of the actual jump, we used projectile motion to model the athlete's trajectory. Note that we do not consider drag or any other such forces in our analysis. Our results clarify why taller jumpers have advantages over shorter jumpers. Because hips are the center of athlete's gravity, the increase in the height of the jump with the length of the hips can be readily computed using the equation of projectile motion.

DOI: http://dx.doi.org/10.5038/2326-3652.5.1.4

CONCLUSION AND RECOMMENDATIONS

Our analysis suggests that during the first two stages of the jump, the athlete should attain the correct angular momentum to clear the bar during the jump. Thus the jumper should not try to be at his or her full velocity. A high velocity of the run is good as long as it is controllable and paired with an even greater vertical velocity generated by the jump. Another important recommendation for high jumpers is to kick their feet up after they reach the maximum height.

NOMENCLATURE

Symbol	Meaning	Units
y	vertical position of athlete	meters (m)
y_0	initial vertical position	meters (m)
v_h	horizontal velocity	meters per second (m/s)
v_v	vertical velocity	meters per second (m/s)
v_i	initial velocity	meters per second (m/s)
t	time	seconds (s)
g	gravitational constant	meters per second squared (m/s^2)
F	centrifugal force	Newtons (N)
m	mass of athlete	kilograms (kg)

REFERENCES

"Amazing Blanka Vlasic." n.d. Croatian World Network. 4 May 2012 <http://www.croatia.org/crown/articles/9196/1/>.

Larson, Ron, Robert Hostetler and Bruce Edwards. Calculus. 8th Edition. Boston, MA: Houghton Mifflin Company, 2005.

Pettibone, Steve. "Student dies of injuries suffered in high jump accident." 2012. News Canada. 4 May 2012 <http://www.calgarysun.com/2012/04/05/>.

Plunkett Research Ltd. "Sports Industry Overview." 2011. 4 May 2012 <www.plunkettresearch.com/sports-recreation-leisure-market-research/>.

Schexnayder. "Applied Kinesiological Concerns for Athletes." 1998. Track and Field News. 4 May 2012 <http://www.trackandfieldnews.com/technique/145_Schexnayder.pdf>.

Shustov, Aleksandr. "All-Athletics." n.d. 4 May 2012 <http://www.all-athletics.com/en-us/photos/aleksandr-shustov-2>.

DOI: http://dx.doi.org/10.5038/2326-3652.5.1.4