

2011

Average Light Intensity Inside a Photobioreactor

Herby Jean
University of South Florida

Advisors:

Arcadii Grinshpan, Mathematics and Statistics
Daniel Yeh, Civil & Environmental Engineering

Problem Suggested By: Daniel Yeh

Follow this and additional works at: <https://scholarcommons.usf.edu/ujmm>



Part of the [Mathematics Commons](#)

UJMM is an open access journal, free to authors and readers, and relies on your support:

[Donate Now](#)

Recommended Citation

Jean, Herby (2011) "Average Light Intensity Inside a Photobioreactor," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 4: Iss. 1, Article 9.

DOI: <http://dx.doi.org/10.5038/2326-3652.4.1.9>

Available at: <https://scholarcommons.usf.edu/ujmm/vol4/iss1/9>

Average Light Intensity Inside a Photobioreactor

Abstract

For energy production, microalgae are one of the few alternatives with high potential. Similar to plants, algae require energy acquired from light sources to grow. This project uses calculus to determine the light intensity inside of a photobioreactor filled with algae. Under preset conditions along with estimated values, we applied Lambert-Beer's law to formulate an equation to calculate how much light intensity escapes a photobioreactor and determine the average light intensity that was present inside the reactor.

Keywords

Light Intensity, Alternative Energy, Lambert Beer's Law

Creative Commons License



This work is licensed under a [Creative Commons Attribution-Noncommercial-Share Alike 4.0 License](https://creativecommons.org/licenses/by-nc-sa/4.0/).

TABLE OF CONTENTS

Problem Statement.....	3
Motivation.....	3
Mathematical Description and Solution Approach.....	4
Discussion.....	5
Conclusion and Recommendations.....	6
Nomenclature.....	7
References.....	7

PROBLEM STATEMENT

After researching the potential use of algae as a renewable resource, we concluded that the energy for algae to grow is essential in order to control algae production in labs and other facilities. Given an algae concentration of 1 g/L in a photo bioreactor with length 0.05 m, and a sunlight supply of light with an intensity of 1000 mmol/m²s, calculate the light intensity inside the photo bioreactor and the amount of light that leaves the reactor.

MOTIVATION

Algae can grow on almost any surface where water is present: ponds, pools, lakes, rivers, and even fountains. Though algae may be unappealing to the eye, the environmental applications are truly amazing. A photo bioreactor has shown to be the best reactor for mass production of algae. Practically any container that is transparent is considered a photo bioreactor, e.g., an open tank or pond. The bioreactor uses some type of light source and converts the light into photonic energy inside the reactor. The device houses and cultivates algae, controlling the rate of growth simultaneously. Before starting this project we predict that algae growth will increase with higher light intensities. With an application of calculus methods we can find the average light intensity that passes through any photo bioreactor, as well as calculate the amount of light algae uses for growth. It is well known that algae needs water to grow, we attempt to determine what can be said about the light energy needed in order for them to grow. This project will attempt to explore how light effects the growth of algae.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

We begin by examining Figure 1; the concept of light attenuation is used to determine how fast light intensity decreases with distance from an object. The figure illustrates the effect on light intensity when the light comes in contact with a reactor or in our case a photobioreactor. Initially, the light strikes the reactor with a constant intensity but when the light is inside the reactor it is converted into photonic energy fueling the algae concentration to grow. As the light continues to move through the reactor the intensity decreases and when the light passes fully through the photo bioreactor less light exits the photo bioreactor than initially entered. The question remains as to what happens with the light intensity as it passes through the photo bioreactor and how could we calculate such a phenomenon.

We apply Lambert-Beer's law which states that

$$I_{out} = I_{in} \cdot e^{-a_c c_{alg} b} \quad (1)$$

where I_{in} (I_{out}) is the light intensity entering (exiting) the photobioreactor, a_c is the spectral-averaged absorption coefficient, c_{alg} is algae concentration, and b is the length of the photobioreactor.

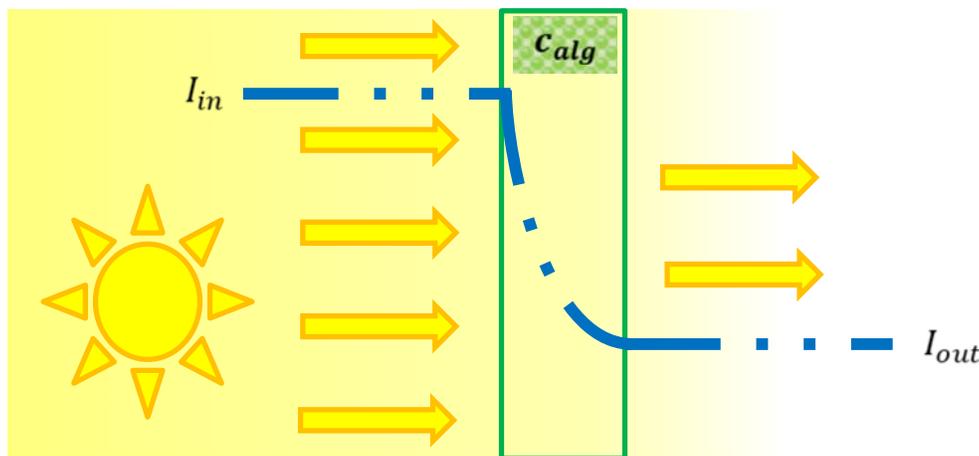


Figure 1: Light attenuation in a photobioreactor.

We let $a_c = 200 \text{ m}^2/\text{kg}$ and assume the initial conditions $I_{in} = 1000 \text{ mmol}/\text{m}^2\text{s}$, $c_{alg} = 1 \text{ g/L}$, and $b = 0.05 \text{ m}$. Substituting these parameters into equation (1) yields,

$$I_{out} = 1000 e^{(-200) \cdot (1) \cdot (0.05)} = 0.0454 \text{ mmol}/\text{m}^2\text{s}. \quad (2)$$

Applying Lambert-Beer's formula once again, this time over the length of the reactor, we calculate the average light intensity to be

$$I_{ave} = \frac{\int_0^b I_{in} e^{-a_c c_{alg} x} dx}{\int_0^b dx} = \frac{I_{in} (1 - e^{-a_c c_{alg} b})}{b a_c c_{alg}} = 99.99 \frac{\text{mmol}}{\text{m}^2\text{s}}. \quad (3)$$

DISCUSSION

Lambert-Beer's law became crucial in deriving an equation to find the average light intensity inside the photo bioreactor. Inside the photo bioreactor, the average intensity of sunlight came out to be $100 \text{ mmol}/\text{m}^2\text{s}$. Also, Lambert-Beer's law allowed us to calculate the light intensity leaving the reactor, $0.0454 \text{ mmol}/\text{m}^2\text{s}$. We predicted the amount of light intensity leaving the reactor would be less than the amount of light intensity entering the reactor. However, we did not expect the result to be as low as we calculated. Our calculations suggest almost all of the light intensity from sunlight was taken and used as photonic energy by the photo bioreactor. These results raised the possibility of increasing or decreasing the length of the photo bioreactor. How much of a difference would a reactor with a path length of 0.05 m be in comparison with a reactor of 0.02 m ? In algae production, the size of the reactor needs to be determined carefully in order to receive the maximum input of the light source. The concentration of algae will also determine how much light intensity leaves the reactor. In this

project, we assumed the sunlight was supplying the reactor with 1,000 mmol/m²s in light intensity. In future applications, the outcome and the light source may be different.

CONCLUSION AND RECOMMENDATIONS

With the presence of algae inside the photobioreactor the conclusions we derived regarding light intensity are straightforward: the light intensity entering the reactor was 1000 mmol/m²s, the light intensity inside the reactor averaged out to 100 mmol/m²s, and light intensity leaving the reactor was calculated to be 0.0454 mmol/m²s. These results illustrate the use of a particular light source by the algae concentration; we notice nearly all of the light intensity was used by the algae as less than 1% of the intensity of the light entering the reactor exited the reactor. The amount of algae growth was not calculated in this project and could be the topic of future research.

NOMENCLATURE

Symbol	Description	Units
a_c	Spectral-averaged absorption coefficient	m^2/kg
b	Length of photobioreactor	m
c_{alg}	Concentration of algae	g/L
I_{in}	Light Intensity entering photobioreactor	$\text{mmol}/\text{m}^2\text{s}$
I_{out}	Light Intensity leaving photobioreactor	$\text{mmol}/\text{m}^2\text{s}$
I_{ave}	Average light intensity inside photobioreactor	$\text{mmol}/\text{m}^2\text{s}$

REFERENCES

Larson, Ron, Robert Hostetler and Bruce Edwards. Calculus. 8th Edition. Boston, MA: Houghton Mifflin Company, 2005.

Hermanto, Mochamad Bagus. "Thesis Systems and Control: Identification of Algae Growth Kinetics." Wageningen University, Agrotechnology and Food Sciences (2009).
<http://edepot.wur.nl/9247>

Hash, Martin. "Light Attenuation." (2004) Web. 9 Dec. 2011
<http://www.hash.com/am2004/Choreography/Attenuation/Light%20Attenuation.htm>

Runkle, Erik. "Principles Of Light." Orchids May 2008: Vol. 77, No. 5. Print.