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Model for Facial Cooling

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Model for Facial Cooling

Abstract
The goal of this project is to estimate the ambient temperature and the wind speed that cause painful sensation in a cheek. Our findings confirm that the higher the wind speed, the less cold air is required to cause cheek pain.

Keywords
Wind Chill Factor, Newton's Law of Cooling, Cheek Pain

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PROBLEM STATEMENT

A person in a cold environment loses heat more rapidly when the wind is blowing – a phenomena which is described by the term *wind chill factor*. The goal of this project is to quantify this effect to the specific case of heat loss through the cheeks of a person whose face is exposed to a low ambient temperature. In this event, there is heat *conduction* from the inside of the mouth to the outer surface of the cheek and heat *convection* from the outer surface of the cheek to the ambient temperature. At steady state, which is the case examined here, these two heat flows must equal each other.

For conduction through the cheek, the human head is modeled as a cylinder of radius $R_s = 0.07$m and the cheek thickness t is taken to be 0.01m. Heat conduction through the cheek is modeled by Fourier's Law:

$$Q = -2\pi rLk\,\frac{dT}{dr} \tag{1}$$

where T is temperature, Q is the heat flow and varies neither with time or radius at steady state, k is the thermal conductivity of the tissue in the cheek (equal to $2W/m\,K$), r is the distance coordinate in the radial direction and L is the length of the cylinder. The inside temperature of the cheek is $T_i = 34°C$ and the temperature T_O of the outer surface will vary with both ambient temperature and wind speed.

The heat transfer from the cheek outer surface (at temperature T_O) to the ambient temperature T_A is given by Newton's Law of Cooling:

$$Q = 2\pi R_S Lh(T_O - T_A) \tag{2}$$

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where h is a convective heat transfer coefficient, which depends on the wind speed. For heat loss from a human head, researchers have used the correlation below to estimate the heat transfer coefficient:

$$\frac{2R_S h}{k_{\text{air}}} = 0.945\text{Re}^{0.5}\text{Pr}^{0.4} \tag{3}$$

where k_{air} is the thermal conductivity of air $(0.024W/mK)$, Pr is the Prandtl number for air $(\text{Pr} = 0.7)$ and Re is the Reynolds Number given by

$$\text{Re} = \frac{2R_S V \rho_{\text{air}}}{\mu_{\text{air}}} \tag{4}$$

where ρ_{air} and μ_{air} are the density and viscosity of air (which can be taken to be $1.3kg/m^3$ and $1.7 \times 10^{-5}kg/m \cdot s$, respectively) and V is the wind speed in m/s.

Research shows that heat loss from the face becomes painful when the outer cheek temperature falls to $10°\text{C}$. The goal of this project is to compute the ambient temperatures T_A at which the outer cheek temperature will reach $10°\text{C}$ for different wind speeds. Examine wind speeds ranging from 2 to 20 m/s and present your results as a graph of T_A versus wind speed.

MOTIVATION

Understanding the effects of wind on cooling of various objects is important and has many applications. One such application is weather forecasting where it is often more important to predict the temperature a person would feel rather than the ambient temperature. Here we present a case study concerning weather conditions which may cause painful sensations in cheeks, an area of the face often left exposed even in a cold weather.

Our analysis is based on an observation by Peter Tikuisis and Randall J. Osczevski (Tikuisis and Osczevski) who found that the cheeks start to experience pain when their outer temperature reaches 10°C. We conclude with a graph describing the ambient temperature and the corresponding wind speed required to lower the cheek temperature to 10°C.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

We begin by rearranging the equation (1) from the problem statement and subsequently integrating it:

$$Q = -2\pi r L k \frac{dt}{dr} \quad \text{and} \quad -\int_{T_i}^{T_o} \frac{2\pi L k}{Q}\, dT = \int_{.06}^{.07} \frac{dr}{r} \tag{5}$$

The bounds of integration in the above equation are determined by the internal and external temperatures and the thickness of the cheek respectively. Observe that (5) implies that

$$-2\pi \frac{Lk}{Q}(T_o - T_i) = \ln \frac{0.07}{0.06}. \tag{6}$$

Now we eliminate the variable Q from (6) using (2) from the problem statement.

$$\frac{-2\pi L k (T_O - T_i)}{\ln \frac{0.07}{0.06}} = Q = 2\pi R_s L h (T_O - T_A) \tag{7}$$

After simplifying (7) we get

$$-k(T_O - T_i) = R_s h (T_O - T_A) \ln \frac{0.07}{0.06} \tag{8}$$

and substituting the values of T_O, T_i, R_s, and k given in the statement of the problem results in

$$-2(10° - 35°) = h(0.07)(0.1542)(10° - T_A). \tag{9}$$

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Next we determine the relationship between h and V. To do this, first substitute all values given in the problem statement into equation (4) as

$$Re = \frac{2R_s V \rho_{\text{air}}}{\mu_{\text{air}}} = \frac{2\,(0.07)\,(1.3)}{(0.000017)} V = 1.0706 \times 10^4 V \quad (10)$$

after simplifications. Combining (10) with equation (3) results in an equation relating h and V:

$$h = 0.140461\,(1.0706 \times 10^4 V)^{\frac{1}{2}}. \quad (11)$$

Substituting (11) into (9) gives the required relationship between the wind's speed and the ambient temperature (V and T_A) as

$$31.878613 = V^{\frac{1}{2}}\,(10° - Ta). \quad (12)$$

Figure 1 depicts this relationship for the wind speeds between 2 and 20 m/s.

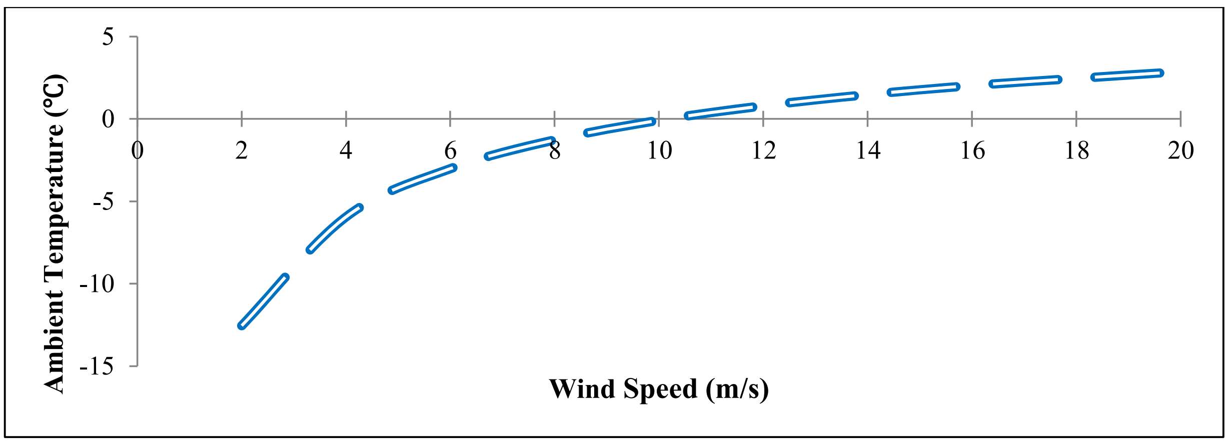

Figure 1: Ambient temperature and the corresponding wind speed needed to lower the temperature of the cheeks to 10°C.

DISCUSSION

Our findings confirm that the more wind there is on a cold day, the more likely a person to experience a painful cheek sensation. Although our results are consistent with intuition, it is still important to obtain accurate numeric estimates of the chill factor under various weather conditions. For example such estimates could serve as a basis for a more meaningful weather forecast or help to determine an appropriate safety measures for individuals working in a harsh environmental conditions.

CONCLUSION AND RECOMMENDATIONS

Our calculations have shown that the colder it is outside, the less wind that is needed to harm the cheeks. These estimates have a number of potential applications which we have discussed earlier. This project could be extended by surveying larger range of wind speeds and considering a more accurate model of the cheek cooling. If they were to take that approach, they could show how sailing on a hot summer's day can have the same effect as walking through the snow in the dead of winter.

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NOMENCLATURE

Symbol	Description	Units	Symbol	Description	Units
k	convective heat transfer	$W/m\ K$	ρ_{air}	density of air	kg/m^3
k_{air}	air's thermal conductivity	W/mK	V	wind speed	m/s
Pr	Prandtl's number	none	μ_{air}	viscosity of air	$kg/m \cdot s$
$\mathrm{R_s}$	cylinder's radius	m	L	length	m
T	temperature	℃	P_{air}	density	kg/m^3
Q	rate of heat loss	J/s	T_O	cheek's outside temperature	℃
T_A	cheek's ambient temperature	℃	T_i	cheek's inside temperature	℃

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