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Optimal Operation of a Concentrator

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Optimal Operation of a Concentrator

Abstract

Concentrators are used in the industrial world to remove water from a chemical or substance by heating the liquid until the water evaporates thereby concentrating the remaining substance. The goal of this project is to find the optimum cycle time, number of cycles per year, and the heating element area that would minimize the total annual cost and hours of operation of an industrial concentrator. It was found that the specified concentrator would achieve a minimum total cost of \$48,720.50 per year and take 993.8 hours to meet the production goal of evaporating one million kilograms of water.

Keywords

Evaporator, Cycle Time, Minimum cost

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PROBLEM STATEMENT

Many industrial processes are run in cycles - for instance a cycle might run for T hours, then set up to run again. The number of cycles per year N and the cycle time T dictate the total hours of operation per year of the process. Processes might be run in cycles for any number of reasons but one reason is that a component of the process degrades over time and must be periodically replenished.

In this process, you will determine the optimum operating strategy for an industrial concentrator. The concentrator raises the concentration of dissolved solids in a liquid product by evaporating water from the product. To meet production goals, the concentrator must evaporate 1×10^6 kg of water every year. Because of fouling, the evaporating capacity changes over time. Specifically, the instantaneous rate of evaporation Q (in kg/hr) is:

$$Q = A (20 - 0.01 t)$$

where A is the surface area of the heating element (in m^2) and t is the time since the beginning of the current cycle in hours.

At the end of each cycle, there is a cost of \$1,000 to clean the heating element. When the cycle is in operation, there is a labor cost of \$20/hr. The cost of the evaporator, prorated over its useful life, is \$400/year per m^2 of surface area. Cost of electricity to generate the heat is \$0.004/kg of water evaporated. The optimal operating strategy is the one that minimizes the total annual cost. Please determine:

- a) The optimum cycle time T , number of cycles per year N and heating element surface area A .
- b) The corresponding annual cost and total hours per year of operation.

MOTIVATION

A concentrator is an industrial machine used to purify water or oxygen by boiling out the solvent so that, “The recovered end product should have an optimum solids content consistent with desired product quality and operating economics” (SPX). Concentrators can be used to condense purified water for drinking, isolate minerals in a liquid by evaporating all of the water out, or both processes simultaneously. Concentrators are extensively used variety of industries which include, “processing foods, chemicals, pharmaceuticals, fruit juices, dairy products, paper and pulp, and both malt and grain beverages” (SPX) and smaller scale concentrators are commonly used to purify oxygen for medical uses.

In this mathematical engineering project, an industrial engineer is needed to determine the optimal operating strategy for an industrial concentrator. The industrial concentrator evaporates water in cycles and also a component of the process degrades over time and must be periodically replenished. The concentrator must evaporate one million kilograms of water every year and the industrial engineer must solve for the optimum cycle time, number of cycles per year, the heating element surface area and total hours of operation per year that minimize the total annual cost. The demands and goals of this project are certainly relevant and realistic in that industrial concentrators do degrade over time and also do need to be cleaned after each cycle especially if they process beverages or corrosive materials. “All evaporators for hygienic duties must be capable of being frequently cleaned in Place” (SPX). Also as stated in the SPX Evaporator Manual, “Corrosion is often a major problem with chemical duties and some hygienic applications.” Every successful production facility wants to produce a quality product at the cheapest cost to them and industrial engineers design and ensure achieving those expectations.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

When first approaching to solve this problem, we recognized that the given equation $Q = A (20 - 0.01 t)$ is the instantaneous rate of evaporation, i.e., the derivative of evaporation E over time t . To get the amount of water evaporated in one cycle, the equation integrated from 0 to T :

$$E(T) = \int_0^T Q(t) dt = \int_0^T A (20 - 0.01 t) dt = \left[20 A t - \frac{0.01 A t^2}{2} \right]_0^T = 20 A T - \frac{0.01 A T^2}{2} \quad (1)$$

We know from the problem the concentrator must evaporate 1×10^6 kg of water per year. Thus

$$\left(20 A T - \frac{0.01 A T^2}{2} \right) N = 1 \times 10^6 \quad (3)$$

where N is the number of cycles per year. Total number of cycle per year multiplied by the amount of water evaporated in time of one cycle (T) and it must equal the total amount of water to be evaporated per year. Now that there is an understanding of the given constraints, we can consider the Total Annual Cost (C) equation,

$$C = \$1000 N + \$20 N T + \$400 A + \$0.004 (1 \times 10^6). \quad (4)$$

In equation (4), the $1000 N$ accounts for the \$1000 cleaning cost at the end of each cycle for N cycles, $\$20 N T$ represents the \$20/hour labor cost when the cycle is in operation for T hours and N cycles per year. The $\$400 A$ is the \$400 per year per square meter of surface area (m^2) and represents the cost of the evaporator prorated over its useful life. The last term of the equation $\$0.004 (1 \times 10^6)$ represents the cost of electricity to generate heat which is \$0.004/kg water evaporated and we have a total of 1×10^6 kg evaporated in one year. The number of

cycles (N) multiplied by the cycle time (T) gives the Total Hours of Operation (H) of the concentrator,

$$H = N T. \quad (5)$$

Now that we have C and H equations, it would be ideal to eliminate the variable A so that (4) and (5) could then only have two unknown variables. To eliminate A , we can use (3) and solve for A in terms of N and T :

$$\left(20 A T - \frac{0.01 A T^2}{2}\right) N = 1 \times 10^6 \quad \Rightarrow \quad A = \frac{1 \times 10^6}{(20 T - 0.005 T^2) N}. \quad (6)$$

Now we can substitute A from (6) into (4) to get

$$C(N, T) = 1,000 N + 20 N T + 400 \frac{1 \times 10^6}{(20 T - 0.005 T^2) N} + 4000. \quad (7)$$

We can now take the partial derivatives of C with respect to N and T and set those equations equal to 0 to minimize C and isolate N and T . This gives the cycle time and number of cycles which minimizes the overall cost.

The partial derivative of (7) with respect to N ,

$$\frac{\partial C}{\partial N} = 1000 + 20 T - \frac{4 \times 10^8}{(20 T - 0.005 T^2) N^2} = 0 \quad (8)$$

The partial derivative of (7) with respect to T :

$$\frac{\partial C}{\partial T} = 20 N - \frac{4 \times 10^8 (20 - 0.01 T)}{N (20 T - 0.005 T^2)^2} = 0 \quad (9)$$

Solving for N^2 gives

$$N^2 = \frac{4 \times 10^8 (20 - 0.01 T)}{20 (20 T - 0.005 T^2)^2}. \quad (10)$$

The N^2 value can substituted into (8) and solved for T ,

$$20 T - \frac{20 T (20 - 0.005 T)}{(20 - 0.01 T)} = -1000 \quad (11)$$

Solving (11) quadratic equation, we get

$$T = 400 \text{ or } -500 . \quad (12)$$

There are two solutions for the equation (11). Since the solution of the variable is time in hours, the value should be a positive number. Therefore the time which minimizes the total cost is 400 hours. Now we can solve for N using (10) and substitute 400 for T :

$$N^2 = \frac{400 \times 10^6 (20 - 0.01 \times 400)}{20(20 \times 400 - 0.005 \times 400^2)} \Rightarrow N = 2.48 \text{ cycles/year.} \quad (13)$$

Now knowing N and T , = 55.9 m² from (6), $H = 993.8$ hours/year from (5), and by (4)

$$C(N, T) = \$48,720.50 . \quad (14)$$

DISCUSSION

Something to consider is that the number of cycles per year and perhaps the concentrator could only run in full cycles. In that case one would have to round the theoretic results to the nearest cycle. This could be either 2 or 3 cycles per year. This changes the optimal values of cycle time, area of the heating element, total cost and total production time of the job. These results would provide the most efficient means of operation for an industrial concentrator that could only run an integer number of cycles. Although these new parameters provide an interesting interpretation to the data, all of the original calculations in the mathematical solutions section are correct under the initial parameters of the project.

One could further explore this problem by entering the various equations and constraints into Mathematica (Wolfram) and graphing for example the integral the instantaneous rate of evaporation. This provides the engineer with the opportunity to further investigate the relationships between the variables in this problem and what happens to the annual cost and operation hours when the relevant parameters are altered.

CONCLUSION AND RECOMMENDATIONS

The results of this project proved to be successful in that we were able to find the number of cycles, cycle time and surface area of the heating element that minimize the total annual cost of the concentrator. For this particular concentrator, 2.48 cycles of 400 hours for a total of 993.8 hours with a heating element having area an area of 55.9 m² will minimize the overall annual production cost at \$48,720.50. Overall these results provide an optimum operating strategy for this industrial concentrator that will provide it to operate at top efficiency.

Engineers may find our analysis helpful when dealing with industrial concentrators of any kind. The goal of an industrial engineer is always to come up with the most viable, efficient and cheapest solution to a problem given the initial parameters or restrictions of the process. In this problem the parameters happened to be that the concentrator had to evaporate one million kilograms of water per year and there were several other costs such as a labor cost, cleaning cost, fixed cost of the evaporator and the cost of electricity power the heating element. Engineers in the field that have parameters similar to these and could use our calculations under their specifications to take a similar approach on achieving the optimum result of minimizing the total annual cost of a concentrator.

Our approach can also provide engineers the flexibility to compare various industrial concentrators. They could compare the specifications for any new concentrator and instantly see how much that concentrator is costing the company per year and how long it is in operation per year. Further, the engineer could check that a concentrator's annual cost coincides with the theoretical cost obtained through calculus methods to ensure absolute accuracy and efficiency.

NOMENCLATURE

Symbol	Description	Units
N	Number of cycles per year	cycles/year
T	Cycle time	hrs
A	Surface area of heating element	m ²
Q	Instantaneous evaporation rate	kg/hr
C	Total cost of operation	\$/year
H	Total hours of operation	hours/year

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APPENDIX

COST INFORMATION:

Starting Parameter	Value
Amount of water to evaporate	1×10^6 kg/year
Cost of cleaning	\$1,000/cycle
Cost of labor	\$20/hour
Prorated cost of evaporator	\$400/(m ² · year)
Cost of electricity	\$0.004/kg of water