
2011

Optimization of a Pressure-Treating Process

Josean Velez
University of South Florida

Advisors:

Arcadii Grinshpan, Mathematics and Statistics
Scott Campbell, Chemical & Biomedical Engineering

Problem Suggested By: Scott Campbell

Follow this and additional works at: <https://scholarcommons.usf.edu/ujmm>



Part of the [Mathematics Commons](#)

UJMM is an open access journal, free to authors and readers, and relies on your support:

[Donate Now](#)

Recommended Citation

Velez, Josean (2011) "Optimization of a Pressure-Treating Process," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 4: Iss. 1, Article 3.

DOI: <http://dx.doi.org/10.5038/2326-3652.4.1.3>

Available at: <https://scholarcommons.usf.edu/ujmm/vol4/iss1/3>

Optimization of a Pressure-Treating Process

Abstract

A company that pressure-treats wood wants to minimize its annual cost without using more than 250 days of operation per year. In addition, they want to find the corresponding value of time, batches and cost for each category. We develop an expression in terms of boards per batch to model the total cost of the treatment process. We then take the derivative and use Newton's Method to find the number of boards per batch that minimizes total cost.

Keywords

Optimization, Newton's Method, Operations Research

Creative Commons License



This work is licensed under a [Creative Commons Attribution-Noncommercial-Share Alike 4.0 License](https://creativecommons.org/licenses/by-nc-sa/4.0/).

TABLE OF CONTENTS

Problem Statement.....	3
Motivation.....	4
Mathematical Description and Solution Approach	4
Discussion	6
Conclusion and Recommendations	7
Nomenclature	8
References.....	9
Appendix	10

PROBLEM STATEMENT

A company pressure – treats wood and wishes to produce 500,000 treated boards per year using a batch process. The time t required for each batch is proportional to the number of boards X treated in each batch and is 4 hours for a batch size of 500 boards. The set up and discharge times for each batch are negligible. Costs associated with the process are:

- Labor: \$40/hr during the batch processing time
- Set-up costs: $100x^{0.2}$ (\$/batch)
- Annual cost of the processing equipment: $800x^{0.7}$ (\$/yr)
- Cost of chemicals: $(0.25x + 100)$ (\$/batch)

A maximum of one batch per day can be run and a maximum of 250 days per year of operation are available.

1. What is the number of boards per batch that will minimize the annual cost? What is the corresponding batch time and number of batches per year?
2. What are the corresponding annual labor, set-up, equipment and chemical costs, and the total annual cost (\$/yr)?



Figure 2: Images of a wood pressure treating facility.

MOTIVATION

The problem we consider falls under the category of industrial engineering and focuses on conducting manufacturing and managing commercial enterprises with efficiency. It is a problem that focuses on the total annual cost of a pressure-treating company who produces treated boards of wood. The company needs to find the optimal amount of labor, set-up, equipment and chemical costs to minimize its total annual expenditures.

It is essential to recognize that cost is a result of a specific decision made. Every benefit has a cost and it is not always easy to determine a company's total operating cost. There are different types of cost, such as relevant cost (arise from decisions) and irrelevant cost (occur regardless of the decision) and short term and long-term costs.

The objective of this project is to analyze every cost that a pressure treatment company may incur and minimize this cost in order for the company to work at its maximum efficiency.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

We first develop an expression for the total cost the company incurred over the year. We are given that the time needed for the pressure treating process is proportional to the number of board per batch, thus

$$t = n x \tag{1}$$

where X represents the number of boards per batch, t represents time in hours, and n is some constant. Since we know it takes four hours to complete a batch of 500 boards we can conclude:

$$t = \frac{4}{500}x = 0.008x. \quad (2)$$

It is known that the company wishes to produce 500,000 treated boards per year. Therefore, we use the following relationship to obtain an equation for the number of batches, b , that will minimize the annual cost. We have:

$$b = \frac{500,000}{x}. \quad (3)$$

Combining all the costs the company sustains yields an expression for the total annual cost:

$$C(x) = 800x^{0.7} + 40tb + 100x^{0.2}b + (0.25x + 100)b. \quad (4)$$

We use (2) and (3) to rewrite (4) as,

$$C(x) = 800x^{0.7} + (5 \times 10^7 x^{-0.8}) + (5 \times 10^7 x^{-1.0}) + 285,000. \quad (5)$$

We differentiate (5) to see:

$$C'(x) = \left(\frac{560}{x^{0.3}}\right) - \left(\frac{4 \times 10^7}{x^{1.8}}\right) - \left(\frac{5 \times 10^7}{x^{2.0}}\right). \quad (6)$$

Now to find the value of X that minimizes total annual cost we need to find a root of equation (6). To accomplish this task we use Newton's Method, which states:

$$C'(x + \Delta x) \approx C'(x) + C''(x) \Delta x. \quad (7)$$

We are interested in the value of x that gives $C'(x) = 0$, so we have,

$$\Delta x = -\frac{C'(x)}{C''(x)} = -\frac{\left(\frac{560}{x^{0.3}}\right) - \left(\frac{4 \times 10^7}{x^{1.8}}\right) - \left(\frac{5 \times 10^7}{x^{2.0}}\right)}{-\frac{168}{x^{1.3}} + \frac{7.2 \times 10^7}{x^{2.8}} + \frac{1 \times 10^8}{x^{3.0}}}. \quad (8)$$

We conclude that the total annual cost is minimized when $x = 2,021.82$. Refer to the Appendix to view the calculations for Newton's Method and its findings; the answer is highlighted for an enhanced view.

Now that we know the optimal value of x we are in position to answer the initial questions. Equation (2) gives the time per batch, $t = 16.2$ hrs, while equation (3) gives the total number of batches $b = 247.3$. From these values we summarize the optimal costs for this company in Table 1 below.

Cost	Formula	Amount
Annual Labor Cost	$40 t b$	\$160,000.00
Equipment Cost	$800 x^{0.7}$	\$164,857.89
Set-up Cost	$100 x^{0.2} b$	\$113,337.24
Chemical Cost	$(0.25 x + 100) b$	\$149,729.02
Annual Total Cost	$C(x)$	\$587,924.15

Table 1: Breakdown of optimal operating costs.

DISCUSSION

The company's primary goal was to minimize its annual cost. However, the company also needed to identify their corresponding batch time and number of batches per year that minimized the annual cost. For this to take place, it was beneficial to find the number of boards per batch that contributed to minimizing the annual cost.

The objective of the company was accomplished as cost was minimized. The total annual cost was calculated to be \$587,924.15 and the company minimized its cost in 247 days of operation. In addition, the corresponding batch time was 16 hours per batch. It was necessary to

analyze this information in order to calculate the corresponding cost for each category in the company.

Every cost that the company incurred over the year was presented by a category and every category was presented by an equation in different units. It was important to develop a relationship and therefore an expression for total annual cost, where we would be able to use time per batch and number of batches, to eventually obtain total annual cost.

CONCLUSION AND RECOMMENDATIONS

In conclusion, it is acceptable to say that this project touched on an essential aspect of engineering; being a creative thinker and solving problems in an efficient way, which in industrial engineering mostly deals with cost and the optimization of systems and enterprises.

The company's goal was to minimize cost in a maximum of 250 days of operation. The goal was accomplished in 247 days and met the objective of the company. The purpose in minimizing the cost in a maximum of 250 days could be a result of numerous reasons such as, rental fees or also a contract agreement. Other goals the company proposed were to produce 500,000 treated boards per year and this was the reason for implementing this in the equation for number of batches.

This project requires a very clear understanding of calculus. Our approach involved developing an expression for the total annual cost and using Newton's method to find the root of the expression; it is an easy and efficient method to find a root of a real valued function.

NOMENCLATURE

Symbol	Definition	Units
$C(x)$	Total Annual Cost	Dollars/Year
b	Number of batches	N/A
t	Time per batch	Hours/Batch
x	Number of boards per batch	Boards/Batch
Δx	Change in x	Boards/Batch

REFERENCES

- Larson, Ron, Robert Hostetler and Bruce Edwards. Calculus. 8th Edition. Boston, MA: Houghton Mifflin Company, 2005.
- Giandomenico, Majone, and Edward S. Quade. Pitfalls of Analysis. 1st Edition. New York: John Wiley & Sons, 1980. Print.
- Sivazlian, B.D., and L.E. Stanfel. Optimization Techniques In Operations Research. 1st Edition. New Jersey: Prentice-Hall Inc, 1975. Print.
- Blumenfeld, Dennis. Operations Research Calculations Handbook. Florida: CRC, 2003. Print.
- Pardalos, Panos M., Yannis Siskos, and Constantin Zopounidis. Advances In Multicriteria Analysis. Netherlands: Springer, 1995. Print.
- Farnham, Dwight T. Scientific Industrial Efficiency. Easton Hive Publishing Company, 1974. Print.
- Blair, Raymond N., and C. Wilson Whitston. Elements of Industrial System Engineering. New Jersey: Prentice-Hall Inc, 1971. Print.
- Barnes, Ralph Mosser. Industrial Engineering And Management, Problems And Policies. New York: Hive Pub Co, 1931. Print.
- Reisman, Arnold. Management Science Knowledge, Its Creation, Generalization, And Consolidation. Connecticut: Quorum Books, 1992. Print.
- Drechsler, Frank, and John Bateson. Management, Information, And Expert Systems. Britain: Irish Academic Press, 1986. Print.
- Stewart, James. Essential Calculus, Early Transcendentals. Brooks/Cole Pub Co, 2010. Print.
- Grinshpan, A. "Project." Project Discussion. USF, Tampa. Lecture.

APPENDIX – NEWTON’S METHOD

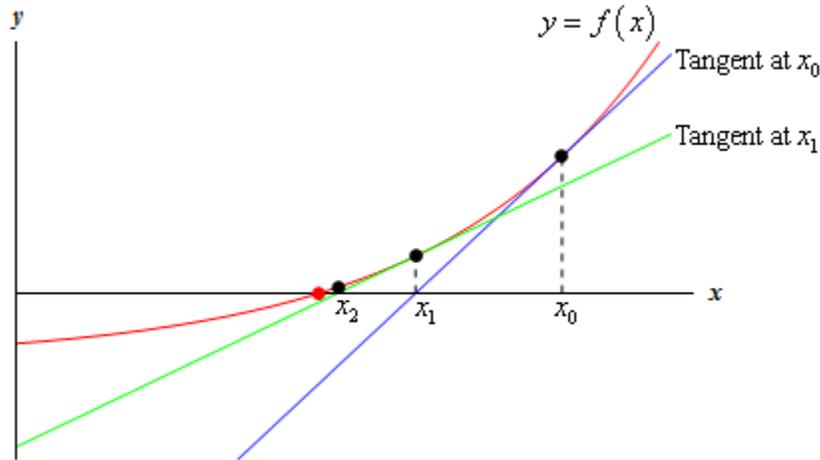


Figure 1: Graphical (above) and numerical (below) representations of Newton’s Method for finding the root of $f(x)$.

x_i	$f(x_i)$	$f'(x_i)$	Δx_i	x_{i+1}
1.0000	-89999440.00	171999832.00	0.5232	1.5232
1.5232	-40301424.42	50453265.81	0.7987	2.3220
2.3220	-18052782.09	14793155.8	1.2203	3.5423
3.5423	-8089286.59	4335550.358	1.8658	5.4081
5.4081	-3625903.92	1270116.149	2.8547	8.2629
8.2629	-1625763.99	371934.4129	4.3711	12.6340
12.6340	-729162.98	108874.9701	6.6972	19.3313
19.3313	-327114.91	31860.808	10.2667	29.5983
29.5983	-146774.99	9321.9368	15.7451	45.3434
45.3434	-65859.40	2727.5865	24.1457	69.4891
69.4891	-29544.61	798.4931	37.0005	106.4895
106.4895	-13243.49	234.0809	56.5765	163.0661
163.0661	-5925.72	68.8344	86.0864	249.1525
249.1525	-2641.26	20.3722	129.6496	378.8022
378.8022	-1168.06	6.1079	191.2365	570.0388
570.0388	-508.38	1.8788	270.5771	840.6160
840.6160	-214.15	0.6079	352.2686	1192.8846
1192.8846	-84.20	0.2170	387.9671	1580.8518
1580.8518	-28.39	0.0931	304.7296	1885.5814
1885.5814	-6.62	0.0541	122.2657	2007.8472
2007.8472	-0.62	0.0445	13.8231	2021.6704
2021.6704	-0.01	0.0435	0.1514	2021.8219
2021.8219	-7.8E-07	0.0435	1.78E-05	2021.8219
2021.8219	-2.31E-14	0.0435	5.30E-13	2021.8219