Undergraduate Journal of Mathematical Modeling: One + Two

Volume 3 | 2011 Spring

Issue 2 | Article 9

Optimization of a Wall Built on a Slope

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Recommended Citation

Hanna, John (2011) "Optimization of a Wall Built on a Slope," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 3: Iss. 2, Article 9.
DOI: http://dx.doi.org/10.5038/2326-3652.3.2.9
Available at: https://scholarcommons.usf.edu/ujmm/vol3/iss2/9

Optimization of a Wall Built on a Slope

Abstract

This project focuses on optimizing the construction of a wall with identical segments built on a slope. Faced with certain variable and fixed cost limitations, one can find the width of a segment that will minimize the total production cost. In this case, the lowest total cost comes from using 10 segments of width 3m.

Keywords

Civil Engineering, Cost Analysis, Wall Construction

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PROBLEM STATEMENT

A wall is to be built on a slope as shown below by placing rows of identical wall segments. The thickness of each segment is $t = 0.01\ m$. The wall has a slope of $\theta = 25°$ and has length in the horizontal direction $L = 30\ m$. The apparent height of the wall above the slope is fixed at $h = 2\ m$. The goal of this problem is to find the dimension x of each segment that will minimize the total cost of building the wall and the resulting minimum cost.

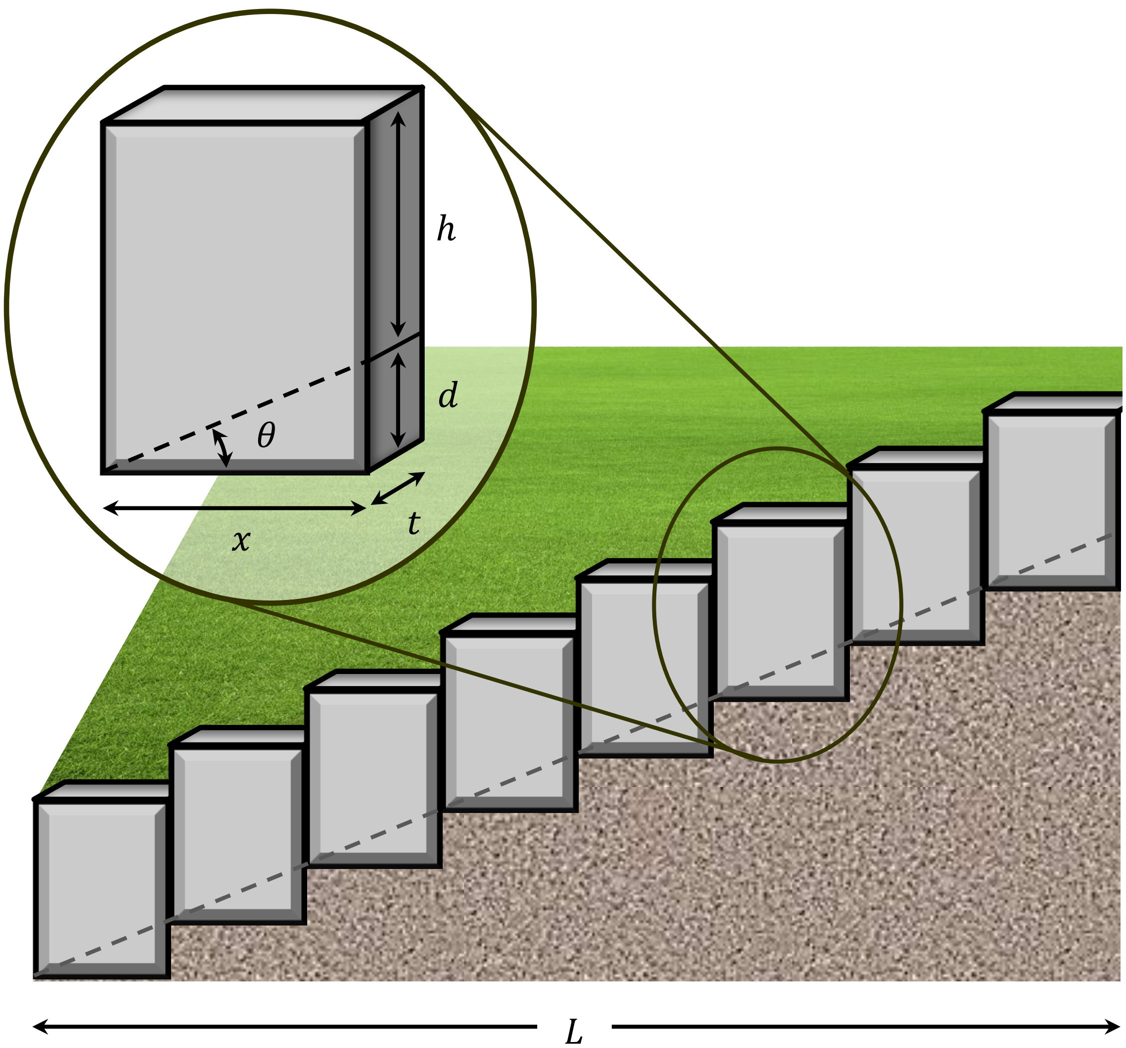

Figure 1: Layout of the brick wall along a slope of $\theta = 25°$.

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There are two costs associated with building this wall: 1) the excavation cost and 2) the price of each wall segment. The excavation cost C_E is a fixed rate of $\$150/m^3$ of ground removed. The price of each wall segment is given as $C_S = C_F + C_V$ where C_F is a fixed rate of $\$200$ per segment and C_V is based on the volume of each segment set at $\$500/m^3$.

MOTIVATION

In any construction project, engineers must work within constraints to reach their goal. Optimization allows an engineer to solve a goal with a minimal amount of time, energy, or money. In this case, we wish to calculate the width of a wall that saves the most money by minimizing the total cost of the building. For many businesses, minimizing costs translates to maximizing profits.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

The width of the wall will be determined using the following steps. The first step is to use the geometry of Figure 1 to relate the known and unknown parameters. The second is expressing the overall cost in terms of x, the width of each wall segment. The third step is minimizing this cost by setting the derivative of that function equal to zero and solving for the optimal width of each wall segment. Lastly, we will use this information to find the best whole number solution since partial segments are not permitted.

In Figure 1 we are given that the slope of the wall is $\theta = 25°$, the total wall length is $L = 30m$, the height above ground is $h = 2m$, and the depth below ground is depicted is d. Since we wish to express the measurements in terms of x we note that

$$d = x \tan\theta = x \tan 25°. \tag{1}$$

The number of wall segments N is simply

$$N = \frac{L}{x} = \frac{30}{x}. \tag{2}$$

Note that N must ultimately be constrained to be an integer. Let V_S represent the volume of each segment and V_E be the volume of excavation required for each segment. Thus for each segment we have that

$$V_E = t \times \left(\frac{1}{2} \times x \times d\right) = \frac{\tan 25°}{20} x^2. \tag{3}$$

and

$$V_S = t \times x \times (d + h) = \frac{1}{10} x(x \tan 25° + 2) = \frac{\tan 25°}{10} x^2 + \frac{1}{5} x \tag{4}$$

We shall now express each of these constituent costs as a function of the width x of each segment. According to the problem statement, the cost of exaction is $\$150/m^3$. The total volume of excavation required is NV_E so (3) gives

$$C_E = \left(\frac{30}{x}\right)\left(\frac{\tan 25°}{20} x^2\right) = \left(\frac{3}{2} \tan 25°\right) x. \tag{5}$$

The total cost of the wall segments C_W is given by $NC_S = N(C_F + C_V)$. Thus

$$\begin{aligned} C_W &= N(200 + 500\, V_s) \\ &= \left(\frac{30}{x}\right)(200 + (50 \tan 25°)x^2 + 100x) \\ &= \frac{6{,}000}{x} + (1{,}500\ \tan 25°)\, x + 3{,}000 \end{aligned} \tag{6}$$

Therefore, the total cost of building the wall is

$$C_{Total}(x) = C_E + C_W = \frac{6{,}000}{x} + \left(1{,}500 + \frac{3}{2}\right)(\tan 25°)\, x + 3{,}000. \tag{7}$$

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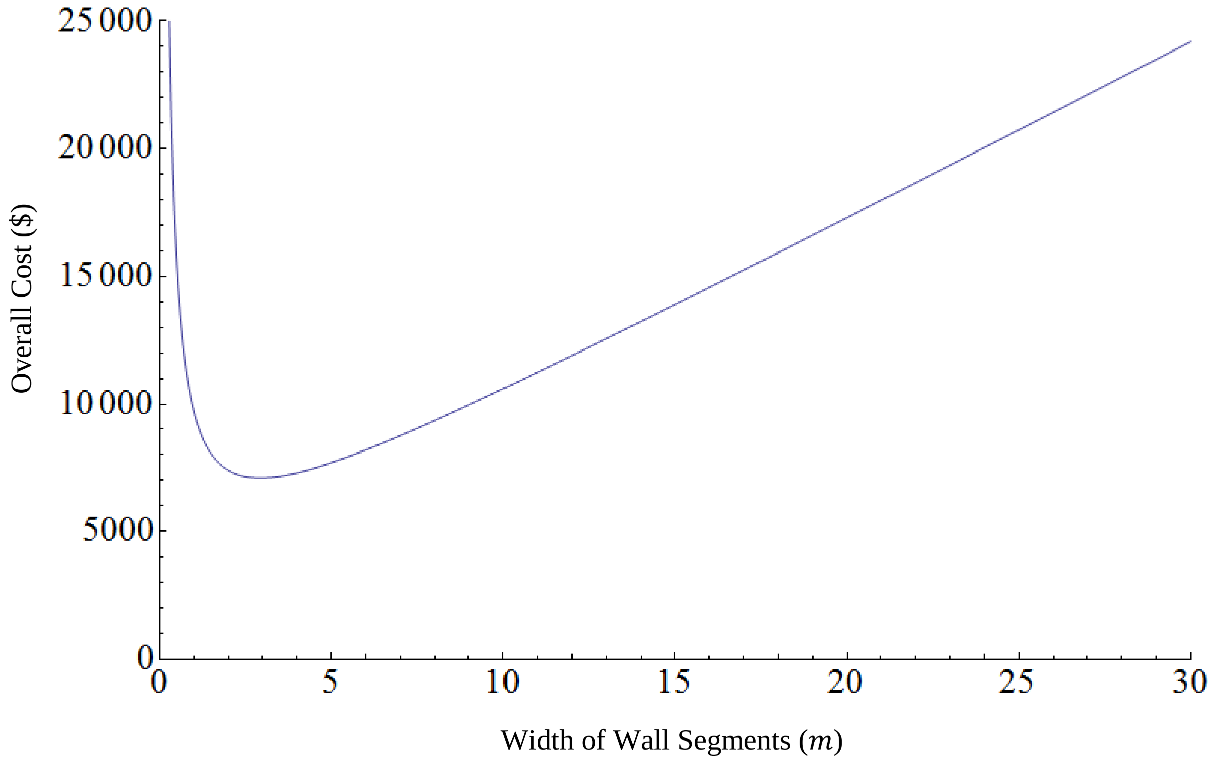

Figure 2: Overall cost of building the wall $C_{Total}(x)$ in terms of the width of each segment x.

In light of (7), we wish to minimize this overall cost in terms of x. To accomplish this, we find the corresponding x for which $C_{Total}'(x) = 0$. We compute this directly as

$$C_{Total}'(x) = \left(1{,}500 + \frac{3}{2}\right)(\tan 25°) - \frac{6{,}000}{x^2} = 0 \tag{8}$$

which implies that

$$x = \sqrt{\frac{6000}{\left(1{,}500+\frac{3}{2}\right)(\tan 25°)}} \approx 2.927. \tag{9}$$

DISCUSSION

Equation (9) gives the width of each segment such that the overall cost of the wall is minimized. However, we must now interpret this answer to make sure that it works physically. As given in the problem statement, the wall segments have to be identical and the total length is set to be $30m$. If we divide 30 by the x-value given in (9), then we would have the number of segments to be built $N = 10.248$ segments. Since the problem asks for the number of identical segments, we know that the ideal number of wall segments is between 10 and 11 which corresponds to $x = 3$ and $\frac{30}{11}$ respectively.

From (7) we see that $C_{Total}(3) = \$7{,}100.48$ and $C_{Total}\left(\frac{30}{11}\right) = \$7109.53$, so it is cheapest to use 10 wall segments of dimension $3\ m\ \times 3.4\ m\ \times 0.1\ m$.

CONCLUSION AND RECOMMENDATIONS

After formulating the total cost of the wall in terms of each wall segment x, we minimized the cost by setting the derivative of the cost function equal to zero. This resulted in a minimal cost when $x = 2.927$ which corresponds with $L = 10.248$ segments. Since partial segments are not permitted, we compared the cost of having 10 segments to having 11. In the end, we found that purchasing 10 wall segments with width $3m$ will minimize the overall cost of the wall at $\$7{,}100.48$.

Future projects may consider incorporating knowledge of the wall material to determine the wall's weight, friction factor, so that one can calculate the best angle to building the wall.

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NOMENCLATURE

Symbol	Desription	Value
$\boldsymbol{N}$	Number of segments	Eq. (1)
$\boldsymbol{L}$	Horizontal length of overall wall	$30\ m$
$\boldsymbol{t}$	Thickness of each segment	$0.1\ m$
$\boldsymbol{h}$	Height of each segment fixed above ground	$2\ m$
$\boldsymbol{x}$	Width of each segment	Eq. (9)
$\boldsymbol{d}$	Height of each segment fixed below ground	Eq. (2)
$\boldsymbol{\theta}$	Slope of the wall	$25°$
$\boldsymbol{V_E}$	Volume of excavated area below ground	Eq. (3)
$\boldsymbol{V_S}$	Volume of one segment	Eq. (4)
$\boldsymbol{C_F}$	Fixed cost of each segment	$\$200$
$\boldsymbol{C_V}$	Cost of each segment by volume	$\$500/m^3$
$\boldsymbol{C_S}$	Cost of each segment	$C_F + C_V$
$\boldsymbol{C_W}$	Cost of all wall segments	NC_s
$\boldsymbol{C_E}$	Excavation cost	$\$150/m^3$
$\boldsymbol{C_{Total}(x)}$	Overall cost of the wall in terms of x	Eq. (7)

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