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Travelling Distance of a Skimboard

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Travelling Distance of a Skimboard

Abstract

The purpose of this project was to determine the optimum conditions to maximize the distance travelled on a skimboard. Given the differential equation that governs the changes in velocity of the skimboard, we were able to integrate and find an expression for the distance travelled by the skimboard. Using this expression we substitute different values for variables, such as angle of attack and resistance caused by the surface area of the rider, to explore the effect on the total distance travelled by the skimboard.

Keywords

Skimboarding, Kinematics, Separable Differential Equations

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PROBLEM STATEMENT

The equation for the acceleration of a skimboarder is

$$\frac{dv}{dt} = -g\alpha - \frac{1}{2m}\rho_w v^2 S_B c_f - \frac{1}{2m}\rho_a v^2 S_H C_D \tag{1}$$

where v is velocity, g is the acceleration due to gravity, α is the angle of attack, m is the mass of the rider, ρ_w is the density of the water, ρ_a is the density of the air, S_B is the surface area of the board, S_H is the surface area of the rider, c_f is the skin friction coefficient, and C_D is the aerodynamic drag coefficient (Sugimoto). Integrate this equation to find an expression that will yield the total traveling distance of a skimboard. Once the expression is found, substitute values for some of the variables to observe the effect on the total traveling distance of the skimboard. In particular determine the effect of changes in the angle of attack, α, and the surface area of the rider.

MOTIVATION

There is a lot of engineering that is put into the designing of a high performance skimboard. Skimboarding has become a very popular sport over the years and there are a number of manufacturers that produce skimboards. In a competitive market, the company that makes the best skimboard will profit the most. Looking at each of the variables, such as the angle of attack or the ratio of the surface of area of the board to the weight of the rider will provide key information for the design of an optimal skimboard. Designers can also create a curvature of the board that will assist the rider in minimizing the angle of attack.

This project creates the possibility for skimboard manufacturers to create specialized boards, which are boards that have a surface area that is proportional to rider weight. This means that companies can push the limits on board size without creating a product that is too cumbersome for riders to use. This specialization will create boards that will handle better as well. Boards will be able to respond better to rider input and create an overall better experience.

As for the rider's experience, this project will also point out the most efficient way for the rider of a skimboard to control their board. The rider is directly responsible for controlling the angle of attack and the area of their body exposed to drag caused by the air. Knowing the effect of different angles of attack and body areas exposed to drag, one can manipulate both board and body to the optimum conditions to maximize distance. Examples of this include distribution of weight between the feet of the rider and the stance of the rider.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

In order to integrate (1) we first use the chain rule to derive the simple fact:

$$\frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds} \cdot v = v \cdot \frac{dv}{ds}. \tag{2}$$

Replacing the left hand side of (1) with (2) yields:

$$v \cdot \frac{dv}{ds} = -g\alpha - \frac{1}{2m}\rho_w v^2 S_B c_f - \frac{1}{2m}\rho_a v^2 S_H C_D. \tag{3}$$

Since a majority of the variables in (3) represent constants it is convenient to set $a = g\alpha$ and $b = \frac{\rho_w S_B c_f}{2m} + \frac{\rho_a S_H C_D}{2m}$, thus (3) becomes:

$$v \cdot \frac{dv}{ds} = -a - bv^2. \tag{4}$$

We separate the variables in (4) to obtain,

$$-ds = \frac{dv}{\frac{a}{v}+bv} \tag{5}$$

which can be easily integrated to find an explicit expression for s_t, the distance travelled by the skimboard. Integrating (5) gives

$$\int_0^{s_t} -ds = \int_{v_0}^{v_t} \frac{vdv}{a+bv^2}\,. \tag{6}$$

Evaluating the definite integral in (6) yields

$$[-s]_0^{s_t} = \left[\frac{\ln(a+bv^2)}{2b}\right]_{v_0}^{v_t}. \tag{7}$$

Thus substituting for a and b in (7) and simplifying reveals

$$s_t = \frac{m}{\rho_w S_B c_f + \rho_a S_H C_D} \cdot \ln\left|\frac{m\ g\ \alpha + [\rho_w S_B c_f + \rho_a S_H C_D]\frac{v_0^2}{2}}{m\ g\ \alpha + [\rho_w S_B c_f + \rho_a S_H C_D]\frac{v_t^2}{2}}\right|. \tag{8}$$

DISCUSSION

To determine the effect of the angle of attack on the total distance travelled by the skimboard, we test different values for α while fixing all the other values. For the remainder of this discussion, we set

$$\rho_w = 1025\frac{kg}{m^3}, \quad \rho_a = 1.225\frac{kg}{m^3}, \quad \text{and} \quad c_f = .005.$$

The results from changing the angle of attack are summarized in Figure 1.

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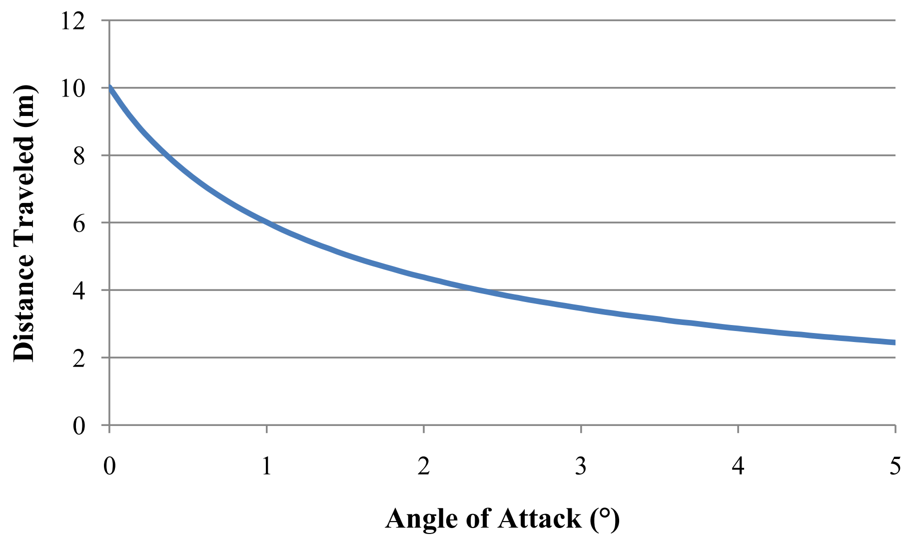

Figure 1: Skimboarding distance traveled by a $29kg$ child with a surface area of $42cm^2$. The child enters the water on a circular skimboard of radius $50cm$ and aerodynamic drag coefficient 1.3 with an initial velocity $2.7m/s$ before reaching a terminal velocity of $1.2\ m/s$.

These results show that the smaller the angle with respect to the horizontal, the greater the total distance traveled by the skimboard. This is expected as the shallower the angle of attack, the smaller the drag that is created on the board by the water. The flatter the skimboard rides on the water, the less water it displaces, thus creating less drag.

We perform the same analysis again, this time letting $\alpha = 2.0^{\circ}$ while changing the surface area of the rider, S_H. These results are summarized in Figure 2.

The results show that the greater the surface area of the rider, the more aerodynamic drag is created, thus decreasing the total distance traveled by the skimboard. It is noticeable that the aerodynamic drag has much less of an effect on the total distance than that of the angle of attack. However, the trend is again negative which suggests that as drag increases, distance decreases.

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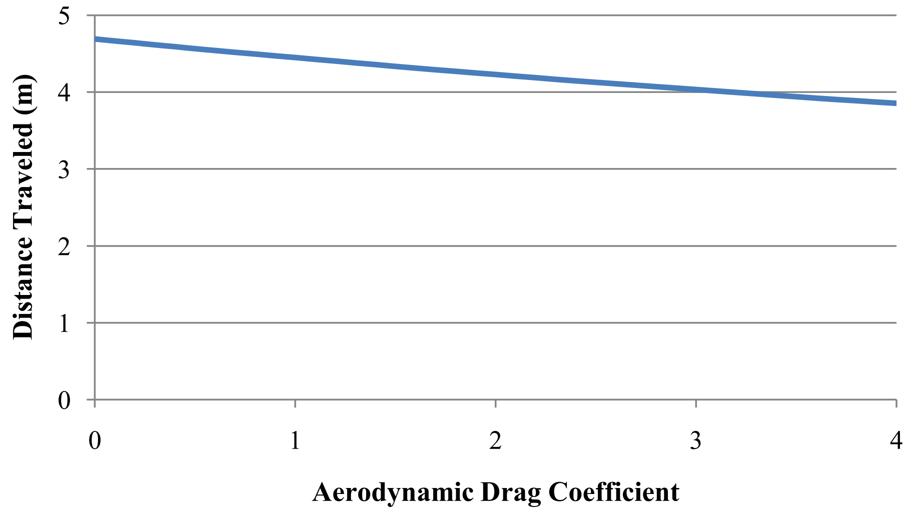

Figure 2: Skimboarding distance traveled by a $29kg$ child with a surface area of $42cm^2$. The child enters the water at a $2°$ angle on a circular skimboard of radius $50cm$ with initial velocity $2.7m/s$ before reaching a terminal velocity of $1.2\ m/s$.

From personal experience with the sport, the results are as expected. A small angle of attack displaces less water which will reduce hydrodynamic drag. Also, the smaller the surface area of the body, the less aerodynamic drag is created which will result in an increased traveling distance of the skimboard.

CONCLUSION AND RECOMMENDATIONS

As we have shown, there are two key things that both skimboard riders and manufacturers must take into consideration. One of these main ideas is that the angle of attack must be as small as possible. Keeping the angle small reduces the pressure drag created by the water, thus allowing the board to travel further. However, it is also crucial that the attack angle is greater than 0° in order for the board to stay afloat and not submarine into the water.

Another key element that should be taken into consideration is the aerodynamic drag. Our analysis shows that the smaller surface area of the body exposed to the drag of the surrounding air, the further the total distance traveled by the skimboard. One way to achieve a lower surface

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area is to keep a low center of gravity by taking a low stance. This may also help the rider create a good stance for maneuvering the skimboard.

Although this project firmly establishes relationships between the angle of attack, the aerodynamic drag, and the total distance traveled, it should be noted that equation (1) relies on some estimation. There are other elements that factor into the equation. Other factors that will affect the total distance traveled include waves the board may encounter, windy conditions, and even the slope of the shore of the beach. Each of the aforementioned can create constructive or destructive interference and elongate or curtail the total distance traveled by the skimboard.

Another factor that is estimated is the skimboard itself. In the equation above the board is assumed to be a circle, but in reality the board is more oval shaped. The board is also assumed to be flat. However, a high performance skimboard is curved up at the nose and also at the tail. This is to try and keep the angle of attack small. By curving the nose of the board up, the area of the board that has a large angle of attack is small. This reduces the drag of the water and makes the board more efficient. Another parameter of the board that could be included is the stiffness of the board. A carbon fiber or fiberglass board will have a longer total distance traveled than a wooden board because the former two are stiffer, thus reducing flex and the hydrodynamic drag associated with the flex of the board.

Lastly, the force associated with the buoyancy of the skimboard was not accounted for. Many high performance skimboards today are made with foam cores covered in fiberglass. The cores range from high-density foam to a polyvinylchloride foam core, each with an associated buoyancy force associated with it. However, it is unclear if the force is large enough to substantial affect performance. Overall we think the equation presented is a good approximation, but could be revisited for accuracy.

NOMENCLATURE

Symbol	Definition	Units
α	Angle of Attack	Degrees
g	Acceleration due to Gravity	$\frac{m}{s^2}$
m	Mass of Rider	kg
$v\ (v_0, v_t)$	Velocity (initial, terminal)	$\frac{m}{s}$
$\rho_w(\rho_a)$	Density of Water (Air)	$\frac{kg}{m^3}$
$S_H(S_B)$	Surface Area of Rider (Skimboard)	m^2
$s(s_t)$	Distance (terminal)	m
C_D	Drag Coefficient	
c_f	Skin Friction Coefficient	

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