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Calculating Optimal Inventory Size

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Abstract
The purpose of the project is to find the optimal value for the Economic Order Quantity Model and then use a lean manufacturing Kanban equation to find a numeric value that will minimize the total cost and the inventory size.

Keywords
Economic Order Quantity, Kanban Scheduling System, Lean Manufacturing

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PROBLEM STATEMENT

A traditional problem in Industrial Engineering is to find the inventory size while minimizing the cost of ordering (set up) and the cost of keeping the inventory. Those models are named Economic Order Quantity Models. However, nowadays manufacturing is evolving into Lean Manufacturing, and it is interested in the use of Kanban systems. To solve the following questions you need to apply your tools learned in your Calculus courses:

a) Find the Economic Order Quantity of a product by minimizing the set up inventory and set up costs. Assume the model is deterministic and no backorders are allowed.

b) Imagine now that you need to calculate the Kanban size for a product, and you have n technologies that you can use to move that product around your manufacturing process. How do you calculate the Kanban size based on minimizing the inventory costs? Which technology should you select? Give a numeric example.

MOTIVATION

Economic Order Quantity (EOQ) (Economic order quantity) is the quantity that minimizes the total cost associated with inventory ordering and storing. Hence EOQ helps to determine the right amount of inventory to order. Inventory model usually consider related costs, demands, order cycles, time period, re-stocking, and leading time which determines if the problem will is deterministic or probabilistic. We consider a deterministic version of the problem with no backorders allowed.

Lean manufacturing (Lean manufacturing, Feld) is a production technique that improves management of inventory, developed by Toyota Motors in Japan. Lean manufacturing technique

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helps the manufacturing process to run quickly and smoothly with little or no delay. The technique is related to just-in-time production (JIT) which is another inventory management strategy. Lean manufacturing aims to make a system flexible, eliminate waste such as overproduction and interference. Lean manufacturing has push and pull systems. A push system moves a completed work output to the next area or station. An example of a pull system is a Kanban (Kanban). Pull systems move output by pulling it from the earlier station as desired. Kanban is a Japanese term that means "sign." Kanban uses cards that include: an item number, item name, product type, container capacity, box type, issue number, and issue date.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

We start by computing the Economic Order Quantity of a product by minimizing the set up inventory and set up costs. Recall that the model is assumed to be deterministic and no backorders are allowed. Note that

$$\text{Total Cost} = \text{Purchase Cost} + \text{Ordering Cost} + \text{Holding Cost}$$

and hence

$$TC = P \cdot D + \frac{D \cdot S}{Q} + \frac{H \cdot Q}{2} \tag{1}$$

where TC is the total cost, P is the unit price, D is the demand quantity, S is the fixed cost associated with each order, Q is the quantity of items in each order, H is the holding cost of each unit.

Differentiating both sides of (1) with respect to Q gives

$$0 = 0 - \frac{D \cdot S}{Q^2} + \frac{H}{2} \tag{2}$$

and solving (2) for Q yeilds

$$Q = \sqrt{\frac{2D\cdot S}{H}} \tag{3}$$

which is the value of Q minimizing the total cost, i.e. Q^*.

Thus EOQ formula is obtained from Total Cost formula by taking the partial derivative with respect to Q. Note that because TC, P, and D are assumed to not depend on Q, the corresponding terms vanish after differentiation.

Now we calculate the Kanban size for a product based on minimizing the inventory costs. Note that

$$\left(\text{Capacity of container1} + \text{capacity of container } 2\cdot\frac{\text{demand}}{\text{total\# of containers}}\right) + \frac{\text{holding cost(total containers}-1)}{2}$$

$$= \text{minimize}_j \left(C_{1ij} + C_{2ij}\cdot\frac{D_i}{n_i}\right) + \frac{h_i\cdot\left(n_j - 1\right)}{2} \text{ subject to: } n_i \le N_j$$

and

$$\frac{\partial}{\partial n_i}\left(\left(C_{2ij}D_i\right)n_i^{-1}\right) = -\frac{C_{2ij}D_i}{n_i^2};\ \frac{\partial}{\partial n_i}\left(\frac{h_i\left(n_j-1\right)}{2}\right) = \frac{h_i}{2};\ \frac{\partial \text{Cost}}{\partial n_i} = 0 - \frac{C_{2ij}D_i}{n_{ij}^2} + \frac{h_i}{2}$$

Thus after taking a partial derivative of the above equation with respect to n_i (keeping in mind that C_{ij} is a constant and therefore vanishes) we get

$$-\frac{C_{2ij}D_i}{n_{ij}^2} + \frac{h_i}{2} = 0\ \left(\text{assuming that } \frac{\partial\ \text{Cost}}{\partial\ n_i} = 0\right).$$

Solving the above equation for n_{ij} gives the optimal size (denoted by n_{ij}^*)

$$n_{ij}^* = \sqrt{\frac{2C_{ij}D_i}{h_i}} \tag{4}$$

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Numerical example:

A large baking factory undergoes 3 procedures to bake cakes: Mixing the flour, baking and frosting. Each of these three steps occurs in different rooms of the bakery. Table 1 shows the annual fixed cost for three options to transport the cakes to different rooms. The factory produces 100,000 cakes per year. The annual inventory holding cost is $1.50 per cake.

Option	Annual Cost	Cost per round	Max container size
Manual labor	$13,500	$0.19	2
Handcart	$14,000	$0.20	15
Motorized lift	$25,000	$0.45	250

Table 1: The annual cost, cost per round, and maximum container size for each step of the production.

Now we find the optimal container size for each option and compare the costs. Note that

$$n_{ij}^* = \sqrt{\frac{2C_{2ij}D_i}{h_i}} = \sqrt{\frac{2\,C_{2ij}(100{,}000)}{1.50}} = \sqrt{\frac{200{,}000C_{2ij}}{1.50}} = 365.14\sqrt{C_{2ij}}$$

and hence

$$n_{\text{manual}} = (365.14)\cdot\sqrt{(3)(0.19)} = 275.67$$

$$n_{\text{cart}} = (365.14)\cdot\sqrt{(3)(0.20)} = 282.83$$

$$n_{\text{lift}} = (365.14)\cdot\sqrt{(3)(0.45)} = 424.25$$

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The above calculations imply that

$$\text{Manual labor: } \$13{,}500 + \$0.19 \cdot \frac{(3)(100{,}000)}{2} = \$42{,}000$$

$$\text{Handcart: } \$14{,}000 + \$0.20 \cdot \frac{(3)(100{,}000)}{15} = \$18{,}000$$

$$\text{Motorized lift: } \$25{,}000 + \$0.45 \cdot \frac{(3)(100{,}000)}{250} = \$25{,}540$$

The best choice is the handcart with the above container size because it is the cheapest.

DISCUSSION

Although our analysis is not new and the considered problem is a classical industrial engineering problem, the solution helps to understand the difficult computational problems that companies must overcome in order to improve the operation of their production lines. Furthermore, the numerical example presented at the end of the solution section shows how the theory can be applied to solve real-world problems.

CONCLUSION AND RECOMMENDATIONS

This work reviews theoretical models that can be useful for solving real-world problems arising in manufacturing and related fields. Furthermore, calculus turns out to be an important tool for studying these models. The numerical example presented in this work shows how the results derived from mathematical analysis of the model can give an insight into the solution of the original problem.

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NOMENCLATURE

Symbol	Name
TC	total cost
P	unit price
D	demand quantity
S	fixed cost of an order
Q	quantity of an order
H or h	unit holding cost
n	container capacity
N	size limit

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