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Stormwater Management System Drawdown

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Stormwater Management System Drawdown

Abstract
This project concerns the computations required to determine the drawdown for the retention/detention of ponds. Drawdown refers to the volume of water in a pond that decreases as the water flows out. The falling head equation has many applications and can be used to calculate the drawdown of a pond through various shaped openings. In particular, we analyze four outflow structures: a rectangular-notch weir, a v-notch weir, a round orifice, and an underdrain. For each instance, we modify the falling head equation to reflect the shape of the respective orifice.

Keywords
Outflow, Drawdown, Weir Design, Falling Head Equation

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PROBLEM STATEMENT

Calculate the amount of time it takes for a retention/detention pond to draw down after a storm event using different shaped openings and structures.

MOTIVATION

This problem is useful in civil engineering; especially in the field of storm water management. According to state law, almost every developed piece of property must abide by water quantity criteria. This means that the rate at which water flowed off the property before development must be equal or greater than the flow rate after development. To ensure this, retention and detention ponds are widely used with the aid of control structures such as weirs and underdrains. These control structures are customized to allow only a certain volume of water to flow out of the pond per second.

As the water from a pond flows out, the total volume of water in the pond naturally decreases. Because of this, the flow rate is constantly decreasing, and the water leaves the pond at a slower rate as time goes on. To take this into account, we must use an integral which can tell us the flow rate at any given second. We consider the height of the water in the pond from the surface to a given point which varies according to the type of outflow structure. This height multiplied by the surface area of the pond gives us the volume of the water in the pond. Since the water height is constantly decreasing, it must be integrated with respect to time.

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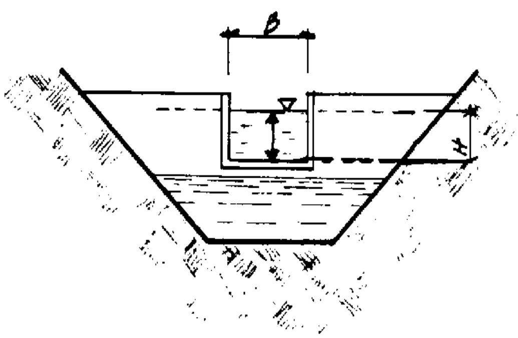
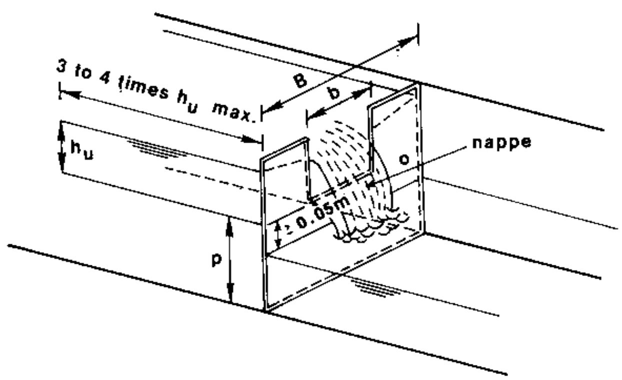
MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

A weir is a structure built into a retention or detention pond to regulate the outflow of water. In this project we will consider two of the different types of weirs; the first is a rectangular weir and the second is a triangular weir. We will also break down the formulas used to calculate drawdown for an orifice and an underdrain. The orifice we describe is found in wet detention ponds and feeds into the control structure. It is simply a pipe usually located underneath the pond's surface that connects to the control structure. An underdrain is a perforated pipe usually located underneath a dry retention pond. This pipe is surrounded by a filter media and carries water away from the pond after it has percolated through the soil.

I. DRAWDOWN CALCULATIONS

Rectangular-Notch Weir.

The base equation used for calculating flow rate, Q, through a rectangular weir is,

$$Q_{rectangular} = c\,L\,h^{3/2} = 3.13\,L\,h^{3/2} \quad (1)$$

where L is the length of the weir, h is the distance from the bottom of the weir to the surface of the pond and $c = 3.13$ is a constant that has been predetermined based on the shape of the weir. If we let V represent the volume of the pond we can rewrite (1) as,

$$\frac{dV}{dt} = c\,L\,h^{3/2} \tag{2}$$

thus the flow rate is simply the rate at which the volume in the pond changes over time. Since the volume of the pond is just the surface area multiplied by the height of the water in the pond, (2) becomes,

$$A\,\frac{dh}{dt} = c\,L\,h^{3/2} \tag{3}$$

where A denotes the surface area of the pond. The surface area of the pond does not change as a function of time so it is convenient to write:

$$\frac{dh}{dt} = \frac{c\,L\,h^{3/2}}{A} \tag{4}$$

We are interested in solving for time, so we separate the variables in (4) and integrate both sides to see,

$$\int_{h_1}^{h_2} h^{-\frac{3}{2}}\,dh = \frac{c\,L}{A}\int_0^t dt. \tag{5}$$

Evaluating the definite integral in (5) yields:

$$-\frac{2\,A}{c\,L}\left(\frac{1}{\sqrt{h_2}} - \frac{1}{\sqrt{h_1}}\right) = t. \tag{6}$$

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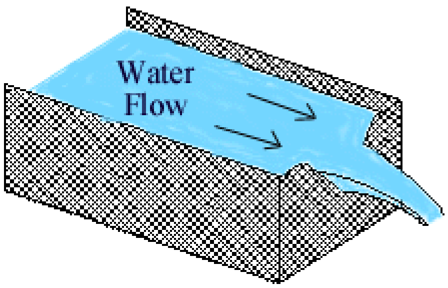
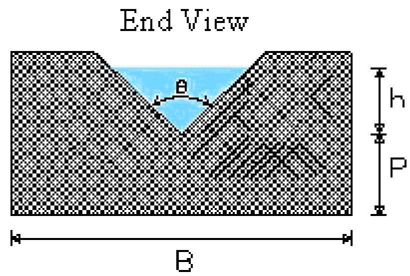
V-Notch Weir.





The equation used to find the flow rate, $Q_{v-notch}$, is,

$$Q_{v-notch} = 2.5\left(\tan\frac{\theta}{2}\right)h^{5/2} \tag{7}$$

where h and θ are as pictured above. Using methods similar to the rectangular notch case we write:

$$Q = \frac{dV}{dt} = A\frac{dh}{dt} = 2.5\left(\tan\frac{\theta}{2}\right)h^{5/2}. \tag{8}$$

Again we are interested in solving for time so we separate the variables in (8), set $K = 2.5\left(\tan\frac{\theta}{2}\right)$, and integrate as before to obtain,

$$\int_{h_1}^{h_2} h^{-5/2}\,dh = K\int_0^t dt. \tag{9}$$

Evaluating the definite integral in (9) yields:

$$t = \frac{-2}{3K}\left[h_2^{-3/2} - h_1^{-3/2}\right]. \tag{10}$$

Orifice.

To calculate the flow rate, $Q_{orifice}$, through an orifice with area a we use the following equation,

$$Q_{orifice} = \frac{dV}{dt} = c\, a \sqrt{2\, g\, h} \tag{11}$$

here g is the acceleration due to gravity. Using methods outlined in the previous calculations we rewrite (11) as:

$$Q_{orifice} = A \frac{dh}{dt} = c\, a \sqrt{2\, g\, h}\,. \tag{12}$$

Now we are interested in solving for time, so as before we separate the variables in (12), set $K = \frac{c\, a \sqrt{2\, g}}{A}$ and integrate to obtain:

$$\int_{h_1}^{h_2} \frac{1}{\sqrt{h}} dh = K \int_0^t dt. \tag{13}$$

Evaluating the definite integral in (13) yields,

$$t = \frac{2\left(\sqrt{h_2} - \sqrt{h_1}\right)}{K}\,. \tag{14}$$

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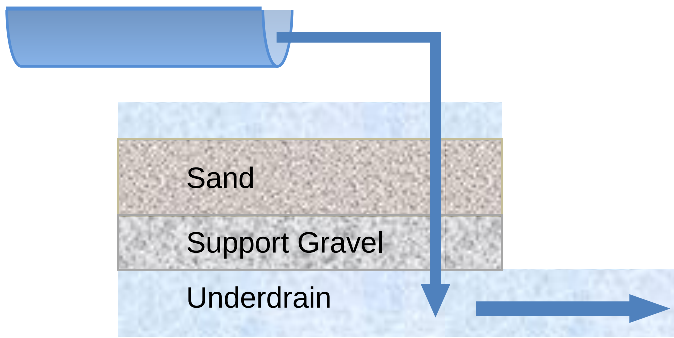
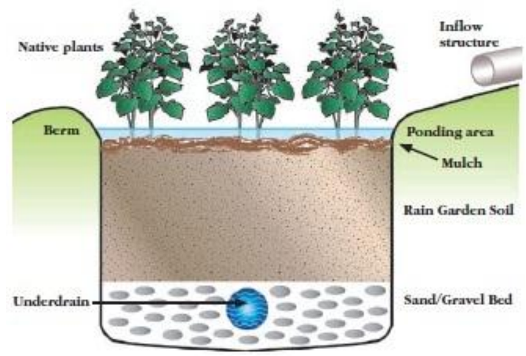
Underdrain.





The base formula for the drawdown of an underdrain with surface area a is,

$$Q_{underdrain} = \frac{dV}{dt} = A\frac{dh}{dt} = k\frac{h}{L}a \tag{15}$$

where k is the pore constant of the filter media, h is the distance from the surface of the water to the centerline of the underdrain pipe and L is the average flow length through the filter media. Again using methods similar to the previous sections we separate the variable in (15) and integrate to obtain the expression,

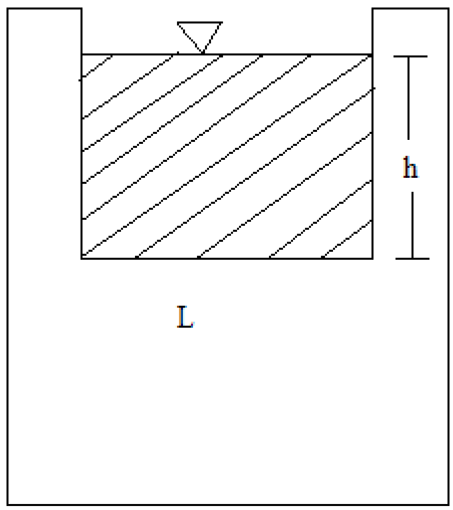
$$\frac{L\,A}{k\,a}\int_{h_1}^{h_2}\frac{1}{h}dh = \int_0^t dt. \tag{16}$$

Evaluating the integral in (16) yields:

$$t = \frac{L\,A}{k\,a}(\ln h_2 - \ln h_1) = \frac{L\,A}{k\,a}\ln\left[\frac{h_2}{h_1}\right]. \tag{17}$$

II. FALLING HEAD EQUATION EXAMPLES

Rectangular-Notch Weir.

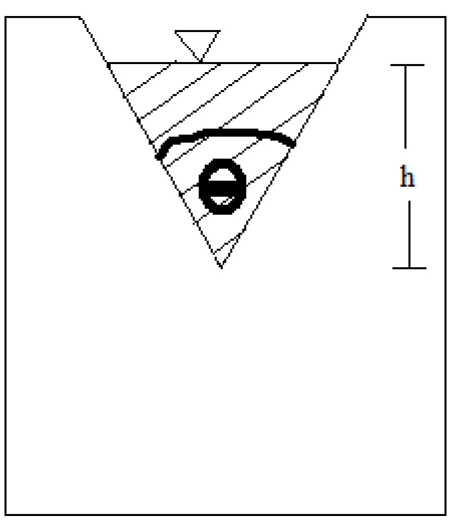


Here we have a standard rectangular-notch weir. The inverted triangle points to the water level, or “head”. We must figure out the amount of time it will take for the head to fall halfway down the weir. We are given $A = 20{,}000\ ft^2$, $h = 1\ ft$, and $L = 1.5\ ft$. Using (6) we conclude,

$$t = \frac{-2\ (20{,}000)}{3.13\ (1.5)}\left(\frac{1}{0.5^{\frac{1}{2}}} - 1\right)$$

$$= -3{,}528.8\ seconds\ \approx 1\ hour. \qquad (18)$$

Triangular-Notch Weir.



Here we have a triangular-notch weir. The inverted triangle points to the water level. We must figure out the amount of time it will take for the head to fall halfway down the weir. Here we are given $A = 20{,}000ft^2$, $h = 1ft$, and $\theta = 60°$. This time using (10) we determine,

$$t = \frac{-2}{3\ (7.22\times10^{-5})}\left[0.5^{-\frac{3}{2}} - 1\right]$$

$$= -16{,}882.98\ s = 4.69\ hr. \qquad (19)$$

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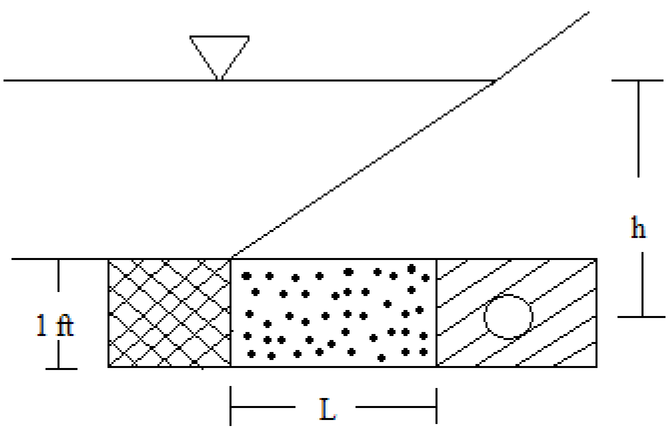
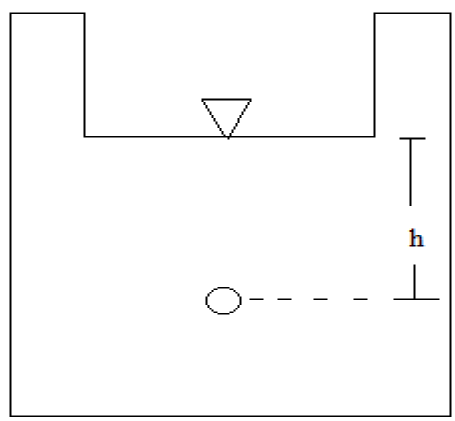
Orifice.

Above we have a rectangular-notch weir. The inverted triangle points to the water level, which is currently at the very bottom of the weir. We must figure out the amount of time it will take for the head to fall halfway to the centerline of the orifice. We are given $A = 20{,}000\ ft^2$, $h = 1\ ft$, $a = \pi\,(.25)^2 = 0.196\ ft^2$, and the acceleration due to gravity is $g = 32.2\ ft/s^2$. Putting these values into (14) reveals,

$$t = \frac{2(\sqrt{0.5}-1)}{4.719\times10^{-5}} = -12{,}413.36\ s\ = 3.45\ hr. \tag{20}$$

III. UNDERDRAIN





Above we have a simple underdrain filter system. The inverted triangle points to the water level. In order for the water to reach the underdrain it must pass through the soil under the pond, filter media of length L, and finally the rock that surrounds the pipe. We must calculate the amount of time it will take for the head to fall halfway to the centerline of the underdrain. We are given $A = 40{,}000 ft^2$, $h_1 = 5\ ft$, $h_2 = 0.5\ ft$, $k = 1.5\ \frac{ft}{hr}$, $L = 4\ ft$, and $a = 50\ ft^2$. Inserting these values into (17) we determine,

$$t = (2133.3)\ \ln\left[\frac{.5}{5}\right] = \ -4912.1\ s = 1.36\ hr.$$

DISCUSSION

After breaking down each formula, we worked through an example problem for each of the four systems. For the rectangular-notch weir, we calculated how long it would take for a 20,000 square foot pond to draw down half a foot flowing out through a 1.5 foot wide weir. We found that it would take about 1 hour. For the v-notch weir, we calculated how long it would take for a 20,000 square foot pond to drop half a foot flowing out through a weir cut at 60 degrees. We found that it would take 4.69 hours to draw down. For the orifice, we calculated how long it would take for a 20,000 square foot pond to draw down half a foot through an orifice with area 0.196 square feet. We found that it would take 3.45 hours. For the underdrain, we calculated how long it would take for a 40,000 square foot pond to drop 4.5 feet flowing through a filter medium of length 4 feet and an area of 50 square feet. The filter medium used had a coefficient of 1.5 feet per hour. We found that it would take 1.36 hours for the pond to draw down completely. Our results suggest that each of the four methods we investigated is an efficient way of drawing down the water level of retention and detention ponds.

CONCLUSION AND RECOMMENDATIONS

We started this project with a goal in mind. That goal was to provide anyone who may read this with a solid understanding of the falling head equation, how civil engineers use it in real life, and how it can be altered to conform to a variety of situations or water management systems. Aside from this goal, we wanted to gain something a bit more personal from constructing this report. Storm water management is a topic we find interesting, so this project was a great way to do research on the subject and gain some knowledge in the field.

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For anyone interested in this topic, a good way to build upon this report would be to delve even further into the falling head equation. Notice that each formula has its own coefficient which stays constant while working through the entire equation. While researching the subject, we often wondered how these constants were calculated. This would serve as a great topic for another report in the same field.

NOMENCLATURE

Rectangular-Notch Weir

c	Constant	3.13
Q	Flow Rate	ft^3/s
h	Height	ft
t	Time	s
A	Area	ft^2
V	Volume	ft^3
L	Length of Weir	ft

Underdrain

k	Pore Rate	ft/hr
Q	Flow Rate	ft^3/s
h	Height	ft
t	Time	s
A	Area	ft^2
g	Acc. Of Gravity	ft/s^2
a	Area (Filter Media)	ft^2
L	Length (FM)	ft

Orifice

c	Discharge Coefficient	0.6
Q	Flow Rate	ft^3/s
h	Height	ft
t	Time	s
A	Area	ft^2
V	Volume	ft^3
g	Acc. Of Gravity	ft/s^2
a	Area (Orifice)	ft^2
K	Constant	$\frac{ca\sqrt{2g}}{A}$

Triangular –Notch Weir

Q	Flow Rate	ft^3/s
h	Height	ft
t	Time	s
A	Area	ft^2
V	Volume	ft^3
K	Constant	$\frac{2.5\tan\frac{\theta}{2}}{A}$

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