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Dosimetry Calculations of Radiopharmaceuticals

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Dosimetry Calculations of Radiopharmaceuticals

Abstract

The goal of this project is to determine the absorbed dose of radiation to a patient using ^{99m}Tc-MAA as a tracer for a lung scan using the MIRD formula accounting only for the radiation dose contributed by the target organ. The radiation dose for other organs to the target organ is considered negligible as the uptake of ^{99m}Tc-MAA from other organs is less than 1%. In addition to calculating the absorbed dose of radiation, the initial dose rate was also calculated by back solving to determine the total mass of the lungs. The radiation dose to the patient by the target organ was calculated to be 269.44 mrads and the initial dose rate was determined to be 156 mrads/hr.

Keywords

Radiopharmaceuticals, Absorbed Radiation Dose, Positron Emission Tomography

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PROBLEM STATEMENT

Calculate the absorbed dose to the lungs of an adult patient who received 3 mCi (111 MBq) ^{99m}Tc-MAA, assuming 99% uptake and uniform distribution of the radioactivity in the lungs. Pertinent data are: $T_b = 1.5\ hr$ and $S = 5.25 \times 10^{-5}\ \frac{rad}{\mu Ci \cdot hr} = 0.0142\ \frac{G_y}{GBq \cdot hr}$. (Saha 199)

1. Calculate the absorbed dose of radiation from the target organ.

2. Calculate the initial dose rate.

MOTIVATION

The calculation of absorbed radiation is crucial for the safety of both the patient and the nuclear medical technician. Radiation overdose risks include vomiting, damage to the hematopoietic system, and death (Saha 199). These risks associated with the stages and dose ranges are outlined in Table 1 in the Appendix. Radiopharmaceuticals have multiple uses. They can be used as tracers for Positron Emission Tomography (PET), studies to determine factors such as certain metabolisms, and absorption and as treatment for certain diseases and conditions. The problem listed above would be a typical dose to a male patient undergoing a PET scan of the lungs to detect a pulmonary embolism. However, the hand calculation of radiation dosing has become essentially obsolete due to the use of computer programs that account for additional variables that differ from patient to patient.

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MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

The Medical Internal Radiation Dose (MIRD) formula is an equation that can be used to determine the dose of radiation to a patient using physical and biological factors such as the radioactivity of the source, the source's effective half-life, target organ mass, as well as many other factors determined by the Society of Nuclear Medicine. The effective half-life T_e of the ^{99m}Tc source must first be calculated to determine the dose formula. This is done by considering that the effective decay constant is given by $\lambda_e = \lambda_p + \lambda_b$, where λ_p and λ_b are the physical and biological decay constants respectively. Since any decay-constant is equal to the factor ln 2 times the reciprocal of the half-life, the effective decay constant formula may be rewritten as

$$\lambda_e = \lambda_p + \lambda_b \rightarrow \frac{1}{T_e} = \frac{1}{T_p} + \frac{1}{T_b}. \quad (1)$$

The physical half-life of ^{99m}Tc is $T_p = 6\ hours$ and the proposed biological half-life of the ^{99m}Tc source is $T_b = 1.5$ hours. This gives an effective half-life T_e for the ^{99m}Tc source of 1.2 hr. Next, the dose rate is given in units of rad/hr as

$$R_i = 2.13\left(\frac{A}{m}\right) N_i E_i \phi_i (v \leftarrow r), \quad (2)$$

where ϕ_i is the absorbed fraction of radiation from the source organ r to the target organ v and the value 2.13 is a conversion factor so that equation (2) is will be expressed in rad/hr. Since the equilibrium dose is given by $\Delta_i = 2.13\ N_i E_i$ equation (2) becomes

$$R_i = \left(\frac{A}{m}\right) \Delta_i \phi_i (v \leftarrow r). \quad (3)$$

Due to physical and biological decay of the radioactivity A, the dose rate will decrease accordingly. Supposing that A decays at an exponential rate, equation (3) may be rewritten as

$$R_i = \left(\frac{A_o e^{-\lambda_e t}}{m}\right) \Delta_i \phi_i (v \leftarrow r), \tag{4}$$

where A_o is the radioactivity in the target organ. To obtain the dose of radiation absorbed, equation (4) is integrated over the interval $0 \leq t < \infty$ giving

$$D_i^{rad} = \frac{A_o}{m} \Delta_i \phi_i (v \leftarrow r) \int_0^\infty e^{-\lambda_e t}\, dt = \frac{1}{\lambda_e} \frac{A_o}{m} \Delta_i \phi_i (v \leftarrow r). \tag{5}$$

Because the effective decay constant is related to the effective half-life by $\frac{1}{\lambda_e} = \frac{T_e}{ln2} = 1.44\, T_e$, equation (5) becomes

$$D_i^{rad} = 1.44\, T_e\, \frac{A_o}{m}\, \Delta_i\, \phi_i (v \leftarrow r). \tag{6}$$

The sum of all the products of the equilibrium dose Δ_i and the absorbed fraction ϕ_i is takes into account the multiple types of radiation that a radionuclide may give off. Thus total radiation dose is expressed as

$$D^{rad} = 1.44\, \frac{A_o}{m} T_e\, [\, \textstyle\sum_{i=1}^{n} \Delta_i\, \phi_i (v \leftarrow r)\,]. \tag{7}$$

Results for the major radiation types that ^{99m}Tc releases can be found in Table 2. After the sum has been determined, $\tilde{A}$ can be substituted for $1.44\, T_e A_o$, and S for $\frac{1}{m} \sum_{i=1}^{n} \Delta_i\, \phi_i$, finally giving

$$D^{rad} = \tilde{A}\, S. \tag{8}$$

The values of $\tilde{A} = 5{,}132.16\, \mu Ci$ and $S = 5.25 \times 10^{-5} \frac{rad}{\mu Ci}$ yield a dose of $269.44\, mrads$.

Now the mass of the lungs must be found to determine the dose rate. This is done by using the calculations of Table 2 in the Appendix and exploiting the fact that $\frac{1}{m} \sum_{i=1}^{n} \Delta_i\, \phi_i = S = 5.25 \times 10^{-5} \frac{rad}{\mu Ci}$, i.e., $m = 1{,}535.24\, g$. After substituting all values into equation (4), the initial dose rate is determined to be 156 mrads/hr.

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DISCUSSION

The MIRD formula has improved the accuracy of dosimetry calculations by reducing the number of assumptions needed to perform dosimetry calculations as opposed to the classic method of dosimetry (Early 90). However, the MIRD formula has proved to be limited by other factors. One of these limitations is the coefficient of variation, "This is a statistical limitation. A coefficient of variation of 50% or greater represents a considerable uncertainty in the estimate of the absorbed fraction" (Early 90). Other limitations include biological calculation factors such as the kidney model not being divided into cortex and medulla, the bladder and stomach being a fixed size, (Early 90) and the fact that the MIRD formula presupposes that the source is uniformly distributed within a standard-sized organ, which is subject to much patient variation" (Early 92). It is imperative that these factors are taken into account, and that the dosimetrist be knowledgeable of the assumptions and limitations (Early 92). With the invention of more powerful computers and additional formulas to take limitations such as the coefficient of variation, organ size and shape, and the uniform distribution of the radionuclide dosimetry, calculations have become far more accurate than the MIRD formula used in this project.

CONCLUSION AND RECOMMENDATIONS

The project was successful in the calculation of the absorbed dose of radiation from the target organ and the initial dose rate. The calculated dose of radiation was $269.44\ mrads$ and the calculated initial dose rate was $156\ mrads/hr$. This problem however, does not account for other dose contributions from other source organs to the target organ, and only accounts for the dose contribution of the target organ to itself. These additional dose contributions would have proved negligible as the remaining 1% of uptake would have been distributed among the organs

in which ^{99m}Tc-MAA implants. These organs would have included the kidneys, liver, ovaries or testes, and the whole body. Although negligible in this project, future projects could account for these additional dose contributions from organs other than the target organ.

NOMENCLATURE

Symbol	Description	Value/Unit
t	Time	Hours (hr)
T_e	Effective half-life	1.2 hr
T_p	Physical half-life	6 hr
T_b	Biological half-life	1.5 hr
A	Radioactivity	Micro-Curie (μCi)
A_o	Radioactivity in target organ	2,970 μCi
$\tilde{A}$	1.44 $T_e A_o$	5,132.16 μCi
S	$\frac{1}{m}\sum_{i=1}^{n} \Delta_i \phi_i$	$5.25 \times 10^{-5} \frac{rad}{\mu Ci}$
m	Mass of target organ	1,535.24 g
N_i	Fraction of quanta emitted per disintegration for the ith radiation	-
E_i	Energy of the emitted quanta for the ith radiation	Mega electron volts (MeV)
$\phi_i(v \leftarrow r)$	Absorbed Fraction from source organ (r) to target organ (v) for the ith radiation	-
Δ_i	Equilibrium Dose for the ith radiation	$\frac{g \cdot rad}{\mu Ci \cdot hr}$
λ_e	Effective decay constant	0.5775
D_i	Cumulative radiation dose for the ith radiation	rad
D	Total radiation dose	rad

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Symbol	Name / Unit	Conversion
Ci	Curie / Radioactivity	$3.7 \times 10^7\ dps$ (disintegrations per second)
Bq	Becquerel / Radioactivity	$2.703 \times 10^{-11} Ci$
MeV	Million electron volts / Energy	$1.602 \times 10^{-6} erg$
rad	Rad / Energy	$100\ erg/g$
Gy	Gray / Energy	$100\ rad$

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Saha, Gopal B. Fundamentals of Nuclear Pharmacy. 4th ed. New York: Springer, 1998.

APPENDIX: TABLES

Stage	Dose Range $rads\ (Gy)$	Symptoms
1	0-200 (0-2)	Usually unobservable
2	150-400 (1.5-4)	Transient nausea and vomiting; some evidence of damage to hematopoietic system, recovering in 1 to 2 months
3	350-600 (3.5-6)	Severe damage to hematopoietic system; bone marrow transplant essential; survival chances, moderate
4	550-1000 (5.5-10)	Gastrointestinal damage; severe nausea, vomiting and diarrhea; very small chance of recovery; death follows in 10-24 days
5	1000 and above (10 and higher)	Confusion, shock, burning sensation; death follows within hours

Table 1: Stages of radiation overdose with corresponding dose ranges and symptoms from Ref. (Early).

Radiation	Δ_i	ϕ_i	$\Delta_i\phi_i$
$140-keV$ photon	0.2640	0.160	0.0422
X rays ($20\ keV$)	0.0031	0.784	0.0024
Electrons	0.0360	1.000	0.0360
		Total $\left(\frac{g \cdot rad}{\mu Ci}\right)$:	0.0806

Table 2: Major forms of radiation associated with ^{99m}Tc-MAA and their associated equilibrium dose and absorbed fraction from Ref. (Saha).

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