

Canal Lock Displacement

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Abstract

In this project we determine the time needed for a pump to raise the water level in a canal lock in order for a boat to continue upstream. Using calculus methods and elementary physics, it was determined that it would take roughly 5 minutes for a single 60 horsepower pump to raise the water level the required 10 feet. We conclude that the lock is fairly efficient but offer some suggestions to increase the time efficiency of the lock system.

Keywords

Work, Water Displacement, Water Pump

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PROBLEM STATEMENT

Located on a canal is a lock with dimensions of 100 *ft* by 30 *ft* and a standing water level of 10 *ft*. This lock must raise the water level to 20 *ft* in order for boats to continue upstream. If the water is pumped in by a 60 *HP* engine, how long will it take the raise the water level to 20 *ft*?

MOTIVATION

This problem is merely a daily occurrence on many major waterways, including those vital to trade such as the Panama Canal and Mississippi River. When locks were first created they revolutionized trade, making it much easier for ships to travel upstream. This problem is useful to engineering as it allows engineers to estimate the time needed for a ship to pass through each lock.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

In order to solve the problem, we first needed to calculate work necessary to raise the water level from 10 *ft* to 20 *ft*. To accomplish this we break the water in the lock into infinitesimally thin sheets and calculate the weight of each sheet using:

$$Force = Weight(sheet) = Volume \times Density. \quad (1)$$

Knowing the density of water (62.4 *lbs/ft*³), the dimensions of the lock, and letting *dD* represent the height of each sheet (1) yields:

$$F = Weight(sheet) = 3,000ft^2 \times dD \times 62.4 \frac{lb}{ft^3} = 187,200 \times dD. \quad (2)$$

To calculate the work done on each sheet we used,

$$Work(sheet) = Weight(sheet) \times (D - 10) = 187,200 dD \times (D - 10) \quad (3)$$

where $(D - 10)$ represents the height each sheet was raised. Thus to calculate the total work done on the water in the lock we summed over all sheets which became the definite integral:

$$W = \int F dD = \int_{10}^{20} [187200 \times (D - 10)] dD = 9,360,000 \text{ ft lbs.} \quad (4)$$

Then to determine how fast the water would fill the lock we used the formula,

$$P = W/T \quad (5)$$

and the conversion factor:

$$1 \text{ HP} = 33,000 \text{ ft lbs/minute.} \quad (6)$$

Given that the power was supplied by a single 60 HP engine combining (4), (5) and (6) gives:

$$T = W/P = 9,360,000 / (60 \times 33,000) \approx 4.73 \text{ minutes.}$$

Thus we concluded approximately 4.73 minutes are required to raise the water level in the lock 10 ft.

DISCUSSION

While completing this project we determined that 9,360,000 ft lbs of work were required to raise the water level in the lock from 10 ft to 20 ft. We also found that using a single 60 HP engine it would take approximately 4.73 minutes to fill the lock to the desired level so a boat could continue upstream. Had this been a field study it could have created

incentive to work towards faster pumping times to ultimately allow more ships to pass through the lock system.

CONCLUSION AND RECOMMENDATIONS

These results suggest that the lock is rather efficient, but there is room for improvement. One way to make this lock more efficient is to reduce the dimensions of the lock to 80 *ft* by 30 *ft*. However, powering the lock with two pumps is probably the most efficient option. This would cut the fill time down to nearly 2 minutes. Similar to this problem, it would be interesting to do a study to calculate the time necessary for the lock to lower a ship from 20 *ft* to 10 *ft* of water.

NOMENCLATURE

Symbol	Description	Unit
F	Force	lbs
D	Distance	ft
W	Work	$ft\ lbs$
P	Power	$ft\ lbs/min$
HP	Horsepower	$ft\ lbs/min$
T	Time	min

REFERENCES

Halliday, Resnick, Walker. Fundamentals of Physics 7th Edition. Hoboken, New Jersey: John Wiley & Sons, 2005.

Larson, Ron, Robert Hostetler and Bruce Edwards. Calculus. 8th Edition. Boston, MA: Houghton Mifflin Company, 2005.