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A Generalized Decision Model for Naval Weapon Procurement:

Multi-Attribute Decision Making

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
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A Generalized Decision Model for Naval Weapon Procurement: Multi-Attribute Decision Making

Jin O Chang

ABSTRACT

For any given reason, every year many countries spend a lot of money purchasing at least one weapon. Due to the secret character of the military, the decision process for specific weapon procurement is shrouded. Moreover, there are several funds loss cases due to mistakes in weapon contractions. Weapon procurement requires very large amounts of money which comes from tax payers. Therefore, an effort to reduce a possible monetary loss is needed.

A decision process based on an analytic model can present a better chance to decision makers for better weapon decisions. In general, weapon procurement decision is a multi criteria environment. Decision making in such environments is defined as Multi-Criteria Decision Making (MCDM). MCDM is broadly classified into two areas: Multi-Attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM).

MADM methods are used for selecting an alternative from a small explicit list of alternatives. MODM methods are used for designing problems involving an infinite number of alternatives implicitly defined by mathematical constraints. This research is intended to be used by the South Korean Navy when there is a need to select one weapon type among several candidate types. Therefore, MADM methods are used in this research.

Many researches for developing an analytical model for better decision-making have been done. However, there is no research for a generalized weapon procurement decision model that is easy to implement. For this reason, whenever there is a need for weapon procurement decision, the Navy has to spend a lot of effort in determining the best weapon. These efforts can be reduced with a generalized model that is proposed in this research for naval weapon procurement.

MADM methods determine alternatives' ranking orders and the highest ranked alternative is the best one. Various MADM methods are used in computing the alternative's ranking scores. However, there is no MADM method which can compensate individual values for an overall value. Our new MADM model can compensate for that. We also provide a sensitivity analysis to the solutions obtained by the proposed model. This new model is applied to a real problem in the South Korean Navy.

Chapter One

Introduction

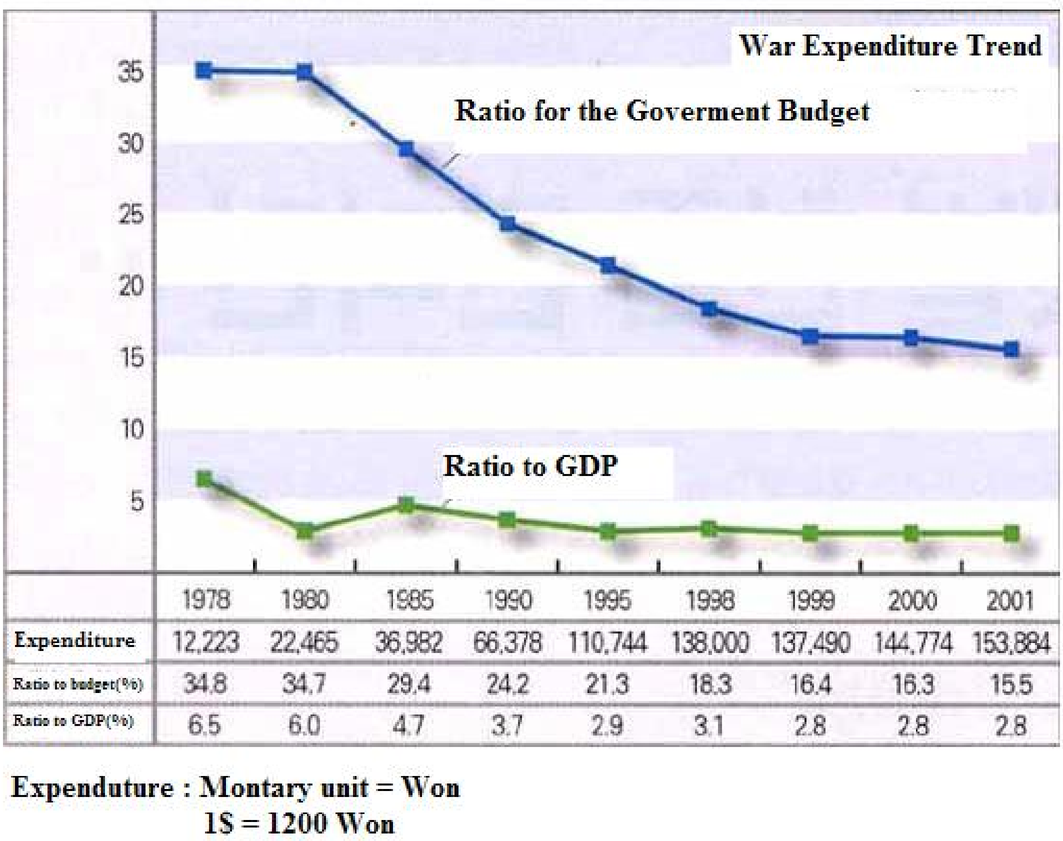
1.1 Problem Definition

For most countries, the portion of the national budget allocated to defense spending is a significant value. In 2002, 14 billion dollars of South Korea's budget was spent on national defense. And this was ranked 11th in the world. Table 1.1 shows the rank based on national defense money spent in 2002 (MND, 2004).

Table 1.1 A Rank for Defense Expenditures in 2002

Rank	Nation	Expenditure (million $)
1	U.S.A	399,900
2	Russia	65,000
3	China	47,000
4	Japan	42,000
5	England	38,000
6	France	29,000
7	Germany	24,000
8	Saudi Arabia	21,000
9	Italy	19,000
10	India	15,000
11	South Korea	14,000

For South Korea, this expenditure is due to its political and geographical situation. South Korea is still in conflict with North Korea. Also it is surrounded by many countries that have a strong military power, such as China, Japan and Russia. However, there is a trend in the world to try to reduce war expenditures to provide more benefits for its citizens. South Korea could not be an exception in that. In fact, as shown in figure 1.1, there was a continuous decrease in defense expenditure ratio to government and GDP (Kwon, 2003).



	1978	1980	1985	1990	1995	1998	1999	2000	2001
Expenditure	12,223	22,465	36,982	66,378	110,744	138,000	137,490	144,774	153,884
Ratio to budget(%)	34.8	34.7	29.4	24.2	21.3	18.3	16.4	16.3	15.5
Ratio to GDP(%)	6.5	6.0	4.7	3.7	2.9	3.1	2.8	2.8	2.8

Expenduture : Montary unit = Won
1$ = 1200 Won

Figure 1.1 Ratio Defense Expenditure to Government Budget and GDP

Due to its situation, it is very important for South Korea to keep its military strength. Reducing costs is the best way to maintain ability when there is not much increase or decrease in budget. Reducing cost can be obtained by scientific and systematic management. Therefore, research for the scientific management for the military is required.

South Korea spends millions of dollars for weapon procurements. And there are many funds lost due to managers' mistakes during weapon procurement decisions. These factors do the opposite of reducing costs. Therefore, efforts to reduce the loss caused by these factors can play a role to reduce costs in weapon procurement and saved money can be used in other important areas. Kim (2000), the author of "Arms Procurement Decision Making", analyzed the defense funds loss and presents the reasons in Table 1.2.

Table 1.2 Military Funds Loss Cases in South Korea (Source: Kim, 2000)

Reasons for the loss	Weapon system	Funds loss (million $)
Excessive payments for the weapons	K-1 tank, UH-60 hello, K200 Armored Vehicle	132
Excessive transaction fees	UH-60 hello, P3-C	48.7
Money exchange late loss	M-60, M-16, Howitzer	263
Total		443.7

In general, the public are restricted in accessing data for weapon procurement decisions for a military secrete purpose. This access is only possible for only few authorized people. It is impossible even to look at the items for other peoples. Every year, there are requirements for new weapon procurements dependant to a new strategy or a replacement for a life cycle ended weapons. Because of the secret nature the military, the decision process for specific weapon procurement is shrouded. In general, weapon procurement requires large sums of money. Therefore, decision making in weapon procurement can be easily affected by political pressures. Kim (2000) presents an interaction of political factors in a decision process changed decisions regardless of weapon performance and its cost effectiveness. In many cases, those decisions changed by politicians lost a lot of money.

Kim (2000) also points out the lack of any systematic decision support system in weapon procurements and suggests that with a systematic decision support system for weapon procurement those problems can be resolved.

1.2 A Generalized Decision Model for Weapon Procurement

A generalized decision model for weapon procurement is defined as follows: it is a scientific and systematic model designed to help senior officers of South Korean Navy (hereafter called SKN) to make the best decision for weapon procurement with a quantitative ranking score. To be scientific and systematic, a model must be supported by an analytical procedure and consistent for the same problem for every calculation. In the SKN Regulation Book 2 (hereafter called NR 2), there are several criteria for selecting weapon. Our model considers these criteria and presents quantitative rank information of the candidate weapons to a Decision Makers (DM).

Multi Criteria Decision Making (MCDM) is broadly classified into two categories: Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM) (Yoon, 1980). MADM methods are used for selecting one alternative from a small, explicit list of alternatives, while MODM methods are used for designing a problem involving an infinite number of alternatives implicitly defined by mathematical constraints. Table 1.3 displays the comparisons between MODM and MADM.

Table 1.3 Comparisons between MODM and MADM

	MODM	MADM
Goal	- Finding optimal solution by using mathematical model under some constraints - Can take several alternatives	- Find only one alternative among several of candidate ones - Each alternative has a same level of identification
Objective Function	- A function has decision variables representing amounts of acquisition of each alternative	- No such objective function
Attributes	- Functions as constraints	- Functions as giving alternative's numerical scores
Supporting research	Yoon (1980), Yoon and Hwang (1995), Pomerol and Romero (2000), Saaty (1986)	

To understand the difference between these two methods, let us consider two example problems. Hall et al (1992) presents a model for making project funding decisions at the National Cancer Institute (hereafter called as NCI). NCI has funded a series of studies for reducing smoking prevalence. DMs of NCI want maximize the budget ability to fund the states with the most highly evaluated proposals. In addition to budget availability, DMs have to consider political pressure as well. Hall et al. (1992) introduce a preference function and propose a decision model such as

$$\text{Max } 1.9x_1 + \ldots + 1.0x_8$$

$$\text{s.t. } 6.3x_1 + \ldots + 7.7x_8 \geq 17 \text{ : minimum number of preference points}$$

$$9.8x_1 + \ldots + 5.6x_8 \leq 37 \text{ : maximum available budget}$$

and some other constraints.

In this example, DMs want to maximize NCI's budget ability by supporting as many as they can while meeting constraints. This is a typical MODM problem.

Let us assume that NCI can only support one proposal. In addition, DMs of the NCI want to review every proposal, regardless of their rank. From the first assumption that NCI can only support one proposal, the above model can be rewritten as

$$\begin{aligned}
&\text{Max } 1.9x_1 + \ldots + 1.0x_8 \\
&\text{s.t. } 6.3x_1 + \ldots + 7.7x_8 \geq 17 \\
&\quad 9.8x_1 + \ldots + 5.6x_8 \leq 37 \\
&\quad \sum_{i=1}^{8} x_i = 1,\, x_i \in \{0,1\} \ \ \forall i\,.
\end{aligned}$$

This model will allow only one proposal selection. However, DMs can not see other proposals that are not selected. Therefore, the second assumption that DMs want to review each proposal is not met by the MODM method.

MADM method presents a set of alternative ranks based on their ranking scores. Based on this rank information, DMs can select the best alternative that has the highest ranking score. In addition, DMs can see all other alternatives' ranking scores. An alternative ranking score is computed by a given set of attributes that are used for constraints in the MOMD method. A detailed procedure for computing an alternative ranking score is presented in Chapter 2.

If the two methods are different, various studies for mutual improvement have been done. For example, Saaty (1986), Hughes (1986) and Rahman (2003) propose that MODM method can be simplified by using MADM method.

In this research, we assume that the analysis is of a single weapon type selection problem. In other words, the SKN can not select multiple weapon systems. In addition, we assume that DMs want to review each candidate weapon regardless of their ranking orders. Based on these two assumptions, the appropriate solution belongs to MADM method.

1.3 Research Objective and Contributions

The objective of this research is to develop a new decision model that can identify the proper best alternative in the presence of extreme alternatives, which may be caused by possible political pressures, on weapon procurement decision. The existing MADM methods in the open literature can not handle such extreme alternatives.

The contributions of this research are summarized as follows.

1. Since the SKN does not have any hierarchy of attributes for evaluating weapon systems, the suggested hierarchical structure can be used for every weapon procurement decision.
2. Since the current MADM methods only consider either an alternative overall ranking score or individual attribute values, they cannot address any political pressure during weapon procurement decision. However, our new decision model can address this pressure by compensating an alternative overall ranking score for its individual attribute values.
3. Proposed sensitivity analysis can present what-if analysis for both the SKN and weapon suppliers.
4. Our new model can be used for any other non-military decision process especially when a decision can be easily affected by some political powers.

1.4 Thesis Overview

The rest of this research is organized into six chapters. Chapter 2 reviews the methodologies as well as the relevant researches to our problem. In chapter 3, a new MADM method is suggested with numerical example problems. A sensitivity analysis to the solutions obtained by the proposed model is also presented in this chapter. In Chapter 4, we develop a hierarchy of attribute for evaluating weapon systems. In Chapter 5, attribute weights are given to this hierarchy by using the AHP method. In Chapter 6, our new model is applied into a real problem in the SKN. The conclusion and further research is followed in Chapter 7.

Chapter Two

Literature Review

2.1 Introduction

Every other year, SKN procures at least one weapon either for improving its defense ability or replacing its old weapons. It ranges from thousands of dollars worth to billions of weapon systems. SKN selects a weapon among several candidate weapons in terms of various requirements defined as Requirement of Operational capability (ROC), cost effectiveness and political situations. Each consideration is conflicting and sometimes requires compensation with each other (e.g., better performance weapon is usually expensive than worse so that in order to save money we may need to choose one that is not best in performance). In other word, DMs deal with decision problems that involve multiple and usually conflicting criteria. MADM procedures can be applied to a wide range of human choices, from the professional to the managerial to the political.

Pomerol and Romero (2000) present the historical background of MADM. From a scientific view point, the research into economics which took place at the end of the nineteenth century and the beginning of the twentieth is one of the sources of inspiration for the MADM. At that time economists were beginning to look for links between the behavior of economic agents and the economy itself. One of the basic factors governing behavior, applying to both producer and consumer, is the way choices are made in consumption and production. Later this is developed as a consumer theory with utility function. By 1960, multi-criterion analysis was acquiring its own vocabulary and problem formulations (i.e., the problem of choosing alternatives in the presence of multiple criteria called attributes in MADM). In 1976, Keeney and Raiffa proposed Multi Attribute Utility Theory (MAUT). Pomerol and Romero classify scholars into two groups: one group is the supporters of the utility and the other is pragmatists using other methods like AHP, TOPSIS, and ELECTRE. The latter group's methods are called MADM methods.

MADM refers to making preference decisions (e.g., evaluation, prioritization, and selection) over the available alternatives that are characterized by multiple, usually conflicting, attributes (Hwang and Yoon, 1981). MADM methods are management decision aids used in evaluating competing alternatives defined by multiple attributes. Starr and Zeleny (1977), Zionts (1978), and Yoon and Hwang (1995), and Saaty and Vargas (2000) are representative researchers in MADM area.

In this chapter, we present numerous MADM methods as well as researches done in decision making problems for both military and non-military areas.

2.2 MADM Methods

MADM methodology tries to obtain a meaningful index from multidimensional data to evaluate competing alternatives. Pioneering surveys on MADM methods were carried out by MacCrimmon (1973). Since then many methods have been developed by researchers in disciplines as diverse as management science, economics, psychometrics, marketing research, applied statistics, and decision theory. All MADM methods can be classified compensatory or noncompensatory, ordinal or cardinal, and quantitative or qualitative. In this section, general steps for MADM and several MADM methods are discussed.

2.2.1 General Steps for MADM

An analysis begins by defining attributes that can measure the relevant goal of accomplishments. These attributes are set up by constructing a problem structure. Then alternatives are contrasted over the chosen attributes. Often all attributes are not of equal importance to the DM. Thus, the rendering of appropriate weights among attributes is of prime concern to the DM. Suppose that there are two types of attributes: qualitative and quantitative. Also each quantitative attribute has a different unit of measurement (e.g., number of people and amounts of dollars). We need a homogenous data type for a DM to compare each alternative. Homogenous data sets can be obtained through the normalization procedure.

There are three possible ways of defining attributes: reviewing literatures, using possible documents, and asking experts' opinions (Keeney and Raiffa, 1976). The Delphi technique and the AHP method are widely used for the latter purpose.

Pardee (1969) suggests that a desirable list of attributes should be complete and exhaustive, contain mutually exclusive items, and be restricted to performance attributes of the highest degree of importance. Again, attributes are developed as a result of constructing a problem structure.

Weights represent the relative importance of each attribute with respect to an overall goal. Therefore, we can define that weights that can play a key role in MADM problems. Moreover, the weights themselves can be useful information to those concerned with the program or project management, since they indicate what the DM is most concerned about in a quantitative way (Edwards and Newman, 1982). Sometimes, the weights themselves are useful tools for management purposes (Chang, 1997).

A DM may use either an ordinal or a cardinal scale to express his or her preference among attributes. Although it is usually easier for a DM to assign weights by an ordinal scale, most MADM methods require cardinal weights. Cardinal weights are normalized to sum to 1, that is $\sum w_j = 1$, where w_j represents weight of the jth attribute .

The simplest way of assessing weights is to arrange the attributes in a simple rank order, listing the most important attribute first and the least important attribute last. When 1 is assigned to the most important attribute, and n (the number of attributes at hand) to the least important, the cardinal weights can be obtained from one of the following formulas (Stillwell et al, 1981):

$$w_j = \frac{\frac{1}{s_j}}{\sum_{k=1}^{n} \frac{1}{s_k}}, \tag{2.1}$$

$$w_j = \frac{(n - s_j + 1)}{\sum_{k=1}^{n} (n - s_k + 1)}, \tag{2.2}$$

where s_j is the rank of the jth attribute. Equation 2.1 is called as rank reciprocal weight method, while the Equation 2.2 is called as rank sum weight method (Yoon and Hwang, 1995). If attributes are tied in the ranking, their mean ranking can be used.

Ranking all attributes at the same time may place a heavy cognitive burden on the DM. Therefore, a method by which a complete ranking can be obtained from a set of pairwise judgments is the preferred approach (Morris, 1964; Saaty and Vargas, 2000).

Attribute ratings are normalized to eliminate computational problems caused by differing measurement units in a decision matrix. It is not always necessary but is essential for many compensatory MADM methods. Normalization aims at obtaining comparable scales, which allow inter-attribute as well as intra-attribute comparisons. Consequently, normalized ratings have dimensionless units and, the larger the rating becomes, the more preference it has. There are two types of normalization methods: linear and vector normalizations.

Linear normalization is simple procedure that divides the ratings of a certain attribute by its maximum value. The normalized value of x_{ij} is given as

$$r_{ij} = \begin{cases} x_{ij} / x_j^*, \text{ when } x_{ij} \text{ is a value for benefit attribute} \\ \dfrac{1/x_{ij}}{1/x_j^-}, \qquad \text{when } x_{ij} \text{ is a value for cost attribute} \end{cases} \tag{2.3}$$

where x_{ij} is the response of alternative i on attribute j, x_j^* is the maximum value of the j th benefit attribute, x_j^- is the minimum value of the j th cost attribute, and r_{ij} is the normalized value of x_{ij}.

Vector normalization divides the rating of each attribute by its norm, so that each normalized rating of x_{ij} can be calculated as

$$r_{ij} = \begin{cases} \dfrac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \text{when } x_{ij} \text{ is a value for benefit attribute} \\ \dfrac{1/x_{ij}}{1/\sqrt{\sum_{i=1}^{m} (1/x_{ij})^2}}, \text{when } x_{ij} \text{ is a value for cost attribute} \end{cases} \tag{2.4}$$

Saaty and Vargas (1993) show that there is only a minor difference between these two normalization methods by using simulation results. The proposed model uses the linear normalization method.

2.2.2 Noncompensatory Methods

Yoon and Hwang (1995) present the taxonomy of 13 methods as shown in Figure 2.1 and they classify methods in the bottom box under Major Class of Method as compensatory methods and others as noncompensatory method.

A compensatory or noncompensatory distinction is made on the basis of whether advantages of one attribute can be traded for disadvantages of another or not. A choice strategy is compensatory if trade-offs among attribute values are permitted, otherwise it is noncompensatory.

Noncompensatory methods are relatively easy to facilitate compared to compensatory methods. However, this approach considers only one attribute at a time and can miss overall good alternatives. For example, when we consider buying a new car, there might be several important factors to be considered like cost, gas mileage, and performance measure. Suppose that we have three cars (A_1, A_2, and A_3, respectively) for consideration. A_1 and A_2 may have one outstanding attribute. While A_3 does not, but in overall score it is first. Noncompensatory may lose A3 for selection (for more detail, see Yoon and Hwang, 1995).

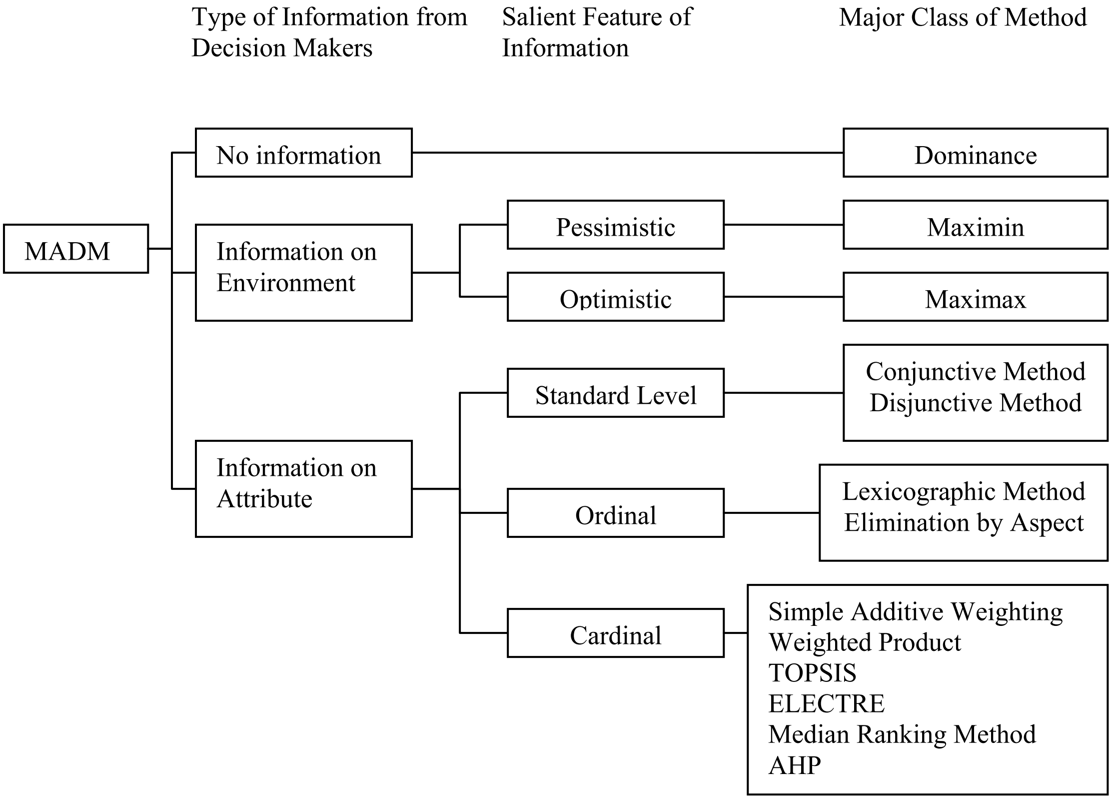


Figure 2.1 A Taxonomy of MADM Methods (Source: Yoon and Hwang (1995))

Weapon procurement decision should not be simply decided because of its monetary big scale and huge impact to the country. Therefore, for this research compensatory methods should be applied.

2.2.3 Compensatory Methods

In compensatory methods, all attributes are considered to make a final decision. For example, let us consider a military fighter selection decision. A country decides to reinforce its air force by purchasing sophisticated jet fighters. Five competing models are available for purchase from the market. The huge acquisition cost and long-term impact on national security force the acquisition officers to make circumspect decisions. They proceed to generate selection criteria by way of a goal hierarchy. The hierarchy for a good fighter is shown in the following figure.

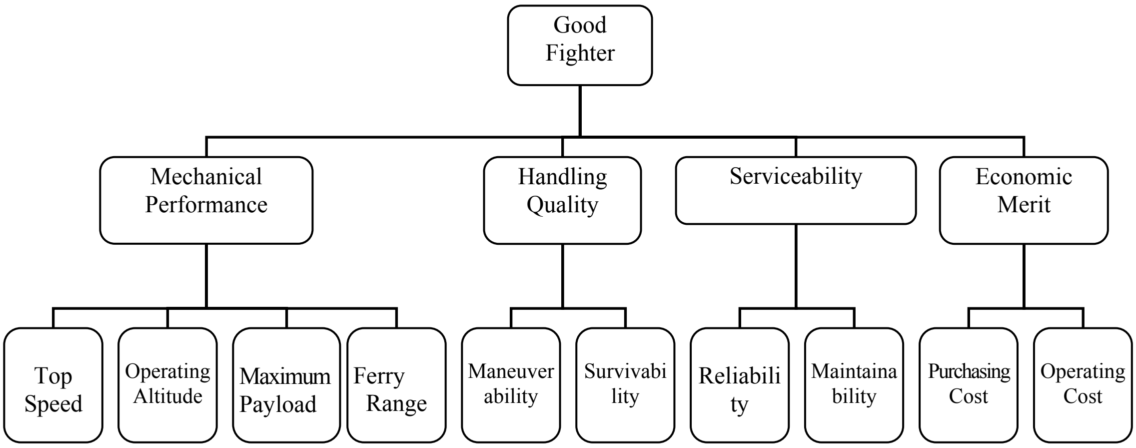


Figure 2.2 A Hierarchy for Fighter Evaluation (Source: Yoon and Hwang (1995))

DMs want a good fighter and they define that a good fighter is dependent on four important factors (mechanical performance, handling quality, serviceability, and economic merit). Let us define factors under good fighter as attributes. Then trade-offs among attribute values should be done in compensatory method. There may be several cases such as: one fighter may have the highest speed but not as good in other attributes, one fighter may have overall good score but not ranked first in any attributes, and so on. If this situations happen, how DM can decide to purchase which fighter. There are several compensatory MADM methods called “Simple Additive Weight (SAW)”, “TOPSIS”, “ELECTRE”, and “AHP” method.

In the SAW method, each alternative's ranking score is obtained by adding all attributes' scores. Formally the value of an alternative in the SAW method can be expressed as

$$V(A_i) = \sum_{j=1}^{m} w_j v_j(x_{ij}), \quad i = 1,2,\ldots,n, \tag{2.5}$$

where $V(A_i)$ is the value function of alternative A_i, and w_j is j th attribute weights, and $v_j(x_{ij})$ is the value of response of alternative i on attribute j. Through the normalization process, each incommensurable attribute becomes a pseudo-value function, which allows direct addition among attributes. The value of alternative A_i can be rewritten as

$$V(A_i) = \sum_{j=1}^{m} w_j r_{ij}, \quad i = 1,2,\ldots,n, \tag{2.6}$$

where r_{ij} is the comparable scale of x_{ij}, which can be obtained by Equation 2.3.

The underlying assumption of the SAW method is that attributes are preferentially independent. Less formally, this means that the contribution of an individual attribute to the total score is independent of other attribute values. Therefore, DM's preference regarding the value of one attribute is not influenced in any way by the values of the other attributes (Fishburn, 1976). Fortunately, studies (Edwards, 1977; Farmer, 1987) show that the SAW method yields extremely close approximations to "true" value functions even when independence among attributes does not exactly hold.

In addition to the preference independence assumption, the SAW has a required characteristic for weights. That is, the SAW presumes that weights are proportional to the relative value of a unit change in each attribute's value function (Hobbs, 1980). For instance, let us consider a value function with two attributes: $V = w_1 v_1 + w_2 v_2$. By setting the amount of V constant, we can derive the relationship of $w_1 / w_2 = -\Delta v_2 / \Delta v_1$. This relationship indicates that if w_1=0.33 and w_2=0.66, the DM must be indifferent to the trade between 2 units of v_1 and 1 unit of v_2. This is the same as utility function's marginal utility (MU) and marginal rate of substitution (MRS) (Sher and Pinola, 1981).

Let us apply this ASW into the example problem in the previous figure 2.2. Recall that there are ten attributes and five alternatives. Four attributes (i.e., mechanical performance, handling quality, serviceability, and economic merit) are under overall goal (i.e., selection of good fighter) and they have four or two sub-attributes (i.e., top speed, operating altitude, maximum payload, ferry range, maneuverability, survivability, reliability, maintainability, purchasing cost, and operating cost). Henceforth first level of attribute is represented as X_i standing first level of i th attribute. From the second level of attribute, the number of subscript ciphers represents the level of its attribute. For example, top speed which is the first sub-attribute is represented as X_{11} and if there is sub-sub-attributes than it can be written as X_{11j} and means j th attribute under top speed.

There are five alternatives and represented as A_1, A_2, A_3, A_4, and A_5. Henceforth alternative is represented as A_i that means i th alternative. Table 2.1 in the following page shows data for evaluation of these fighters.

Table 2.1 Data for Evaluation of Fighters (Source: Yoon and Hwang, 1995)

		Alternatives				
Attribute	Weight	A_1	A_2	A_3	A_4	A_5
1. Mechanical performance						
1.1 Top speed (Mach)	0.20	2.0	2.0	2.5	2.0	1.8
1.2 Operating altitude (1,000 ft)	0.04	60	50	60	50	50
1.3 Maximum payload (1,000 lbs)	0.04	23	20	18	20	21
1.4 Ferry range (NM)	0.12	1,900	2,000	3,500	2,400	2,300
2. Handling quality						
2.1 Maneuverability(*)	0.09	7	8	8	9	9
2.2 Survivability (*)	0.21	8	9	7	8	8
3. Serviceability						
3.1 Reliability (*)	0.12	8	7	9	8	8
3.2 Maintainability (*)	0.08	9	7	8	7	7
4. Economic merit						
4.1 Purchasing cost ($M/ea)	0.06	4.5	5.0	6.5	5.5	5.0
4.2 Operating cost ($1,000/year)	0.04	90	90	100	80	70

Note that weights are assumed given in this table and * units are from a 10-point scale, from 1 (worst) to 10 (best). The normalized decision matrix from the above data by Equation 2.3 is given as

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccccc} \mathrm{X}_{11} & \mathrm{X}_{12} & \mathrm{X}_{13} & \cdots & \mathrm{X}_{41} & \mathrm{X}_{42} \end{array} \\ \left[\begin{array}{cccccc} 0.80 & 1.00 & 1.00 & \cdots & 1.00 & 0.78 \\ 0.80 & 0.83 & 0.87 & \cdots & 0.90 & 0.78 \\ 1.00 & 1.00 & 0.78 & \cdots & 0.69 & 0.70 \\ 0.80 & 0.83 & 0.87 & \cdots & 0.82 & 0.88 \\ 0.72 & 0.83 & 0.91 & \cdots & 0.90 & 1.00 \end{array}\right] \end{array}$$

where columns of $X_{14}, X_{21}, X_{22}, X_{31}$, and X_{32} are not shown here. X_{41} and X_{42} are cost attributes. Therefore, their normalized values of r_{ij} are computed as

$$r_{ij} = \frac{\frac{1}{x_{ij}}}{\frac{1}{x_j^-}}, i = 1,2,3,4,5; \; j = 1,2,\ldots,10, \tag{2.7}$$

where x_j^- is the smallest value of x_{ij}. Other values of r_{ij} are calculated by Equation 2.3 and detailed calculation processes are not present here. For the purpose of graphical view, weights of each attribute are shown in the following figure.

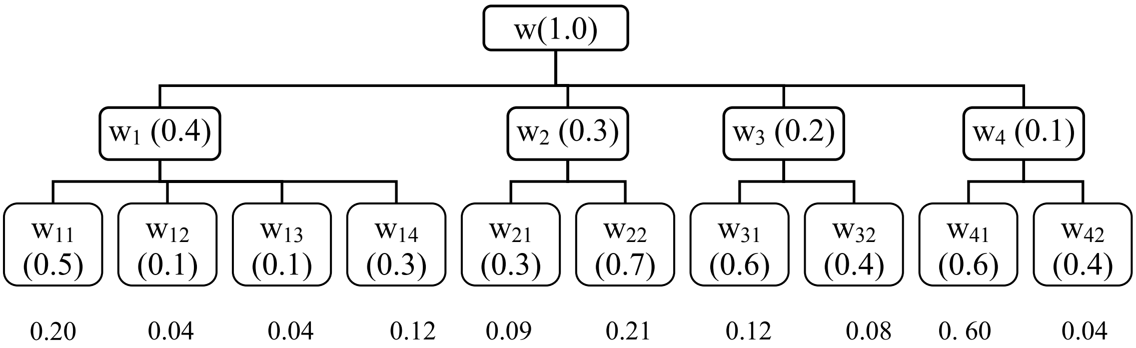


Figure 2.3 Weight Assessment for Fighter Evaluation

The value of alternative A_1 is then computed by SAW as below.

$$V(A_1) = \sum_{j=1}^{10} w_j r_{ij} \, ,$$

$$= 0.2(.8) + 0.04(1.0) + \ldots + 0.04(.78) = 0.8396.$$

The other alternatives have values of $V(A_2)$=0.8274, $V(A_3)$=0.8953, $V(A_4)$=0.8400, and $V(A_5)$=0.8323. The preference order is $[A_3, A_4, A_1, A_5, A_2]$, where A_3 is the first rank and A_2 is the last.

Even though this method has easy computational merit, it has somewhat of a weakness, that it may not consider extreme data: some of alternatives may have higher overall values because of some extreme high values in some attributes but low scores in the other attributes. It is the same problem that we have when we use mean value itself in statistics. In addition to this problem, to be able to use the ASW method we should have weight information.

In the SAW method, addition among attribute values was allowed only after the different measurement units were transformed into a dimensionless scale by a normalization process. However, this transformation is not necessary if attributes are connected by multiplication (see Brauers (2001) for more detailed example). When we use multiplication among attribute values, the weights become exponents associated with each attribute value: a positive power for benefit attributes, and a negative power for cost attributes. Formally, the value of alternative A_i is given by

$$V(A_i) = \prod_{j=1}^{m} x_{ij}^{w_j}, i = 1,2,\ldots,n. \tag{2.8}$$

Because of the exponent property, this method requires that all ratings be greater than 1. For instance, when an attribute has fractional ratings, all ratings in that attribute are multiplied by 10^m to meet this requirement. Since there is no fractional number in our example in Figure 2.2, we can plug all the x_{ij} values in the previous table 2.1 into Equation 2.15 and values of alternatives become such as: $V(A_1) = 8.1716$, $V(A_2) = 8.0707$, $V(A_3) = 8.7689$, $V(A_4) = 8.2565$, and $V(A_5) = 8.1485$. The final rank order is $[A_3, A_4, A_1, A_5, A_2]$ and this rank is the same as previous SAW method.

Saaty and Vargas (2000) show that multiplicative and additive syntheses are related analytically through the approximation as below:

$$\prod x_{ij}^{w_i} = \exp\log\prod x_{ij}^{w_i} = \exp\left(\sum \log x_{ij}^{w_i}\right) = \exp\left(\sum w_i \log x_{ij}\right)$$

$$\approx 1 + \sum w_i \log x_{ij} \approx 1 + \sum \left(w_i x_{ij} - w_i\right) = \sum w_i x_{ij} \ . \tag{2.9}$$

Therefore, we can say that there is no difference between SAW method and weight product method from a point of final rank order.

A MADM problem with m alternatives that are evaluated by n attributes may be viewed as a geometric system with m points in the n-dimensional space. Hwang and Yoon (1981) develop the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution and the longest distance from the negative-ideal solution. This principle is also suggested by Zeleny (1982), and Hall (1989). Loerch et al (1998) apply this method to their research. Recently this method is enriched by Yoon (1987) and Hwang et al. (1993).

TOPSIS starts from the concept of an ideal solution. An ideal solution is defined as a collection of ideal levels (or ratings) in all attributes considered. However, the ideal solution is usually unattainable or infeasible. Then to be as close as possible to such and ideal solution is the rational of human choice (Yoon and Hwang, 1995). Coombs (1958, 1964) also claimed that there is an ideal level of attributes for alternatives of choice and that the DM's utilities decrease monotonically when an alternative moves away from this ideal (or utopia) point (Yu, 1985). Formally the positive-ideal solution is denoted as

$$A^* = \left(x_1^*, \ldots, x_j^*, \ldots, x_n^*\right), \tag{2.10}$$

where x_j^* is the best value for the jth attribute among all available alternatives. While the negative-ideal solution is composed of all worst attribute ratings attainable.

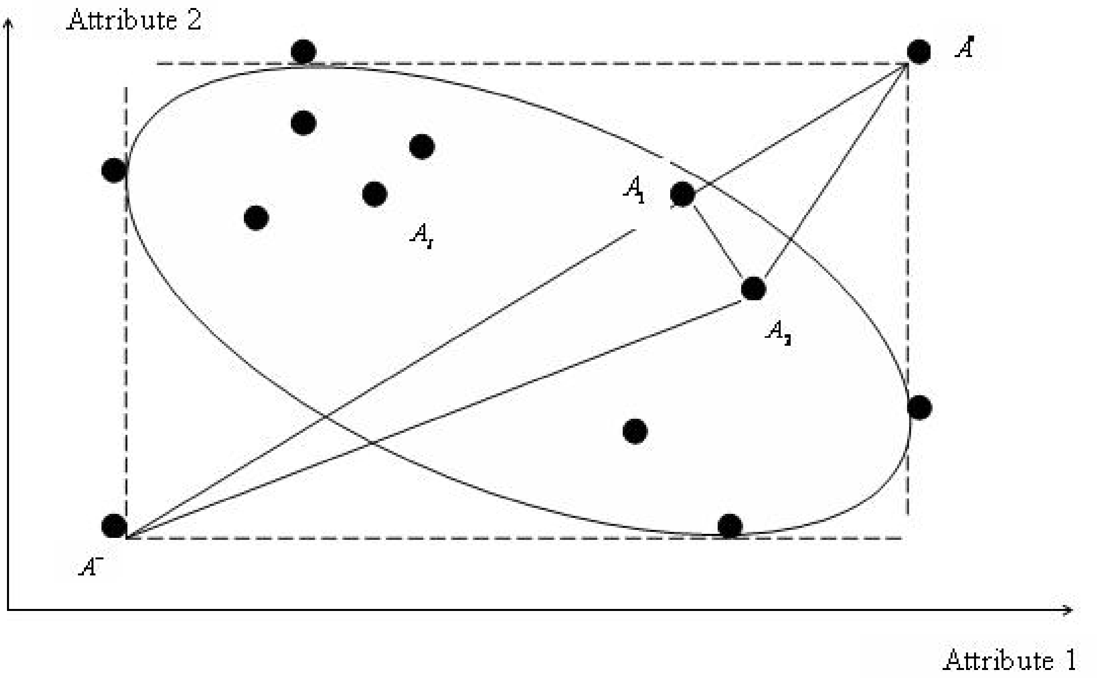


Figure 2.4 Euclidean Distances to Positive-Ideal and Negative-Ideal Solutions in Two-Dimensional Space (Yoon and Hwang, 1981)

The negative-ideal solution is given as

$$A^- = \left(x_1^-,\ldots,x_j^-,\ldots,x_n^-\right), \tag{2.11}$$

where x_j^- is the worst value for the jth attribute among all available alternatives. Figure 2.4 graphically shows the two ideal solutions. For example, consider two alternatives A_1 and A_2 with respect to two benefit attributes in Figure 2.4. A_1 is the closest to A^* but A_2 is the farthest from A^-.

TOPSIS defines an index called similarity (or relative closeness) to the positive-ideal solution by combining the proximity to the positive-ideal solution and the remoteness from the negative-ideal solution. Then the method chooses an alternative with the maximum similarity to the positive-ideal solution. TOPSIS assumes that each attribute takes either monotonically increasing or monotonically decreasing utility. That is, the larger the attribute outcome, the greater the preference for benefit attributes and the less the preference for cost attributes. The method is presented as a series of successive steps:

Step 1. Calculate normalized ratings. We use the ideal normalization is used for computing r_{ij}, which is given as Equation 2.3.

Step 2. Calculate weighted normalized ratings. The weighted normalized value is calculated as

$$v_{ij} = w_j r_{ij},\ \ i = 1,\ldots,n; j = 1,\ldots,m, \tag{2.12}$$

where w_j is the weight of the jth attribute.

Step 3. Identify positive-ideal and negative-ideal solutions.

$$A^* = \{v_1^*, v_2^*, \ldots, v_j^*, \ldots v_n^*\}$$
$$= \left\{\left(\max_i v_{ij} \middle| j \in J_1\right), \left(\min_i v_{ij} \middle| j \in J_2\right) \middle| i = 1, \ldots, n\right\} \tag{2.13}$$

$$A^- = \{v_1^-, v_2^-, \ldots, v_j^-, \ldots v_n^-\}$$
$$= \left\{\left(\min_i v_{ij} \middle| j \in J_1\right), \left(\max_i v_{ij} \middle| j \in J_2\right) \middle| i = 1, \ldots, n\right\} \tag{2.14}$$

where J_1 is a set of benefit attributes and J_2 is a set of cost attributes.

Step 4. Calculate separation measures. The separation (distance) between alternatives can be measured by the n-dimensional Euclidean distance. The separation of each alternative from the positive-ideal solution, A^*, is then given by

$$S_i^* = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^*\right)^2}, i = 1, \ldots, n. \tag{2.15}$$

Similarly, the separation from the negative-ideal solution, A^-, is given by

$$S_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, i = 1, \ldots, n. \tag{2.16}$$

Step 5. Calculate similarities to positive-ideal solution.

$$V(A_i) = S_i^- / \left(S_i^* + S_i^-\right), i = 1, \ldots, n\ . \tag{2.17}$$

Note that $0 \le V(A_i) \le 1$, where $V(A_i) = 0$ when $A_i = A^-$, and $V(A_i) = 1$ when $A_i = A^*$.

Step 6. Rank preference order. Choose an alternative with the maximum C_i^* in descending order.

Let us solve the previous example problem given in the Figure 2.2.

Step 1. Normalization

We use normalized decision matrix under table 2.2 and show all the ratings that were abbreviated. The decision matrix is shown as:

$$\begin{array}{cc}
 & \begin{array}{cccccccccc} X_{11} & X_{12} & X_{13} & X_{14} & X_{21} & X_{22} & X_{31} & X_{32} & X_{41} & X_{42} \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \left[\begin{array}{cccccccccc}
0.80 & 1.00 & 1.00 & 0.543 & 0.778 & 0.889 & 0.889 & 1 & 1.00 & 0.78 \\
0.80 & 0.83 & 0.87 & 0.571 & 0.889 & 1 & 0.778 & 0.778 & 0.90 & 0.78 \\
1.00 & 1.00 & 0.78 & 1 & 0.889 & 0.778 & 1 & 0.889 & 0.69 & 0.70 \\
0.80 & 0.83 & 0.87 & 0.686 & 1 & 0.889 & 0.889 & 0.778 & 0.82 & 0.88 \\
0.72 & 0.83 & 0.91 & 0.657 & 1 & 0.889 & 0.889 & 0.778 & 0.90 & 1.00
\end{array}\right]
\end{array}$$

Step 2. Weighted Normalization. The weights of (0.2, 0.04,…,0.04) from the table 2.2 are multiplied with each column of the normalized rating matrix:

$$\begin{array}{cc}
 & \begin{array}{cccccccccc} X_{11} & X_{12} & X_{13} & X_{14} & X_{21} & X_{22} & X_{31} & X_{32} & X_{41} & X_{42} \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} & \left[\begin{array}{cccccccccc}
0.16 & 0.04 & 0.04 & 0.065 & 0.070 & 0.187 & 0.107 & 0.08 & 0.06 & 0.031 \\
0.16 & 0.033 & 0.035 & 0.069 & 0.080 & 0.21 & 0.093 & 0.062 & 0.054 & 0.031 \\
0.2 & 0.04 & 0.031 & 0.12 & 0.080 & 0.163 & 0.12 & 0.071 & 0.041 & 0.028 \\
0.16 & 0.033 & 0.035 & 0.082 & 0.09 & 0.187 & 0.107 & 0.062 & 0.049 & 0.035 \\
0.144 & 0.033 & 0.036 & 0.079 & 0.09 & 0.187 & 0.107 & 0.062 & 0.054 & 0.04
\end{array}\right]
\end{array}$$

Note: Since we already normalized by using Equation 2.3, there is no cost attributes in the above matrix. Therefore, we only need to consider benefit attributes.

Step 3. Positive ideal solution and negative ideal solutions are

$A^* = (0.2, 0.04, 0.04, 0.12, 0.09, 0.21, 0.12, 0.08, 0.06, 0.04)$

$A^- = (0.144, 0.033, 0.031, 0.065, 0.07, 0.163, 0.093, 0.062, 0.041, 0.028)$

Step 4. The separation measures from A^* are computed first:

$$S_{A_1}^* = \sqrt{\sum_{j=1}^{10} \left(v_{1j} - v_j^*\right)^2}$$

$$= \left[\left(0.16 - 0.2\right)^2 + \ldots + \left(0.031 - 0.04\right)^2\right]^{1/2} = 0.075$$

Separation measures from A^* of all alternatives are

$$\left(S_{A_1}^*, S_{A_2}^*, S_{A_3}^*, S_{A_4}^*, S_{A_5}^*\right) = (0.076, 0.074, 0.055, 0.064, 0.077)$$

The separation measures from A^- are computed as

$$S_{A_1}^- = \sqrt{\sum_{j=1}^{10} \left(v_{1j} - v_j^-\right)^2}$$

$$= \left[\left(0.16 - 0.144\right)^2 + \ldots + \left(0.031 - 0.028\right)^2\right]^{1/2} = 0.043$$

All separation measures from are

$$\left(S_{A_1}^-, S_{A_2}^-, S_{A_3}^-, S_{A_4}^-, S_{A_5}^-\right) = (0.043, 0.053, 0.084, 0.043, 0.041)$$

Step 5. Similarities to positive ideal solution are computed as

$$V(A_1) = S_{A_1}^- /(S_{A_1}^* + S_{A_1}^-)$$

$$= 0.043/(0.076+0.043) = 0.361$$

All similarities to the positive ideal solution are

$$\left(V(A_1), V(A_2), V(A_3), V(A_4), V(A_5)\right) = (0.361, 0.414, 0.607, 0.396, 0.349).$$

Step 6. Preference rank. Based on the descending order of $V(A_i)$, the preference order is given as $[A_3, A_2, A_4, A_1, A_5]$, which selects alternative 3 fighter to purchase. Three sets of preference rankings are shown at the following table.

Table 2.2 Three Sets of Preference Rankings

	S^*		S^-		$V(A_i)$	
Fighter	Value	Rank	Value	Rank	Value	Rank
A_1	0.076	4	0.043	3.5	0.361	4
A_2	0.074	3	0.053	2	0.414	2
A_3	0.055	1	0.084	1	0.607	1
A_4	0.064	2	0.043	3.5	0.396	3
A_5	0.077	5	0.041	5	0.349	5

The idea of this method can be summarized as follows: an alternative has to be close to the positive ideal solution and this represents a preference of the shorter distance; an alternative has to be located far away from the negative ideal solution which prefers longer distance; the final rank is based on the compensation of these two distances. We observed that this idea is exactly the same as variance information in the statistics. The detailed discussions are presented in Chapter 3.

Like problems found from the ASW method, this TOPSIS method also requires the weight information. Moreover, this method has weakness that it only considers distance from the ideal solutions and hence does not consider the alternative's overall values. However, DMs may want to see an individual alternative's overall scores in addition to the distance information.

The ELECTRE (Elimination et choix traduisant la réalité) method is originated from Roy (1971) in the late 1960s. Since then Nijkamp and van Delft (1977) and Voogd (1983) have developed this method to its present state. The method dichotomizes preferred alternatives and nonpreferred ones by establishing outranking relationships. This method is most popular in Europe, especially among the French-speaking community.

When a DM feels that A is better than B, then it is defined that A outranks B and the notation is (A R B) or (A→B). For the utility theory, the SAW method, and the AHP method, the transitive assumption (i.e., if A is better than B and B is better than C, then A must be better than C) is important. However, this ELECTRE method does not need this transitive assumption. Therefore, the relation of (A R B) and (B R C) do not necessarily imply (A R C). The outranking relationships are determined by concordance and discordance indexes (Yoon and Hwang, 1995).

Figure 2.5 shows one example of a relationship of preferred alternatives in the ELECTRE method.

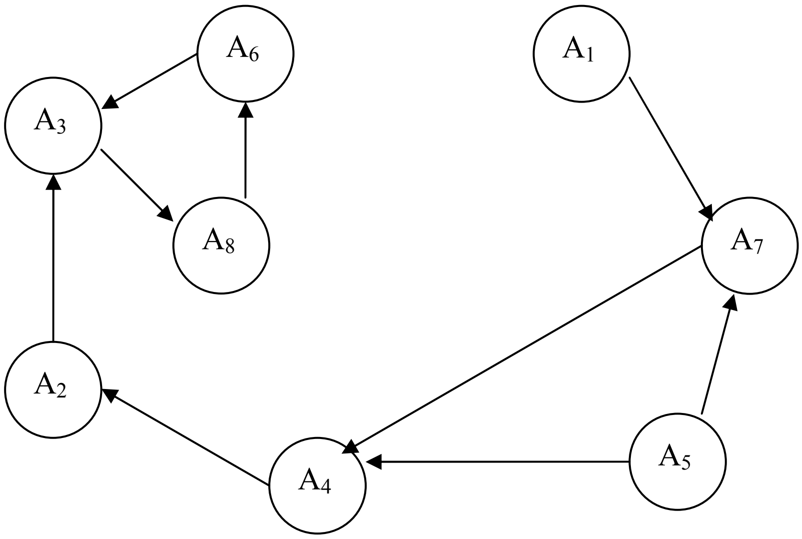


Figure 2.5 A Digraph for Eight Alternatives (Yoon and Hwang, 1995)

Nine outranking relationships from this figure are given as: (A_1→A_2), (A_2→A_3), (A_3→A_8), (A_4→A_2), (A_5→A_4), (A_5→A_7), (A_6→A_3), (A_7→A_4), and (A_8→A_6). When a directed path begins in a node and comes back to this very node, this path is called as a cycle. All nodes in a cycle are considered to have an equivalent preference. In the above figure, $A_3 \rightarrow A_8 \rightarrow A_6 \rightarrow A_3$ is a cycle.

The kernel (or core) of an acyclic digraph is a reduced set of nodes that is preferred to the set of nodes that do not belong to the kernel. Kernel (K) is defined as a set of preferred alternatives by ELECTRE. The K should satisfy the following two conditions:

1. Each node in K is not outranked by any other node in K.

2. Every node not in K is outranked by at least one node in K.

Figure 2.6 shows the Kernel of this example.

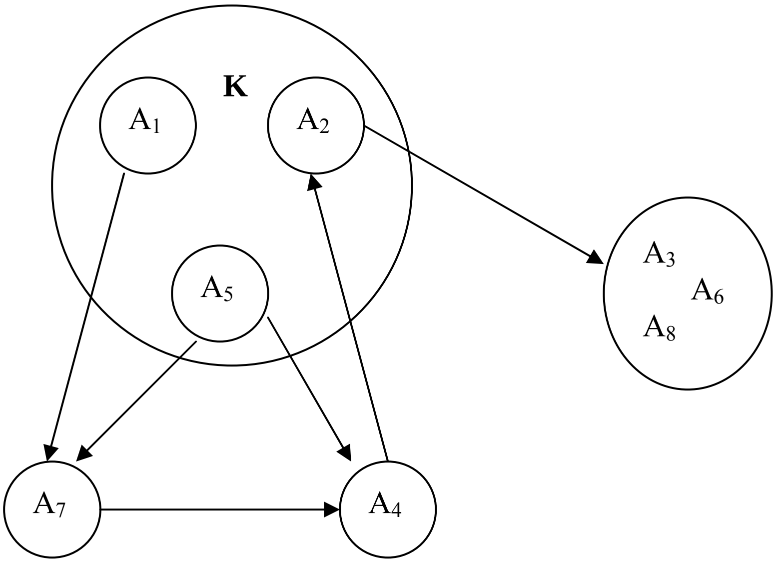


Figure 2.6 The Kernel of Figure 2.5

The set of preferred alternatives defined by the kernel is K=$\{A_1, A_2, A_5\}$.

The ELECTRE method formulates concordance and discordance indexes in order to obtain outranking relationships, and renders a set of preferred alternatives by forming a kernel. Concordance and discordance indexes can be viewed as measurements of satisfaction and dissatisfaction that a DM feels on choosing one alternative over the other.

Let us imply this ELECTRE method into the same problem given in the Figure 2.2 and follow each step given by Yoon and Hwang (1995).

For the convenience, we rewrite the weighted normalization matrix such as below. One thing that we change is the subscripts of attributes and this is done for a notational convenience (i.e., X_i is used instead of X_{ij}).

$$\begin{array}{c|cccccccccc}
 & X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} \\
\hline
A_1 & 0.16 & 0.04 & 0.04 & 0.065 & 0.070 & 0.187 & 0.107 & 0.08 & 0.06 & 0.031 \\
A_2 & 0.16 & 0.033 & 0.035 & 0.069 & 0.080 & 0.21 & 0.093 & 0.062 & 0.054 & 0.031 \\
A_3 & 0.2 & 0.04 & 0.031 & 0.12 & 0.080 & 0.163 & 0.12 & 0.071 & 0.041 & 0.028 \\
A_4 & 0.16 & 0.033 & 0.035 & 0.082 & 0.09 & 0.187 & 0.107 & 0.062 & 0.049 & 0.035 \\
A_5 & 0.144 & 0.033 & 0.036 & 0.079 & 0.09 & 0.187 & 0.107 & 0.062 & 0.054 & 0.04
\end{array}$$

Step 1. Construct concordance and discordance sets. For each pair of alternatives A_p and A_q $(p, q = 1, \ldots, 5)$, the set of attributes is divided into these two distinct subsets. The concordance set, which is composed of all attributes for which alternative A_p is preferred to alternative A_q, can be written as

$$C(p,q) = \{j | v_{pj} \geq v_{qj}\} \tag{2.18}$$

where v_{pj} is the weighted normalized rating of alternative A_p with respect to the j th attribute. In other words, $C(p,q)$ is the collection of attributes where A_p is better than or equal to A_q. The complement of $C(p,q)$, which is called the discordance set, contains all attributes for which A_p is worse than A_q. This can be written as

$$D(p,q) = \{j | v_{pj} < v_{qj}\}. \tag{2.19}$$

Note that $C(p,q)$ is not equal to $D(p,q)$ when tied ratings exist. The concordance and discordance sets are obtained as

C(1,2) = {1, 2, 3, 7, 8, 9, 10}   D(1,2) = {4, 5, 6}
C(1,3) = {2, 3, 6, 8, 9, 10}   D(1,3) = {1, 4, 5, 7}
C(1,4) = {1, 2, 3, 6, 7, 8, 9}   D(1,4) = {4, 5, 10}
C(1,5) = {1, 2, 3, 6, 7, 8, 9}   D(1,5) = {4, 5, 10}
C(2,1) = {1, 4, 5, 6, 10}   D(2,1) = {2, 3, 7, 8 ,9}
C(2,3) = {3, 5, 6, 9, 10}   D(2,3) = {1, 2, 4, 7, 8}
C(2,4) = {1, 2, 3, 6, 8, 9}   D(2,4) = {4, 5, 7, 10}
C(2,5) = {1, 2, 6, 8, 9}   D(2,5) = {3, 4, 5, 7, 10}
C(3,1) = {1, 2, 4, 5, 7}   D(3,1) = {3, 6, 8, 9, 10}
C(3,2) = {1, 2, 4, 5, 7, 8}   D(3,2) = {3, 6, 9, 10}
C(3,4) = {1, 2, 4, 7, 8}   D(3,4) = {3, 5, 6, 9, 10}
C(3,5) = {1, 2, 4, 7, 8}   D(3,5) = {3, 5, 6, 9, 10}
C(4,1) = {1, 4, 5, 6, 7, 10}   D(4,1) = {2, 3, 8, 9}
C(4,2) = {1, 2, 3, 4, 5, 7, 8, 10}   D(4,2) = {6, 9}
C(4,3) = {3, 5, 6, 9, 10}   D(4,3) = {1, 2, 4, 7, 8}
C(4,5) = {1, 2, 4, 5, 6, 7, 8}   D(4,5) = {3, 9, 10}
C(5,1) = {4, 5, 6, 7, 10}   D(5,1) = {1, 2, 3, 8, 9}
C(5,2) = {2, 3, 4, 5, 7, 8, 9,10}   D(5,2) = {1, 6}
C(5,3) = {3, 5, 6, 9, 10}   D(5,3) = {1, 2, 4, 7, 8}
C(5,4) = {2, 3, 5, 6, 7, 8, 9, 10}   D(5,4) = {1, 4}

Step 2. Compute concordance and Discordance Indexes. The relative power of each concordance set is measured by means of the concordance index. The concordance index C_{pq} represents the degree of confidence in the pairwise judgments of $(A_p \rightarrow A_q)$. The concordance index of $C(p,q)$ is defined as

$$C_{pq} = \sum_{j^*} w_{j^*} \tag{2.20}$$

where j^* are attributes contained in the concordance set $C(p,q)$. The concordance indexes of this example are

C_{12}= 0.58 C_{13}=0.47 C_{14}=0.75 C_{15}=0.75 C_{21}=0.66

C_{23}=0.44 C_{24}=0.63 C_{25}=0.59 C_{31}=0.57 C_{32}=0.65

C_{34}=0.56 C_{35}=0.56 C_{41}=0.78 C_{42}=0.73 C_{43}=0.44

C_{45}=0.86 C_{51}=0.58 C_{52}=0.59 C_{53}=0.44 C_{54}=0.68

where C_{12}= 0.58 was obtained from

$$C_{12} = \sum_{j^*} w_{j^*} = w_1 + w_2 + w_3 + w_7 + w_8 + w_9 + w_{10}$$

$$= 0.2+0.04+0.04+0.12+0.08+0.06+0.04 = 0.58.$$

The discordance index, on the other hand, measures the powers of $D(p,q)$. The discordance index of $D(p,q)$, which represents the degree of disagreement in $(A_p \to A_q)$, can be defined as

$$D_{pq} = \frac{\left(\sum_{j^\circ} \left| v_{pj^\circ} - v_{qj^\circ} \right|\right)}{\left(\sum_{j} \left| v_{pj} - v_{qj} \right|\right)} \tag{2.21}$$

where j° are attributes that are contained in the discordance set $D(p,q)$. The discordance indexes of this example are

D_{12}=0.425 D_{13}=0.648 D_{14}=0.500 D_{15}=0.457 D_{21}=0.575

D_{23}=0.667 D_{24}=0.594 D_{25}=0.530 D_{31}=0.352 D_{32}=0.333

D_{34}=0.331 D_{35}=0.337 D_{41}=0.500 D_{42}=0.406 D_{43}=0.669

D_{45}=0.367 D_{51}=0.543 D_{52}=0.470 D_{53}=0.663 D_{54}=0.633

where D_{12}= 0.58 was obtained from

$$D_{12} = \frac{\left(\sum_{j^\circ} \left|v_{pj^\circ} - v_{qj^\circ}\right|\right)}{\left(\sum_{j} \left|v_{pj} - v_{qj}\right|\right)}$$

$$= \frac{\left(\left|v_{14} - v_{24}\right| + \left|v_{15} - v_{25}\right| + \left|v_{16} - v_{26}\right|\right)}{\sum_{j=1}^{10} \left|v_{1j} - v_{2j}\right|}$$

$$= \frac{\left(\left|0.065 - 0.069\right| + \left|0.07 - 0.08\right| + \left|0.187 - 0.21\right|\right)}{0.087}$$

$$= \frac{0.037}{0.087} = 0.4253\,.$$

Step 3. Outranking Relationships. The dominance relationship of alternative A_p over alternative A_q becomes stronger with a higher concordance index C_{pq} and a lower discordance index D_{pq}. The method defines that A_p outranks A_q when $C_{pq} \geq \overline{C}$ and $D_{pq} < \overline{D}$, where $\overline{C}$ and $\overline{D}$ are the averages of C_{pq} and D_{pq}, respectively.

For this problem,

$\overline{C} = \dfrac{0.58 + \cdots + 0.68}{20} = 0.62$, and $\overline{D} = \dfrac{0.425 + \cdots + 0.633}{20} = 0.50$.

Table 2.3 Determination of Outranking Relationship

C_{pq}	Is $(C_{pq} \geq \overline{C})$? $\overline{C}$ =0.62	D_{pq}	Is $(D_{pq} < \overline{D})$? $\overline{D}$ =0.50	Is $(A_p \rightarrow A_q)$?
C_{12}=0.58	No	D_{12}=0.43	Yes	No
C_{13}=0.47	No	D_{13}=0.65	No	No
C_{14}=0.75	Yes	D_{14}=0.50	No	No
C_{15}=0.75	Yes	D_{15}=0.46	Yes	Yes
C_{21}=0.66	Yes	D_{21}=0.57	No	No
C_{23}=0.44	No	D_{23}=0.67	No	No
C_{24}=0.63	Yes	D_{24}=0.59	No	No
C_{25}=0.59	No	D_{25}=0.53	No	No
C_{31}=0.57	No	D_{31}=0.35	Yes	No
C_{32}=0.65	Yes	D_{32}=0.33	Yes	Yes
C_{34}=0.56	No	D_{34}=0.33	Yes	No
C_{35}=0.56	No	D_{35}=0.34	Yes	No
C_{41}=0.78	Yes	D_{41}=0.50	No	No
C_{42}=0.73	Yes	D_{42}=0.41	Yes	Yes
C_{43}=0.44	No	D_{43}=0.67	No	No
C_{45}=0.86	Yes	D_{45}=0.37	Yes	Yes
C_{51}=0.58	No	D_{51}=0.54	No	No
C_{52}=0.59	No	D_{52}=0.47	Yes	No
C_{53}=0.44	No	D_{53}=0.66	No	No
C_{54}=0.68	Yes	D_{54}=0.63	No	No

Table 2.3 illustrates the determination of outranking relationships. Four outranking relationships are obtained: ($A_1 \rightarrow A_5$), ($A_3 \rightarrow A_2$), ($A_4 \rightarrow A_2$), and ($A_4 \rightarrow A_5$). The Kernel of this problem is shown in Figure 2.7.

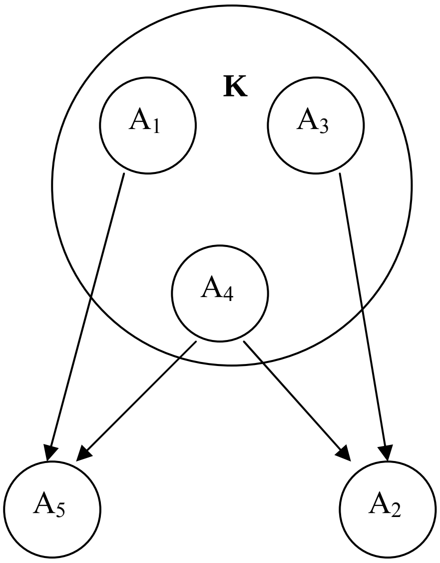


Figure 2.7 The Kernel of the Example Problem

Yoon and Hwang (1995) state that a weakness of ELECTRE might lie in its use of the critical threshold values of $\overline{C}$ and $\overline{D}$. These values are rather arbitrary, although their impact upon the ultimate result may be significant. They also notice that there is no rank order for alternatives that are inside of K. They introduce the net outranking relationship into the ELECTRE method to address these problems. By suing this relationship they can transform the current ELECTRE's ordinal rank into cardinal rank, and hence DMs can see the preference between alternatives among in the K.

Complementary ELECTRE defines the net concordance index C_p, which measures the degree to which the dominance of alternative A_p over competing alternatives exceeds the dominance of competing alternatives over A_p. Similarly, the net discordance index D_p

measures the relative weakness of alternative A_p with respect to other alternatives. These net indexes are mathematically denoted as

$$C_p = \sum_{\substack{k=1 \\ k \neq p}}^{m} C_{pk} - \sum_{\substack{k=1 \\ k \neq p}}^{m} C_{kp} \text{ , and} \tag{2.22}$$

$$D_p = \sum_{\substack{k=1 \\ k \neq p}}^{m} D_{pk} - \sum_{\substack{k=1 \\ k \neq p}}^{m} D_{kp} \text{ .} \tag{2.23}$$

Obviously, an alternative A_p has a greater preference with a higher C_p and a lower D_p. Hence the final selection should satisfy the condition that its net concordance index should be at a maximum and its net discordance index at a minimum. If both these conditions are not satisfied, the alternative that scores the highest average rank can be selected as the final solution.

For our example problem, the net concordance and discordance indexes are shown in Table 2.4.

Table 2.4 The Net Concordance and Discordance Indexes of the Alternatives in the K

Net concordance index	value	rank	Net discordance index	value	rank	Final rank
C_1	-0.04	3	D_1	0.06	3	3
C_3	0.55	1	D_3	-1.29	1	1
C_4	0.19	2	D_4	-0.12	2	2

Based on this complementary method, we can present alternatives ranking order which are in the Kernel. However, this complementary method causes that the Kernel of ELECTRE method is not required: we can present all the alternatives ranking order followed by this complementary method, and hence there is no need for us to define the Kernel. Moreover, both the ELECTRE and the complementary ELECTRE method do not consider an overall ranking score of each alternative.

2.2.4 Analytic Hierarchy Process (AHP)

Previous Methods assume that attributes' weights are already given. Therefore, there was no need to assign weights for each attribute. However, most of real life decision problems are different from this assumption. Therefore, we need to assign each attributes' weight with one of following three methods: the AHP method, the Delphi technique, and the Utility theory. These three methods are also used for a problem not only with quantitative data, but also with qualitative data.

In 1980, Saaty presented the AHP method. This method is widely used for many different areas such as political, economic, sociology, and even in medical areas because of these superiorities: 1) This method can handle both quantitative and qualitative data at the same time; 2) This method uses the eigenvector and eigenvalue property and this property presents a computational merit; 3) Saaty already proved the advantages of this method with many case studies; and 4) This method gives less cognitive burden to DMs compared to the other two methods.

The AHP method has two important theoretical backgrounds: the fundamental scale, and eigenvector and eigenvalue property. The Saaty's fundamental 1-9 scale has its origin on the Weber-Fechner's sensation (response) equation (i.e., $M = a \log s + b, a \neq 0$, where M denotes the sensation and s the stimulus) (Fechner, 1966). While the noticeable ratio stimulus increases geometrically, the response to that stimulus increases arithmetically. In making pairwise comparisons, nearest integer approximation from the fundamental scales are used. This scale has been validated for effectiveness, not only in many applications by a number of people, but also through theoretical justification of what scale one must use in the comparison of homogeneous elements (Saaty and Vargas, 2000). The upper limit of 9 is defined following Miller (1956)'s "Magical number theory".

Alternatives are compared by DMs with respect to those fundamental scales of Table 2.5. For example, if a DM decides that alternative A_i is strongly important than alternative A_j, then he or she assigns the value of 5 into the corresponding cell of a decision matrix. From these pairwise comparisons, a decision matrix is composed. This decision matrix is defined as A, or A = (a_{ij}), where a_{ij} denotes the number which indicates the preference strength of an alternative A_i over an alternative A_j. By the reciprocal property of the AHP (i.e., $a_{ij} = 1/a_{ji}$), the matrix A has the form

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{bmatrix}$$

If a DM's judgment is consistent over all the comparisons, then there is a transitivity of the preferred relationship such as $a_{ik} = a_{ij} \bullet a_{jk}$. These two properties (i.e., reciprocal and transitivity) are important assumptions in this AHP method.

Table 2.5 The Fundamental Scales (Saaty and Vargas, 2000)

Intensity of importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
2	Weak	
3	Moderate importance	Experience and judgment slightly favor one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgment strongly favor one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

An obvious case of the consistent matrix is one in which the comparisons are based on exact measurements: the weights $w_1,\ldots,w_n$ are already known, and hence

$$a_{ij} = \frac{w_i}{w_j} \quad i, j = 1,\ldots,n. \tag{2.24}$$

And thus

$$a_{ij} a_{jk} = \frac{w_i}{w_j} \bullet \frac{w_j}{w_k} = a_{ik}.$$

Also, of course,

$$a_{ji} = \frac{w_j}{w_i} = \frac{1}{w_i / w_j} = \frac{1}{a_{ij}}.$$

Note that the notation w_i used in above development is different from previous example: in previous example, w_i denotes the relative importance of each attribute; while, w_i used in equation 2.24 denotes the absolute importance of an alternative A_i. Let us consider this paradigm case further. The matrix equation of the homogeneous equation of

$$\sum_{j=1}^{n} a_{ij} x_i = y_i \quad i = 1, \ldots, n \tag{2.25}$$

is denoted by $A \bullet x = y$, where $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$.

From the equation 2.24, we obtain

$$a_{ij} \bullet \frac{w_j}{w_i} = 1 \quad i, j = 1, \ldots, n$$

and consequently

$$\sum_{j=1}^{n} a_{ij} w_j \frac{1}{w_i} = n \quad i = 1, \ldots, n$$

or

$$\sum_{j=1}^{n} a_{ij} w_j = nw_i \quad i = 1, \ldots, n$$

which is equivalent to

$$Aw = nw \tag{2.26}$$

There is an infinite number of ways to derive the vector of priorities from that matrix. But emphasis on consistency leads to the eigenvalue formulation $Aw = nw$ such as

$$\begin{bmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \cdots & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \cdots & \frac{w_2}{w_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & \cdots & \frac{w_n}{w_n} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

It is known that matrix A=(a_{ij}) is said to be consistent if and only if its principal eigenvalue is equal to n. The sum of the eigenvalues of a matrix is equal to its trace (i.e., the sum of its diagonal elements). In this case the trace of A is equal to n.

However, since a DM is a human, he or she cannot give the precise values of w_i / w_j, but only an estimate. Therefore, Saaty replaces $\lambda_{\max}$ for the n, and $Aw = nw$ becomes

$$Aw = \lambda_{\max} w \tag{2.27}$$

where $\lambda_{\max}$ is the largest or principal eigenvalue of matrix A. Saaty defines the difference between $\lambda_{\max}$ and n as a Consistency Index (CI). CI is calculated as

$$CI = \frac{(\lambda_{\max} - n)}{(n-1)}. \tag{2.28}$$

Saaty and his colleagues generated an average random index (RI) for matrices of order 1-15 using a sample size of 100 at Oak Ridge National Laboratory developed average random consistency index (for more detail, see Saaty,1980). RI increases as the order of the matrix increases and is shown in the following table.

Table 2.6 Average Random Consistency Index (RI)

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	.58	.90	1.12	1.24	1.32	1.41	1.45	1.49	1.49	1.51	1.56	1.57	1.59

Consistency Ratio (CR) is used to check the consistency of comparisons and is computed by

$$\mathrm{CR} = \mathrm{CI} / \mathrm{RI}. \tag{2.29}$$

The less value of CR represents the more consistent. Saaty suggests that we have a consistency if CR is less than 0.1. If CR is more than 0.1, that comparison is considered as inconsistent and should be excluded to calculate weight because that DM is considered to have no rationality.

A pairwise comparison is simple and convenient for both DM and analyst. Moreover, any qualitative data can be easily handled. However, because pairwise comparison is done by human being, there can be any inconsistency or irrational response. AHP method uses this powerful pairwise comparison and solves any possible human's irrational responses by using CR.

To show the procedure of a hierarchical composition of priorities, let us imply example problem in the Figure 2.2 with the assumption that no weights are assigned yet.

Step 1. Proceed with pairwise judgments for the first level attribute. Each questionnaire given to DMs is designed to compare two attributes at a time under the consideration that DMs need to achieve a goal and they need to decide which attribute or alternative is more important and how much. For example, a DM may compare mechanical performance and handling quality in terms of achieving a good fighter, and between mechanical performance and serviceability, and so on. Let us assume that we did pairwise comparisons then the pairwise judgment matrix is as in Table 2.7.

Table 2.7 Comparison of First Level of Attributes with Respect to Satisfying a Good Fighter

	Mechanical performance	Handling quality	Serviceability	Economic Merit
Mechanical performance	1	4	6	7
Handling quality	1/4	1	3	4
Serviceability	1/6	1/3	1	2
Economic Merit	1/7	1/4	1/2	1

From Table 2.7, the attribute's weights are obtained as in Table 2.8.

Table 2.8 Attribute Weights

	Mechanical performance	Handling quality	Serviceability	Economic Merit
weight	0.62	0.22	0.10	0.06
	$\lambda_{\max} = 4.1$, C.R. = 0.03			

Since C.R. < 0.1, we can consider this comparison is consistent and hence acceptable for being used to present weight information.

Step 2. Precede the pairwise comparison for the second level of attributes in terms of the first level of attributes. If there is some lower level of attributes, we need do the same step until we reach the very bottom level of attribute. The process is same and not present here.

Step 3. Develop a decision matrix based on these pairwise comparisons and compute each alternative's relative importance with respect to each attribute. Since the alternative's value can be obtained from the lower level of attributes, we only need to compute based on the lowest level of attributes. In our example, we have ten lowest attributes. Therefore, we need to develop ten individual pairwise judgment matrices. Table 2.9 shows one of these ten matrices.

Table 2.9 Example of Pairwise Comparison Matrix with Respect to Each Attribute

	With respect to maintainability					
	A_1	A_2	A_3	A_4	A_5	Eigenvector
A_1	1	1/3	1/2	1/2	7	0.140
A_2	3	1	1	2	9	0.343
A_3	2	1	1	1	7	0.259
A_4	2	1/2	1	1	7	0.226
A_5	1/7	1/9	1/7	1/7	1	0.031
	$\lambda_{\max}$ =5.11, C.I. = 0.03, C.R. = 0.03					

If the information is quantitative such as each fighter's top speed, there is no need for pairwise comparison. Instead, the normalization procedures are used (see Section 2.2.1 and 2.2.2).

Step 4. Compute alternatives' cardinal rank scores by synthesizing all attributes' values. For the synthesis, we can use additive or multiplicative function. However, there is no difference between these two methods (see Equation 2.9).

Since a different weight is used for our example in compare to other methods, we do not present the alternative rank scores and rank order.

2.2.5 Utility Theory for Decision Making

Utility theory has its origin on consumer behavior in Microeconomics. The behavior of a consumer in the market (i.e., what choice a consumer makes) is influenced by numerous factors, including individual preference and purchasing power such as budget availability. The relationship between the amount of commodities and/or services that an individual consumes and the satisfaction called utility derived from them can be likened to the relationship between inputs and output in production (Sher and Pinola, 1981). The alternative value function in the utility theory is given by

$$V(A_i) = \sum_j w_j U(x_{ij}), \tag{2.30}$$

where $U(x_{ij})$ is a utility value for an x_{ij}, $0 \le U(x_{ij}) \le 1$, $w_j > 0$, and $\sum w_j = 1$.

Suppose that we need to buy a car from the market and we decide a car in terms of price (in thousands of dollars), comfort and fuel consumption (in miles per gallon), these are called attributes. Let us assume that we have ten alternatives, which means ten car types are available to buy from the market, and three attributes namely price, comfort, and fuel consumption. The detailed data are given as Table 2.10.

Table 2.10 Decision Matrix for Ten Cars (Source: Pomerol et al. 2000)

	Price (K$) X_1	Comfort X_2	Miles per gallon X_3
A_1	70	10	14
A_2	60	9	20
A_3	50	9	18
A_4	45	8	22
A_5	40	9	16
A_6	40	7	14
A_7	30	6	20
A_8	30	7	22
A_9	20	5	24
A_{10}	20	4	26

Note that A_i is the car type i in the market and a comfort is quantified based on a 10-point scale, from 1 (worst) to 10 (best). Utility theorists assume that there is indifference curve and any two points on this curve are indifferent in terms of DM's satisfaction. This indifferent relationship is written such as $(x_i, y_i, z_i) \approx (x_k, y_k, z_k)$, where the parenthesis represents a set of consumption and elements in the parenthesis are value from X_1, X_2, and X_3 respectably. Because of the dimensional restriction for drawing, we only present two-dimensional graphs with two values from X_1 and X_2 in the following figure. This indifference relationship is the core of the utility theory because DMs can determine utility function as well as attribute weights based on this assumption.

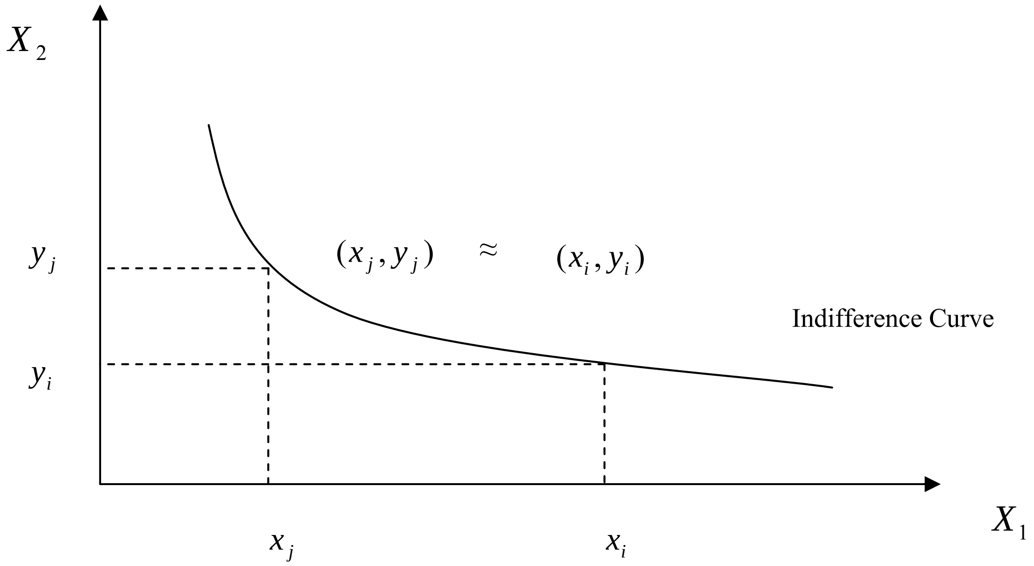


Figure 2.8 Graphic Representation of Indifference Curve

We assume that DMs in the SKN decide attribute weights and alternative values with certainty. In other words, they do not answer such as “I prefer twice A than B with 90% confidence”. Therefore, our problem is deterministic and this figure is for a deterministic problem case. However, if an uncertainty of preferences exists, we need to add a probability to Equation 2.30 (for more detail, see Keeney and Raiffa, 1976).

Like other MADM methods, utility theory depends on experts’ opinion for determining the attribute weight and alternative utility. Pomerol and Romero (2000) show the decision procedure for determining the attribute weights and the value of $U_j(x_{ij})$ given in the previous car selection problem. This procedure is as follows:

Although we are in the discrete case, we can fictitiously consider that we are reasoning under $X = [20,70] \times [4,10] \times [14,26]$ such as the topological assumption. This is equivalent to saying that any triplet of X represents a possible choice. We proceed to the dialogue stage. One protagonist is the analyst (A) and the other the DM.

We start off with $U_1(70) = 0$ and $U_2(20) = 1$, because 70 is the most not preferred value under attribute X_1 while 20 is best on that attribute.

A. What gas per mileage can you assign for this question mark in (45,8,?) to make yourself feel indifferent to (20,7,20) in terms of satisfaction? This question is simplified as "What gas per mileage can you give to make you indifferent between (20,7,20) and (45,8,?)".

DM. I would be indifferent between (20,7,20) and (45,8,26).

A. Where does the price have to lie so that (20,7,20) ≈ (?,8,26) and (?,7,20) ≈ (70,8,26)?

DM. 50.

A. Where does the price lie such that (20,7,20) ≈ (?,8.5,26) and (?,7,20) ≈ (50,8.5,26)?

DM. 40.

A. Where does the price lie such that (50,7,20) ≈ (?,8.5,26) and (?,7,20) ≈ (70,8.5,26)?

DM. 60.

With more intermediate points from this conversation, we can construct the cure of U_1, U_2 and U_3 (see Figure 2.8).

We can note that $V(20,4,14) = w_1U_1(20) + w_2U_2(4) + w_3U_3(14) = w_1$, and likewise $V(70,10,14) = w_2$ and $V(70,4,26) = w_3$. To determine $V(20,4,14)$ we can ask DM what value of comfort (70,?,14) is indifferent to (20,4,14). If the response is 9, this will lead to the equation:

$w_1 = w_2U_2(9)$ or $w_1 = 0.9w_2$, since $U_2(9) = 0.9$ from the Figure 2.8.

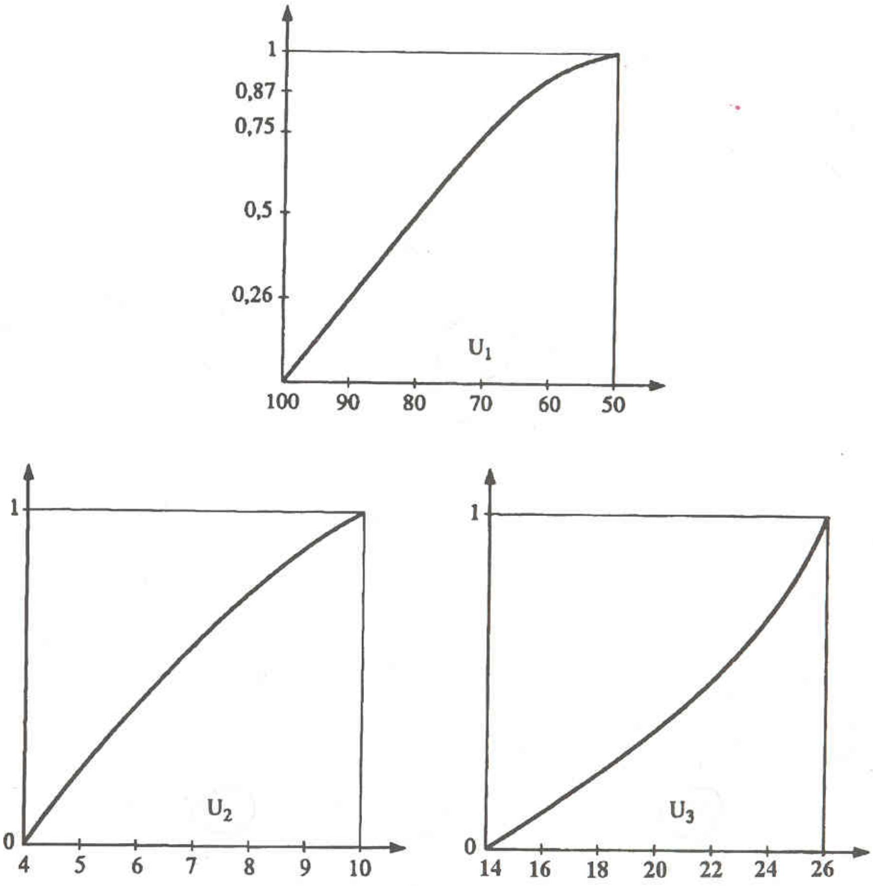


Figure 2.9 Utility Functions for Car Selection Example

(Source: Pomerol and Romero, 2000)

Similarly, we ask the question: for what consumption is (70,4,?) indifferent to (20,4,14)? If the response is 26, this will lead to:

$w_1 = w_3 U_3(26)$ or $w_1 = w_3$.

Finally since we have $w_1 + w_2 + w_3 = 1$, then $w_1 = w_3 = 0.32$ and $w_2 = 0.36$, which completes the determination of V. The final alternatives' rank scores are given as Table 2.11.

Table 2.11 The Final Alternatives' Rank Scores (Source: Pomerol et al. 2000)

	Price	U_1	Comfort	U_2	Fuel consumption (miles/gallon)	U_3	$V(A_i)$
A_1	70	0	10	1	14	0	0.36
A_2	60	0.25	9	0.9	20	0.35	0.516
A_3	50	0.5	9	0.9	18	0.25	0.564
A_4	45	0.62	8	0.75	22	0.5	0.628
A_5	40	0.75	9	0.9	16	0.1	0.596
A_6	40	0.75	7	0.58	24	0.75	0.688
A_7	30	0.9	6	0.4	20	0.35	0.544
A_8	30	0.9	7	0.58	22	0.5	0.656
A_9	20	1	5	0.2	24	0.75	0.632
A_{10}	20	1	4	0	26	1	0.64

Since this procedure requires DMs to determine one value based on all other attribute values are given, DMs must consider all attribute values at the same time. In contrast to this utility theory, the AHP method asks DMs to compare only two attributes at a time. Therefore, this procedure gives lots of cognitive burden to DMs compared to the AHP method especially when the problem size becomes big. Moreover, Bard (1992) shows that there is no significant difference between these two method if and only if the same questions are given to the same DMs. Therefore, we decide not to use the utility theory for attribute weighting decision, but to use AHP method.

2.2.6 The Delphi Method

In the Delphi method, DMs are directly asked about each attribute's relative weight by using questionnaires. An analyst continues surveying until his or her desired variance is achieved. Figure 2.10 shows an example that the second survey has a small variance compared to the first survey.

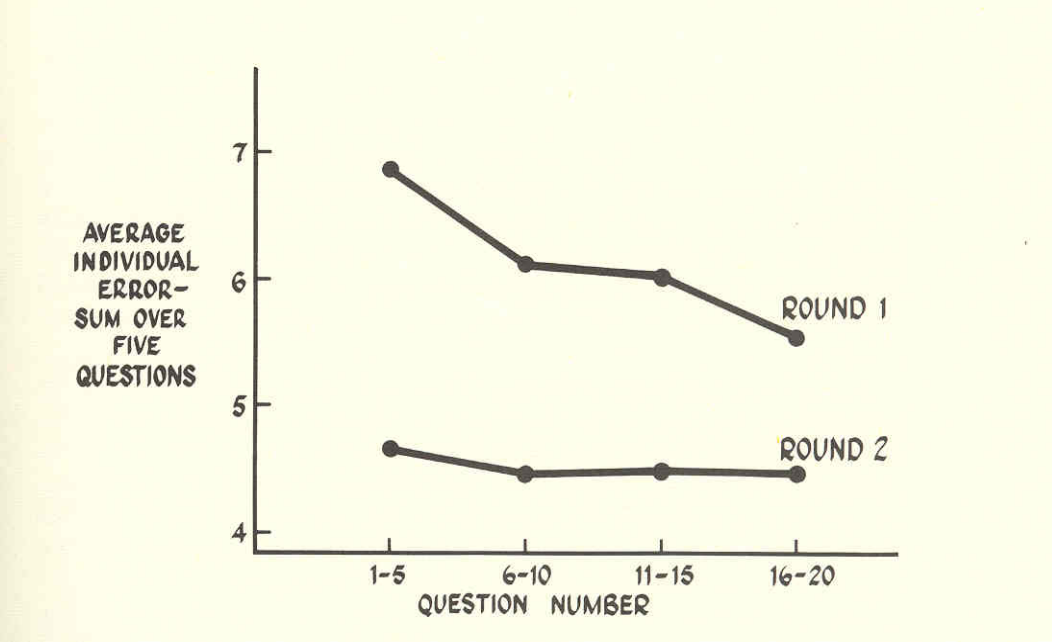


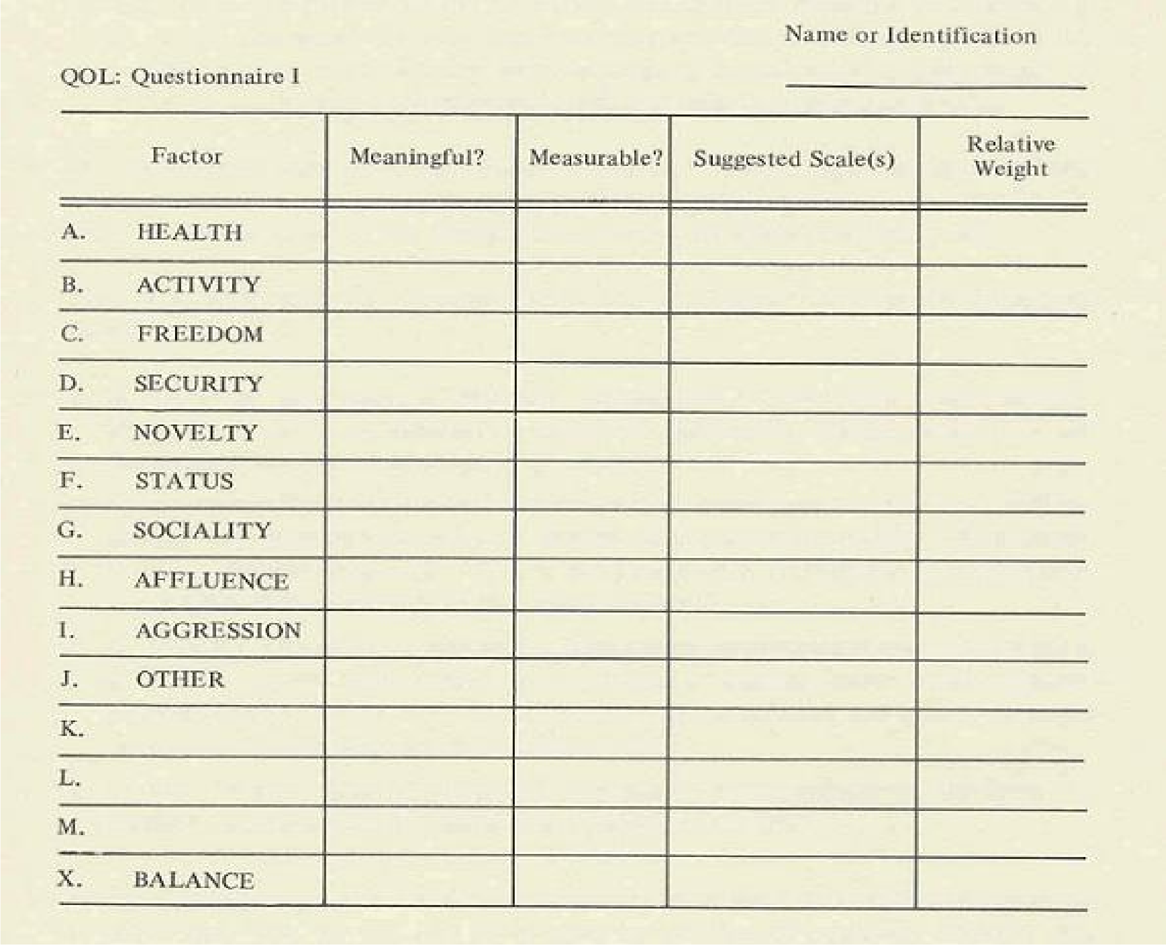
Figure 2.10 Two Variances from the First and Second Round in the Delphi Method

(Source: Dalkey at el. 1972)

Therefore, DMs tend to move to majority opinions regardless of the quality of opinions. Moreover, the quality of results is dependent on the quality of the questionnaires. Table 2.12 shows a sample questionnaire which is used in this method.

The Delphi method repeats questions to the DMs until the errors between answers goes to allowable ranges predefined by analysts. This method shows the survey results as well as the variance of the result. DM may change his or her answer by looking at the common opinions as well as his or her deviation from them. This is the way to decrease errors in the Delphi method. However, by showing other's opinions to DMs, and asking them to answer again can force each DM to move to a common idea. From a statistical perspective, the mean value does not always represent the best information.

Table 2.12 A Sample Questionnaire of the Delphi Method (Source: Dalkey at el. 1972)



Name or Identification

QOL: Questionnaire I

Factor	Meaningful?	Measurable?	Suggested Scale(s)	Relative Weight
A. HEALTH				
B. ACTIVITY				
C. FREEDOM				
D. SECURITY				
E. NOVELTY				
F. STATUS				
G. SOCIALITY				
H. AFFLUENCE				
I. AGGRESSION				
J. OTHER				
K.				
L.				
M.				
X. BALANCE				

Based on these two weaknesses (i.e., dependence on the quality of questionnaire and preference of mean value) of this Delphi method, we decide this method is not proper for our research.

2.3 Research on Similar Problems Related to this Problem

Research in decision making for weapon procurement has been done using two different approaches: weapon selection among several similar candidate weapons and decisions of budget allocation for weapon procurements. The latter approach is an overall and strategic plan which covers all weapons procurements, while the first approach can be classified as a tactical and specific weapon procurement decision. If the latter presents a proposed weapon set for procurement, the first approach suggests only one weapon. Budget allocation typically requires optimization techniques because the solution varies depending on various combination sets of weapons. The determination of one weapon does not require optimization techniques because of the assumption that the candidate weapons are already fit within budget and one type of weapon is required.

Various research efforts in decision making models for both military and non-military areas have been reviewed. Table 2.13 displays the main approaches along with the strengths and weaknesses of these efforts.

Kim (1987) develops a model called Weapon Acquisition Support System (WASS). This model is aimed at helping DMs to decide between developing and buying in terms of weapon procurement. Therefore, we suggest that this model can be placed before our model because our model is aimed at how we can buy a best weapon.

Loerch et al. (1998) use Corps Battle Analyzer (CORBAN), which is an U.S. army combat simulation model. CORBAN develops a response surface model and this model is used as an objective function of an army weapon procurement budget allocation problem. The formulation of their model is summarized as follows: maximize force effectiveness subject to budget ceiling, production limitations, force structure requirements, and other decision constraints. In this case the decision variables are the quantities of each weapon procured in each year. This model assists the army leadership in evaluating and prioritizing competing weapon system alternatives during the process of building the army budget. This model evaluates two different combat scenarios with two terms (i.e., long and short term) and presents an optimal weapon procurement set.

The strength of Loerch et al (1998)'s model is that it provides an optimal weapon combination for the army as they prepare for any combat. Since their model depends on combat simulation, DMs can have confidence in their weapons without actual war testing. However, because their model's emphasis is on combat operations, it does not work well as a generalized weapon procurement decision model. Moreover, their model requires sophisticated combat simulation model which can provide all detailed combat information.

Hall et al. (1992) present a project funding decision model at the National Cancer Institute (NCI). They use the Delphi method to construct a decision structure as well as attribute's weight. The preliminary results select the highest twenty proposals and assigns each a rank score. These eight proposals are added into a decision model as decision variables. Through a maximization model under specified constraints, DMs select proposals to be funded. Like other decision making problems, NCI is subject to some political pressures. Therefore, a preference function is developed so that decision makers' preferences can function as a constraint.

Brauers (2001) develops a fighter decision model for Belgian Air force. His model is categorized as a MADM method. His model has five attributes (i.e., opinion of the Secretary of Defense, an increase of employment, deficit of balance of payments, a fighter price, and risk related with contraction. He uses Delphi method for developing attributes and weights.

Table 2.13 Summary of Similar Researches

Method	Area	Research (Author, Year)	Main approaches	Strengths	Weaknesses
MODM	Military	Loerch et al. (1998)	-Allocating army budget for weapon procurement - Response surface model by using combat simulation - Use as an objective function - Solve optimization problem for a budget allocation for army weapon procurements	-Improvement of a combat performance based on the best weapons combination	- Need a sophisticate combat simulation model which can give DMs detailed data - Not enough to be a general decision model
	Non-military	Hall et al. (1992)	- NCI funds allocation decision (funds for reducing smoking rate researches) - Develop a rank function of candidate researches based on quality of that - Applied as an objective function in maximization model	- Compensate qualities of researches with DMs preferences	-Atypical MODM process therefore it is not proper to be used for our research
MADM	Military	Brauers (2001)	- Rankings of candidate fighters for Belgium Air force - Use a multiplicative form as an alternative value function	- Exclude a normalization procedure	- Since additive function is no different from multiplicative form, his approach can be modified more easy
	Non-military	Dyer (1990)	- Analyze flaw of AHP method - Focus on AHP's arbitrary ranking method	- Propose a possible solution for AHP method (Applied to our research)	- Propose a possible solution without any detailed procedures
		Bard (1992)	- A comparison between AHP and Utility theory	- Presented with a numerical problem	- An analysis based on one example - Not enough to be used as an generalized comparison
		Hughes (1986), Saaty (1986), Rahman (2003)	- Some approaches to develop a GLM by using MADM approaches	- Possible to reduce a computational requirement of MODM	- More applicable budget allocating types of problems than our problem definition

In MADM method, each alternative is assigned values with respect to all attributes which can be obtained by summation of all attribute values. This function is called an alternative value function. Since each attribute is different from other attributes in terms of its relative importance, the value function has a form of product sum of each attribute score and the attribute's weight such as Equation 2.6.

Brauers presents multiplicative form of alternative value function given by

$$V(A_i) = \frac{B_i}{C_i} \tag{2.31}$$

with

$$B_i = \prod_g \left(\alpha_g + x_{ij}\right)^{\varepsilon g},$$

α_g = DMs preference for attribute g,

εg = weight of attribute g,

B_i = sum of benefit attribute value of alternative i,

and

$$C_i = \prod_k \left(x_{ik} - \alpha_k\right)^{\varepsilon k},$$

C_i = sum of cost attribute value of alternative i.

This model was seen to be an improvement, since normalization is not required. However, this normalization step is less cumbersome with computer advancements and its omission is not suggested. By using normalization procedure, Brauers' model can be simplified to Equation 2.6. Let us show the simplification procedures as follows. In addition to the reasoning of Equation 2.9, we suggest that there is no reason to use two different types of weights for one attribute. Two different weights can be combined as one by some a proper method such as AHP. Braurers's model has two weights for one attribute, therefore, each benefit and cost value function can be changed into a form given by

$$B_i = \prod_g \left(x_{ij}\right)^{w_g} . \tag{2.32}$$

$$C_i = \prod_k \left(x_{ik}\right)^{w_k} . \tag{2.33}$$

Through normalization, we do not need to divide into two equations (i.e., benefits and costs attributes). By this reason, we can modify Braurers' alternative value function, which is Equation 2.32, into a form as

$$V(A_i) = \prod_j (x_{ij})^{w_j} \tag{2.34}$$

By the reasoning of Equation 2.9, the Equation 2.31 has a form of Equation 2.6.

In addition to the computational complexity of Braurers' model, this model only allows five attributes, which is not sufficient for all the important factors in weapon procurement decision making. Moreover, his model does not consider extreme alternatives which can be covered in our model.

Dyer (1990) reviews the AHP method and shows this method's weakness of rank reversal by using an example problem. A rank reversal may occur when a new alternative is added or deleted from the candidate list; we discuss this problem in more detail in Chapter 3.

He states that the fundamental problem of the AHP method is its subjective approach from DMs. In other words, attribute weights and alternatives' ranking are dependent on DMs' subjective opinions.

Dyer proposes two possible methods to avoid this rank reversal problem: using a absolute measurement and using an empirical method such as utility theory. Since utility theory has computational and cognitive difficulty when a problem size becomes large, we suggest using his first suggestion. This is applied into our research as an indexation procedure which is a categorization of each attribute; we explain this indexation procedure for more detail in Chapter 4. Additionally, Dyer also does not consider any extreme alternatives. Chapter 3 presents the definition of the extreme alternative as well as our solution for that.

Bard (1992) compares AHP method and utility theory by using an example problem. He uses the exact same approach presented before for both AHP and utility theory. The example problem is composed of twelve attributes and three alternatives. By chance, the results of AHP and utility theory give almost the same ranking scores. Theoretically speaking, the AHP weights and the MAUT (multi-attribute utility theory) scaling constants measure different phenomena, and hence, cannot be given the same interpretation (Kamenentzky, 1982).

Bard analyzes the reason for the same solutions as follows: the same questions are given to each DM for both AHP and utility theory, therefore their responses would be the same even if they are answering for two different question types (i.e., pairwise comparisons for AHP method, and determining indifferent score of each attributes based on other attribute are given as maximum values for utility theory).

Based on Bard's analysis, we decided to use AHP method for assigning attribute weights even though this method has a possible ranking reversal weakness. Both utility theory and AHP rely on experts' opinion. However, the AHP method is for both computational and cognitive simpler for DMs. Due to weakness of the AHP, we should only use AHP method to decide attributes weights. For the alternative ranking scores, we will use an indexation procedure which is an absolute measurement.

Even if MADM methods are not proper for MODM problems, these MADM methods can help MODM problems become easier to solve. Hughes (1986), Saaty (1986), and Rahman (2003) present some approaches to develop a GLM (general linear model) for MODM problem by using MADM approaches. Their approaches depend on experts or DMs opinions which are also used to decide attribute weights. Therefore, DMs can avoid analyzing empirical data to figure out the coefficient of each decision variable called as alternatives in MADM problems.

2.4 Summary

In Chapter 2, we have shown various MADM methods as well as weighting methods. Even though, these weighting methods are classified as MADM methods, since these methods are applied into our research only for defining weights, we define these methods as weighting methods. In this chapter, we summarize this literature review in two ways: one is for MADM methods and the other is for the weighting methods.

The following table provides comparisons within MADM methodologies in terms of strengths and weaknesses. From this table, we identify that the alternative ranking order is different between the SAW method and other two methods. We suggest this difference occurs because the SAW method uses overall value of each alternative, while the other two methods use different information (i.e., TOPSIS use the distance information and ELECTRE uses outranking information). However, none of these methods can be used without a weighting method because they require weight information.

We provide three different weighting methods and the Table 2.14 and 2.15 provide comparisons among these methods in terms of advantages and disadvantages.

Table 2.14 Comparison of MADM Methods

Method	Rank order (from example problem)	Strengths	Weaknesses
SAW	A_3, A_2, A_4, A_1, A_5	- A Simple calculation procedure - Present overall alternatives' rank scores	- Individual data information is not given by this method
TOPSIS	A_3, A_4, A_1, A_5, A_2	- Variance information is available by using two ideal solutions - Information how close to the ideal solution	- Overall rank scores are not given by this method - When a $C_i^* < 0.5$, this method can choose an alternative which has an overall good score but individually bad scores
ELECTRE	A_3, A_4, A_1	- Transitive assumption is not required	-Do not present all alternatives' rank scores

As Bard's (1992) suggestion, we assume that there is no difference among these three methods in terms of weighting information. However, the AHP method can give the smallest cognitive burdens to a DM. With this merit, AHP method is applied for a various decision making procedure specially when there are large number of qualitative attributes. A weapon procurement decision making contains a large amounts of qualitative attributes, therefore, AHP method is used for determining attribute weights.

The one thing that we have not found in this literature review is a consideration of extreme alternatives. An extreme alternative is defined as an alternative which has an overall good score but some poor individual scores. In other words, this alternative may have a good overall score due to some extremely high scores in some attributes and in other attributes it has a very low score. The navy does not want this type of weapon. Therefore we introduce a new alternative ranking method which can compensate alternative's overall ranking scores and individual attribute's values. This new method is explained in detail in Chapter 3.

Table 2.15 Comparisons Within Several Weighting Methods

Methods	Descriptions	Strengths	Weaknesses
Utility theory	- Empirical modeling procedure - Also use experts' opinion for a qualitative problem structure	- Can present a preference function called utility function - An utility function can be used as an objective function in MODM environment	- When a problem size becomes big, this method gives more cognitive burden than AHP method - For that reason, right decision is more challenge than AHP method
Delphi technique	- Use experts opinion with several times of interviewing or surveying - Gives DM a chance to see what other DMs opinions are and how his/her opinion is different from them	- Relatively convenient than utility theory because an analyst dose not need any conditional types of question used in utility theory - Good quality of results	- Asking several times for the same problem by showing other DMs opinions can forces DMs to move into median or mean values which does not need to be best solution
AHP method	- Pairwise comparison is used - Eigenvector and eigenvalue approach are used	- By using pairwise comparison, this method has less cognitive burdens than other two methods - By using consistence index, any irrational response can be filtered to determine weights	- Still subjective as other two methods because this method is also dependents of experts opinion

Another thing that we have found in this literature review is that there is no general decision model for weapon procurement. Based on these two things that we have found our model can be defined as uniqueness as opposed to any other MADM methods.

Chapter Three

Problem Statement and Methodology

3.1 Introduction

There are a given number of alternative weapon systems, from which one weapon system will be selected to procure, based on a given set of attributes. The problem related to weapon procurement is complex. A MADM model is developed for naval weapon procurement decision. This model is expected to give DMs a better weapon selection. There are several MADM methods but none of them provides a solution in terms of compensating individual values for an overall value. This research suggests a new MADM method which can provide an alternative ranking score by compensating these two values. We also provide a sensitivity analysis to the solutions obtained by the proposed model.

3.2 Problem Statement

An extreme alternative is defined as an alternative which has an overall good score but some poor individual scores. In other words, this alternative may have a good score due to some extremely high scores in some attributes and in other attributes it has very low scores. For example, let us consider two alternatives A_1 and A_2, where A_1 is considered as an extreme alternative. Table 3.1 presents attributes values for these two alternatives.

Table 3.1 Example for an Extreme Alternative

Attributes		X_1	X_2	X_3	X_4	X_5	
Weights (w_j)		0.6	0.1	0.1	0.1	0.1	$\sum w_j = 1$
Alternatives	A_1	1000	1	1	1	1	
	A_2	200	200	200	200	200	

Note that in this example, all attributes are assumed to be benefit attributes. For candidate weapons given in Table 3.1, SKN probably does not want to procure A_1. As we described in the previous chapter, there are several MADM methods designed to determine alternative rank scores. However, there is no MADM method which can consider both an alternative's overall rank score and individual attribute values. Most MADM methods would select A_1 to be the best alternative, which will be seen in Section 3.3. This ranking result is evidently not appropriate as alternative A_2 is clearly better than A_1. Therefore, the development of a new method that can deal with such extreme alternatives is well justified.

3.3 Best Selection Method (BSM)

Our new MADM method called BSM can present an alternative rank score by compensating overall rank score for individual attribute values. Therefore, DMs can avoid selecting an extreme alternative which is bad. To be able to compensate an overall score for individual attribute values, we need to have two types of value functions: one is for an overall value and the other is for an individual value.

From a statistical point of view, there are two types of information that we can use for overall and individual scores: mean and variance. The mean is computed by

$$\bar{x} = \frac{1}{n}\sum x_i \,, \tag{3.1}$$

where $\bar{x}$ is the mean of random variable X from n samples. The sample variance is generally computed by

$$s^2 = \frac{1}{n-1}\sum \left(x_i - \bar{x}\right)^2 . \tag{3.2}$$

Let us compare mean and the SAW method first. If we define that $x_j = w_j r_{ij}$, then mathematical term of SAW method is given as Equation 2.6. When we multiple by $1/m$ on both sides, then Equation 2.6 can be written as

$$\frac{1}{m}V(A_i) = \frac{1}{m}\sum_{j=1}^{m} x_j \,. \tag{3.3}$$

Since we assume that all alternatives should be considered with respect to all the attributes of a problem structure, multiplication by $1/m$ does not affect the final rank score in SAW method. By this reasoning, we can rewrite Equation 3.3 as

$$V(A_i) = \frac{1}{m}\sum_{j=1}^{m} x_j \,. \tag{3.4}$$

Equation 3.4 has the same mathematical term as mean given by (3.1). Therefore, we can use SAW rank scores to represent mean information.

In comparison between TOPSIS and variance, we note that both equations have same function (i.e., sum of individual data point from a constant value): $\bar{x}$, v_j^*, and v_j^- are constant. Therefore, we can say that TOPSIS is an arithmetic combination of variances. We also know that arithmetic combination of variance is also variance. Therefore, we can use TOPSIS as information of variance of alternatives.

However, TOPSIS is a non-linear function and this causes mathematical computational difficulty. Therefore, an effort making a linear function is required. Modification to a linear function can be accomplished by defining new ideal solutions: called natural positive solution $\overline{1}$ and natural negative solution $\overline{0}$.

In TOPSIS, the ranking score of an alternative is close to 1 if it is close to the positive ideal solution A^*, and close to 0 if it is close to A^-, as computed by Equation 2.17. However, if we replace our two solutions for TOPSIS's ideal solutions, Equation 2.17 can be simplified as a linear function given by

$$V(A_i) = \sum_{j=1}^{m} \left| 1 - r_{ij} \right| . \tag{3.5}$$

This modification is possible because of the following reasons:

1. Since r_{ij} ranges from 0 to 1, we can define two new ideal solutions as $\overline{1}$ and $\overline{0}$ instead of A^* and A^-.
2. Because $\overline{1}$ is a replacement for a positive ideal solution, by minimizing the deviation from $\overline{1}$ we can choose an alternative which has maximum closeness to the positive ideal solution which is the same as the TOPSIS.

3. Maximizing the deviation from the negative ideal solution is the same as minimizing the deviation from the positive ideal solution, and hence we do not need to use two deviation terms used in TOPSIS.

For Equation 3.5, there is an absolute value term. However, this term causes mathematical computational difficulty and hence should be removed. Since $r_{ij} - 1 \leq 0$ for all i and j, $1 - r_{ij} \geq 0$. Therefore, Equation 3.5 can be rewritten as

$$V(A_i) = \sum_{j=1}^{m} \left(1 - r_{ij}\right). \tag{3.6}$$

From Equation 3.6, we notify that as $A_i \to \overline{1}, V(A_i) \to 0$, and as $A_i \to \overline{0}, V(A_i) \to m$. Up to now, it is defined that $V(A_i)$=1 as the best, and $V(A_i)$=0 as the worst. In addition,

$$\min \sum_{j=1}^{m} (1 - r_{ij}) = \min\left(m - \sum_{j=1}^{m} r_{ij} \right) = m + \min\left[- \sum_{j=1}^{m} r_{ij} \right] = m - \max \sum_{j=1}^{m} r_{ij} .$$

That is, $\min \sum_{j=1}^{m} (1 - r_{ij})$ is equivalent to $\max \sum_{j=1}^{m} r_{ij}$. Therefore, we modify Equation 3.6 as

$$V(A_i) = \frac{1}{m} \sum_{j=1}^{m} \left(r_{ij}\right). \tag{3.7}$$

Now Equation 3.7 can present an alternative ranking scores such that when $A_i \to \overline{1}, V(A_i) \to 1$, and when $A_i \to \overline{0}, V(A_i) \to 0$. We define Equation 3.7 as an individual value function (IV).

In addition to simplifying the TOPSIS, there is one weakness in this method for addressing an extreme alternative: TOPSIS uses v_{ij} (see Equation 2.12) and this value is not proper to measure how an alternative has individually good attribute values. For example, let us consider the same example shown in Table 3.1. The following table presents both r_{ij} and v_{ij} for this example.

Table 3.2 r_{ij} and v_{ij} for the Extreme Alternative Problem

Attributes		X_1	X_2	X_3	X_4	X_5	
Weights (w_j)		0.6	0.1	0.1	0.1	0.1	$\sum w_j = 1$
A_1	x_{1j}	1000	1	1	1	1	
A_2	x_{2j}	200	200	200	200	200	
A_1	r_{1j}	1	0.005	0.005	0.005	0.005	$\sum r_{1j}$ =1.02
A_2	r_{2j}	0.2	1	1	1	1	$\sum r_{2j}$ =4.20
A_1	v_{1j}	0.6	0.0005	0.0005	0.0005	0.0005	$\sum v_{1j}$ =0.602
A_2	v_{2j}	0.12	0.1	0.1	0.1	0.1	$\sum v_{2j}$ =0.52

From this table, one can see that A_2 has individually better attribute values than A_1, for 4 of 5 attributes. However, since $(v_{11} - v_1^-)^2$ >> $(v_{2j} - v_j^-)^2$ and $(v_{11} - v_1^*)^2$ << $(v_{2j} - v_j^*)^2$ for $j = 2 \sim 5$, the TOPSIS presents A_1 as a better alternative and this is not a proper solution in terms of individual attribute values. The detailed results are presented in Table 3.4. In contrast to the TOPSIS, the IV uses r_{ij} and hence can present A_2 as a better alternative in terms of individual attribute values. This is computed as

$$V(A_1) = \frac{1}{5}\sum_{j=1}^{5} r_{1j} = \frac{1.02}{5} = 0.204,$$

$$V(A_2) = \frac{1}{5}\sum_{j=1}^{5} r_{2j} = \frac{4.2}{5} = 0.84.$$

As we can see, A_2 has higher rank score than A_1. Therefore, using r_{ij} is more proper than using v_{ij} to measure how individually good attribute values.

By compensating mean type of value for variance type of value, we can compensate an alternative's overall ranking score for individual attribute values. As explained earlier in this section, we use the value from the SAW method representing an alternative's overall ranking score. For the variance type of information, we use our new function of IV.

To compensate two different values, each value must have same scale factors. For example, if one value ranges from 0 to 100 while the other value ranges 1 to 2, the former value absorbs the latter value. The SAW method ranges from 0 to 1: since $\sum_{j=1}^{m} w_j = 1$ and Max (r_{ij}) =1, therefore, $V(A_i) = \sum_{j=1}^{m} w_j r_{ij}$ ranges from 0 to 1. And we already showed that IV ranges from 0 to 1.

Since these two values (i.e., the SAW function and IV) can be added, the sum of these two values can present information of compensating an alternative's overall score and individual attribute values. In BSM, the alternative ranking score is given by

$$V(A_i) = \frac{1}{2}\sum_{j=1}^{m}\left[\left(w_j + \frac{1}{m}\right) r_{ij}\right]. \tag{3.8}$$

The ranking score from the BSM also ranges from 0 to 1.

3.3.1 A Numerical Example

From Table 3.2 and the BSM ranking function, we can compute an alternative ranking score shown in Table 3.3. The value of A_1 is computed as

$$V(A_1) = \frac{1}{2}\left(\sum_{j=1}^{5} w_j r_{ij} + \frac{1}{5}\sum_{j=1}^{5} r_{ij} \right)$$

$$= \frac{1}{2}\left(0.602 + \frac{1.02}{5} \right) = \frac{1}{2}(0.602 + 0.204)\text{=0.403, likewise } V(A_2) = 0.680\,.$$

Table 3.3 Ranking Scores for the Extreme Alternative Problem

Alternatives	Values from the SAW method	IV values	BSM ranking scores	Ranking
A_1	0.602	0.204	0.403	2
A_2	0.520	0.840	0.680	1

In this example, A_1 can have better score in terms of an overall value that is given by the SAW method while individually bad scores presented by the IV. In other words, even though A_2 has lower overall score by SAW, since this alternative has individually good attribute values, A_2 can be ranked first in our method. Therefore, we can avoid selecting an extreme alternative.

3.3.2 Comparison with Current MADM Methods

The most common MADM methods are the SAW, TOPSIS, and ELECTRE methods. However, as explained in the previous chapter, the ELECTRE method does not provide all alternatives' ranking scores. For this reason, in this research we compare the BSM with the SAW and TOPSIS methods.

First let us consider the extreme alternative case which is shown in previous section. Table 3.4 shows each ranking scores as well as ranking orders by each method.

Table 3.4 Ranking Scores for the Example from the Three MADM Methods

Alternatives	SAW	Ranking	TOPSIS	Ranking	BSM	Ranking
A_1	0.602	1	0.707	1	0.403	2
A_2	0.520	2	0.293	2	0.680	1

From this example, the SAW and TOPSIS are shown to be inappropriate in the preference of an extreme alternative.

Table 3.5 Ranking Scores for the Fighter Selection Problem

Alternative	ASW	Rank	TOPSIS	Rank	BSM			Rank
					ASW	IV	BSM	
A_1	0.840	2	0.361	4	0.840	0.868	0.854	2
A_2	0.827	5	0.414	2	0.827	0.820	0.823	5
A_3	0.895	1	0.607	1	0.865	0.873	0.883	1
A_4	0.840	2	0.396	3	0.840	0.844	0.842	4
A_5	0.832	4	0.349	5	0.832	0.857	0.845	3

Let us consider another example problem shown in Table 2.1 which does not include any extreme alternatives. Table 3.5 contains all five alternatives ranking scores as well as ranking information by SAW, TOPSIS and BSM. From this example, all three methods present A_3 as the best alternative because this alternative has an overall good score as well as individual good attribute values.

3.4 Sensitivity Analysis

DMs may want to see if the ranking order will change if some attribute value changes. For instance, in the previous fighter example A_2 is ranked second. If one of the ten attributes values for this fighter changes, this alternative may improve its rank from the second to the first.

Sensitivity analysis is concerned with how outcomes change when inputs changes. In this research, the sensitivity analysis is defined as follows: how changes in an attribute's value affect the current ranking. To do this, it is necessary to consider the proposed alternative value function (henceforth called as BSM function) presented in the previous section.

The alternative ranking is determined in terms of each alternative's ranking value calculated by Equation 3.8. Therefore, if we calculate the difference between any two alternatives' ranking values, this difference can be considered as a critical value for the case of a ranking change with each alternative. For example, let us consider any two alternatives, A_i and A_k, which has a ranking relationship of $V(A_i) > V(A_k)$. This relationship is denoted by

$$V(A_i) - V(A_k) = \frac{1}{2}\left\{\left(\sum_{j=1}^{m} w_j r_{ij} + \frac{1}{m}\sum_{j=1}^{m} r_{ij}\right) - \left(\sum_{j=1}^{m} w_j r_{kj} + \frac{1}{m}\sum_{j=1}^{m} r_{kj}\right)\right\} > 0 \qquad (3.9)$$

From this example, we can consider two possible critical values: changes in A_i such that $V(A_i) = V(A_k)$, and changes in A_k such that $V(A_i) = V(A_k)$.

To be able to process the sensitivity analysis, we change one attribute's value of some alternative at a time and let all other attribute values remain the same. To be able to change the ranking of A_i and A_k, the value of $V(A_i) - V(A_k)$ should be at least zero and this is given by

$$V(A_i) - V(A_k) = \frac{1}{2}\left\{\left(\sum_{j=1}^{m} w_j r_{ij} + \frac{1}{m}\sum_{j=1}^{m} r_{ij}\right) - \left(\sum_{j=1}^{m} w_j r_{kj} + \frac{1}{m}\sum_{j=1}^{m} r_{kj}\right)\right\} = 0. \qquad (3.10)$$

Let r_{kl}^c be the critical attribute value such that when the current r_{kl} is changed to r_{kl}^c, A_i and A_k will have the same rank. Since all other attributes' values remain the same, the critical value r_{kl}^c can be computed by

$$V(A_i) - \frac{1}{2}\left(\sum_{j \neq l}^{m} w_j r_{kj} + \frac{1}{m}\sum_{j \neq l}^{m} r_{kj}\right) - \frac{1}{2}\left(w_l r_{kl}^c + \frac{1}{m} r_{kl}^c\right) = 0,$$

and when we solve this equation for r_{kl}^c, then

$$r_{kl}^c = \frac{m}{m w_l + 1}\left\{2V(A_i) - \left(\sum_{j \neq l}^{m} w_j r_{kj} + \frac{1}{m}\sum_{j \neq l}^{m} r_{kj}\right)\right\}. \qquad (3.11)$$

If we are interested in a critical value in A_i called r_{il}^c, it is easily computed by switching A_i and A_k.

3.4.1 Current Basis and Allowable Ranges

Since r_{ij} ranges from 0 to 1, r_{kl}^{c} is valid only within this range. Therefore, the allowable range for r_{kl}^{c} is defined as

$$0 \leq r_{kl}^{c} \leq 1. \qquad (3.12)$$

However, since r_{ij} is computed as the proportion between x_{ij} and x_{j}^{*} or x_{j}^{-}, if the value of x_{j}^{*} and x_{j}^{-} are changed, all r_{ij} has to be computed again (see Equation 2.3). Therefore, to be able to remain the current ranking scores valid, we need to make sure that x_{j}^{*} and x_{j}^{-} are not changed. Therefore, the current basis can be defined as

$$\text{Set}\{r_{ij} = 1 \text{ and } r_{kj} < 1 \text{ for } \forall k \text{ that } k \neq i\}. \qquad (3.13)$$

For example, for the fighter example, the current basis can be defined by (3.13) as follows:

$$\text{Current basis} = \{r_{311}, r_{113}, r_{314}, r_{314}, r_{222}, r_{331}, r_{132}, r_{141}, \text{and } r_{542}\}.$$

If critical values fall in the range given by (3.12), the ranking change within the two alternatives is possible. On the other hand, the case of out of range does not provide a chance of ranking change by sensitivity analysis. Based on the critical values and the allowable ranges, the sensitivity analysis of this research provides two possible scenarios for any two alternatives A_i and A_k.

Scenario 1 (changes in A_k). If $V(A_i) > V(A_k)$, then there are three cases:

1. $r_{kl}^c \in [0,1]$. Then $V(A_i) > V(A_k)$ for all $r_{kl} \in [0, r_{kl}^c)$, and $V(A_i) < V(A_k)$ for all $r_{kl} \in (r_{kl}^c, 1]$.
2. $r_{kl}^c \notin [0,1]$. Then sensitivity analysis does not apply.
3. $r_{kl} \in$ Current basis. Then sensitivity analysis does not apply.

Scenario 2 (changes in A_i). If $V(A_i) > V(A_k)$, then there are three cases:

1. $r_{il}^c \in [0,1]$. Then $V(A_i) < V(A_k)$ for all $r_{il} \in [0, r_{il}^c)$, and $V(A_i) > V(A_k)$ for all $r_{il} \in (r_{il}^c, 1]$.
2. $r_{il}^c \notin [0,1]$. Then sensitivity analysis does not apply.
3. $r_{il} \in$ Current basis. Then sensitivity analysis does not apply.

The next section provides a numerical example of this sensitivity analysis.

3.4.2 A Numerical Example

Let us consider the fighter selection problem. Table 3.6 shows the alternative ranking values and ranking orders.

Table 3.6 The Alternative Values and Ranking Information

Alternative	BSM ranking scores	Rank
A_1	0.854	2
A_2	0.823	5
A_3	0.883	1
A_4	0.842	4
A_5	0.845	3

Recall the normalized decision matrix used to calculate each alternative ranking score.

Weight: 0.2 0.04 0.04 0.12 0.09 0.21 0.12 0.08 0.06 0.04

$$\begin{array}{c|cccccccccc|}
 & \mathrm{X}_{11} & \mathrm{X}_{12} & \mathrm{X}_{13} & X_{14} & X_{21} & X_{22} & X_{31} & X_{32} & \mathrm{X}_{41} & \mathrm{X}_{42} \\
A_1 & 0.80 & 1 & 1 & 0.543 & 0.778 & 0.889 & 0.889 & 1 & 1.00 & 0.78 \\
A_2 & 0.80 & 0.83 & 0.87 & 0.571 & 0.889 & 1 & 0.778 & 0.778 & 0.90 & 0.78 \\
A_3 & 1 & 1 & 0.78 & 1 & 0.889 & 0.778 & 1 & 0.889 & 0.69 & 0.70 \\
A_4 & 0.80 & 0.83 & 0.87 & 0.686 & 1 & 0.889 & 0.889 & 0.778 & 0.82 & 0.88 \\
A_5 & 0.72 & 0.83 & 0.91 & 0.657 & 1 & 0.889 & 0.889 & 0.778 & 0.90 & 1
\end{array}$$

From this matrix, the current basis is defined as previous, i.e., current basis equals to $\{r_{311}, r_{113}, r_{314}, r_{314}, r_{222}, r_{331}, r_{132}, r_{141}, \text{and } r_{542}\}$. Sensitivity Analysis for r_{3j} in terms of $V(A_3) = V(A_i)$ is as follows. Since DMs are only interested in the first ranked alternative (i.e, A_3), every alternative has to be compared with respect to the first ordered alternative. For this reason, the considerable change in A_3 is loosing its first ranking order. If A_3 loose its first order position, the only possible alternative which can be ranked first is A_2. In other words, intermediate ranking orders are not important to DMs. Therefore, only A_1 and A_3 are considered for the sensitivity analysis of r_{3j}.

Let us solve each critical value of r_{3j}^c which provide the condition that $V(A_3) = V(A_2)$. r_{312}^c for $V(A_3) = V(A_1)$ is computed by Equation 3.11.

$$r_{312}^c = \frac{10}{10w_{12}+1}\left\{2V(A_1) - \left(\sum_{j\neq 12} w_j r_{3j} + \frac{1}{10}\sum_{j\neq 12} r_{3j}\right)\right\}$$

$$= \frac{10}{0.4+1}\{2(0.854) - (0.855 + 0.773)\} = 0.574$$

Since r_{312}^{c} = 0.574, if $r_{312} \in [0,0.574)$, then $V(A_3) < V(A_1)$, and if $r_{312} \in (0.574,1]$, then $V(A_3) > V(A_1)$. The critical value of x_{ij}^{c} can be computed by solving Equation 2.3 for x_{ij}^{c}, that is given by

$$x_{ij}^{c} = \begin{cases} r_{ij}^{c} * x_j^{*}, & \text{when } x_{ij} \text{ is a value for benefit attribute} \\ \\ r_{ij}^{c} * x_j^{-}, & \text{when } x_{ij} \text{ is a value for cost attribute} \end{cases} \tag{3.14}$$

By Equation 3.14, the critical value of x_{312} (denoted as x_{312}^{c}) can be computed as

$x_{312}^{c} = r_{312}^{c} * x_{12}^{*}$ = 0.574 * 60 = 34.44 (1,000 ft).

If A_3 has more than 34,440 ft of operating altitude, this alternative can keep its first rank order. In other words, if A_3 has its operating altitude less than 34,440 ft, then this alternative can loose its first rank order, and A_1 can be ranked first.

Through the same procedure, the rest r_{3j}^{c} are computed as in the following table.

Table 3.7 Critical Values for A_3 in terms of $V(A_3) = V(A_2)$

	Attributes (j)									
	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{31}	X_{32}	X_{41}	X_{42}
r_{3j}^{c}	c.b.	0.574	0.354	c.b.	0.575	0.585	c.b.	0.557	0.317	0.274

Note that c.b. denotes that $r_{ij} \in$ Current basis and hence no sensitivity analysis is applied for this value.

Sensitivity analysis for A_1, A_2, A_4 and A_5 in terms of $V(A_3) = V(A_i)$ is as follows. Since the only first rank order can be chosen, it is reasonable to consider the case of being improved as rank one. For this reason, A_1, A_2, A_4 and A_5 should be considered in terms of $V(A_3) = V(A_i)$. Table 3.8 shows critical values for A_1, A_2, A_4 and A_5 in terms of $V(A_3) = V(A_i)$.

Table 3.8 Critical Values for A_1, A_2, A_4 and A_5 in terms of $V(A_3) = V(A_i)$

	Attributes (j)									
	X_{11}	X_{12}	X_{13}	X_{14}	X_{21}	X_{22}	X_{31}	X_{32}	X_{41}	X_{42}
r_{1j}^c	1.002	1.433	c.b.	0.818	1.097	1.084	1.164	c.b.	c.b.	1.211
r_{2j}^c	1.203	1.693	1.733	1.12	1.525	c.b.	1.327	1.449	1.655	1.643
r_{4j}^c	1.077	1.424	1.464	1.064	1.438	1.157	1.267	1.240	1.340	1.474
r_{5j}^c	0.9812	1.390	1.470	1.013	1.412	1.142	1.245	1.121	1.390	c.b.

From Table 3.8, it is shown that A_1 can be ranked first if r_{114} can be increased more than 0.818 (this alternative has current value of 0.543). However, the results show that the sensitivity analysis does not apply in many r_{ij} for this example.

3.5 Summary

In this chapter, we present a new ranking score method called BSM with a sensitivity analysis. By introducing this method, we expect that DMs can choose an alternative which has an overall good ranking score as well as individual good attribute values. In addition to this method, the provided sensitivity analysis can give what if analysis for both SKN and weapon companies.

Chapter Four

Construction of A Hierarchical Structure

4.1 Introduction

Like Saaty and Vargas's (2000) suggestion, the most creative task in making a decision is to determine what factors to include in a hierarchical problem structure. SKN follows NR 2 for a weapon procurement decision. However, there is no problem structure which can be used for analytic decision model. Therefore, there must be a work for developing a problem structure which can be used for an analytic decision model.

NR 2 contains five principles for weapon procurement decision and these are shown in the following table.

Table 4.1 Principles of Weapon Procurement (Source: NR 2)

Principles	Explanation
Operational Performance	Maintain the level of performance to meet the operational requirements
Readiness on Time	Weapons should be ready on time for a specific purpose
Technical Merits	Try to get technologies from weapon procurement if it is come from other countries. Give some priority for the domestic products
Cost Effectiveness	Acquire the best products with the lowest price
Sustainment	Keep the required performance within the life

From Table 4.1, we can construct hierarchical problem structure with five attributes. This problem structure is graphically displayed in Figure 4.1. In this figure, we can see that there are five level 1 attributes which are defined as “Principles for weapon procurement” by NR 2.

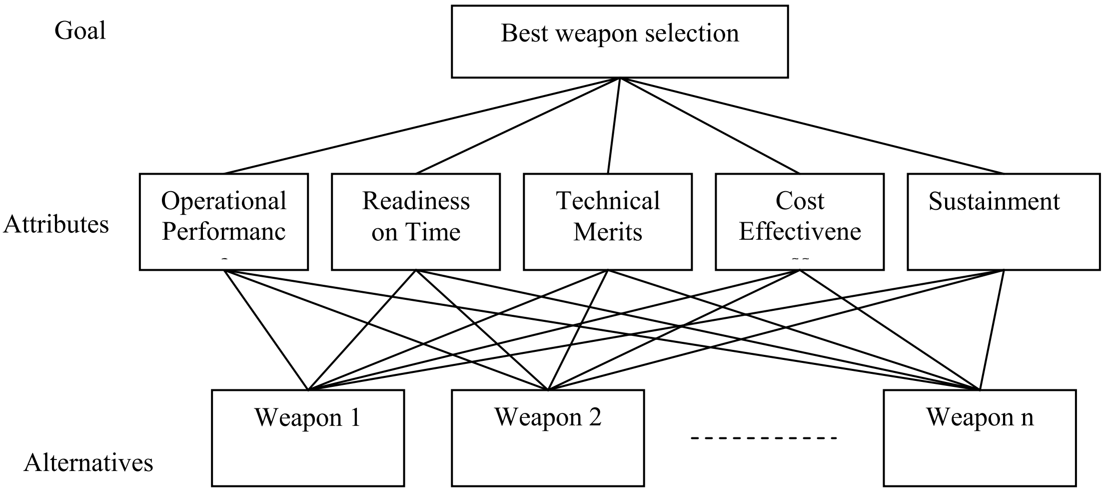


Figure 4.1 A Hierarchy for Best Weapon Selection

The indexation is defined as a categorization of an individual qualitative attribute to present each alternative’s value of the attribute. For example, we can categorize an attribute into five indexes such as outstanding, above average, average, below average, and unsatisfactory. With this categorization, we assign value of 5, 4, 3, 2, or 1 to each index. Based on this index, each alternative’s value can be determined (e.g., if an alternative is classified as an outstanding, the value of 5 is assigned to the alternative for that attribute value). We use these indexes for assigning alternatives’ values for qualitative attributes. In this chapter, we develop a more detailed problem structure with indexation procedures.

4.2 Operational Performance

SKN defines operational performance as “maintaining the level of performance to meet the naval operational requirements” (NR 2). Naval operations can be classified into two types: An actual naval warfare operation and peace-time regular operation. It is difficult to decide which operation is more important. However, during a peaceful time, people may consider regular operations (e.g., sea patrol, ensuring freedom of the seas so that merchant ships can bring the vital raw materials into Korea, collecting information surrounded in Korea, and so on) as more important. In contrast to this, actual combat effectiveness can be considered as more important than effectiveness in regular operations if DMs stress on an actual warfare. Therefore, an operational performance is measured in terms of sum of two sub attributes’ scores: a combat operational performance and a regular operational performance. Figure 4.2 shows a hierarchical structure of an operational performance.

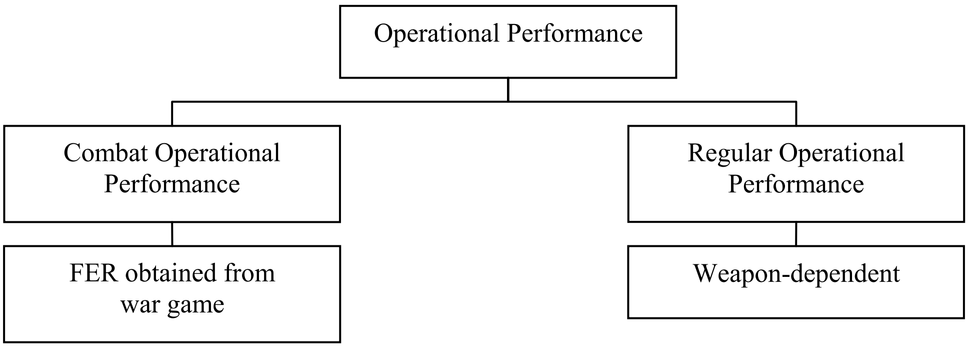


Figure 4.2 Hierarchical Structure of Operational Performance

The sub-attribute items under a regular operational performance are weapon dependant. That is because each weapon has different purposes and hence should be evaluated differently in terms of each purpose. For example, the purpose of the missile is to hit an opponent's object, while radar has a purpose to detect an opponent's object. In terms of different purposes of these two weapon types, the measure of effectiveness is defined differently. For example, the attack range, attack precision, and penetration are appropriate measure of effectiveness for missile and the detection range, accuracy, and performances regarding electrical warfare are good for radar. Therefore, detailed structures under a regular operational performance can be defined according to weapon type.

Combat effectiveness, which can measure the performance of weapon on naval warfare, can not be tested unless there is a real war. However, a simulation game called War Game is widely used in many countries on a real combat's behalf for several purposes such as training people in case of a combat, evaluating task forces in terms of real warfare, and improving commanders' tactical abilities.

Loerch et al. (1998) propose a Fractional Exchange Ratio (FER) as a measure of combat effectiveness. FER is computed by (note that red represents enemy force and blue represents friendly force)

$$\Psi = \frac{\sum_i a_i r_i / \sum_i r_i}{\sum_i a_i b_i / \sum_i b_i} \tag{4.1}$$

where

Ψ = FER

$$b_i = \begin{cases} 1, \text{when blue force i joins a combat} \\ 0, \qquad\qquad\qquad\qquad \text{otherwise} \end{cases}$$

$$r_i = \begin{cases} 1, \text{when red force i joins a combat} \\ 0, \qquad\qquad\qquad \text{otherwise} \end{cases}$$

a_i= remaining availability of blue or red force i after the combat,

d_i= damage rate of blue or red force i after the combat,

$0 \le d_i \le 1 \quad \forall i$, $a_i = 1 - d_i$, and $b_i, r_i \in \{0,1\}$.

In Equation 4.1, $d_i = 0$ means that a blue or red force i does not have any damage while $d_i = 1$ represents entire loss. However, FER is only applied for a unit combat system such as a ship, a submarine, or an aircraft. That is because a component weapon system (e.g., gunnery, missile, radar, sonar, and radio) can not be measured separately from its host system (i.e, a unit weapon system).

4.3 Readiness on Time

The terminology of readiness on time is defined as weapons should be ready on time for a specific purpose by NR 2. The attribute of readiness on time is composed of three sub attributes as follows.

1. Readiness of operators. Operators should be trained. Therefore, training time as well as number of people of both trainers and trainees should be considered.
2. Readiness of weapons. If any supplier can not meet demand (i.e., be on time, and to meet amounts of weapons), that company can not be attractive to DMs. Being on time represents that weapons should be prepared for use before or at the time when SKN requires. SKN requires a certain number of weapons to be able for use and it is represented by the attribute of meeting amounts of weapons.

3. Readiness of supporting systems. Supporting systems can be defined such as department of administration, logistics, and maintenance. If SKN can use current supporting systems to operate a candidate weapon, the weapon should have advantages under this attribute. In many cases, more efforts than buying a weapon itself are required to develop supporting systems. These three factors are shown as sub-attributes of readiness on time in Figure 4.3.

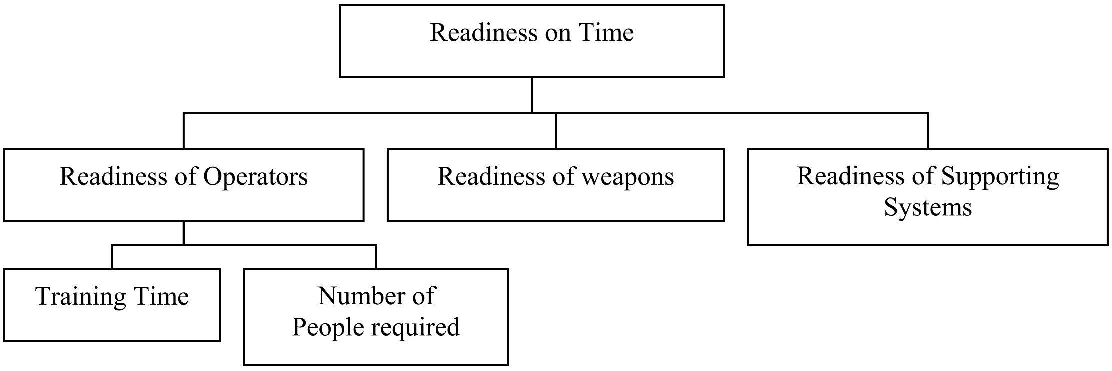


Figure 4.3 Hierarchical Structure of Readiness on Time

Sub-attributes of readiness of operators are cost attributes. Therefore, value of each alternative should be plugged as a form of $1/x_{ij}$ as explained in Section 2.2.1.

Data for readiness of weapons and supporting systems are both qualitative and hence an indexation process is required. Table 4.2 and 4.3 shows, respectably, indexation of readiness of weapons and supporting systems.

Table 4.2 Indexation of Readiness of Weapons

Scale	Criteria	Scores
Outstanding	All two factors can be met before due date	5
Above Average	Weapons are ready but not all of them	4
Average	Weapons can be ready shortly after due date but with all of weapons	3
Below Average	Weapons can be ready shortly after due date but still not meeting the amounts of weapons	2
Unsatisfactory	Preparing weapons take longer time than SKN's expectation	1

Note that the two factors are being on time, and meeting the amounts of weapons.

If SKN can use current supporting systems for operating any candidate weapon, that weapon has an advantage of smaller cost required in building a supporting system. This advantage is considered by the attribute of readiness of supporting systems.

Table 4.3 Indexation of Readiness of Supporting Systems

Scale	Criteria	Scores
Outstanding	Current three supporting systems are available to use for using a candidate weapon	5
Above Average	Two of three current supporting systems are available to use for using a candidate weapon	4
Average	One of three current supporting system is available to use for using a candidate weapon	3
Below Average	None of systems are available to use, but these three systems can be built with no difficulty	2
Unsatisfactory	None of systems are available to use, and building these three systems are challenging	1

Note that the three supporting systems are administration, logistics, and maintenance.

4.4 Technical Merits

NR 2 defines technical merits as “Try to get technologies from weapon procurement if it is coming from other countries. However, some priorities are given to the domestic products. One should also consider giving a benefit on contributions toward a domestic economic. By this definition, two sub attributes under the attribute of technical merits are presented in the following figure.

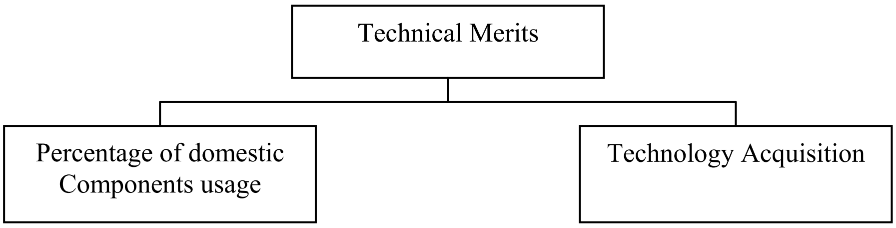


Figure 4.4 Hierarchical Structure of Technical Merit

Percentage of domestic components used is provided by each candidate company. This attribute is considered as a benefit attribute since the more domestic components are used, the better for Korean economics.

Korea has recently developed and hence there are not enough technologies available for not only developing weapons, but also producing a competitive product in world wide markets. For this reason, Korean government considers obtaining technologies from outside of Korea as an important strategy. SKN classifies the degree of technology acquisition into three categories: core technologies, important technologies, and general technologies. The indexation of acquisition of technologies is presented in Table 4.4.

Table 4.4 Indexation of Technology Acquisition

Scale	Criteria	Scores
Excellent condition	Core technologies can be obtained	4
Very good condition	Important technologies can be obtained	3
Good condition	General technologies can be obtained	2
Poor condition	No technologies can be obtained	1

SKN defines these four categories used in the above table as follows.

1. A core technology is a very important technology which has a critical impact on domestic economies especially for a domestic weapon industry. The classification about what should be the core technologies is decided before weapon procurement is issued by SKN.
2. An important technology is not considered as a core technology, but still considered as an important technology. This technology is also classified by SKN before the weapon procurement decision is processed.
3. A general technology is neither critical nor important. However, if a company can give us any technologies which Korea lacks, this is good for both domestic companies and the SKN.

If any contract cannot present any technologies to Korea, this weapon has a value of one on this attribute.

4.5 Cost Effectiveness

There are two types of costs that are involved in weapon procurement: the first acquisition cost and operational costs. To show how these two different costs affect decision making, let us consider two extreme cases.

1. Low acquisition price, but high operational cost. In this case, the weapon cannot be attractive to buyers.
2. Reasonable operational cost, but high acquisition cost. In this case, DM can hesitate to decide to buy the weapon.

Therefore, a trade off between acquisition and operational cost is necessary. The attribute of cost effectiveness is designed for this trade-off. Figure 4.5 shows corresponding hierarchical structure.

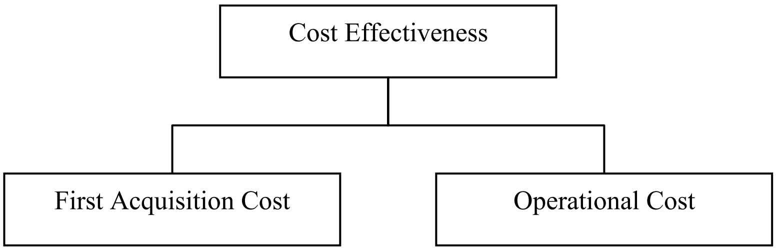


Figure 4.5 Hierarchical Structure of Cost Effectiveness

4.6 Sustainment

To sustain means to supply with necessities or provide for. SKN defines three important factors in the sustainment of equipment as logistics, maintenance, and reliability. These three factors are presented in Figure 4.6.

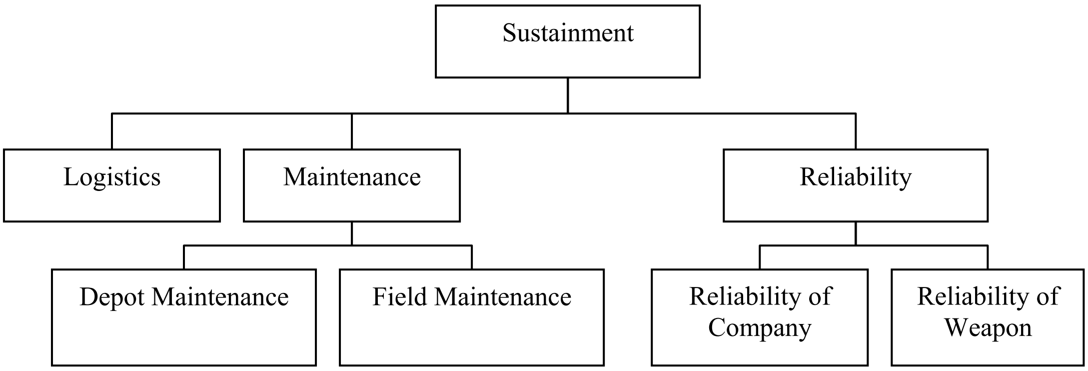


Figure 4.6 Hierarchical Structure of Sustainment

Web dictionary defines logistics as “handling an operation that involves providing labor and materials be supplied as needed” (http://www.wordwebonline.com). From a user point of view, good logistics means that any supplement and maintenance parts should be ready within an expected period of time. And how the logistics works depends on supplier types. For military contracts, there are typically three different types of suppliers: a domestic company, a foreign company, and a foreign government. The latter one is defined as Foreign Military Sale (FMS) by NR 2. The table below shows an indexation of logistics.

Table 4.5 Indexation of Logistics

Scale	Criteria	Scores
Best supplier type	Supplement and parts are distributed by a domestic company	3
Normal supplier type	Supplement and parts are distributed by a foreign company	2
Worst supplier type	Supplement and parts are distributed by the contraction type of FMS	1

A domestic company can be considered as the best supplier in terms of a quick response time. In addition to this response time, SKN can have various advantages such as small transport costs, no affects from foreign exchange rate, and increasing rate of employment by contracting a domestic company.

A contract with a foreign company can lead to long response time and high costs because all products are transported to Korea from a foreign country. However, this contraction type is better than the FMS. FMS is a contraction between two countries. A supplying country needs to get an approval from its Congress to export products to other countries. For this reason, it normally takes more than a year to receive a product supplied by FMS contraction. In contrast to FMS, direct contraction between SKN and a foreign company does not need an approval from Congress. Therefore, SKN prefer a contraction with a foreign company compared to FMS.

SKN operates two types of maintenance systems: field and depot maintenance. Field maintenance is generally routine maintenance and is conducted by individual operators. Depot maintenance is conducted by special technicians and most of maintenances are repeated more than once per year. Both field and depot maintenances include pre-maintenance as well as after failure maintenance. Table 4.6 shows an indexation of depot maintenance.

Table 4.6 Indexation of Depot Maintenance

Scale	Criteria	Scores
Best depot maintenance	SKN can conduct a depot maintenance	3
Normal depot maintenance	Depot maintenance needs to be conducted by a domestic company	2
Worst depot maintenance	Depot maintenance needs to be conducted by a foreign company	1

Depot maintenance is composed of works beyond weapon operator's abilities. Moreover, it is a big job such as an overhaul and hence requires a facility with many special tools. This maintenance is conducted either by SKN or a private company. From a SKN point of view, navy facility is the best and a foreign company is the worst in terms of depot maintenance.

Since individual weapon operators conduct field maintenance, how they feel in terms of easiness can be important criteria. If operators feel difficulty in maintaining a weapon, this weapon can not be considered as good in terms of field maintenance and vice versa. Table 4.7 presents indexation of field maintenance. The decision regarding which scale should be assigned to each candidate weapon is done by consensus from operators.

Table 4.7 Indexation of Field Maintenance

Scale	Criteria	Scores
Outstanding	Very easy for conducting field maintenance	5
Above Average	Easy for conducting field maintenance	4
Average	Commonly difficult for conducting field maintenance	3
Below Average	Difficult for conducting field maintenance	2
Unsatisfactory	Very difficult for conducting field maintenance	1

Reliability is a characteristic of an item, expressed by the probability that the item will perform its required function under given conditions for a stated time interval (Birolini, 1999). Normally reliability is considered only for a product. Even though SKN buys a best weapon which has best performance in terms of reliability, if a company which produces the weapon is unstable and disappeared in the market, SKN can not maintain the weapon anymore. For this reason, we expand reliability into each company.

SKN defines mean time between failures (MTBF) as an index for representing weapon's reliability. Since a longer MTBF is more appropriate for SKN, this index is considered as a benefit attribute. SKN obtains this information from each candidate company.

Company ranking information can be obtained by reliable periodicals such as *DefenseNews, Business.com and Fortune.com*. Since high ranking is represented as low number index, this index is considered as a cost attribute (i.e., a value of each alternative has a form of $1/x_{ij}$).

In addition to these two reliabilities, warranty condition can play an important role in terms of reliability. Even if a weapon has a short MTBF, but the company can offer a good warranty condition, SKN can buy the weapon offered from this company. From a company's point of view, a good warranty condition can increase its competition ability. From a SKN point of view, a good warranty condition can be considered as a merit. However, an indexation for this condition is difficult. Therefore, we assume that each company offers the same conditions in terms of warranty condition. This assumption is based on the reasoning that (1) SKN requires a certain limit of warranty conditions and each company has to meet the limit to be eligible as a candidate company, (2) companies will try to lower their weapon's prices to increase their competition edge, and (3) consequently, companies are unable to offer better warranty condition than required by SKN.

4.7 Summary

Since SKN follows NR 2 for weapon procurement decision, this problem structure can be considered as non creative works. However, since there is no structured procedure like this hierarchical structure, the right decision of selecting the best weapon has been always challenging for SKN. We expect this hierarchical structure can be used as a general structure for a weapon procurement decision. Figure 4.7 shows a hierarchy of attributes for the weapon procurement. This hierarchical structure has three levels of attributes: five first-level attributes, twelve second-level attributes, and six third-level attributes.

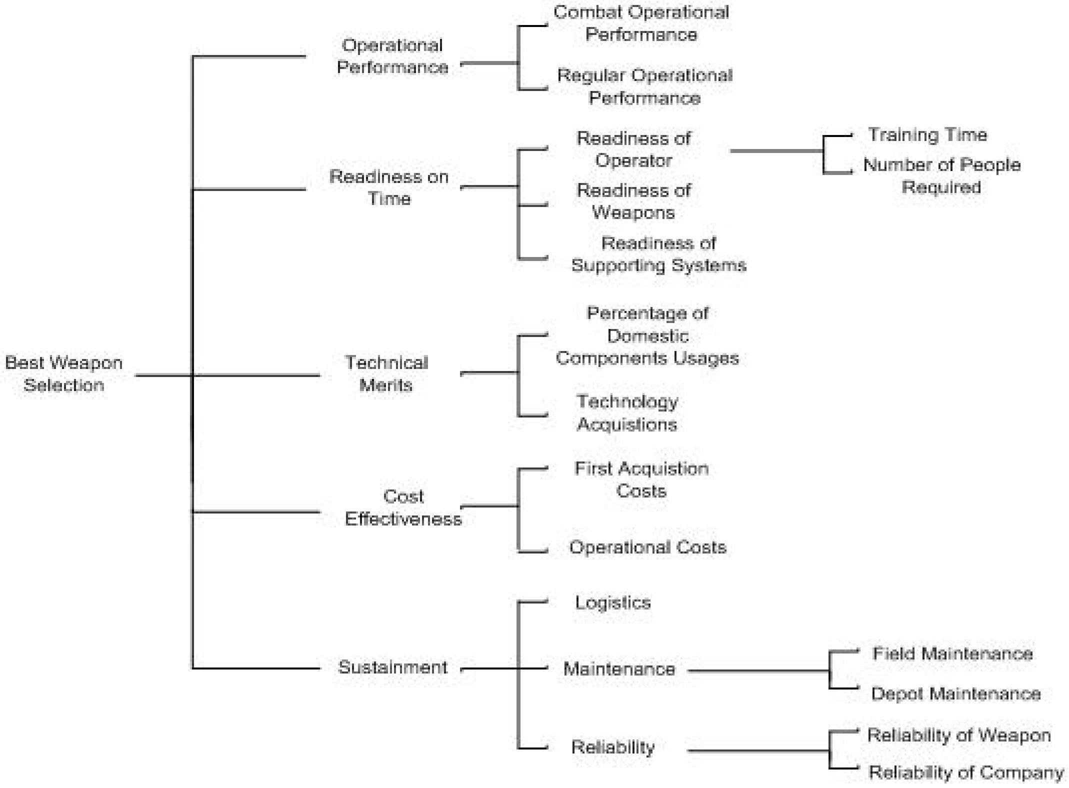


Figure 4.7 A Hierarchy of Attributes For the Best Weapon Procurement

BSM function presents alternatives' raking scores in terms of this hierarchical structure. However, as presented in the previous example, weights should be assigned into all attributes. The attribute weighting for this problem structure is followed in the next chapter.

Chapter Five

Attribute Weighting

5.1 Introduction

Since all attributes do not have the same importance, we need to assign a degree of importance to each attribute. As explained earlier, these are called weights. The weights are the core of compensation methods. In most cases, it is difficult to determine relative importance among attributes, especially for qualitative ones. In this research, the AHP method is used, based on the comparison results shown in Section 2.4

The AHP method uses pairwise comparisons from Experts. In this research, Experts are defined as SKN senior officers. The sample size of n is computed by Equation 5.1 (Lee and Park, 1995). Computation results are not presented in detail due to military secrets.

$$n = \frac{NZ_{\alpha/2}^{2} pq}{NB^{2} + Z_{\alpha/2}^{2} pq}. \tag{5.1}$$

In this equation, N is the population size, B the significance level, and p and q are expectation values of population proportion, i.e., p represents proportion of senior officers in the SKN and q the other SKN personal. The sample size is calculated as 50 and mailing interviews will be used.

A survey form is composed of two parts. Part 1 is composed of questionnaires for personal and general parts which aims to help an individual officer feels comfortable and can answer with ease for questionnaires given in part 2. Part 2 is an actual pairwise comparison designed to assign attribute weights. These two parts of survey forms are presented in Appendix 1.

As explained in Chapter 2, Saaty (1980) develops the CR presented in Equation 2.29 and suggests that the answer is not consistent if CR is more than 0.1 and such answers should be excluded in calculating an attribute weight. However, this suggestion has no support for the value of CR=0.1. Sin (1988) suggests using a value of 0.2 instead of 0.1 is more practical than Saaty's suggestion because we can increase the number of answers which can be used to calculate an attribute weight. By following Sin's suggestion, we define the critical value for consistency as 0.2. The actual calculation is computed by the weighting program developed by using C++ (Chang, 1997). Since this program is not used in Visual C program, we modify this program to be used in Visual C program. We present this modified program in Appendix 2.

We obtain total 450 pairwise comparison matrices (i.e., 9 matrices for each DM and we have fifty DMs; therefore, $9 * 50 = 450$). Among these matrices, 34 matrices are excluded in weighting computation due to $CR \geq 0.2$. Weight for each attribute is computed by the following steps.

Step 1. From each decision matrix, we obtain individual DM's weighting values. This follows the five steps in Section 2.2.4.

Step 2. Each decision matrix presents fifty or fewer weighting values depend on the state of $CR \geq 0.2$.

Step 3. From these weighting values, each attribute's mean weight is computed.

Step 4. Attribute weights are defined as this mean values.

We also compute 95% confidential intervals for each attribute weight. This interval is presented as a half width with an attribute weight value. Half width is computed by

$$\text{Half width} = Z_{0.975}\frac{S}{\sqrt{n}} = 1.96\frac{S}{\sqrt{n}}, \tag{5.1}$$

where S is a standard deviation of the n weight values whose $CR < 0.2$. Note that since $n > 30$ for all matrices whose $CR < 0.2$, we use Z-distribution instead of t-distribution. Detailed results are described in the following sections.

5.2 Weights for the First Level Attributes

Among the fifty decision matrices for five first-level attributes, fourteen matrices are excluded in computing weights due to $CR \geq 0.2$. Table 5.1 shows the weights of these five first-level attributes.

From this table, sustainment and operational performance are drawn as important attributes while readiness on time is considered as less important decision factor with respect to the best weapon procurement.

Table 5.1 Weights for the Five First-Level Attributes

Results	Attributes with respect to best weapon selection					
	Operational performances	Readiness on Time	Technical merits	Cost effectiveness	Sustainment	Total
Mean (weights)	0.294	0.073	0.177	0.126	0.330	1
Standard deviation	0.127	0.044	0.057	0.054	0.087	
Half width	0.042	0.015	0.019	0.018	0.029	

5.3 Weights for the Second Level Attributes

Operational performance has two second level attributes: combat and regular operational performance. In this case, since we have two attributes that we need to compare, only one comparison is required to determine weights. In addition, when we compare two attributes, we have CI = 0 because $\lambda_{\max} = n$ for n =2 (see Equation 2.28). Due to CI = 0, $CR = 0$ (see Equation 2.29). Therefore, we can compute mean value for each of the fifty decision matrices. Table 5.2 shows weights for combat and regular operational performance with respect to operational performance. In this table, combat operational performance is considered as two times important than regular operational performance.

Table 5.2 Weights for the Two Second-Level Attributes of Operational Performance

Results	Attributes with respect to operational performance		
	Combat operational performance	Regular operational performance	Total
Mean (weights)	0.693	0.307	1
Standard deviation	0.192	0.192	
Half width	0.053	0.053	

Readiness of operators, weapon and supporting systems are three second-level attributes of readiness on time. In this case, eleven decision matrices are excluded in computing weights due to $CR \geq 0.2$. Table 5.3 shows weights for these three second-level attributes. From this table, readiness of supporting system is considered as more important than the other two attributes.

Table 5.3 Weights for the Three Second-Level Attributes of Readiness on Time

Results	Attributes with respect to readiness on time			
	Readiness of operators	Readiness of weapons	Readiness of supporting systems	Total
Mean (weights)	0.283	0.309	0.408	1
Standard deviation	0.105	0.184	0.198	
Half width	0.033	0.058	0.062	

Table 5.4 shows weights for the two second-level attributes under the attribute of technical merits (i.e., percentage of domestic components usage and technology acquisitions). In this case, as in Table 5.2, all of fifty decision matrices are used for computing weights. In this table, the attribute of technology acquisitions is considered as more important decision factor than the attribute of percentage of domestic components usage with respect to technical merits.

Table 5.4 Weights for the Two Second-Level Attributes of Technical Merits

Results	Attributes with respect to technical merits		
	Percentage of domestic components usage	Technology Acquisitions	Total
Mean (weights)	0.375	0.625	1
Standard deviation	0.216	0.216	
Half width	0.060	0.060	

Table 5.5 shows weights for the first acquisition costs and operational costs with respect to cost effectiveness. From this table, one can see that operational costs are more important than the first acquisition costs.

Table 5.5 Weights for the Two Second-Level Attributes of Cost Effectiveness

Results	Attributes with respect to cost effectiveness		
	First acquisition costs	Operational costs	Total
Mean (weights)	0.341	0.659	1
Standard deviation	0.217	0.217	
Half width	0.060	0.060	

Logistics, maintenance, and reliability are second-level attributes which compose the attribute of sustainment. In this case, nine matrices are excluded in computing weights due to $CR \geq 0.2$. Table 5.6 presents weights for these three second-level attributes. In this table, logistics is shown as the most important attribute with respect to sustainment.

Table 5.6 Weights for the Three Second-Level Attributes of Sustainment

Results	Attributes with respect to sustainment			
	Logistics	Maintenance	Reliability	Total
Mean (weights)	0.429	0.341	0.230	1
Standard deviation	0.174	0.123	0.144	
Half width	0.055	0.039	0.045	

5.4 Weights for the Third Level Attributes

Three of the second-level attributes (i.e., readiness of operators, maintenance, and reliability) have their sub-attributes (i.e., third level attributes). Weights for these third level attributes are presented in the following table.

Table 5.7 Weights for the Third Level Attributes

Second level attributes	Third level attributes	Weights	Standard deviation	Half width
Readiness of operators	Training Time	0.530	0.170	0.047
	Number of people required	0.470	0.170	0.047
Maintenance	Field maintenance	0.617	0.193	0.053
	Depot maintenance	0.383	0.193	0.053
Reliability	Reliability of weapon	0.540	0.173	0.048
	Reliability of weapon company	0.460	0.173	0.048

From this table, there is no big difference between training time and the number of people required in terms of weights. Field maintenance appears to be about two times more important than depot maintenance. Weapon's reliability is considered as slightly more important than company's reliability.

5.5 Summary

Up to now, weights of the same level attributes having the same upper level attribute sum to one as shown in the previous tables. However, a final weight should be computed in terms of its upper level attribute's weight and hence all the bottom level attributes can sum to one. A tree structure is used to obtain the final weights (Yoon and Hwang, 1995). The final weights for attributes at each twig of the tree of Figure 4.7 are obtained by multiplying through the branches. Figure 5.1 shows the entire weight assessment process.

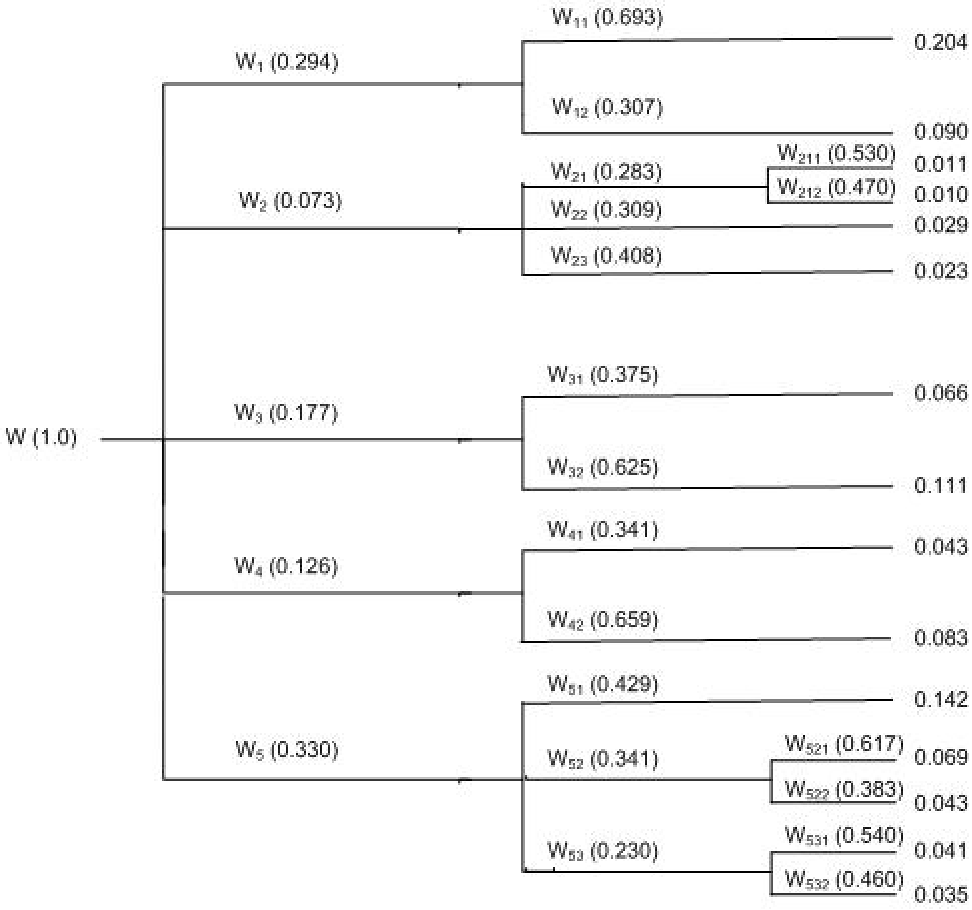


Figure 5.1 Weight Assessments for Best Weapon Procurement

Based on the above figure, we present attribute weightings for our problem structure in Table 5.8. In this table, numbers prefixed in each attribute are the same as subscripts in Figure 5.1. There are fifteen bottom level attributes in this table. These attributes are assumed to be independent with each other. This is the basic assumption of MADM methods (see Yoon and Hwang (1995), Saaty (1980) for more detail). In this table, combat operational performance is determined as the most important attribute while training time and number of people required are the least important attributes.

Table 5.8 Attribute Weightings for the Best Weapon Procurement

Attribute (weight)	Weight
1. Operational Performances	
1.1 Combat operational performances	0.204
1.2 Regular operational performances	0.090
2. Readiness on Time	
2.1 Readiness of operators	
2.1.1 Training time	0.011
2.1.2 Number of people required	0.010
2.2 Readiness of weapons	0.029
2.3 Readiness of supporting systems	0.023
3. Technical Merits	
3.1 Percentage of domestic components usages	0.066
3.2 Technology acquisitions	0.111
4. Cost Effectiveness	
4.1 First acquisition costs	0.043
4.2 Operational costs	0.083
5. Sustainment	
5.1 Logistics	0.142
5.2 Maintenance	
5.2.1 Field maintenance	0.069
5.2.2 Depot maintenance	0.043
5.3 Reliability	
5.3.1 Reliability of weapons	0.041
5.3.2 Reliability of company	0.035
Total	1

In the next chapter, we apply this hierarchical problem structure into the real problem in the SKN with the BSM. This application will validate the BSM.

Chapter Six

Case Study

6.1 Introduction

In 1998, South Korea considered procuring a few submarines from Russia. At that time, Russia had borrowed approximately 0.18 billion dollars from South Korea and could not return that money. Instead, Russia wanted to refund the debts with their military weapons.

The type Kilo submarine was offered to South Korea by Russia for this reason. The payment condition for this submarine was considered 30% from the debts and 70% from cash (*Dongailbo*, 8.3.2000). Since the price of this submarine was 0.2 billion dollars (*Chosunilbo*, 12.21.2004), South Korea could buy this submarine for only 0.14 billion dollars. This price was less than half price of the type 214 submarine made by Germany. Type 214 submarine was considered as the same type of type Kilo submarine and its price was 0.3 billion dollars (*Shin-donga*, July 2001).

Since Russia could return their debts and South Korea could obtain important weapon systems for a good price, this offer was attractive for the South Korean government.

During this time, SKN was going to develop submarine power and already had some submarines from Germany. Therefore, all the supporting and operating systems were setup with respect to German submarines. In order to be able to operate this Russian submarine, SKN would have to expend a lot of effort to construct all the supporting and operating systems, which could be considered a double invest. Therefore, type Kilo submarine was not an attractive plan for the SKN. As a result, SKN did not agree with the Government's plan. Instead, SKN asked the Government to procure type 214 submarines for the following reasons.

1. Many military weapon analysis organizations such as *Military review* and *Naval technology* have data that clearly show that type 214 has better performance measures than type Kilo submarines do.
2. Type 214 submarine has advantages in terms of logistics and maintenance because SKN has used similar type submarines from Germany. Therefore, SKN can use current logistics and maintenance systems. However, if type Kilo submarines are used, SKN has to construct all these systems and costs would rise.
3. Germany suggests giving core technology (i.e., submarine design technology) to the SKN if their submarines are accepted by the SKN. However, Russia does not suggest this core technology transfer.

As explained by the above reasons, type 214 submarine has many advantages over type Kilo submarine. Therefore, the Government cancelled the plan of procuring type Kilo submarines. However, since this submarine could be obtained for a good price, the offer from Russia was attractive to the South Korean Government.

In this chapter, we compute these two submarines' ranking scores by the BSM in terms of two cases.

1. The SKN's view point. In this case, weights in Table 5.8 are used in computing these two submarines ranking scores because these weights came from the SKN.
2. The Government's view point. In this case, weights that are artificially assigned for the purpose of aiming to represent Government's intention are used in computing these two ranking scores.

Since type 214 submarine is reviewed as a more proper decision for the SKN, a good decision model must select this submarine as the best alternative in both cases. The BSM can select type 214 submarines as the best alternative in both cases while current MADM methods can not. This result can be considered as a justification for our new model, the BSM.

6.2 Data Collection for Evaluating Submarines

Before we compute alternative ranking scores, we need to collect data for each alternative with respect to all the attributes. In this section, data for these two submarine types are collected. Various sources of information such as *Military review* and *Naval technology* are used for this purpose. For the qualitative data, we use indexation tables presented in Chapter 4.

6.2.1 Data for Operational Performance

As shown in Table 5.8, combat operational performance is the most important attribute. However, there is a restriction in collecting data for this attribute (i.e., conducting actual war game for this research is not allowed due to military secrete purpose). Therefore, responses for this attribute are assumed to be equal between type 214 and type Kilo submarine.

Regular operational performance can be measured by submarine speed, diving depth, cruse range, attack ability, and mission endurance. Table 6.1 presents these performance data for both submarines.

Note that *Naval technology* is an internet resource for navy ship technology, information on naval projects, conferences, exhibitions and suppliers as well as a detailed manufacturer directory.

Table 6.1 Data for Regular Operational Performances (Source: *Naval technology*)

Regular operational performance measures (units)	Type Kilo Submarine	Type 214 Submarine
Maximum submerged speed (knots)	17	20
Maximum surface cruise range (NM)	6,000	12,000
Maximum submerged cruise range (NM)	400	420
Maximum diving depth (meters)	300	400
Attack ability (anti submarine, anti surface, and anti air)	Similar	Similar
Mission endurance (days)	45	50

6.2.2 Data for Readiness on Time

From interviews with some SKN submarine officers, we obtain that there is no significant difference between the two submarines in terms of training time and number of people required.

For the readiness of weapons, both submarines can be considered as outstanding (see Table 4.2). This consideration is possible because they can be supplied within the time required by the SKN. Therefore, score 5 is given to both alternatives.

By the reasoning in the previous section and Table 4.3, type 214 and type Kilo submarines are classified respectably as outstanding and unsatisfactory in terms of readiness of supporting systems. Therefore, score 5 and 1 are given to type 214 and type Kilo submarine respectably. Table 6.2 presents data for readiness on time.

Table 6.2 Data for Readiness on Time

Attributes	Type Kilo Submarine	Type 214 Submarine
Training time	No difference	No difference
Number of people required	No difference	No difference
Readiness of weapons	5	5
Readiness of supporting systems	1	5

6.2.3 Data for Technical Merits and Cost Effectiveness

Howaldtswerke Deutsche Werft (HDW) is a German company which builds and exports type 214 submarines. HDW offered that they can give submarine design technology for their submarines. This technology is very important, especially for developing a new type of submarine. Therefore, type 214 submarine can be considered as an excellent condition in terms of technology acquisition (see Table 4.4). However, type Kilo submarine does not have any technical merits and hence is classified as poor condition for the same attribute. Therefore, score 4 and 1 are given to type 214 and type Kilo submarine, respectively.

The following table shows the responses of these two submarines on the attribute of technical merits. Since no domestic components are used in both alternatives, value 0 is given for the percentage of domestic component usage.

Table 6.3 Data for Technical Merits

Attributes	Type Kilo Submarine	Type 214 Submarine
Percentage of domestic component usage	0	0
Technology acquisition	1	4

Table 6.4 shows data for cost effectiveness. First acquisition costs are explained in Section 6.1. However, we could not obtain data for the operational costs for military secretes.

Table 6.4 Data for Cost Effectiveness (Source: *Shin-donga*, July 2001)

Attributes	Type Kilo Submarine	Type 214 Submarine
First acquisition costs (million dollars)	140	300
Operational costs	NA	NA

6.2.4 Data for Sustainment

Both companies of type 214 and type Kilo submarines can be considered as normal suppliers in terms of logistics (see Table 4.5). Therefore, score 2 is given to both alternatives on the attribute of logistics.

For the depot maintenance, type 214 and type Kilo submarines are classified respectively as the best and worst alternatives for the following reason. Type 214 submarines are supposed to be built and maintained by a Korean domestic company based on technologies given by HDW, while type Kilo submarines should be sent to Russia for the depot maintenance. Therefore, score 3 and 1 are given to type 214 and type Kilo submarines respectively with respect to the attribute of depot maintenance.

Type 214 submarine can be classified as the above average in terms of field maintenance because of the following reasons. SKN already has some submarines from Germany and they are not much different than type 214 submarines in terms of field maintenance. Therefore, operators can maintain this submarine with ease.

Contrast to type 214 submarine, type Kilo is considered as below average in terms of field maintenance because SKN has never used Russian submarines before. Therefore, based on Table 4.7, score 4 and 2 are given to type 214 and type Kilo submarines respectively for the attribute of field maintenance. Table 6.5 shows data for logistics and maintenance.

Table 6.5 Data for Logistics and Maintenance

Attributes	Type Kilo Submarine	Type 214 Submarine
Logistics	2	2
Depot maintenance	1	3
Field maintenance	2	4

Since 1991, *Defense News* has published the Defense News Top 100, a ranking and report about the world's leading defense companies. The highlight of this report is the annual list of the world's top 100 defense companies based on defense revenues. Table 6.6 shows 2004 defense company rankings. In this table, one can see that ThyssenKrupp Werften (i.e., the parents company of HDW) is ranked 39th. However, Rosvoorouzhenie (i.e., Russian state owned company which makes type Kilo submarines) has not been ranked in 100 ranking list since 2000. Table 6.7 shows Rosvoorouzhenie's last ranking within 100 ranking list.

Table 6.6 2004 Defense Company Rankings (Source: *Defense News*)

Rank	Company	Country	2002 Rank	2003 Defense Revenue (million $)	2003 Total Revenue (million $)	Percent of Revenue from Defense (%)	2002 Defense Revenue (million $)
1	Lockheed Martin	U.S.	1	30,097.0	31,824.0	94.6	23,337.0
2	Boeing	U.S.	2	27,360.0	50,500.0	54.2	22,033.0
3	Northrop Grumman	U.S.	5	18,700.0	26,200.0	71.4	12,278.1
4	BAE Systems	U.K.	4	17,159.0	22,359.3	76.7	15,036.4
39	ThyssenKrupp Werften	Germany	NR	1,110.0	6,152.9	18.0	955.0

Table 6.7 2000 Defense Company Rankings (Source: *Defense News*)

Rank	Company	Country	1998 rank	1999 Defense Revenue (million $)	1999 Total Revenue (million $)	Percent of Revenue from Defense (%)	1999 Net Income (million $)
1	Lockheed Martin	U.S.	1	17,800.00	25,500.00	69.80	382.00
2	Boeing	U.S.	2	16,250.00	58,000.00	28	1,120.00
3	BAE Systems	U.K.	4	15,200.00	19,400.00	78.4	491.5
4	Raytheon Co.	U.S.	3	14,489.00	19,841.00	73	404
12	Rosvoorouzhenie	Russia	14	2,830.00	2,830.00	100	NA

Since Rosvoorouzhenie is not listed in the 2004 company ranking list, we arbitrary define this company's ranking as 101st in the ranking list. 101st ranking is the first ranking out of 100 ranking list. Data for reliability of weapons are not available to obtain for the military secrets. However, within the military this data can be obtained and hence can be applied for computing alternative ranking orders. In this case, we assume that there is no difference between two alternatives in terms of reliability of weapons. Table 6.8 shows the responses of these two alternatives on the attribute of reliability.

Table 6.8 Data for Reliability

Attributes	Type Kilo Submarine	Type 214 Submarine
Reliability of weapons	NA	NA
Reliability of companies	101	39

6.2.5 Summary

Let us define two alternatives: A_1 for type 214 submarine and A_2 for type Kilo submarine. The values of these alternatives on each attribute described in previous sections can be summarized in Table 6.9.

From Table 6.9, values for combat operational performance, operational costs, and the reliability of weapon are given as 1 for both alternatives because these values are not available to obtain at this time. In addition, it is known that there is no difference between the two alternatives in terms of attack ability, training time, and the number of people required. Therefore, values for these attributes are given as 1 for both alternatives.

Note that training time, number of people required, first acquisition costs, operational costs, and reliability of company are cost attributes and the others are benefit attributes. Their normalized values are computed in the following section.

Table 6.9 Data for Evaluation of Submarines

Attribute (weight)	Weight	Alternatives	
		Type 214 (A_1)	Type Kilo (A_2)
1. Operational Performances			
1.1 Combat operational performances	0.204	1	1
1.2 Regular operational performances			
1.2.1 Maximum submerged speed (knots)	0.015	20	17
1.2.2 Maximum surface cruise range (NM)	0.015	12,000	6,000
1.2.3 Maximum submerged cruise range (NM)	0.015	420	400
1.2.4 Maximum diving depth (meters)	0.015	400	300
1.2.5 Attack ability	0.015	1	1
1.2.6 Mission endurance (days)	0.015	50	45
2. Readiness on Time			
2.1 Readiness of operators			
2.1.1 Training time	0.011	1	1
2.1.2 Number of people required	0.010	1	1
2.2 Readiness of weapons	0.029	5	5
2.3 Readiness of supporting systems	0.023	5	1
3. Technical Merits			
3.1 Percentage of domestic components usages	0.066	0	0
3.2 Technology acquisitions	0.111	4	1
4. Cost Effectiveness			
4.1 First acquisition costs (million $)	0.043	300	140
4.2 Operational costs	0.083	1	1
5. Sustainment			
5.1 Logistics	0.142	2	2
5.2 Maintenance			
5.2.1 Field maintenance	0.069	4	2
5.2.2 Depot maintenance	0.043	3	1
5.3 Reliability			
5.3.1 Reliability of weapons	0.041	1	1
5.3.2 Reliability of company	0.035	39	101
Total	1		

6.3 Alternatives Rankings Based on SKN's View Points

From Table 6.9, we can compute alternatives' normalized and weighted normalized values shown in Table 6.10. Each alternative ranking score is computed based on Table 6.10. Following the definition in Section 6.2.5, x_{1j}, r_{1j} and v_{1j} represent the values of type 214 submarines for an attribute j. Like wise, x_{2j}, r_{2j} and v_{2j} represent the values of type Kilo submarines. Subscript numbers of each attribute are the same as numbers prefixed in each attribute in Table 6.9.

Table 6.10 Alternatives' Normalized and Weighted Normalized Values

Attributes (X_j)	Weights (w_j)	Alternative values (x_{ij})		Alternative's normalized values (r_{ij})		Alternative's weighted normalized values (v_{ij})	
		x_{1j}	x_{2j}	r_{1j}	r_{2j}	v_{1j}	v_{2j}
X_{11}	0.204	1	1	1	1	0.204	0.204
X_{121}	0.015	20	17	1	0.85	0.015	0.013
X_{122}	0.015	12,000	6,000	1	0.5	0.015	0.008
X_{123}	0.015	420	400	1	0.952	0.015	0.014
X_{124}	0.015	400	300	1	0.75	0.015	0.011
X_{125}	0.015	1	1	1	1	0.015	0.015
X_{126}	0.015	50	45	1	0.9	0.015	0.014
X_{211}	0.011	1	1	1	1	0.011	0.011
X_{212}	0.010	1	1	1	1	0.010	0.010
X_{22}	0.029	5	5	1	1	0.029	0.029
X_{23}	0.023	5	1	1	0.2	0.023	0.005
X_{31}	0.066	0	0	0	0	0	0
X_{32}	0.111	4	1	1	0.25	0.111	0.028
X_{41}	0.043	300	140	0.167	1	0.020	0.043
X_{42}	0.083	1	1	1	1	0.083	0.083
X_{51}	0.142	2	2	1	1	0.142	0.142
X_{521}	0.069	4	2	1	0.5	0.069	0.035
X_{522}	0.043	3	1	1	0.333	0.043	0.014
X_{531}	0.041	1	1	1	1	0.041	0.041
X_{532}	0.035	39	101	1	0.386	0.035	0.014
Total	1			18.467	14.622	0.911	0.732

Alternatives' ranking scores are computed from Table 6.10. Table 6.11 presents the two alternatives' ranking scores computed by three MADM methods (i.e., SAW, TOPSIS, and the BSM). From this table, one can see all three MADM methods select A_1 as the best alternative. Therefore, we can say that SKN's opposition to the Government's plan about type Kilo submarine is reasonable.

Table 6.11 Ranking Scores for the Submarine Selection Problem

Alternatives	ASW	Rank	TOPSIS	Rank	BSM		Rank
					IV	BSM	
Type 214 submarine (A_1)	0.911	1	0.812	1	0.923	0.917	1
Type Kilo submarine (A_2)	0.732	2	0.188	2	0.731	0.732	2

There are three data that we could not obtain (i.e., data for combat operational performance, operational costs, and reliability of weapon). Even if A_1 is considered to have better values for these attributes, since no exact data are available, we need to present what if analysis for these attribute values. This analysis can be done by the sensitivity analysis explained in Chapter 3. An alternative ranking score (i.e., $V(A_i)$) is defined as the BSM score for this sensitivity analysis.

Since $V(A_1) > V(A_2)$ and each $r_{ij} = 1$ for $i = 1$ and 2, and j = 11, 42, and 531, the ranking change is only possible within the changes (i.e., decrease in these three attribute values) in A_1. This is the scenario 2 of Section 3.4.1. r_{111}^c, r_{142}^c and r_{1531}^c are computed by Equation 3.11. These critical values are presented in Table 6.12.

Table 6.12 Critical Values for A_1 in terms of $V(A_1) = V(A_2)$

	Attributes (X_j)		
	X_{11}	X_{42}	X_{531}
r_{1j}^c	-0.462	-1.792	-3.080

From Table 6.12, since $r_{1j}^{c} \notin$ allowable range (see Equation 3.12), sensitivity analysis does not apply for these attribute values. In other words, regardless of how bad scores A_1 has for these attributes, this alternative can be ranked first in terms of sensitivity analysis.

6.4 Alternative Rakings From the Government's View Points

To represent the Government's intention, weight values can be modified such as in the following table. In this case, we arbitrarily assume that all attribute weights are equal except the attribute of first acquisition cost. That is because the Government considers that the first acquisition cost is much more important than any other decision factors, even though they do not know exactly what factors are the most important. Table 6.13 shows the case that the Government considers the first acquisition cost is important as much about 40% of the entire decision factors.

Table 6.13 Data for Evaluation of Submarines with Modified Weight Values

Attributes (X_j)	Weights (w_j)	Alternative values (x_{ij})		Alternative's normalized values (r_{ij})		Alternative's weighted normalized values (v_{ij})	
		x_{1j}	x_{2j}	r_{1j}	r_{2j}	v_{1j}	v_{2j}
X_{11}	0.03	1	1	1	1	0.204	0.204
X_{121}	0.03	20	17	1	0.85	0.015	0.013
X_{122}	0.03	12,000	6,000	1	0.5	0.015	0.008
X_{123}	0.03	420	400	1	0.952	0.015	0.014
X_{124}	0.03	400	300	1	0.75	0.015	0.011
X_{125}	0.03	1	1	1	1	0.015	0.015
X_{126}	0.03	50	45	1	0.9	0.015	0.014
X_{211}	0.03	1	1	1	1	0.011	0.011
X_{212}	0.03	1	1	1	1	0.010	0.010
X_{22}	0.03	5	5	1	1	0.029	0.029
X_{23}	0.03	5	1	1	0.2	0.023	0.005
X_{31}	0.03	0	0	0	0	0	0
X_{32}	0.03	4	1	1	0.25	0.111	0.028
X_{41}	0.43	300	140	0.167	1	0.020	0.043
X_{42}	0.03	1	1	1	1	0.083	0.083
X_{51}	0.03	2	2	1	1	0.142	0.142
X_{521}	0.03	4	2	1	0.5	0.069	0.035
X_{522}	0.03	3	1	1	0.333	0.043	0.014
X_{531}	0.03	1	1	1	1	0.041	0.041
X_{532}	0.03	39	101	1	0.386	0.035	0.014
Total	1			18.467	14.622	0.911	0.732

Based on this assumption, we can compute the two alternatives ranking scores as shown in Table 6.14.

Table 6.14 Ranking Scores Based on Modified Attribute Weights

Alternatives	ASW	Rank	TOPSIS	Rank	BSM		Rank
					IV	BSM	
Type 214 submarine (A_1)	0.741	2	0.175	2	0.923	0.832	1
Type Kilo submarine (A_2)	0.839	1	0.825	1	0.731	0.785	2

As the results shown in Table 6.14, only the BSM selects A_1 as the best alternative and others do not. The reason how the BSM can select A_1 as the best alternative is that this alternative has better IV value than A_2. However, since the difference of these IV values is not as big as in the extreme alternative problem in Section 3.3.1 (see Table 3.3), the BSM will not select A_1 as the best when $w_{41} > 0.55$. One example of this case is shown in Table 6.15.

Table 6.15 Ranking Scores Based on Modified Attribute Weights (when $w_{41} = 0.56$)

Alternatives	ASW	Rank	TOPSIS	Rank	BSM		Rank
					IV	BSM	
Type 214 submarine (A_1)	0.675	2	0.111	2	0.923	0.799	2
Type Kilo submarine (A_2)	0.873	1	0.889	1	0.731	0.802	1

The results shown in Table 6.15 can be explained by the following reason: even if A_2 does have very few good attribute values compared to A_1, each value's gap is not as serious as in the extreme alternative case. However, when there is a political pressure such as in Table 6.13, only the BSM can work properly in terms of best weapon decisions.

6.5 Summary

From this submarine selection problem, one can see how seriously a DM can affect the final decision. A DM can be any person with power such as a political leader, a group of DMs who decide a final decision, and so on. We showed that the current MADM methods did not work properly under this political power. However, the BSM could avoid this political pressure by compensating an overall value for individual attribute values.

Chapter Seven

Summary and Further Research

7.1 Summary

At the end of the nineteenth century and the beginning of the twentieth, the research for better decision making has begun by applying economic theory into human decision process. By 1960, MCDM acquired its own vocabulary (Pomerol and Romero, 2000). From 1975, numerous researches were made in MCDM areas. During this time, MCDM was divided into two different areas (i.e., MODM and MADM). Fishburn (1970), and Keeney and Raiffa (1976) are representative researchers in MODM area. Zionts (1978), Saaty (1980), Yoon (1980), and Zeleny (1982) are representative researchers in MADM area. By 1985, these methods had been recognized with many countries contributing (Pomerol and Romero, 2000).

In this research, we reviewed many MADM methods and found that there is a drawback in the current methods (i.e., lack of ability addressing political pressures which can be obstacles for the best weapon procurements). Therefore, the idea of compensating an alternative's overall value for its individual attribute values is suggested for overcoming this drawback. This idea is based on the following reasons: DMs can change an alternative's overall value by changing some weights but can not change alternatives' attribute values.

For compensating these two values, the concept of two statistics (i.e., mean and variance) was introduced. Then we found that the SAW method and the TOPSIS can be used for representing mean and variance type of information, respectively. However, since the TOPSIS use a non-linear function, we developed a new value function, which is linear and is called IV. Based on the concept of compensating mean type value for a variance type value, the SAW and IV are added and the sum is used as an alternative ranking score. This new method is referred to as the BSM.

The BSM can compensate an alternative's overall value for its individual attribute values. To further strengthen the proposed BSM, we presented a sensitivity analysis for what-if analysis for both consumers and suppliers. Computation results on several numerical examples indicate that the BSM can work properly as a generalized decision making model for naval weapon procurement, even when there are any political pressures that lead to extreme alternatives. We expect this method can also work well in other decision making situations, especially when the decision can be easily affected by some political powers.

7.2 Further Research

The BSM value function has the same weight for both overall and individual value functions (i.e., 0.5 for both the SAW and IV). However, one can ask that 0.5 may or may not be the best weight. And this question can be answered by determining two parameter values, α and β, shown in Equation 7.1. This equation came from the BSM value function with the consideration of different weights for the SAW and IV.

$$V(A_i) = \sum_{j=1}^{m} \left[\left(\alpha w_j + \frac{\beta}{m} \right) r_{ij} \right], \ \alpha, \beta \geq 0 \text{ and } \alpha + \beta = 1. \tag{7.1}$$

From this equation, alternatives rank can be changed by assigning different values of α and β. For example, let us consider the same problem in Table 3.1. There are two alternatives A_1 and A_2, and A_1 is considered as an extreme alternative because it has overall good score due to one extremely high attribute value. In other words, A_2 is ranked second by current MADM methods even if it has individually good attribute values. However, these alternatives have different ranking scores as well as different rank order by Equation 7.1 with different α and β. The table shows their ranking scores as well as rank order with respect to different α and β.

Table 7.1 Alternative Ranking Scores for Different α and β

	α	1	0.9	0.886	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
	β	0	1	0.114	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	$V(A_1)$	0.602	0.562	0.557	0.522	0.483	0.443	0.403	0.363	0.323	0.284	0.244	0.204
	$V(A_2)$	0.520	0.552	0.557	0.584	0.616	0.648	0.680	0.712	0.744	0.776	0.808	0.840
Rank	A_1	1	1	1	2	2	2	2	2	2	2	2	2
	A_2	2	2	1	1	1	1	1	1	1	1	1	1

From this table, one can see that when β >0.1143, $V(A_1) < V(A_2)$. However, determining these parameter values is another decision process, which can be difficult. One possible way is to ask DMs' opinions. This can be done either by an individual interview or brainstorming. This method can be useful because the two parameter values might be depend on weapon procurement environment.

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Appendices

Appendix A. Survey Forms for Pairwise Comparisons

Part 1

This survey form is designed to determine weights for the decision model of naval weapon procurement. Your sincere answers are very important to develop a successful decision model for naval weapon procurement. This model is expected to help the South Korean navy choose the best weapon in terms of performance as well as costs. Please read each question carefully and give your answers.

1. General questions
   a. What is your current rank? __________________
   b. How many years have you been in the Navy? _____________ years
   c. Have you ever worked for weapon procurements? _____ Yes _____ No

2. What is your opinion regarding the current weapon procurement system?
   a. It is a very proper system b. It is a proper system
   c. It should be improved d. It should be improved immediately

3. If you think that the current weapon procurement system is required to be improved, what is the most important problem that you are considering?
   a. No, there is no need to be improvement.
   b. There is no generalized weapon procurement decision model that can help decision makers decide best weapon.
   c. Not enough experts are in the Navy who can decide best weapons.
   d. There is a political pressure which can obstruct best weapon selection.
   e. The decision procedures are not open to the public.

Part 2 to be continued!

Appendix A. (Continued)

Part 2

The criteria below show how to compare two factors at a time. For example, if you think that factor A is very strongly more important than factor B, you should mark on the number 7 placed in A-side. The table right below the criteria shows this example.

▼ Criteria for pairwise comparisons

Intensity of importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one activity over another
5	Strong importance	Experience and judgment strongly favor one activity over another
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
9	Extreme importance	The evidence favoring one activity over another is of the highest possible order of affirmation

*** Intermediate scales such as 2, 4, 6, and 8 are possible to use!

▼ Example

The case that you think that factor A (Economic) is very strongly more important than factor B (Education) in terms of allocating budget.

Factor (A)	Relative Importance	Factor (B)
Economic	9 [✓ ·] 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Education

← A is more important   B is more important →

1. Among the five principles (i.e., operational performances, readiness on time, technical merits, cost effectiveness, and sustainment) which affect the decision for best weapon selection, please determine a relative importance between each principle in terms of the best weapon selection.

Factor (A)	Relative Importance	Factor (B)
Operational performance	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Readiness on Time
Operational performance	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Technical merits
Operational performance	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Cost effectiveness
Operational performance	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Sustainment
Readiness on Time	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Technical merits
Readiness on Time	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Cost effectiveness
Readiness on Time	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Sustainment
Technical merits	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Cost effectiveness
Technical merits	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Sustainment
Cost effectiveness	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Sustainment

← →

A is more important B is more important

2. Among the two factors (i.e., combat operational performances and regular operational performances) which compose the principle of operational performance, please determine a relative importance between these two factors in terms of weapon's operational performance.

Factor (A)	Relative Importance	Factor (B)
Combat operational performances	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Regular operational performances

◄──────── ────────►

A is more important B is more important

3. Among the three factors (i.e., readiness of operators, readiness of weapons, and readiness of supporting systems) which compose the principle of readiness on time, please determine a relative importance between these three factors in terms of readiness on time.

Factor (A)	Relative Importance	Factor (B)
Readiness of operators	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Readiness of weapons
Readiness of operators	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Readiness of supporting systems
Readiness of supporting systems	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Readiness of supporting systems

◄──────── ────────►

A is more important B is more important

4. Among the two factors (i.e., percentage of domestic components usage and technology acquisitions) which compose the principle of technical merits, please determine a relative importance between these two factors in terms of technical merits.

Factor (A)	Relative Importance	Factor (B)
Percentage of domestic components usage	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Technology acquisitions

← →

A is more important B is more important

5. Among the two factors (i.e., first acquisition costs and operational costs) which compose the principle of cost effectiveness, please determine a relative importance between these two factors in terms of cost effectiveness.

Factor (A)	Relative Importance	Factor (B)
First acquisition costs	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Operational costs

← →

A is more important B is more important

6. Among the three factors (i.e., logistics, maintenance, and reliability) which compose the principle of sustainment, please determine a relative importance between these three factors in terms of sustainment.

Factor (A)	Relative Importance	Factor (B)
Logistics	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Maintenance
Logistics	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Reliability
Maintenance	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Reliability

← A is more important   B is more important →

7. Among the two sub-factors (i.e., training time and number of people required) which compose the factor of readiness of operators, please determine a relative importance between these two sub-factors in terms of readiness of operators.

Factor (A)	Relative Importance	Factor (B)
Training time	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Number of people required

← A is more important   B is more important →

Appendix A. (Continued)

8. Among the two sub-factors (i.e., depot maintenance and field maintenance) which compose the factor of maintenance, please determine a relative importance between these two sub-factors in terms of maintenance.

Factor (A)	Relative Importance	Factor (B)
Depot maintenance	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Field maintenance

⟵ A is more important ⟶ B is more important

9. Among the two sub-factors (i.e., reliability of weapon and supplying company) which compose the factor of reliability, please determine a relative importance between these two sub-factors in terms of maintenance.

Factor (A)	Relative Importance	Factor (B)
Reliability of weapons	9 · 7 · 5 · 3 · 1 · 3 · 5 · 7 · 9	Reliability of company

⟵ A is more important ⟶ B is more important

Appendix B. C++ Weight Computation Program

```
#include <stdafx.h>

#include <iostream>

#include <stdlib.h>

#include <stdio.h>

#include <math.h>

#define MAX 16

float RI[MAX]={0, 0, 0, 0.58, 0.90, 1.12,

               1.24, 1.32, 1.41, 1.45, 1.49,

               1.51, 1.48, 1.56, 1.57, 1.59};

float matrix[MAX][MAX];

float row_rate[MAX];

int mtrx_size;

FILE *in;

// ***********************************************************/

void printmatrix(){

      int i,j;

      for(i=1; i<= mtrx_size; i++)

            for (j=1; j<=mtrx_size; j++)

                  scanf("%f",&matrix[i][j]);

      printf("\n\n1. Given matrix is as follows !\n\n");

      for(i=1; i<=mtrx_size; i++){

            for (j=1; j<=mtrx_size; j++)

                  printf("%6.2f", matrix[i][j]);

            printf("\n");

      }

}

//************************************************************/
```

```
void value_added_vector(){
      float row_mul[MAX];
      float row_nsqr[MAX];
      float sum = 0.0;
      int i, j;
      for (i=1; i<=mtrx_size; i++){
            row_mul[i]=1;
            for (j=1; j<= mtrx_size; j++)
                  row_mul[i] *= matrix[i][j];
      }
      for (i=1; i<=mtrx_size; i++){
            row_nsqr[i] = pow(row_mul[i], (float) 1.0/mtrx_size);
            sum +=row_nsqr[i];
      }
      printf("\n2. Value added Vector is as follows ! \n\n");
      for (i=1; i<=mtrx_size; i++){
            row_rate[i] = row_nsqr[i] / sum;
            printf("%7.3f", row_rate[i]);
      }
}
//****************************************************************/
void cal_cr(){
      float prod[MAX] = {0.0, };
      float con_did[MAX];
      float sum1 = 0.0;
      float con_idx;
      float ci;
```

```
      float cr;
      int i, j;
      for(i=1; i<=mtrx_size; i++)
            for (j=1; j<=mtrx_size; j++)
                  prod[i] += matrix[i][j] * row_rate[j];
      for(i=1; i<=mtrx_size; i++){
            con_did[i] = prod[i]/row_rate[i];
            sum1 += con_did[i];
      }
      printf("\n\n3. Consistency : ");
      con_idx=sum1/mtrx_size;
      ci= (con_idx - mtrx_size)/(mtrx_size-1);
      cr=ci/RI[mtrx_size];
      printf("%7.2f", cr);
}
//****************************************************************/
main(int argc, char* argv[]){
      int i,j;
      /*if((in = fopen(argv[1], "rt")) == NULL){
            printf("\n\nError opening the input file !!!\n");
                  exit(0);
            }*/
      //cin>>mtrx_size;
      scanf("%d", &mtrx_size);
      printmatrix();
      value_added_vector();
      cal_cr();
```

```
      //fclose(in);


}
```

About the Author

Jin O Chang received a B.S. degree in Management Science in 1992 from South Korea Naval Academy. He was then commissioned an ensign and worked as a naval officer until 1995. In 1995, he was selected as a navy scholarship student for a Master degree. In 1997, he received a M.S. degree in Aero Space Engineering from Advance Institute of Military Science and Technology, Seoul, Korea. After that he returned to the navy and worked in various areas. In 2002, he was selected as a student to be sent abroad on government support and joined the Ph.D. Program in the Department of Industrial and Management System Engineering at the University of South Florida. His research interests include decision modeling, scheduling, statistical analysis, mathematical programming and simulation.