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A New Hybrid Method for Estimating Hydrologically Induced Vertical Deformation From GRACE and a Hydrological Model: An Example From Central North America

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Abstract Hydrologically induced deformation of Earth’s surface can be measured with high precision geodetic techniques, which in turn can be used to study the underlying hydrologic process. For geodetic study of other Earth processes such as tectonic and volcanic deformation, or coastal subsidence and its relation to relative sea level rise and flood risk, hydrological loading may be a source of systematic error, requiring accurate correction. Accurate estimation of the hydrologic loading deformation may require consideration of local as well as regional loading effects. We present a new hybrid approach to this problem, providing a mathematical basis for combining local (near field) and regional to global (far field) loading data with different accuracies and spatial resolutions. We use a high-resolution hydrological model (WGHM) for the near field and GRACE data for the far field. The near field is defined as a spherical cap and its contribution is calculated using numerical evaluation of Green’s functions. The far field covers the entire Earth, excluding only the near-field cap. The far-field contribution is calculated using a modified spherical harmonic approach. We test our method with a large GPS data set from central North America. Our new hybrid approach improves fits to GPS-measured vertical displacements, with 25% and 35% average improvement relative to GRACE-only or WGHM-only spherical harmonic solutions. Our hybrid approach can be applied to a wide variety of environmental surface loading problems.

1. Introduction

Earth’s surface experiences dynamic loading on a wide variety of temporal and spatial scales. In addition to solid Earth processes, such as earthquakes and volcanoes, mass distribution throughout Earth’s outer fluid envelope, including its atmosphere, ocean, and cryosphere, as well as groundwater movements in the porous uppermost continental crust, occurs with significant seasonal to decadal variability. Loading of the solid Earth displaces the ground surface and changes the gravitational potential. Loading usually causes an elastic response (horizontal and vertical surface displacements), depending on the temporal and spatial scales of the load, and the mechanical properties of the solid Earth. Gravity changes can be divided into the direct effect, related to the gravitational attraction of the load itself, and the indirect effect resulting from Earth deformation.

For more than two decades, continuously operating geodetic networks (e.g., GPS, VLBI, SLR) and satellite InSAR (Interferometric Synthetic Aperture Radar) have provided essentially global observations, and corresponding opportunities to study the elastic response to surface loading. These techniques are also useful for measuring poroelastic and unrecoverable (inelastic) deformation associated with hydrologic phenomena. While loading theory cannot be used to study the porous response of Earth’s surface to groundwater discharge and recharge (typically a short spatial wavelength effect; see section 5.1 and Argus et al., 2014), correct calculation of the loading response (typically a long spatial wavelength effect) may be important when using geodetic data to investigate hydrologic processes, since the space geodetic data incorporate both short and long wavelength effects.

The elastic response of the Earth’s crust to an arbitrary surface load distribution can be estimated through spatial convolution of appropriate Green’s function with a surface mass loading field. The Green’s functions describe the deformation of the Earth under action of a unit surface point mass through the three-dimensionless load Love numbers for a spherical nonrotating elastic layered Earth model (Farrell, 1972; Farrell, 1977).
Longman, 1963) or through the elastic moduli (Poisson’s ratio and Young’s modulus) for a layered elastic half-space model (Farrell, 1972). The half-space model provides a useful analytical solution when the load extends over a small area and Earth’s curvature can be neglected (e.g., for volcanic deformation, for smaller earthquakes, and for smaller aquifers and lakes). For larger areas, the amplitude and phase of the vertical and horizontal components of the deformation are better explained with a spherical nonrotating elastic layered Earth model (Chanard et al., 2014). In addition, if the spatial extent of the load is large, the Young’s modulus may have to be unrealistically increased to include more distant loads in the computation (e.g., Chanard et al., 2014; Fu & Freymueller, 2012; Karegar et al., 2014).

The convolution integral of Green’s functions with a surface mass loading field for a spherical nonrotating elastic layered Earth model can be analytically expressed in terms of spherical harmonics (e.g., Le Meur & Hindmarsh, 2000; Mitrovica et al., 1994). The spherical harmonic approach is commonly used to model Earth’s surface elastic response to environmental loading (e.g., Blewitt, 2003; Chanard et al., 2014; Davis et al., 2004; Fu & Freymueller, 2012; Kusche & Schrama, 2005; van Dam et al., 2007; Wahr et al., 2013; Zou et al., 2014). This approach requires as input the spherical harmonic coefficients of the geoid changes (Stokes coefficients) and the load Love numbers. The major advantage of the spherical harmonic approach is that spectral analyses such as analysis of degree variance spectrum and degree correlations can be readily implemented (e.g., Yan et al., 2016). Furthermore, the same filtering and smoothing procedures that are used for satellite observations (e.g., GRACE) can be applied to the modeled deformation when using hydrological models (e.g., Döll et al., 2014). The spherical harmonic approach is particularly suitable when the modeling spans a large part or all of Earth’s surface so that most of the load energy is concentrated in the low-degree spherical harmonics. But, it does require an accurate prediction of far-field contributions (e.g., Glacial Isostatic Adjustment or global sea level rise). However, the spatial resolution of modeled deformation is independent of position. For example, high spatial resolution in one region requires the same resolution for other regions, which increases processing time (Mitrovica et al., 1994). Furthermore, to properly represent the geometrical properties of the surface mass load, the correct choice of truncation of the spherical harmonic expansion (cutoff value) can be challenging (Le Meur & Hindmarsh, 2000; Yan et al., 2016). The convolution integral can alternatively be computed by numerical techniques in the space domain (often called the Green’s function approach). The Green’s function approach is suitable for regional and basin-scale studies where mass changes adjacent to the location of geodetic sites have the largest influence on the displacements. However, this approach is sensitive to the total number of terms used to calculate the Green’s functions. In practice, the convergence problem can be overcome by taking into account the asymptotic expressions of the load Love numbers and Kummer’s transformation (see section 2).

The Green’s function approach and the spherical harmonic approach have been widely used to model the effect of hydrological loading on geodetic position or displacement time series. Among the various types of geophysical fluid loads (atmosphere, ocean, and cryosphere), hydrological loading often represents the dominant signal in the vertical component, especially at multiannual and shorter periods (e.g., Fritsche et al., 2012; Jiang et al., 2013). Accurate modeling of these effects in position time series is therefore beneficial for reliable long-term surface velocity estimates (Klos et al., 2017; Santamaría-Gómez & Mémín, 2015) and noise analysis (Bogusz & Klos, 2016; Davis et al., 2012; Santamaría-Gómez et al., 2011), as well as studies of tectonic processes (Bennett, 2008; Blewitt & Lavallée, 2002; Bos et al., 2010; Vergnolle et al., 2010), volcanic processes (Henderson & Pritchard, 2017), geodynamics (Chanard et al., 2018; Clarke, 2018), sea level rise (Santamaría-Gómez et al., 2017), geomechanics (Karegar et al., 2015b), and reference frame definition (Collier-leux et al., 2012; Krásná et al., 2015).

Since 2002, the Gravity Recovery and Climate Experiment (GRACE) satellite mission has described monthly changes in Earth’s gravitational potential, reflecting mass redistribution close to the Earth’s surface (Tapley et al., 2004). The raw GRACE data (Level-1) are processed by different research centers, and the time-variable gravitational field is determined either in the form of spherical harmonics, or as mascons (mass concentrations). Noise is removed through filtering the Stokes coefficients during postprocessing or through the introduction of a priori information (geophysical models) in the mascon solution. Although these filters are quite effective, they damp geophysical signals and limit GRACE resolution at spatial scales smaller than the native resolution (300 km). Hence, a rigorous comparison of observed deformation (e.g., from GPS) and GRACE-based modeled deformation should be based on a regionally coherent deformation signal among neighboring sites rather than on individual deformation signals at single sites. One could
spatially filter the GPS time series in a way that is similar to the GRACE products, estimating deformation caused only by those components of the mass distribution that have scales similar to GRACE products (~400–500 km), but this results in information loss, i.e., local hydrological deformation with spatial scales smaller that 400–500 km.

So far, modeling the effects of hydrological loading on geodetic position time series has been limited to water storage data sets from GRACE or hydrological models. In this paper, we describe and evaluate a new hybrid approach for more accurate estimation of the Earth’s elastic response to hydrological loading. We combine water storage data from GRACE with limited spatial resolution, and a hydrological model with more detailed spatial resolution. Our proposed hybrid approach combines the spherical harmonic approach and the Green’s function approach, providing a flexible way that benefits from each method’s strengths. We use this hybrid approach to investigate vertical crustal changes in the continental interior of North America, a region that is undergoing spatially variable water stress. We show that our proposed approach results in a solution with a better fit to the observed deformation at individual GPS sites. The key advantage to our hybrid approach is that it provides a mathematical basis for combining global and regional loading data with different spatial resolutions for regions closer and regions farther away from the point of computation (e.g., geodetic sites). Such a combination is useful for loading studies, as the larger impact of small-scale mass changes in the near field makes it important to include higher resolution data set, while not neglecting effect of large-scale mass changes in the far field (Dill & Dobslaw, 2013). The method can be applied to a wide variety of environmental surface loading problems including hydrologic, ice, ocean, and atmospheric loads.

The remainder of this paper is organized as follows. In section 2, we derive a hybrid formula to model observed vertical surface displacement based on a combination of the Green’s function approach to describe near-field deformation, and the spherical harmonic approach for far-field deformation. In section 3, we describe the study region. In section 4, we describe the GPS, GRACE, and hydrological model data sets and their respective processing. In section 5, we describe the model calculations. Section 6 presents results, section 7 briefly describes potential applications, and Section 8 summarizes the conclusions.

2. A New Hybrid Method for a Spherical Nonrotating Elastic Layered Earth Model

The Green’s function approach establishes a convolution integral relationship between the surface mass density variations $\Delta \sigma(\theta', \lambda', t)$, that is available at integration point $(\theta', \lambda')$ at time $t$ over a certain integration domain, and vertical displacement $\nu(\theta, \lambda, t)$ at a computation point $(\theta, \lambda)$ (Mitrovica et al., 1994; equation (23));

$$\nu(\theta, \lambda, t) = a^2 \int_\sigma \Delta \sigma(\theta', \lambda', t) \ G(\psi) \ d\sigma$$  \hspace{1cm} (1)

where $a$ is the mean Earth’s radius, $\sigma$ is the integration domain which is the surface of a whole sphere, and $d\sigma$ are infinitesimally small surface compartments. The quantity $G(\psi)$ denotes the vertical Green’s function, which is a function of the angular distance $\psi$ between the computation point and the integration point. The need for performing the integration over the entire surface of the Earth in the Green’s function approach makes this method computationally very expensive. Therefore, the integration is generally performed over a limited area or truncated spherical cap. Neglecting loading data outside the spherical cap results in errors which might adversely affect the predicted deformation. We divide the integration domain into a spherical cap with certain radius $\psi_0$ around each computation point and the reminder of the sphere. The contribution of loading within the spherical cap $\nu_{\psi_0}$ is evaluated using discrete numerical integration of Green’s functions and the gridded water storage data from a hydrological model (see section 2.1). This term is called contribution of the near field ($\nu_{nf}$). The contribution of the distant load (the far field, $\nu_{ff}$) for the region outside the spherical cap is computed using the spherical harmonic approach. Our approach is similar to that of Molodenskii (1962), who used for the geoid determination to calculate the contribution of the far field. The vertical deformation can be then written as a sum:

$$\nu(\theta, \lambda, t) = \nu_{\psi_0}(\theta, \lambda, t) + \nu_{nf}(\theta, \lambda, t)$$ \hspace{1cm} (2)

where each term will be formulated for the point-load type of the vertical Green’s functions.
The point-load Green’s function $G(\psi)$ for computation of vertical displacement in equation (1) is given in terms of load Love numbers $h_n$ and the Legendre polynomials of degree $n$, $P_n(\cos \psi)$ (Farrell, 1972):

$$G(\psi) = \frac{a}{m_e} \sum_{n=0}^{\infty} h_n P_n(\cos \psi)$$  \hspace{1cm} (3)

where $m_e$ is the total mass of the Earth. The Green’s function (3) is truncated at a cutoff value $N_{\text{max}}$, causing oscillations whose wavelengths are controlled by the cutoff (known as Gibbs phenomena) and results in an incorrect response (e.g., Le Meur & Hindmarsh, 2000). The load Love numbers $h_n$ reach an asymptotic non-zero value (denoted with $h_\infty$) when $n$ gets large. Thus, the Green’s function does not converge, particularly when the angular distance $\psi$ is very small, i.e., for $\psi = 0$, $P_n(\cos \psi) = 1, \forall n$. To avoid the Gibbs effect and speed up converging series (3), Kummer’s transformation is used. This consists of adding and subtracting the asymptotic value $h_\infty$ from series (3) (Farrell, 1972; Le Meur & Hindmarsh, 2000):

$$G(\psi) = \frac{a}{m_e} \frac{h_\infty}{2 \sin \frac{\psi}{2}} + \frac{a}{m_e} \sum_{n=0}^{\infty} (h_n - h_\infty) P_n(\cos \psi)$$  \hspace{1cm} (4)

where the first term represents the infinite sum of the Legendre polynomials $P_n$. Several load Love numbers have been computed for a spherical nonrotating elastic layered Earth model based on seismic wave travel times. Here we use a set of high-degree load Love numbers produced in the CE frame (a frame whose origin is fixed to the center of mass of the solid Earth) and extended to degree 45,000 determined by Wang et al. (2012) for the PREM (Preliminary Reference Earth Model) (Dziewonski & Anderson, 1981).

2.1. Contribution of the Computation Point and Near Field From the Green’s Function Approach

The point-load Green’s function is singular for angular distance $\psi = 0$. Therefore, an appropriate treatment should be chosen when evaluating the convolution integral (1) at the computation point. We use a similar approach to that of Novák et al. (2001) who treat the singularity of Stokes integral for the geoid determination. We add and subtract the surface mass density at the computation point

$$\nu_{nf}(\theta, \lambda, t) = a^2 \int_{\psi=0}^{\psi=\psi_0} \{ \Delta \sigma_P \ G(\psi) + [\Delta \sigma(\theta', \lambda', t) - \Delta \sigma_P] \ G(\psi) \} \ d\psi \ d\sigma$$  \hspace{1cm} (5)

Note that the integration in equation (5) is performed over a limited spherical cap $\psi_0$. Equation (5) is split into two integrals:

$$\nu_{cf}(\theta, \lambda, t) = a^2 \int_{\psi=0}^{\psi=\psi_s} \Delta \sigma_P \ G(\psi) \ d\psi \ d\sigma$$  \hspace{1cm} (6)

which is the contribution of the computation point and

$$\nu_{sc}(\theta, \lambda, t) = a^2 \int_{\psi=0}^{\psi=\psi_s} [\Delta \sigma(\theta', \lambda', t) - \Delta \sigma_P] \ G(\psi) \ d\psi \ d\sigma$$  \hspace{1cm} (7)

which is the contribution of rest of the spherical cap near the surface load. Equation (7) is no longer singular at the angular distance $\psi = 0$ since $\Delta \sigma (\theta', \lambda', t) = \Delta \sigma_P$ at the computation point and the value of the integral equals zero. To compute the contribution of the computation point, we use the analytical formalism below:

$$\nu_{cp}(\theta, \lambda, t) = 2\pi a^2 \Delta \sigma_P \int_{\psi=0}^{\psi=\psi_s} G(\psi) \ \sin \psi \ d\psi$$  \hspace{1cm} (8)

The integral of Green’s function over the spherical cap $\psi_0$ can be written as the difference between integral over the whole sphere ($0 \leq \psi \leq \pi$) and integral over the area outside the spherical cap ($\psi_0 \leq \psi \leq \pi$):

$$\int_{\psi=0}^{\psi=\psi_s} G(\psi) \ \sin \psi \ d\psi = \int_{\psi=0}^{\psi=\pi} G(\psi) \ \sin \psi \ d\psi \ - \int_{\psi=\psi_0}^{\psi=\pi} G(\psi) \ \sin \psi \ d\psi$$

[Note: The equation and text are continued as in the original document.]
from the orthogonality of the Legendre polynomials over the sphere (Heiskanen & Moritz, 1967; section 1–13):

$$\int_{0}^{\pi} G(\psi) \sin \psi \, d\psi = \int_{0}^{\pi} G(\psi) \sin \psi \, d\psi - \int_{0}^{\pi} G(\psi) \sin \psi \, d\psi$$

The degree-zero load Love number ($h_0$) equals zero to ensure that the Earth’s total mass is conserved. Thus,

$$\int_{0}^{\pi} G(\psi) \sin \psi \, d\psi = -\int_{0}^{\pi} G(\psi) \sin \psi \, d\psi$$

The integral on the right-hand side of equation (11) is then of the form:

$$Q_0(\psi_0) = \int_{0}^{\pi} G(\psi) \sin \psi \, d\psi$$

which has an analytical derivation (see section 2.2 and equation (23)). Finally the contribution of the surface mass density at the computation point to the vertical deformation is given by:

$$v_{cp}(\theta, \lambda, t) = -2\pi a^2 \Delta \sigma_0 G(\psi_0)$$

The convolution integral (1) is generally discretized and numerically evaluated by means of summation of the products $D_r(h_0, k_0, t)$ over compartments within the spherical cap $\psi_0$ (e.g., Heiskanen & Moritz, 1967). After accounting for contribution of computation point, the contribution of the rest of cap is:

$$v_{rc}(\theta, \lambda, t) = \sum_{k} (\Delta \sigma_k - \Delta \sigma_0) G(\psi_k) A_k$$

where $\Delta \sigma_k$ and $A_k$ are the mass density anomaly and the area of $k$th compartment, respectively. $G(\psi_k)$ is the value of Green’s function at the center of $k$th compartment which approximates the mean value of Green’s function across the compartment. Finally, the contribution of near field is a sum:

$$v_{nf}(\theta, \lambda, t) = v_{cp}(\theta, \lambda, t) + v_{rc}(\theta, \lambda, t)$$

2.2. Contribution of the Far Field From the Spherical Harmonic Approach

In this section, we find a spectral formalism based on spherical harmonics for the contribution of surface mass density outside the integration domain $\psi_0$. As mentioned earlier, within the spherical cap $\psi_0$ we use the Green’s function approach to estimate the vertical deformation. This leads to an error due to neglecting the contribution of loading outside the spherical cap $\psi_0$ (the contribution of far field $v_{nf}$):

$$v_{ef}(\theta, \lambda, t) = 2\pi a^2 \int_{0}^{\pi} \int_{0}^{\frac{\pi}{2}} \Delta \sigma(\theta', \lambda', t) G(\psi) \, d\psi \, d\sigma$$

which may be expressed over the whole sphere by:

$$v_{ef}(\theta, \lambda, t) = 2\pi a^2 \int_{0}^{\pi} \int_{0}^{\frac{\pi}{2}} \Delta \sigma(\theta', \lambda', t) G(\psi) \, d\psi \, d\sigma$$

upon the introduction of the truncated Green’s function $G(\psi)$ defined by:
\[ 0 \leq 0 \leq \psi_0 \]
\[ G(\psi) \psi_0 < 0 \leq \pi \] (18)

which is complementary to the integration kernel in equation (5). The truncated Green’s function \( \overline{G}(\psi) \) in now expanded in series of orthogonal Legendre polynomial basis functions:

\[ \overline{G}(\psi) = \frac{a}{m_e} \sum_{n=0}^{\infty} \sum_{k=0}^{2n+1} \frac{1}{2} Q_n(\psi_0) P_n(\cos \psi) \] (19)

where the expansion (truncation) coefficients \( Q_n(\psi_0) \) are derived using properties of series expansions in terms of orthogonal polynomials, analogous to a Fourier series (e.g., Gottlieb & Orszag, 1977):

\[ Q_n(\psi_0) = \frac{m_e}{a} \int_{\psi = 0}^{\pi} \overline{G}(\psi) P_n(\cos \psi) \sin \psi d\psi = \frac{m_e}{a} \int_{\psi = \psi_0}^{\pi} G(\psi) P_n(\cos \psi) \sin \psi d\psi \] (20)

which follow from the orthogonality relationships for Legendre polynomials. The surface mass density at the computation point \( \Delta \sigma(\theta, \lambda, t) \) is given by infinite series of surface spherical harmonics (e.g., Wahr et al., 1998):

\[ \Delta \sigma(\theta, \lambda, t) = a \rho_w \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left( \Delta \tilde{C}_{nm} \cos m \lambda + \Delta \tilde{S}_{nm} \sin m \lambda \right) \overline{P}_{nm}(\cos \theta) \] (21)

where \( \sigma_n \) is the \( n \)th degree surface spherical harmonic of surface mass density, \( \rho_w \) is the density of water (~1,000 kg/m^3), \( \Delta \tilde{C}_{nm} \) and \( \Delta \tilde{S}_{nm} \) are dimensionless spherical harmonic coefficients of surface mass density at time \( t \) which can be related to the GRACE stokes coefficients (e.g., Wahr et al., 1998, equation (13)). \( \overline{P}_{nm}(\cos \theta) \) is a fully normalized associated Legendre function of degree \( n \) and order \( m \). Using the two series, equations (21) and (19), in equation (17), and performing the integrations over the sphere, after some simplification (e.g., Mitrovica et al., 1994) we arrive at the following spectral representation:

\[ \nu_M(\theta, \lambda, t) = \frac{a \rho_w}{2 \rho_{ave}} \sum_{n=0}^{\infty} Q_n(\psi_0) \sigma_n(\theta, \lambda, t) \] (22)

where \( \rho_{ave} = m_e/4\pi a^3 \) is the average density of the Earth (~5.517 kg/m^3). Equation (22) is used to replace the integral (17), which yields a series expansion of the contribution of the far field in terms of the surface spherical harmonics of the surface mass density. It shows that the contribution of the far field is a function of the truncation coefficients \( Q_n(\psi_0) \). By inserting equation (4) into equation (20) and interchanging the order of summation and integration, we obtain:

\[ Q_n(\psi_0) = h_{\infty} \int_{\psi = \psi_0}^{\pi} \frac{\sin \psi}{2 \sin^2 \frac{\psi}{2}} P_n(\cos \psi) d\psi + \sum_{k=0}^{\infty} (h_k - h_{\infty}) \int_{\psi = \psi_0}^{\pi} P_k(\cos \psi) P_n(\cos \psi) \sin \psi d\psi \] (23)

We then can derive the following expression for \( Q_n(\psi_0) \):

\[ Q_n(\psi_0) = h_{\infty} b_n + (h_n - h_{\infty}) R_{n,n}(\psi_0) + \sum_{k=0}^{\infty} (h_k - h_{\infty}) R_{n,k}(\psi_0) \] (24)

where the coefficients \( b_n \) are:

\[ b_n = \int_{\psi = \psi_0}^{\pi} \frac{\sin \psi}{2 \sin^2 \frac{\psi}{2}} P_n(\cos \psi) d\psi \] (25)

which can be computed using the recursive algorithms of Meissl (1971) given in Appendix A. The coefficients \( R_{n,k}(\psi_0) \) are:
which can be evaluated using the recursive algorithms of Paul (1973) given in Appendix B. The truncation coefficients \( Q_n(\psi_0) \) as a function of spherical cap \( (\psi_0, \text{integration domain}) \) is an important parameter that controls the contribution of the far field to the vertical displacement. For example, when the integration domain spans the whole sphere, i.e., \( \psi_0 = 0 \), \( R_n(0) = 0 \) due to orthogonality of Legendre polynomials and \( R_n(0) = \frac{2}{\pi} \) (Heiskanen & Moritz, 1967), therefore \( Q_n(0) = \frac{2}{\pi} \) and \( G(\psi) = G(\psi) \). Thus, the convolution integral (1) and equation (22) reduce to the spherical harmonic representation of the vertical deformation (the spherical harmonic approach). The validity of truncation coefficients can be further verified by varying the angular distance from zero to \( \pi \). Figure 1b shows that the values of truncation coefficients are diminished with increasing integration domain \( (\psi_0) \). Truncation coefficients at higher degrees \( (n) \) decay faster than those at lower degrees, emphasizing the relative importance of the nearby mass load.

3. Study Area

GRACE observations have been extensively used to model hydrological loading in geodetic position time series at global and regional scales. A number of studies have emphasized the significance of GRACE-derived hydrological loading signals in global GPS networks (Döll et al., 2014; Horwath et al., 2010; King et al., 2006; Kusche & Schrama, 2005; Rietbroek et al., 2012, 2014; Tesmer et al., 2011; Tregoning et al., 2009; Yan et al., 2016). Consistency between GPS and GRACE-derived deformation has been observed in large drainage basins where seasonal changes are significant, including Africa (Birhanu & Bendick, 2015; Nahmani et al., 2012), the Amazon River Basin, and South America (Davis et al., 2004, 2008; Fu et al., 2013), Europe (Valty et al., 2013; van Dam et al., 2007), the Himalayan region (Chanard et al., 2014; Fu et al., 2013; Fu & Freymueller, 2012; Liu et al., 2014; Steckler et al., 2010), Southern Alaska (Fu et al., 2012), the Western United States (Argus et al., 2014, 2017; Fu et al., 2015), Australia (Han, 2017), and near the Greenland Ice Sheet (Liu et al., 2017).

Here we compare GPS and GRACE-derived vertical deformation for a large region in the interior of North America, including the Mississippi River basin and Texas. The Mississippi River basin is the fifth largest in the world in terms of water discharge. It drains \( \sim 40\% \) of the contiguous U.S., discharging a yearly average of about 1.5 km\(^3\)/d into the Gulf of Mexico. It is also fifth in drainage basin size, with a drainage area of \( 4.76 \times 10^6 \) km\(^2\). The basin spans three different climate regimes: cold in the north, subtropical in the southeast, and dry in the southwest. The basin is divided into five main subbasins (Figure 2): (1) the Missouri River basin, (2) the upper-Mississippi River basin, (3) the Ohio and Tennessee River basin (4) the Arkansas and Red River basin, and (5) the lower-Mississippi River basin. Most of Texas is hydrologically isolated from the Mississippi Basin, with climate that varies from humid in the east (maximum annual precipitation 140 cm) to arid in the west (maximum annual precipitation 30 cm). Groundwater management is critical in west Texas.
because of diminishing water supplies and frequent droughts, while flood management is critical along the
Gulf Coast from both hurricane storm surge and high rainfall events (Karegar et al., 2015a). The Mississippi
basin and Texas include extensively irrigated regions and some of the world’s significant aquifer systems:
the High Plains aquifer, Atlantic and Gulf Coastal Plains Aquifer, and Cambro-Ordovician Aquifer System
(WHYMAP & Margat, 2008).

GRACE-based studies indicate that soil moisture and groundwater changes comprise the majority of total
water storage (TWS) changes in the Mississippi basin and Texas region (Freedman et al., 2014; Kim et al.,
2009; Long et al., 2013; Rodell et al., 2007). However, river and snow loads are still of important components
of TWS variability in the Mississippi basin. For example, Kim et al. (2009) showed that rivers and snow con-
tribute up to 28% of TWS variability. The TWS change in the southeast Mississippi basin (Ohio and Tennes-
see River basin and the lower-Mississippi River basin) shows a strong seasonal cycle (Figure 2) with
maximum loading and storage in spring and early summer, and minimum loading in late fall and early win-
ter (see Figure 9 in Humphrey et al., 2016). The area is characterized by long humid summers and relatively
short, mild winters. Annual precipitation averages about 100–150 cm but exceeds 150 cm near the
costline.

4. Data Processing

4.1. GRACE Data

We produce monthly TWS and surface mass density ($\Delta r$) estimates based on postprocessing the GRACE
Stokes coefficients (RL05) provided by the Center for Space Research (CSR) at the University of Texas for the
period 2002–2015. The GRACE Stokes coefficients describe changes in the geoid surface. We postprocessed
the nonisotropic filtered Stokes coefficients (Kusche, 2007) provided by ICGEM (http://icgem.gfz-potsdam.
de/ICGEM/TimeSeries.html) by replacing zonal degree 2 Stokes coefficients ($C_{20}$) with the more reliable solu-
tion from analysis of Satellite Laser Ranging (SLR) measurements and adding degree 1 Stokes coefficients
($C_{10}$, $C_{11}$, and $S_{11}$) obtained from oceanic models available at the NASA-JPL Tellus website (ftp://podaac.jpl.
nasa.gov/allData/tellus/L2/degree_1/) (Swenson et al., 2008). The spherical harmonic expansion was trun-
cated at degree and order 90. These coefficients are nonisotropic filtered Stokes coefficients (DDK2) where
the isotropic part resembles a Gaussian filter with a half width of 340 km (Kusche et al., 2009). The filtered
Stokes coefficients are used in equation (21) to calculate the surface spherical harmonic of surface mass
density ($r_{n}$), which is then applied in equation (22) to estimate the contribution of the far field in our hybrid
approach.

Figure 2. RMS scatters of detrended total water storage (2002–2015) from (a) GRACE DDK2-filtered Stokes coefficients (CSR-RL05). (b) WaterGap Global Hydrologi-
cal Model (WGHM, version 2.2b). (c) North American Land Data Assimilation System (NLDAS2-Noah). The red dots represent the location of GPS stations used in
this study. Also shown are the seven subbasins: (1) the Missouri River basin, (2) the upper-Mississippi River basin, (3) the Ohio and Tennessee River basin, (4) the
Arkansas and Red River basin, (5) the lower-Mississippi River basin, (6) the Rio Grande basin, and (7) the Texas and Gulf basin.
4.2. Hydrological and Land-Surface Models

We use monthly terrestrial water storage estimate from two high spatial resolution models: (1) the Water-Gap Global Hydrological Model (WGHM) (Döll et al., 2003) and (2) the North American Land Data Assimilation System (NLDAS-Noah) (Mitchell et al., 2004). WGHM is a global hydrological model with 0.5° spatial resolution accounting for all main compartments of continental total water storage including groundwater but excluding the polar ice sheets and glaciers. The model has been calibrated by observed long-term average river discharge at river gauges. We use the latest version of WGHM (version 2.2b, Müller Schmied et al., 2014) calibrated for monthly time series of observed precipitation WFDEI (Watch Forcing Data based on ERA-Interim) and GPCC (Global Precipitation Climatology Center) data. WGHM has been widely used for estimating global and regional hydrological flux and storage (e.g., Scanlon et al., 2018), and loading deformation at global scale (e.g., Döll et al., 2014; Fritsche et al., 2012). NLDAS-Noah is a land-surface model that provides monthly changes in soil moisture, snow equivalent water height, and canopy surface water over the U.S. with 0.125° spatial resolution. Unlike WGHM, NLDAS does not explicitly represent groundwater and surface water storage. We chose the NLDAS model for comparison to the WGHM model because it has the highest spatial resolution of the modeled water storage data sets available at continental scale. In addition, the NLDAS model is a physically based land-surface model that integrates more observations at the surface while the WGHM model is a conceptual model with very simplified realization of physical processes of hydrosphere that is only calibrated and validated by river discharge measurements. We use the NLDAS-2 Noah version (Xia et al., 2012) with improved accuracy and consistency of climate data and with upgraded model parameters compared to the first version of NLDAS (NLDAS-1) (Mitchell et al., 2004). TWS changes from hydrological models and land-surface models compared with GRACE TWS observations have shown the limitations of models for resolving large-scale TWS changes (section 6 and supporting information Figure S4) due to errors in climate data and uncertainty in the model structure and hydrologic process representation (e.g., Eicker et al., 2014; Scanlon et al., 2018).

Water storage changes from the models are detrended and are used in equation (15) to compute the contribution of the near field in the hybrid approach. Note that the aim of this study is not to assess performance of different hydrological and land-surface models. Rather, comparison of results from two different models provides information on the reliability and evaluation of our hybrid approach.

4.3. GPS Data

We use publicly available data from continuous GPS sites. Data from a total of 762 GPS sites with nearly continuous observations is available, with durations ranging from 4 to 20 years. The raw GPS data were processed using the software package GIPSY/OASIS II (Release 6.3) of JPL and the precise point positioning technique using 24 h data batches. JPL’s reprocessed precise satellite orbit and clock parameters (Repro 2.1, Desai et al., 2014) together with the absolute GPS receiver and satellite antenna phase calibration model igs08.atx (Schmid et al., 2016) are used to generate nonfiducial daily point position time series. Since the GPS satellites orbit are expressed in Center of Mass (CM) of the Earth’s system (solid Earth and its fluid envelope), the nonfiducial position time series are calculated in the CM frame. We applied the same processing strategy used in JPL’s reanalysis of orbit and clock products (Repro 2.1) to our GPS raw data to isolate effects of inconsistency between orbits/clocks and our solutions. The first-order effects of the ionosphere are corrected using the dual-frequency approach and a linear combination of carrier phase measurements. The second-order effects are corrected using the JPL IONEX model for the data after 1999 and the IGS IRI2012 model for the data prior to 1999. Tropospheric effects are modeled using the troposphere mapping function GPT2 (Lagler et al., 2013) which maps the zenith troposphere delay to the elevation of each observation. A priori zenith tropospheric delays are estimated from GPT2. The elevation cutoff angle is fixed to 7°, a compromise to better constrain tropospheric effects but minimize multipath errors. The single station ambiguity resolution algorithm of Bertiger et al. (2010) is adopted to solve integer ambiguities for each station. The effects of solid Earth tides, solid Earth and ocean pole tides were corrected following the IERS 2010 conventions (Petit & Luzum, 2010). These models are consistent with the background models used in raw GRACE data processing (e.g., Watkins et al., 2015; Wiese, 2015).

The effects of atmospheric and oceanic mass changes should be removed from the GPS time series to be consistent with modeled deformation from GRACE and WGHM. Tidal contributions of ocean loading, which can reach several mm in the vertical component for near-coastal sites, are corrected using 11 tidal constituents from the latest version of Ray’s (1999) global ocean tide model GOT4.8 in the CM frame. The current
version of GIPSY software does not incorporate the total effects of atmospheric (tidal + nontidal) and non-
tidal oceanic mass loading at the observation level. However, these effects are removed at a postprocessing
step using 1° × 1° global daily displacement models provided by GeoForschungsZentrum Potsdam (GFZ).
These displacement fields are produced using the same global ocean bottom pressure data (MPIOM) and
atmospheric surface pressure data (ECMWF) as applied to remove the relevant effects (AOD1B) from GRACE
gravity field. We first detrended the loading displacement field (Jiang et al., 2013) and then subtracted the
detrended daily time series (in the CM frame) from our GPS nonfiducial displacement time series. In our
study area, the nontidal ocean loading effect is small compared to hydrological loading effects but its
amplitude reaches 1.5 mm (supporting information Figure S1) along the Gulf of Mexico and hence is nonne-
gligible for stations near the coastline.

Degree 1 Stokes coefficients represent the geocenter motion (the relative motion between CE and CM).
Similar to GPS satellites, GRACE satellite orbit are expressed in the inertial CM frame and are thus insensitive
to geocenter displacement. Therefore, the GRACE satellites do not deliver degree 1 Stokes coefficients.
However, in calculating TWS, degree 1 Stokes coefficients have been added from the combination of GRACE
data and ocean models. The Green’s function used here is specified in the CE frame which is consistent
with GRACE TWS data. For an appropriate comparison with GPS time series, the nonfiducial daily position
time series are transformed into the IGB08 reference frame (Rebischung et al., 2012) using JPL’s X-files, a
seven-parameter transformation. The GPS time series transformed into the IGB08 are in a frame defined by
the center of surface figure of the Earth (CF) which differs from the CE frame. We therefore transferred
degree 1 load love number (h1) from the CE frame to CF frame using the formula given in Blewitt (2003,
equation (23)). In this way, the predicted vertical displacements will be in a consistent frame (CF) with the
GPS height time series.

5. Evaluation of Modeled Deformations

The daily GPS height time series were detrended and averaged to monthly intervals to be consistent with
the temporal resolution of the GRACE and WGHM data sets. The quality of the modeled deformation was
estimated by calculating the RMS scatter of GPS monthly height time series (uGPS) before and after remov-
ing the detrended modeled vertical deformation (umodel). The reduction in RMS is a common statistic to
quantitatively compare the observed and modeled deformation and assess the impact of hydrological load-
ing in GPS height time series (e.g., Dixon 1991; Fu et al., 2012; Tesmer et al., 2011; Tregoning et al., 2009; van
Dam et al., 2007). The RMS reduction in percentage is defined as:

\[
\text{RMS}_{\text{red}} = \frac{\text{rms}(u_{\text{GPS}}) - \text{rms}(u_{\text{GPS}} - u_{\text{model}})}{\text{rms}(u_{\text{GPS}})} \times 100
\]  

representing the average contribution of the hydrological loading estimated from the hydrological data
sets in the GPS height time series.

5.1. Omitting GPS Sites Responding to Nonloading Changes

Four GPS stations located inside the Yellowstone Caldera in the Missouri River basin were removed from
the data set as they are strongly influenced by volcanic and hydrothermal activity (supporting information
Figure S2). Unlike the Plate Boundary Observatory (PBO) GPS network in the Western U.S. designed for Earth
science studies, the GPS stations in the central U.S. are operated by several agencies and organizations and
are not necessarily focused on Earth science applications. The optimum (low noise) PBO style GPS monu-
ment (a deep-drilled and braced design) is limited in this area (<20 stations). Two hundred and thirty-three
stations have shallow-drilled braced monuments or concrete pillars and masts for monuments, while 257
stations are building roof installations. The remaining stations have unknown monument types description.
The diversity of monument types leads to colored noise in the displacement time series that is site-
dependent. Of the 762 GPS stations used in this study, only 210 stations have a known foundation depth, at
least as described in the publicly available metadata. Figure 3 compares the RMS reduction (e.g., using
hybrid solution) in GPS height time series and the foundation depth of GPS monuments where this is
known. For stations with foundation depths shallower than 3 m, we found a moderate correlation
(\(\rho = 0.55\)). The hydrological loading in GPS stations with deeper foundation depth is more significant rela-
tive to stations anchored at shallower depth. We interpret this as follows. GPS stations with shallower
foundation depths experience additional processes besides mass loading. Local soil expansion and compaction due to shallow water table-level fluctuations can cause shallow poroelastic deformation, a sponge-like behavior of ground, i.e., increase in water table-level causes uplift, while subsequent drying causes subsidence (Argus et al., 2014). These effects can be large and can dominate other signals. For example, GPS stations in Houston, Texas, show poroelastic deformation in response to shallow groundwater changes and heavy municipal and industrial pumpage (supporting information Figure S3) (Bawden et al., 2012). The elastic mass loading theory cannot be applied to estimate shallow hydrological deformation at GPS sites affected by such poroelastic effects. We thus excluded these sites from our study.

Thermoelastic deformation due to surface temperature variation is another process that influences GPS stations, in particular, those with shallow foundation depth. Thermal expansion of GPS monument (structure supporting the antenna and structure supporting the foundation) and thermal expansion of the ground surface through heat conduction into nearby bedrock are two major components that contribute to thermoelastic deformation (Dong et al., 2002; Yan et al., 2009). There is a one-to-one correspondence between thermoelasticity and poroelasticity (e.g., Fang et al., 2014; Tsai, 2011) which differs from elastic mass loading theory. However, the heat conduction from the ground surface to the depth decreases exponentially, hence stations with shallower foundation depth are more likely to be affected. GPS stations with deeper foundation depth are less likely to be affected by thermal expansion, hence elastic mass loading theory and mass changes from GRACE and hydrological models are more likely to fit the observed deformation. As a result of these considerations, of the total 762 processed GPS stations, 36 were eliminated from our database, either because of volcanic deformation, or poroelastic or thermoelastic effects associated with shallow monuments.

5.2. Validating the Hybrid Approach

We validate our hybrid approach by using GRACE data in both near and far field in the following steps. In this case, the hybrid approach should produce a solution equal to the spherical harmonic approach.

1. We use the filtered Stokes coefficients in equation (21) to generate the surface mass density \( D \) on a grid. The gridded surface mass density is then used to calculate the contribution of the near field using the Green's function approach (equations (13–15)), for the spherical cap with radius \( \psi_0 \) varying from 1° to 20° in steps of 1°.

2. We use the filtered Stokes coefficients to calculate the surface spherical harmonic of surface mass density \( (r_n) \), which is then applied in equation (22) to estimate the contribution of the far field for the spherical cap with radius \( \psi_0 \) varying from 0° to 20° in steps of 1°. As mentioned earlier, the truncation coefficients at \( \psi_0 = 0 \) are \( Q_n(0) = \frac{2n}{2n+1} \), thus, equation (22) reduces to the spherical harmonic representation of the vertical deformation (the spherical harmonic approach).

3. We then calculate the hybrid solution by adding the contribution of the near field from step 1 to the contribution of the far field from step 2.

4. The mean GPS and model fit for each aforementioned step is computed by averaging RMS reductions for all of 736 GPS stations.

When our modeling of the vertical deformation is limited to the effect of the near field (blue circles in Figure 4a), the mean RMS reduction increases with increasing radius of the integration cap, reaching a maximum value at \( \psi_0 \approx 5° \) (except at \( \psi_0 = 180° \) which reproduces spherical harmonic approach) and then drops slightly. Note that at \( \psi_0 = 5° \) the overall fit to GPS height time series is slightly better relative to the spherical harmonic approach (red dashed line in Figure 4a). This is primarily due to including a larger number of load Love numbers in the Green’s function approach (equation (4)). Here we chose the upper limit of the Green’s function expansion to be 1,500, while in the spherical harmonic approach the series expansion is limited to the spatial resolution of load, typically maximum degree expansion 90 for GRACE data and 180 for WGHM.
When summing up the contributions of the far field and near field with the hybrid approach, the mean RMS reduction becomes independent of the radius of the integration cap (cyan triangles in Figure 4a), reproducing the spherical harmonic approach (red dashed line) and validating our formulation of the hybrid approach.

5.3. Modeling the Vertical Deformation at the GPS Sites

We produce three sets of solutions using the following three approaches: (1) Green’s function approach (2) spherical harmonic approach; and (3) the hybrid approach. For the Green’s function approach (near-field contribution), we use the detrended surface mass density changes from hydrological models and GRACE in equations (13–15) to estimate vertical displacements at GPS sites. We estimate the vertical deformation for spherical caps with radius \( w_0 \) varying from 1/8 to 20/8. For the spherical harmonic approach, we converted the monthly detrended TWS changes from WGHM and NLDAS to the surface mass density changes \( (D_r = \Delta \rho_s) \) and then to spherical harmonic coefficients (e.g., equation (1.76) in Moritz and Heiskanen, 1967 and Eshagh and Karegar, 2010) of surface mass density. The processed GRACE Stokes coefficients (section 4.1) and the WGHM and NLDAS spherical harmonic coefficients are then used in equation (22) with \( \psi_0 = 0 \) and \( Q_n(0) = \frac{2n!}{(2n+1)!} \) to estimate vertical displacement at GPS sites. For the hybrid approach, we use the processed GRACE Stokes coefficients to calculate the contribution of the far field at various spherical caps. We then add the contribution of the near field estimated from WGHM and NLDAS TWS data (using the Green’s function approach) to the contribution of the far field computed from GRACE. The GPS and model fit for each approach is computed by averaging RMS reductions for all GPS stations.

6. Results and Discussion

6.1. Comparison of the Three Approaches

Estimates of vertical crustal deformation based on GRACE data in both the spherical harmonic approach (Figure 5, dashed lines) and Green’s function approach (Figure 4b, circles) provide a better fit to GPS height time series than the WGHM and NLDAS models. However, combining WGHM and NLDAS TWS data with...
GRACE data through the hybrid approach results in the greatest mean RMS reduction (triangles in Figure 5). The hybrid approach (combining GRACE and WGHM) improves the fit to GPS time series by an average of 25% and 35% relative to, respectively, the GRACE-only and WGHM-only spherical harmonic solutions (Figure 5). The hybrid approach (combining GRACE and NLDAS) improves the fit to GPS time series by an average of 10% and 25% relative to, respectively, the GRACE-only and NLDAS-only spherical harmonic solutions.

When comparing the Green’s function approach (circles in Figure 4b), at the spherical cap $\psi_0 = 4^\circ$ (~450 km from GPS sites) GRACE provides a maximum mean RMS reduction 15.1%, while WGHM and NLDAS models lead to 11.7% and 12.7% RMS reduction. This implies that GRACE can better describe the loading changes adjacent to the GPS stations. As we stated earlier, GRACE data are by-products of filtering processes which damp TWS changes. The GRACE water storage variability is smoother than WGHM and NLDAS models in Central North America (Figure 2). How does the combination of TWS data from hydrological models in the near field with TWS data from GRACE in the far field lead to a better fit to GPS height time series? To assess which data set better models deformation in the near field at GPS sites we should remove the effect of the far field from GPS time series and then compare the solution from the Green’s function approach for each data set. First, we calculate the contribution of the far field using processed GRACE Stokes coefficients (equation (22)). Second, residual GPS time series are calculated by subtracting the far-field contribution for varying spherical cap sizes from the GPS time series. We attribute displacement in residual GPS time series to near-field deformation. Third, the fit of Green’s function approach (near-field contribution using GRACE and hydrological models) to the residual GPS height time series is calculated. We find that WGHM and NLDAS (red and green squares in Figure 4b) significantly better model deformation in the residual GPS height time series compared to GRACE (blue squares). These results are consistent with the known spatial resolution of GRACE TWS data (~500 km). While GRACE accounts for large-scale variabilities ($>500$ km due to filtering and smoothing), WGHM and NLDAS models incorporate higher spatial variability in TWS, better explaining the small-scale deformation near GPS stations.

For spherical integration caps smaller than $7^\circ$, the hybrid solutions (combining GRACE with WGHM; or GRACE with NLDAS) are better than the GRACE-only spherical harmonic approach (Figure 5). For caps with radii larger than $7^\circ$ the hybrid approach does not improve the estimate relative to GRACE and the mean RMS reduction decreases. The rapid drop in mean RMS reduction for the hybrid solution indicates that hydrological models cannot resolve the large-scale deformation in comparison to GRACE. Therefore, hydrological and land-surface models alone are not sufficient to characterize the hydrological deformation at geodetic sites. This result can be further investigated by calculating the degree variance spectrum of the TWS data. The degree variance spectrum gives an indication of the power in the TWS data as a function of spherical harmonic degree. The WGHM and NLDAS models deliver higher power than GRACE in higher spectral degrees (larger than 30–40), i.e., short wavelength signals in the models are contained in higher degrees, as expected (supporting information Figure S4). However, the WGHM TWS data indicate higher power than NLDAS for degrees larger than 30 (supporting information Figure S4b). This suggests that the
WGHM hydrological model better determines hydrological loading deformation relative to the NLDAS land-surface model (Figures 4b and 5). This is consistent with results in Argus et al. (2014, 2017) and Fu et al., (2015) that suggest that the NLDAS model underestimates snow equivalent height in the western United States. Moreover, groundwater and surface water are not explicitly represented in the NLDAS model, which are two important components of TWS changes in the Mississippi River basin and Texas. These comparisons provide some characterization of the uncertainties inherent in the WGHM and NLDAS models, reflecting the spatial heterogeneity of the region, nonlinear processes, and complex interactions and feedbacks between surface and subsurface water cycles that are not fully captured by the models. In the remaining discussion, we adopt the WGHM model to evaluate the hybrid approach and hydrological loading in the GPS height time series.

6.2. Radius of the Near Field in the Hybrid Approach at Individual GPS Sites

We use the maximum RMS reduction to find the corresponding radius of the near field for individual GPS stations in the hybrid approach (GRACE + WGHM) (Figure 6). Integrations caps with sizes between 1° and 3° result in the highest RMS reduction at more than 95% of GPS stations (inset histogram in Figure 6). Given the spatial resolution of the GRACE TWS data (a few degrees), WGHM model adds the surface mass loading at higher spectrum, making the optimum prediction of vertical deformation limited to integration caps between 1° and 3°. Stations with cap size larger than 3° are primarily concentrated in arid regions, including the Rio Grande basin and southwest area of the Arkansas and red River basin. In these regions, GPS data

![Figure 6](image-url)

**Figure 6.** Spherical cap size ($\phi_0$, radius of the near field) for the maximum RMS reduction in GPS monthly height time series. Color circles show the values of maximum RMS reduction and the size of circles represent the size of corresponding integration caps. Black circles indicate that RMS reduction is negative. Lower-right inset is a histogram of the size of integration caps. At 95% of GPS stations, spherical caps smaller than 3° achieve the maximum RMS reduction. Upper-right inset shows RMS reduction at GPS stations as a function of distance from the coastline. The red line is linear regression fit. For distances shorter than 300 km the correlation coefficient between RMS reduction and distance from the coast is 0.40.
indicate relatively small deformation, i.e., RMS scatters of detrended time series are smaller than 3 mm (supporting information Figure S6a).

The hybrid approach provides a fast and flexible way of combining two loading data sets, minimizing the difference between observed and modeled deformation at individual geodetic sites when the size of the spherical cap (the boundary between the near and far field) is optimized. This will be useful in geophysical studies where accurate characterization of deformation due to hydrological loading in geodetic position or displacement time series is required.

6.3. Comparison of the Hybrid Approach With the GRACE-Only Spherical Harmonic Approach at Individual GPS Sites

We show the performance of hybrid approach at individual GPS stations by comparing the maximum RMS reduction from the hybrid solution with those from the GRACE-based spherical harmonic solution (Figure 7). Note that stations with RMS reduction larger than 10% (from the hybrid approach) were included (80% of stations). The hybrid solution substantially improves the scatter of observed vertical displacements compared to GRACE-based spherical harmonic results. Almost half of the GPS stations show an improvement greater than 30% in RMS reduction, and 35% of stations show an improvement larger than 50%. Most significant improvements are in areas where the WGHM model provides small-scale variabilities that are not resolved by GRACE. These areas also include coastal regions where GRACE data are affected by spatial leakage error.

Fifteen percent of stations show no improvement with the hybrid approach. These stations are mainly located near riverbanks (Figure 7 and supporting information Figure S8). One possible explanation is the

![Figure 7](image_url)

**Figure 7.** Comparison of RMS reduction in GPS monthly height time series between GRACE-based spherical harmonic solution and hybrid solution (GRACE + WGHM). Stations with RMS reduction larger than 10% (from the hybrid approach) are shown. The light to dark blue circles indicate sites where the RMS reduction has increased; the light to dark red, where the RMS reduction has decreased.
rather conceptual representation of groundwater and river storage in the hydrological model, which may break down in these areas of high water flux, high temporal variability, and feedback with local groundwater.

6.4. Contribution of Hydrologic Loading to the GPS Height Time Series

The RMS scatters of 94% of the GPS stations are reduced after removing the hydrological loading model using the hybrid approach (Figure 6 and supporting information Figure S6b). Eighty percent of these stations have a RMS decrease greater than 10%. Stations with the greatest RMS decrease (e.g., larger than 30%) are primarily located in the center of the Mississippi River, regions where GRACE and WGHM TWS data indicate higher variability (Figure 2). Application of the hydrologic loading correction to the height time series increases RMS scatter in 6% of studied stations (47 stations). Most of these stations (black circles in Figure 6) exhibit only small deformation, i.e., RMS scatters of displacement time series are smaller than 3 mm (supporting information Figure S7). A few exceptions (RMS scatters significantly larger than nearby stations; supporting information Figure S7) may be affected by local processes such that loading theory is not applicable.

For stations close to the Gulf coast and Great Lakes, there is less improvement than for stations at larger distances from these coasts (Figure 6). The level of agreement is positively correlated ($\rho = 0.40$) with station’s proximity (up to 300 km) to the coastline, i.e., stations farther from the coast have larger RMS reductions (upper-right inset plot in Figure 6). The RMS scatter of nontidal ocean loading displacements reach 1.5 mm at these coastal sites (supporting information Figure S1) comparable to RMS scatters of GPS height time series (corrected for ocean loading) (supporting information Figure S6a). Perhaps small-scale coastal ocean or lake dynamics not resolvable in current models is affecting the vertical displacement of the nearby land stations. Spurious long-period displacements have been shown to arise from mismodeling of tidal ocean loading at subdaily periods (e.g., Penna et al., 2007; van Dam et al., 2007). Also, required postprocessing of GRACE level-2 data (filtering and truncating) can lead to leakage of water mass variabilities in the ocean and Great lakes to neighboring land areas, contaminating the estimate of hydrologic loading displacement.

The hybrid approach described here actually reduces this effect (see Figure 7 and section 6.3).

6.5. Sensitivity to Different Filter Intensities in GRACE Data

We assess the sensitivity of different filter intensities in the GRACE data using different versions of the decorrelation DDK filter developed by Kusche (2007). These coefficients are nonisotropic filtered Stokes coefficients with various degree of smoothing from DDK1 to DDK8, with DDK1 having the strongest smoothing and DDK8 the weakest. The RMS of total water storage shows significant variability among different filter intensities (supporting information Figure S5). Since the optimal signal to noise ratio of GRACE data is unknown, we test the performance of different filters in the hybrid approach. First, we use WGHM TWS data to calculate the contribution of the near field using the Green’s function approach (equation (15)), for a spherical cap with radius $\psi_0$ varying from 1° to 20°. Second, we use DDK-filtered stokes coefficients (DDK1–DDK8) to calculate the contribution of the far field using equation (22), for the same spherical cap size. Third, we compute the mean GPS and model fit for a given solution by averaging RMS reductions for all of the GPS stations. Figure 8 plots the mean RMS reduction as a function of spherical cap size for the various filters.

Different filter intensities cause different GRACE error reduction and leakage effects in TWS changes (supporting information Figure S5). However, the effects of different filters on the estimated deformation are controlled by the size of the spherical cap in the hybrid approach (Figure 8). For spherical caps larger than 6° (600–700 km), different GRACE filter intensities do not alter the estimated deformation, as WGHM TWS data are used for larger areas. Larger differences occur for smaller spherical caps, since GRACE TWS data are used in a larger fraction of the test area. The greatest mean RMS reduction occurs at spherical cap $\psi_0 = 2°$ (~220 km) for all tested solutions. The filter with the most smoothing (DDK1) generates TWS data that greatly damps estimated deformation at the GPS sites. The filter with the least smoothing (DDK8) leads to erroneous TWS data and deformation estimates. Among other applied filters (DDK2–DDK7), no significant differences are observed at $\psi_0 = 2°$. This analysis suggests that when GRACE TWS data are combined with WGHM TWS data using our the hybrid approach, the deformation estimates are relatively insensitive to moderate filter strengths (DDK2–DDK7) for spherical caps with sizes larger than ~2° to 3°. We chose the DDK2-filtered Stokes coefficients to calculate the far-field signal in the hybrid approach.
7. Potential for Broader Applications

Our hybrid solution can be applied to a wide variety of environmental surface loading problems. For example, in situations where it is desirable to measure vertical coastal motions, a global ocean circulation model along with a high-resolution local tide model and storm surge model can be used to improve estimates of tidal and nontidal ocean loading coastal deformation, so that residual signals (e.g., due to local subsidence or isostatic adjustment) are clear. This is particularly important for GPS sites colocated with tide gauges along the coast, where GPS data are used to study sea level change. Similarly, global and local models of atmospheric surface pressure can be used with our method to better predict the atmospheric pressure loading, and high-resolution regional coupled land-surface-groundwater models may be employed for the near field. Our approach also allows the optimal combination of GRACE data with Surface Mass Balance models in Greenland and Antarctica to model annual and inter-annual deformation due to ice mass changes near individual GPS sites. It can be used to effectively separate ice and nontidal loading sources (e.g., Liu et al., 2017). Our approach also has implications for tectonic and volcanic deformation, as the accurate determination of secular rates, postseismic deformation, and detection of slow slip transient deformation requires proper modeling of nontectonic sources of surface deformation. Recent studies typically use statistical signal separation techniques to identify signals of interest in GPS time series (e.g., Bird & Carafa, 2016; Crowell et al., 2016; Voss et al., 2017; Walwer et al., 2016). These methods divide the displacement time series into different modes (or channels) to separate tectonic signal from other sources (e.g., anthropogenic water extraction or drought). However, these techniques do not provide any information about the physical processes governing specific deformation modes. This could lead to misinterpretation of the extracted signal. For example, Karegar et al. (2015b) showed that three GPS sites exhibit surface uplift associated with CO₂ injection and storage in an oil reservoir at depth in coastal Texas, where the timing and magnitude of uplift were also influenced by hydrological seasonal deformation. A principal component analysis was applied to a network of nearby GPS sites to detect and remove the hydrological loading signal. However, a portion of the uplift signal in the excluded principal components was likely related to hydrological loading, affecting the interpretation. The hybrid approach provides tools for investigating the interplay of elastic deformation produced by near-field and far-field mass changes. This could be beneficial for definition of terrestrial reference frame parameters, for example reducing aliasing of load-related deformation into reference frame estimation, and understanding the effects of elastic displacement due to nearby surface loads on the geocenter. Finally, the hybrid approach can be used to assess the quality of hydrological models as well as GRACE and GRACE-Follow products.
8. Summary and Conclusions

We develop a hybrid approach to exploit loading data with different accuracies and spatial resolutions, combining near-field and far-field contributions for hydrologic loading calculations. The hybrid approach consists of two steps: (1) the contribution of the near field (2) the contribution of the far field. Using Green’s functions and mass changes from high-resolution hydrological models, contributions to the near field can be estimated at the location of geodetic sites. To properly treat the singularity of Green’s function at the computation point, the near field is split into the contribution at the computation point and the contributions from the rest of a spherical cap. We then derive a formula based on spherical harmonics to account for far-field contributions using filtered GRACE Stokes coefficients. This effect is added to the near-field effect to calculate total displacement. Monthly modeled mass changes from two high-resolution hydrological models were compared to estimate the near-field information, while mass changes from GRACE contribute far-field information.

Data from 762 high precision GPS stations in the Central U.S. were processed and corrected for tidal and nontidal atmospheric and ocean loading effects, using parameters consistent with GRACE processing, and used to validate the model. Our new hybrid approach achieves a better fit to GPS-measured vertical displacement than the widely used spherical harmonic approach, accounting for local to regional variabilities adjacent to the GPS station. We tested two high-resolution hydrological models and several different GRACE DDK-filtered Stokes coefficients. The WGHM model better resolves smaller scale and local mass variability compared to the NLDAS model. Combining the WGHM model with GRACE DDK-filtered data (DDK2–DDK7) through the hybrid approach results in the highest RMS reduction in GPS height time series in Central North America. The estimated deformations are insensitive to the filter strength (DDK2–DDK7) for spherical caps larger than $2^\circ$ to $3^\circ$.

Appendix A: Recursive Algorithm for Coefficients $B_n$ (equation (25))

Meissl (1971, p. 45) derives a recursive algorithm for the integral:

$$b_n^* = - \int_{\psi = \psi_0}^{\psi = \psi_2} \frac{\sin \psi}{\sqrt{2 \sin \frac{\pi}{2}}} P_n(\cos \psi) \, d\psi$$

Integral (25) can be written in terms of $b_n^*$, as follows:

$$b_n = \frac{\sqrt{2}}{2} b_n^*$$

where

$$b_n^* = \frac{P_{n-2}(\cos \psi_0) - P_n(\cos \psi_0)}{n+\frac{1}{2}} \sqrt{1-\cos (\psi_0)} + \frac{2(n-\frac{1}{2})}{n+\frac{1}{2}} b_{n-1}^* - \frac{n-\frac{3}{2}}{n+\frac{1}{2}} b_{n-2}^*$$

with the initial values:

$$b_0^* = -2 \sqrt{1-\cos (\psi_0)} + 2 \sqrt{2}$$

$$b_1^* = -\frac{2 \sqrt{\left[1-\cos (\psi_0)\right]^3}}{3} - 2 \sqrt{1-\cos (\psi_0)} + \frac{2 \sqrt{2}}{3}$$

Appendix B: Recursive Algorithm for Coefficients $R_{n,k}(\psi_0)$ (equation (26))

Paul (1973) presents recursive algorithms for integral (26) which is used for computing truncation coefficients $Q_n(\psi_0)$. For $k \neq n$:

$$R_{n,k}(\psi_0) = \int_{\psi = \psi_0}^{\psi = \psi_2} P_k(\cos \psi) \, P_n(\cos \psi) \sin \psi \, d\psi$$
where $P_j = P_j(\cos \psi_0)$; and for $k = n$:

$$R_{n,n}(\psi_0) = \frac{(n+1)(2n-1)}{n(2n+1)} R_{n+1,n-1}(\psi_0) - \frac{n-1}{n} R_{n,n-2}(\psi_0) + \frac{2n-1}{2n+1} R_{n-1,n-1}(\psi_0)$$

with the initial values,

$$R_{0,0}(\psi_0) = \cos \psi_0 + 1$$

$$R_{1,1}(\psi_0) = \frac{(\cos \psi_0)^3 + 1}{3}$$

References


