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Equatorial inertia-gravity waves observed in TOPEX/Poseidon sea surface heights

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Abstract. We report an evaluation of the ability of the TOPEX/Poseidon altimeter to observe equatorial inertia-gravity waves in the central Pacific. Aliasing problems limit us to examining meridional mode two, but this mode is an energetic one in the Pacific basin. These estimates of the temporal potential energy modulations are compared with estimates from tide gauges in the tropical Pacific, and we find that the time series obtained at these points show good agreement.

1. Introduction

Inertia-gravity waves comprise the high frequency portion of the theoretical equatorial wave spectrum, typically having periods of less than 5 days. *Wunsch and Gill* [1976] first identified these waves in sea level records, and others [*Weisberg et al.*, 1979; *Luther*, 1980; *Eriksen*, 1982; *Garzoli and Katz*, 1981; *Chiswell et al.*, 1987] have verified their existence in all basins using sea level, dynamic height and velocity records. Evidence suggests that these waves are seasonally modulated [*Garzoli*, 1984], and *Luther* [1980] has provided evidence of zonal modulations as well. These modulations have been difficult to study in detail using the available *in situ* data, which motivates our attempt to use the now decadal long global coverage afforded by satellite altimeters to study these waves.

Here we ask whether the data from a particular satellite altimeter (TOPEX/Poseidon; hereinafter T/P) can be exploited for this purpose. The technique we use is only briefly described, but more detail can be found in *Gilbert* [2001], which is available from the authors. We focus here on the central equatorial Pacific and examine data from late 1992 to present. We attempted descriptions of the first three meridional modes for the first vertical mode. We also developed a random noise model to do significance testing, and the method used to develop this noise model is briefly described below. We then compare out T/P results to estimates from *in situ* tide gauges.

2. T/P estimates of wave energies

An example of the T/P sampling is seen on Figure 1. This example is for a zonal width of 30° and for a single T/P cycle, which is about 10 days. The height along each track is sampled approximately every 7 *km* and there are 28 different tracks within the 30° zonal box. This set of ground tracks is repeated each T/P cycle, so that at any given point on a single pass we have a time series with a 10-day sampling interval. If one were to use the time series at each sample point independently, the Nyquist period would be 20 days, and since the inertia-gravity waves have periods of 3-5 days for the first few meridional modes associated with the first vertical mode, these waves would be aliased in any single time series. It is important to note, however, that the 28 passes shown are all at different times within the 10-day T/P cycle. If we assume that the wave amplitude, period and wavelength are constant over the 30° zonal and 10-day temporal box shown, we have 28 samples in 10 days, which in principle allows us to fit the inertia-gravity waves without temporal aliasing. If the zonal width is too short we cannot obtain stable results due to too few points in the 10-day intervals, and if it is too large we lose the ability to measure zonal energy modulations. From experiments with simulated T/P data we determined that a zonal width of 30° gave stable results, but the width of the temporal window was not critical. We thus do each of our individual fits to all of the T/P data in a 30° by 20-day box.

Since we are fitting to the theoretical structure of the waves, we also need to take into account changes in the theoretical wave structures due to varying stratification. This requires knowing the internal gravity wave speed, or the equivalent depth, as a function of longitude. We used the internal gravity wave speeds from the analysis of *Chelton et al.* [1998] in this study. In the portion of the Pacific that we consider the equatorial values vary from 2.6 to 3.0 *m/s*. For each 30° box we used the average internal gravity wave speed in that box.

To be more specific, we estimated the amplitudes of the inertia-gravity waves by fitting T/P heights in the form

$$h(x,y,t) = Re[A_m e^{\imath(k_m x - \omega_m t)}]F_m(y) \tag{1}$$

where h is the T/P height, A_m is the complex amplitude, k_m is the zonal wavenumber, ω_m is the frequency, which is determined from the zonal wavenumber via the dispersion relation, and $F_m(y)$ is the meridional pressure structure function that is known from theory (an example for meridional mode two is shown on Figure 1). The m subscript refers to the meridional mode being fit. The two unknowns in this fit are the complex amplitude and the zonal wavenumber. The complex amplitude is obtained for a given k_m with a straightforward least squares minimization. This results in a set of $|A_m|^2$ as a function of k_m, and the final A_m and k_m are then determined by maximizing $|A_m|$ with respect to k_m. As noted above, all of the T/P data in a box spanning 30° zonally and 20 days temporally were used in a given fit and the amplitude and wavenumber were assigned to the longitude and time at the center of the box. The 30° by 20-day windows used were centered at 5° and 10-day intervals, so not all estimates are independent. Note also that

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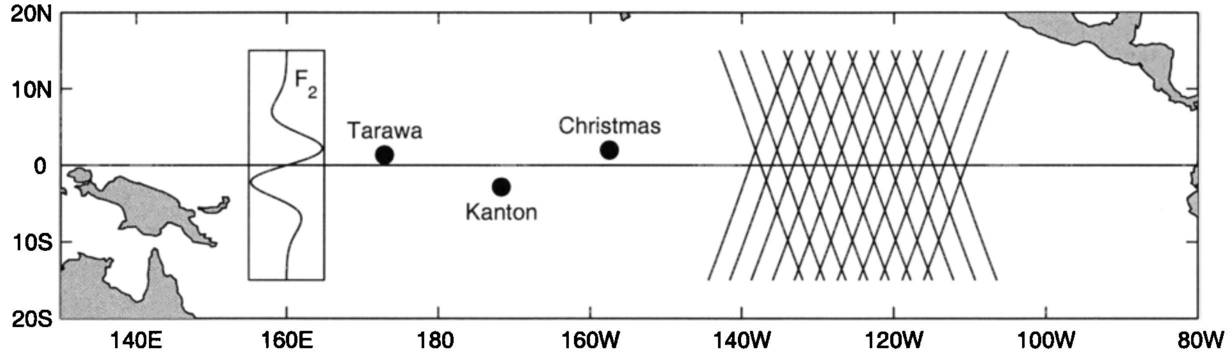

Figure 1. Tide gauge locations are shown. Also shown is an example of the T/P coverage in a 30° zonal bin during a 10-day cycle. The inset on the left shows the meridional pressure function for the first vertical, second meridional inertia-gravity wave mode as a function of latitude.

the search in wavenumber space is performed over a predetermined wavenumber range. This range was estimated by noting that in the near-equatorial tide gauge autospectra (not shown), the location of the peak in frequency for a given mode was always within a few percent of the theoretical minimum frequency for that mode. This frequency range and the dispersion relation allowed us to set the nondimensional wavenumber range of interest to be -0.9 to 0.5. The potential energy in that mode at that longitude and time was then computed as $|A_m|^2/2$.

There is a potentially serious problem with our approach, however, which is that the T/P zonal/temporal sampling at the equator (i.e., the order in which the tracks shown on Figure 1 are observed) is not at all random. This leads to potential aliasing, which we examined in some detail. The simplest way to see the potential problem is to imagine that the T/P tracks were made sequentially from east to west at some constant time interval. This means that the observations would effectively be made in a coordinate system that is propagating westward at a constant speed, and if this speed were to match the zonal phase speed of the waves, then we would not see any phase difference from one T/P observation to the next. In this case the fit would fail to distinguish this wave from a mean sea surface height signal. The actual T/P sampling is more complex than this simple illustration, of course, but aliasing is still possible and we considered it in detail.

Two different aliasing problems were found. First, the T/P sampling is such that meridional mode three aliases in a fashion similar to the example given above and is thus indeterminate. Second, it was found that some low frequency signals could have a small response in the inertia-gravity wave band, which is problematic since low frequency signals in the equatorial region have relatively large amplitude. In particular we were worried about aliasing of Tropical Instability Wave (TIW) variability into our analyses, and we examined this question in some detail. Temporal high-pass filtering the T/P data appears to eliminate these problems, but this high-pass had to be done on the T/P time series at each point in space. The high-pass filter used has a 50% amplitude pass point at a period of 32 days, and signals with periods longer than this are eliminated. Meridional mode one, which has a period near 5 days, aliases to long periods in the 10-day T/P samples obtained at any given point, and is lost in the high-pass filtering. For the internal gravity wave speed range in the central Pacific where we will compare to the tide gauges, meridional mode two aliases to periods less than 25 days and is thus nearly unaffected. The net result of our consideration of potential aliasing problems is that only the mode two waves are accessible to our method. The mode two waves are known, however, to be energetic [*Wunsch and Gill*, 1976; *Luther*, 1980] and

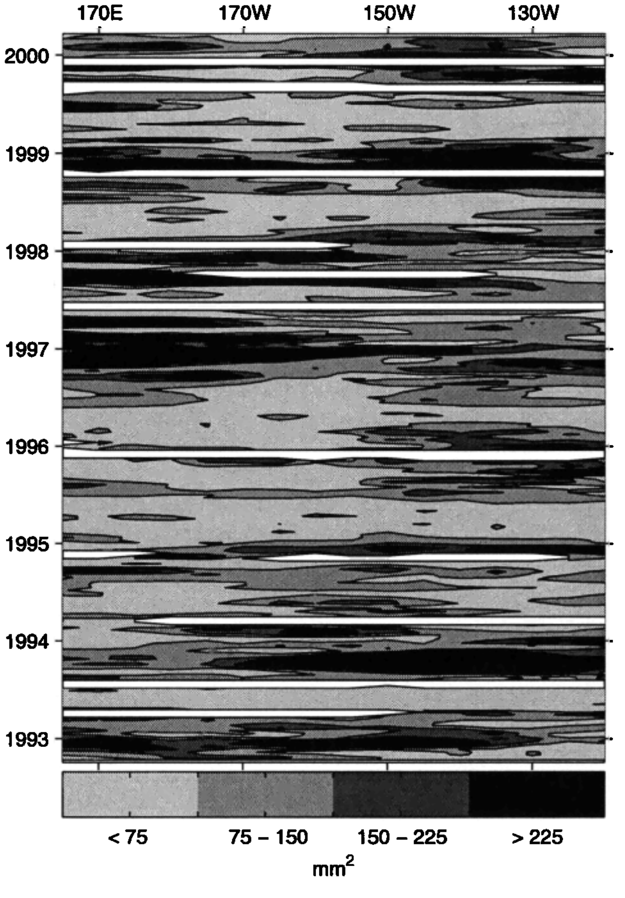

Figure 2. Potential energy estimates were made each 5° of longitude and for each T/P cycle (10 days). These estimates are from a box that is 30° wide and two T/P cycles (20 days) long. White areas indicate where there were not enough data to attempt the fit due to missing data. The zonal range includes only areas where the 30° zonal box did not overlap the Pacific boundaries.

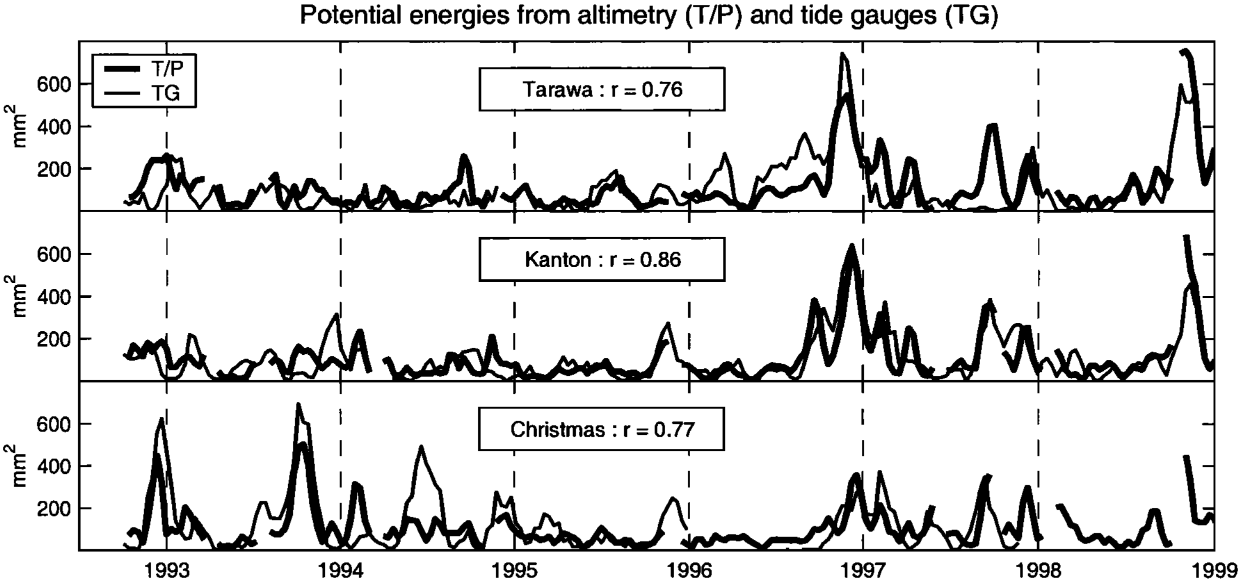

Figure 3. Potential energy time series from T/P (thick) and the central equatorial Pacific tide gauges (thin). The correlations are all significantly different from zero at the 95% confidence level. The method used to derive these time series is described in the text.

are therefore intrinsically interesting. In the future we will examine the possibility of determining modes one and three as well by including data from other satellite altimeters in order to modify the overall sampling characteristics.

From these calculations, we conclude that the T/P altimeter should be capable of accurately estimating meridional mode two inertia-gravity waves. The power of the altimetric analysis is that we can more clearly see the zonal and temporal structure in the inertia-gravity wave energy, which is difficult when working strictly from the available *in situ* data. Figure 2 shows the structure of the meridional mode two inertia-gravity wave potential energy in the central Pacific during the more than 7 years covered by the T/P data. We have restricted this analysis to the portion of the Pacific basin where our 30° box does not overlap with eastern and western boundaries, but this analysis could in principle be extended globally. The ability to make such a description demonstrates the potential power of the altimetric data for the future study of these waves.

As part of the T/P analysis we also tested various noise models in order to form significance estimates for our results. We simulated T/P data from white noise and also from a variety of red noise models. We also included other known signals in these noise models, such as the TIW signals discussed earlier, other equatorial waves (e.g., Rossby or Yanai waves), and El Niño signals. None of these noise models indicated that the results we were obtaining were due to noise. We decided, however, that a more powerful test could be devised by using the T/P data itself. This test is based on the idea that if we use off-equatorial T/P data with the same sampling as the equatorial data, and treat these data in the fit as equatorial data, then any energy in these fits must be due to noise. We chose 4 off-equatorial locations for centering these boxes (15° S, 8° S, 8° N and 15° N). At these latitudes the sampling is identical to the equatorial sampling and the mode two meridional pressure function has near zero correlation with itself when shifted by these amounts. We found that the energies obtained in this fashion were well described as proportional to a χ_2^2 random variable with the proportionality constant being weakly dependent on the wavenumber used in the fit and the internal gravity wave speed. The full details of these calculations are given in *Gilbert* [2001], but for the present purpose we simply note that the central Pacific T/P results were significantly different from zero with >95% confidence.

3. Comparison to *in situ* analyses

The potential energy estimates at the tide gauges (Figure 1) were obtained by fitting sinusoids at the mode two frequency to the data in a temporal window. The mode two frequencies were selected by examining frequency spectra at each tide gauge, and the periods used were 4.00, 3.91 and 4.02 days at Tarawa, Kanton and Christmas, respectively. By sliding the window along the time series, we obtain a time series of the potential energy modulations that can be compared to the T/P potential energy time series. As for the T/P data, the errors in the tide gauge data lead to errors proportional to a χ_2^2 random variable. The mean potential energy in both the T/P and tide gauge analyses are therefore biased upward since the χ_2^2 random errors do not have zero mean. We chose the width of the fitting window for the tide gauge series to be 60 days, which is the window width that makes the bias error for the tide gauge potential energy estimates similar to the bias error estimates for the T/P potential energies, the latter being computed from the meridionally shifted T/P data fits described earlier. These bias errors are difficult to compute precisely, however, and we estimate that the errors in the T/P and tide gauge bias estimates are of order 10-20 mm^2. These bias errors do not affect the correspondence of the temporal variations in the T/P versus tide gauge potential energy time series, but do make it difficult to evaluate the precision of the temporal mean values of the potential T/P energy estimates.

To make an estimate of what T/P should see at a particular gauge we must divide the tide gauge potential energy estimate by the square of the value of the meridional pressure structure function evaluated at the tide gauge latitude. Where this adjustment factor is small (we chose <0.3) we are simply magnifying the error in the potential energy due to noise. The theoretical meridional pressure structure function (F_2) and the tide gauges used here are shown on Figure 1. The time series of potential energy for all three tide gauges along with the corresponding T/P series are shown in Figure 3. The correlations are good, ranging from 0.76 to 0.86 and most of the events seen in the time series at each station are common to both the tide gauge and T/P. There are, however, events that are not in agreement, such as the large event in the Christmas tide gauge in mid-1994 or the event in the T/P series at Tarawa in late 1997. The mean values of the series are all approximately 100 mm^2, and the differences in the mean values are of the same order as the expected errors in the bias errors, as discussed earlier.

Although the magnitudes of the potential energy fluctuations are difficult to compare precisely, it is very encouraging that the time series correlate well. These correlations indicate that about 60% of the variance is accounted for in all three cases. These correlations are considered reasonable given that the series are not expected to agree exactly. This is because the true zonal and temporal potential energy modulations are averaged quite differently in the T/P and tide gauge analyses. T/P gives a value appropriate for a 30°, 20-day average, while the tide gauges give 60-day average values at a point in space.

In summary, although we are presently limited with T/P to analyses of only the meridional mode two inertia-gravity waves, the estimates that we obtain are consistent with tide gauge measurements in the central Pacific. For the second meridional mode the spatial and temporal coverage of the T/P altimeter allows a unique view of the zonal and temporal modulations of these waves in the central Pacific, and it is possible that similar results might be obtained for additional meridional modes if data from other satellite altimeters were to be used as well. In particular we plan to investigate the use of ERS-2 data for this purpose, and to attempt comparable global descriptions of the zonal and temporal modulations of equatorial inertia-gravity waves.

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