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Investigating Levels of Graphical Comprehension Using the LOCUS Assessments

Charlotte A. Bolch
University of Florida, cbolch@ufl.edu
Tim Jacobbe
University of Florida, jacobbe@coe.ufl.edu

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Investigating Levels of Graphical Comprehension Using the LOCUS Assessments

Abstract

Statistical literacy refers to two interrelated components: people's ability to interpret and critically evaluate statistical information, and their ability to discuss or communicate their reactions to statistical information. The ability to read and interpret graphical displays is part of statistical literacy because much of the statistical information people encounter in their everyday lives are graphical displays or tables of data. The purpose of this study was to investigate college-level students' graphical comprehension. Students' graphical comprehension was assessed using items from the Levels of Conceptual Understanding in Statistics (LOCUS) assessments. Much can be learned about students' graphical comprehension based on this study. Results from this study can be used to highlight the importance of incorporating graphical comprehension and/or data visualization into introductory statistics courses.

Keywords

Graphical displays, Graphical comprehension, Statistical literacy, Assessments

Cover Page Footnote

Charlotte Bolch is a doctoral student in Statistics Education at the University of Florida. Her current research focus is on students' understanding and application of data visualization skills. Tim Jacobbe is an Associate Professor of Mathematics and Statistics Education at the University of Florida. His research currently focuses on the assessment of students' and teachers' statistical literacy.

Introduction and Review of Literature

What is the difference between a bar chart and a histogram? This is a question that some students struggle with answering correctly at the end of their semester- long introductory statistics course. Personally, I teach summary statistics and graphical displays during the second and third weeks of my introductory statistics course. I walk the students through the concept that numerical data can be summarized using histograms and that categorical data can be summarized using bar charts. I walk through examples of each graph explaining what the data looks like in a table and then how that data is used to create the graphical displays making sure to clarify the differences in the y-axis for each plot. However, I didn't realize that until the end of semester that some of my students could not differentiate between the two graphical displays.

Friel et al. define graphical comprehension as composed of three different behaviors: translation, interpretation, and extrapolation/interpolation (2001). Translation is the ability to interpret data in graphs and tables at a descriptive level. Interpretation focuses on being able to sort or rearrange components of a graph from the most to least important aspects. The behavior of extrapolation is a continuation of interpretation such as identifying trends within the data or implications (Wood 1968; Jolliffe 1991). In the literature, graphical comprehension has been looked at from many different perspectives with some examples focusing more on cognitive ability (Nicolaou et al. 2007; Lowrie et al. 2012; Lem et al. 2015), the graph as a singular entity (Shah and Hoeffner 2002; Selva and Lima 2010), or a student's ability to translate information between multiple modes (Kosslyn 1985; Espinel Febles and Carrion Perez 2006).

To understand and develop data visualization and graphical comprehension skills, students need to first develop a foundation of statistical literacy skills. There has been a shift in the focus of statistics education from teaching statistics as a discipline focused on techniques to a method of thinking/perspective (Rossman et al. 2013). Statistical literacy is the ability to understand statistical information or research results not only with statistical knowledge, but also with literacy skills, mathematical knowledge, context knowledge, and critical questions (Gal 2004). One aspect of literacy skills is document literacy which requires people to be able to identify, interpret, and use information from lists, tables, charts, and graphical displays (Gal 2004). Gal defined five key parts of the statistical knowledge base that is required for statistical literacy. Two of the five parts of the statistical knowledge component involve familiarity with basic terms and ideas related to data representations such as graphical and tabular displays (Gal 2004). Graphs are often found in media sources such as online or print newspapers and magazines. Individuals need to have the graph comprehension skills to make sense of the information presented and communicated from these external sources (Friel et al. 2001). In the past, before students started college the instruction they received during their K–12 education was usually focused on constructing graphs and identifying the shape, trend, and potential outliers in the graphs (Cooper and Shore 2010). With the implementation of the Common Core Mathematics Standards, specifically the Statistics & Probability standards for grades 6–12, hopefully there will be a shift in overall understanding of statistical concepts such as center and variability, and how the interpretation of these concepts change based on various graphical displays (National Governors Association Center for Best Practices and Council of Chief State School Officers 2010). However, it is possible that current college students could not have experienced statistics (graphical displays in particular) in a way that leads to a deeper understanding of the discipline. Therefore, graphical comprehension skills need to be understood within the larger context of statistical literacy.

Understanding graphical displays is important not only for students at the college-level, but also for adults given that statistical literacy skills can benefit them in their daily life. Adults should understand graphical and tabular displays of data which allow for multiple pieces of information to be organized and facilitate the comparison of trends (Tufte 1997). Graphical displays are pervasive in our society, and adults should be aware that graphs can be created to intentionally misinform or feature/obscure specific trends or differences (Friel et al. 2001; Gal 2004). The importance of graphical comprehension skills in statistical literacy is that everyone needs to be able to read the data values in tables or graphs, understand the conventions of creating graphs and charts, and recognize violations of those conventions (Bright and Friel 1998). Therefore, the ability to understand and interpret graphical displays is an aspect of creating a well-rounded citizen. Graphical comprehension is a component of all citizens acquiring statistical literacy skills and increases their ability to become statistically literate and informed democratic citizens (Ben-Zvi and Garfield 1997; Philip et al. 2016).

There has been a limited amount of research conducted on students' graphical comprehension in relation to statistics compared to research on student graph choice and construction in areas of mathematics, science, and psychology. Curcio (1987) introduced three levels of graphical comprehension questions defined as "read the data," "read between the data," and "read beyond the data." Friel et al. (2001) further explored Curcio's three levels and looked at skills needed to answer questions at each of the levels. "Read the data" involves "lifting information from the graph to answer explicit questions for which the obvious answer is in the graph." "Read between the data" includes the "interpretation and integration of information that is presented in a graph—the reader completes at least one step of logical or pragmatic inferring to get from the question to the answer." Finally, "read beyond the data" requires "extending, predicting, or inferring from the representation to answer questions—the reader gives an answer that requires prior knowledge about a question that is related to the graph" (Friel et al. 2001, 130). Other frameworks have been proposed that focus on graphical comprehension in terms of graphical interpretations with constructed-response questions rather than just multiple-choice questions (Pfannkuch 2006; Kemp and Kissane 2010; Boote 2014).

Given the importance of statistical literacy and graphical comprehension in society today, guidelines for teaching statistics to students at the postsecondary education level have been developed with this goal in mind. At the college-level, students need to cultivate statistical literacy and thinking skills to have a deeper conceptual understanding of statistics (GAISE College Report ASA Revision Committee 2016). Students can use those methods to apply what they have learned to problems and issues they encounter with data, analysis, and inference in their everyday lives (Pfannkuch and Wild 2004).

To develop statistical literacy and thinking skills, students must understand Curcio's three levels of graphical comprehension, so the purpose of this study was to explore college students' level of understanding. Curcio's three levels of graphical comprehension ("read the data," "read between the data," and "read beyond the data") were used as a framework to categorize the questions that included graphs on the beginning/intermediate assessment of the Levels of Conceptual Understanding in Statistics (LOCUS). This framework was used to answer the following research question: What is college students' understanding of graphical comprehension?

Methods

The sample group for the study was college-level students enrolled in an introductory statistics course. All students took the beginning/intermediate assessment of the National Science Foundation-funded LOCUS project (Grant No. DRL-1118168). The LOCUS project was first funded on September 1, 2011. The pilot assessments occurred in 2013 and the operational assessments took place during 2014. The sample of students for this study was first collected starting in 2015 when the LOCUS assessments first became available through the website. The LOCUS project focused on developing statistical assessments in the spirit of the Guidelines for Assessment and Instruction in Statistics Education (GAISE) framework. Evidence Centered Design (ECD) was used to create a beginning/intermediate and an intermediate/advanced statistical literacy assessment. The ECD process begins with the analysis of the domain of interest and adds the following layers of domain modeling, conceptual assessment framework, assessment implementation, and assessment delivery (Jacobbe et al. 2014; Riconscente et al. 2015). Three separate teams of experts (an advisory board and two test development committees) carried out the five layers of the ECD process. The test development committees went through several iterations of writing and revisions of the assessments. As part of this process, the American Statistical Association and National Council of Teachers of Mathematics also reviewed and endorsed the assessments through the Joint ASA/NCTM Committee on Curriculum in Statistics and Probability. Validation and reliability studies have been done with the final report currently under review (Whitaker et al. 2015).

Twenty-three questions of the LOCUS beginning/intermediate assessment contained a graphical display on the assessment. The authors separately assessed and assigned all 23 multiple-choice questions that contained a graph, chart, or table to a level of graphical comprehension based on Curcio's three levels. The graphical comprehension levels of the items were determined by using Table 1: Taxonomy of Skills Required for Answering Questions at Each Level from Friel et al. (2001). The table provides detailed information and examples of questions at each level. The Cohen's kappa for inter-rater reliability for the two raters was 0.875. The two raters discussed and agreed on the level assigned to each question to reach 100% agreement. Table 1 provides the number of questions for each level of graphical comprehension.

Table 1
Number of Questions per Level of Graphical Comprehension

	# of Questions Overall	# of Questions for "Read the Data"	# of Questions for "Read between the Data"	# of Questions for "Read beyond the Data"
Total	23	6	12	5

The example items following are available as sample items that are accompanied by commentaries at the LOCUS website (https://locus.statisticseducation.org/). The example items align with the graphical comprehension levels for "read the data" (Fig. 1(a)) and "read between the data" (Fig. 1(b)). There are five items that assess students' abilities to "read beyond the data." However, due to test security concerns, an example of "read beyond the data" cannot be provided in this article because there is not a sample item from the LOCUS project that addresses this level.

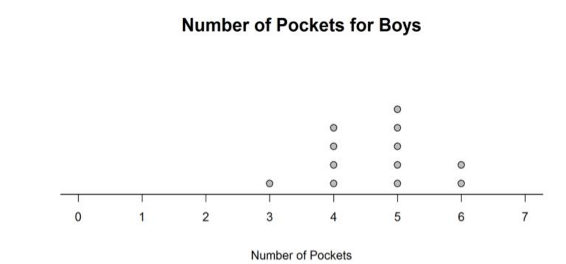
The total number of participants who took the assessment was 519 college-level students. The data collection took place using the online version of the beginning/intermediate assessment (https://locus.statisticseducation.org/). Since the assessment was open to any teacher or professor that was interested in administering the assessment to their students, the test takers are students from various educational settings such as research institutions, liberal arts colleges, or community colleges. Data was not collected about the types of courses, class size,

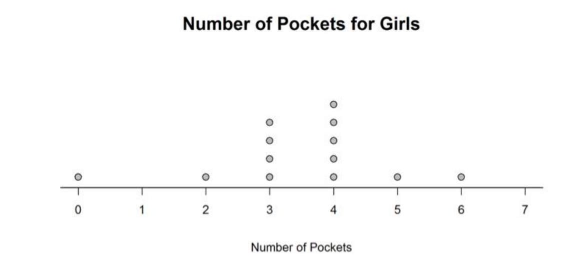
(a) The table below shows the number (to the nearest ten thousand) of cars of different colors in Denmark.

Color of Car	Number
Silver	860,000
Black	600,000
White	370,000
All other colors	620,000

Which of the following is appropriate for representing the data in the table above?
(A) Bar graph
(B) Scatterplot
(C) Histogram
(D) Dotplot

(b) The dotplots below show the distribution of the number of pockets on clothing for a group of 12 boys and for a group of 13 girls.





Based on the graphs, which of the following statements is true?
(A) The data vary more for the boys than for the girls.
(B) The median number of pockets for boys is larger than the median for girls.
(C) The mode for the boys and for girls is 4.
(D) The maximum number of pockets for boys is larger than the maximum number of pockets for girls.

Figure 1. Representative example of a LOCUS question categorized as Curcio's (a) "read the data" and (b) "read between the data"

or the teaching methods the teacher or professor used. The demographics collected on the assessment before the student began the assessment were gender, Hispanic or Latino background, race, and whether or not English is the most common language spoken at home. The "typical" respondent was female, Mexican, Mexican-American, or Chicano, self-identified their race as "other," and English was the most common language spoken at home. See Table 2 for the breakdown of demographic variables for the sample. A graphical comprehension score was calculated for each student at each graphical comprehension level as well as an overall graphical comprehension score. A student's response was counted as correct if the answer given matched the answer key. If the answer did not match or no answer was provided, then the student got the question wrong, so the question for that student was labeled as incorrect. The graphical comprehension score at each level ("read the data," "read between the data," and "read beyond the data") was the sum of the number of correct responses for each subscale. The overall graphical comprehension score was the sum of the correct responses on all graphical comprehension items. Counts, means, and percentages were recorded for the entire sample. All statistical analyses were performed using R (R Core Team 2016).

Table 2
Demographics of the Sample

Characteristics	Count	Percentage
Gender		
Female	334	64.35%
Male	185	35.65%
Hispanic or Latino Background		
Not Hispanic or Latino	115	22.16%
Mexican, Mexican-American, or Chicano	270	52.02%
Other	134	25.82%
Race		
Black or African American	62	11.95%
White	80	15.41%
Other	377	72.64%
Is English the most common language spoken at home?		
No	185	35.65%
Yes	334	64.35%

Results

Boxplots including median graphical comprehension scores for each level and overall are provided in Figure 2. Looking at graphical comprehension scores by level allowed the researchers to assess whether students answered the majority of questions correctly in "read the data," "read between the data," or "read beyond the data" levels. The boxplots for "read the data" and "read between the data" levels overlap and the median graphical comprehension scores for those two levels are equal (1 out of 6 and 2 out of 12, respectively). The boxplots for "read the data" and "read beyond the data" overlap as well with the median graphical comprehension scores for each of those levels being similar (1 out of 6 and 1 out of 5, respectively). The range of overall graphical comprehension scores for students in this study was from 0 to 10 (possible range of scores is 0 to 23). A frequency table of overall graphical comprehension scores is provided in Table 3.

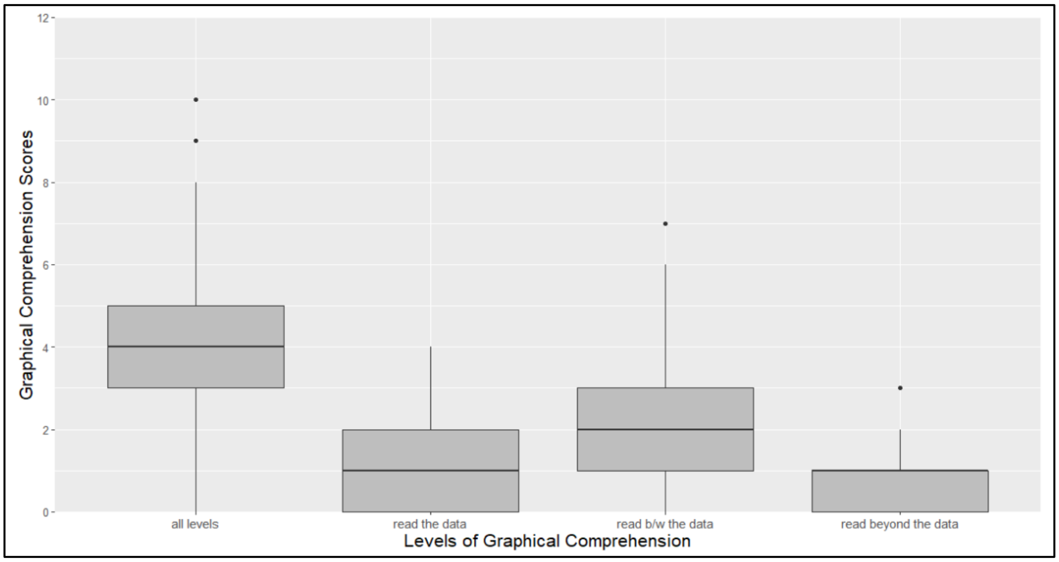


Figure 2. Boxplot of median graphical comprehension scores by level
Max score for Overall is 23; max score for "read the data" is 6; max score for "read between the data" is 12; max score for "read beyond the data" is 5.

Table 3.
Frequency Table of Overall Graphical Comprehension Scores

Overall Graphical Comprehension Score	Count
0	8
1	33
2	75
3	94
4	98
5	89
6	53
7	41
8	19
9	7
10	2

On a "read the data" question from the LOCUS assessment, less than half of the college-level students that took the assessment answered the question correctly (about 40% of students). The question has a frequency table with time intervals in minutes for the amount of time it takes for a student to get to school in the morning along with the number of students that fall into each of the time intervals out of the sample of 51 students. The question required students to assess the histogram along with a frequency table of the data and recognize the graphical error that was made in making the histogram. The graphical error that the students needed to recognize in order to answer the question correctly was that the y-axis of the histogram should not start at 10 but rather 0.

The percentage of students that incorrectly answered that histograms should not be used with numerical data was about 21%. About 13% of students answered that the histogram should not have been created using intervals that were the same width, which indicates a lack of understanding in how to create a histogram from a frequency table. Other college-level students (about 27%) answered that the numerical variable on the x-axis should not be grouped into intervals, which indicates that some students have difficulty with translating data from a frequency table to a histogram.

One item on the LOCUS assessment classified as “read between the data” provided students three dotplots for the top three scorers in a summer basketball league. The dotplots showed the points scored in eleven games for each player. The students were asked which statement best describes additional information about the player’s performances based on the comparison of the dotplots. The correct answer, chosen by about 48% of the students, described the player that had the most variation in their scores and the player that was the least variable. The second most selected answer, chosen by about 25% of students, was that the dotplots do not provide any additional information. This answer choice may indicate that about one-fourth of students in the sample are not able to make comparisons between graphs regarding variability in distributions. Another answer option describes that one player’s distribution of scores had a smaller median but was more variable, which was selected by about 17% of students. This option indicates that students struggle with understanding how medians compared to means are more stable measures of center when a distribution has large variation or is skewed. The last answer choice is a statement that one player always scores more points per game than either of the other two players. This answer choice is an incorrect statement given that the player has a few points on the dotplot below the minimum of the other two players. About 11% of students selected this last option, which suggests that these students had difficulties interpreting and comparing the dotplots in terms of minimum values for the three players.

For the “read beyond the data” question, students were asked to look at a scatterplot that showed the number of games local high school football teams won on the y-axis and the number of points scored by the opposing football teams on the x-axis. There was a negative association between the two variables. There is a single point that was an outlier of the dataset that has zero games won by a football team. The question asks what the effect would be on the association between the two variables if this data point were removed. The majority of students did not select the correct answer that removing the data point would weaken the association

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between the number of games won and the total points scored (about 16% of students answered the question correctly). About 21% of the students said that the effect cannot be determined and about 27% of students answered that the association would stay the same if the outlier was removed from the scatterplot. The most common answer was that the association would become stronger between the two variables (about 36% of students). This answer choice indicates that college-level students struggle with predicting how an association between two variables would change when a data point is removed. The students were more likely to answer the question with the association changing in the opposite way.

Discussion

The results suggest that regardless of the level of graphical comprehension, college-level students struggle on questions that contain a graphical display. Students seem to struggle with basic "read the data" questions along with questions that require comparisons between graphical displays. Given that the Common Core Mathematics Standards for grades 6–12 (National Governors Association Center for Best Practices and Council of Chief State School Officers 2010) involve only a few standards regarding graphical comprehension, this suggests that instructors of college-level courses should be cautious assuming students already comprehend graphical displays based on prior learning experiences. Since students had difficulty with "read the data" graphical comprehension questions, instructors should not assume that students are able to read and interpret the information presented in a graphical display. Instructors may need to change their instruction for mathematics and statistics at the college level to review basic concepts of graphical displays. For example, instructors may need to review concepts such as how to create graphical displays from a table, the difference between the various displays, or what type of data uses a bar chart compared to a histogram. Instructors also may need to focus their instruction on comparing different graphical displays and teaching students about making correct predictions or inferences from a graphical display. To teach those higher-level concepts, students first need to be able to read the data presented in a graph or table.

The results of this study highlight the importance of incorporating both graphical comprehension skills and data visualization skills into the introductory statistics curriculum. The term *data visualization* is a more recent term that developed to encompass graphical displays that are used for complex datasets and are sometimes interactive. During the literature review search for this article, the term *graphical comprehension* was used rather than *data visualization,* likely due to the advances in technology of the past ten years. There is a potential shift in the statistics curriculum moving from graphical comprehension with Curcio's foundational definition to data visualization skills with an emphasis on data science literacy.

Technological advances have had a universal impact on the ability of various industries and research fields to collect, store, visualize, and analyze large amounts of data (Forbes et al. 2014). As a result, data visualization tools have developed over the past ten years to present complex data and information in a succinct way to foster data interpretation in order to discover new relationships or patterns (Keim et al. 2010; Forbes et al. 2014; Mirel et al. 2016). Data visualizations facilitate communication and exploration of the statistical information that saturates daily life (Koparan and Güven 2015; Philip et al. 2016). Data visualizations also build upon traditional graphical displays like the histogram by presenting data in a way that makes the data stand out, provides comparisons between multiple variables or subgroups, may allow the user to interact with the graphic or use words or pictures to create infographics (Nolan and Perrett 2016). Data visualization skills involve what currently have been labeled as information literacy and data literacy practices (Philip et al. 2016). Information literacy involves identifying the need for information, understanding how to access the information, critically evaluating the validity and quality of information, identifying the purpose for the information, and understanding the social, legal, and economic policies and possible consequences of using the information (Philip et al. 2016). Data literacy is a part of information literacy that involves understanding how to use data and the appropriate data representations to support evidence-based thinking that aims to communicate solutions to authentic problems (Calzada Prado and Marzal Miguel 2013; Vahey et al. 2012).

In regards to teaching students about graphical comprehension and data visualization, educators' time might be well spent helping students anticipate how raw information and data will be displayed, what type of data visualization would work best to answer the question of interest, and what modifications may need to be made to the raw data or data visualization to achieve the end result of a clear and concise visualization. One way that educators can improve student graphical comprehension is by applying these concepts to open source data sets that may be of interest to students such as Netflix recommendations, Twitter hashtags, baseball statistics, or marketing information from popular clothing stores. Using data visualization tools, educators can assist students in learning about the different types of data, the purpose of creating graphs, why certain data visualizations are used for certain data types, and how they can judge which visualizations are more useful given the type of data and the purpose of the analysis. In taking this approach to teaching data visualization by incorporating information and data literacy skills, students will hopefully be able to understand the three levels of graphical comprehension in terms of reading information from the graphical display, understanding comparisons between graphical displays, and generalizing, predicting, and identifying trends from graphical displays.

One limitation of this study is that the LOCUS assessments were not created with Curcio's levels of graphical comprehension in mind, so there was not a balanced number of questions within each level. Since the students answered the graphical comprehension questions while completing the entire assessment rather than only the questions that contained a graphical display, this factor may or may not have influenced their overall graphical comprehension score. Also, the LOCUS beginning/intermediate assessment was provided as an online resource for any teacher or professor to use, meaning that information about the institution and the course was not collected. Future research about data visualization and graphical comprehension could be done involving a pre-test/post-test study that includes a specific intervention including teaching students about data visualizations. In addition, the intervention could focus on creating data visualizations using previous research about common graphical comprehension errors in order address those common misconceptions. Then, the pre-test/post-test study could involve a task that requires students to create a graphical display. This change would allow researchers to understand how students learned or did not learn how to address common errors when creating graphical displays.

Conclusion

It should not be assumed that college-level students have a solid understanding of graphical comprehension skills. Statistics education research needs to focus on defining how learning graphical comprehension and data visualization skills can be incorporated into introductory statistics courses. Instructors should be encouraged to incorporate more graphical comprehension at all three levels into their statistics courses along with data visualizations. These graphical comprehension skills can benefit students' ability to read information in the media. The skills taught can improve students' ability to read and understand what is presented to them in the data visualizations used in online magazine articles or other media sources. The students will also be able to question the types of statements made and understand when the media is misinterpreting a visualization. Instructors should not assume that students in their introductory statistics course(s) understand how to "read the data," "read between the data," or "read beyond the data" from their high school courses.

The first few chapters of introductory statistics textbooks usually focus on how to create and read graphical displays involving quantitative and categorical data. Instructors might consider spending adequate time on these chapters to make sure that their students can master the first level of graphical comprehension ("read the data") before moving on in the course to comparing distributions and making informal inferences, which are components of "read between the data" and "read beyond the data." Building a solid foundation of graphical comprehension skills starting with the "read the data" level allows students to develop statistical literacy skills that will benefit them in their everyday lives.

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