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CAVE TEMPERATURES AND GLOBAL CLIMATIC CHANGE

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Abstract
The physical processes that establish the cave temperature are briefly discussed, showing that cave temperature is generally strictly connected with the external climate. The Global Climatic changes can then influence also the underground climate. It is shown that the mountain thermal inertia causes a delay between the two climates and then a thermal unbalance between the cave and the atmosphere. As a consequence there is a net energy flux from the atmosphere to the mountain, larger than the geothermal one, which is deposited mainly in the epidermal parts of caves.

Key words: Global Climatic Change underground, cave temperature, cave climate, cave thermal capacity, underground-atmosphere thermal interaction

Introduction

The temperature inside the caves is quite constant unlike the fluids that flow inside from the external atmosphere. The reason is that in the underground the fluids enter in thermal contact with other substances with much larger thermal capacity.
In very first approximation we can consider a cave as a thermally insulated system in which air and water streams flow at variable temperature.
It is easy to show that its temperature asymptotically approaches the average fluid temperature weighted with the inflowing thermal capacity fluxes (Appendix A). It is also possible to show that the thermal capacity of inflowing water is generally much larger than the thermal capacity of air, and its very near to the local average temperature T_L (Badino, 1995); then in very first approximation we can consider that the cave temperature T is almost equal to T_L, with some important corrections that we are going to see briefly.
The main purpose of this work is to discuss the effects of Global Climatic Changes on the cave temperatures.

Thermal inertia of caves

We have to calculate the thermal inertia of caves, but we can only define the thermal capacity of something that exists, the trouble is that a cave does not exist in itself. Its thermal capacity can be formally defined as the ratio between the thermal energy absorbed by the mountain through the cave and the corresponding temperature increase of the whole system. The cave's lack of materiality determines that the ratio depends on the time characteristics of energy release.
With diffusive material such as massive rock, it is usual to introduce the idea of penetration lengths, defined as

$$x_{sin} = \sqrt{\frac{a}{\pi}\tau} \qquad (1)$$

for sinusoidal fluctuation and

$$x_{step} = \sqrt{4at} \qquad (2)$$

for step changes (Isachenko, 1969), (Holman, 1996). The limestone thermal diffusivity (Lide, 1988) is $a=1.0\times10^{-6}$ m^2 s^{-1}, but for real rocks saturated with water, this value can be for a factor 10 higher.
The equation (1) gives the penetration length of a sinusoidal thermal waves of period τ, whilst equation (2), with a quite crude approximation, gives the depths to which a sudden temperature disturbance at the surface arrives after a time t.

period τ, or time t	x_{sin} in limestone (m)	x_{step} in limestone (m)
1 day	0.17	0.6
1 year	3.2	10
1000 years	100	400

It is possible to see that for sudden temperature changes the "thermal capacity" of the cave is that of stored air and water deposits, but for thermal pulses longer than a few minutes the rock dominates completely. In these cases we can assume that the cave thermal inertia is the rock thermal capacity up to the penetration length of that particular temperature change.
For long time-scale changes the whole mountain (from the aquifer to the top) participates to the total thermal capacity. We can then write

$$C = \rho_R C_R (HA) = 2\times10^6\, HA \qquad (3)$$

Where H is the average mountain altitude above the aquifer and A is the surface.
If we assume alpine karst conditions, i.e. an average infiltration I (precipitation minus evaporation) of 1000 mm y^{-1} (which corresponds to an infiltration $I=3\times10^{-5}$ kg $s^{-1}m^{-2}$) and a specific air flux through the cave system of 10 kg $s^{-1}km^{-2}$, we have thermal capacity rates of 0.13 and 0.01 $Wm^{-2}K^{-1}$. In the Appendix B the concept of equilibration time Dt_{eq} of a cave system is introduced (Eq. B.2): it is the scale of time necessary to a "cave" (in fact: to a mountain) to attain the thermal equilibrium with the flowing fluids. It is there shown that, from another point of view, it is the time during which the *total thermal capacity of flowing fluids equals the total thermal capacity of the mountain* (Eq. B.5).
The time scales of "cave cooling" due to water and air for different mountain "thickness" are given in Table 1 (Eq. B.4 in Appendix B).

Table 1 – Equilibration time scale as function of average mountain size

H (m)	Δt_{eq}, air (y)	Δt_{eq}, water (y)
10	60	5
100	600	50
1000	6000	500

It is clear that in typical alpine situations we can assume that the cave temperature is established by the entering water, and that the typical equilibration time scales are of order of centuries.
In this model, the "cave" thermal capacity acts then as a capacitor in a low-pass filter, that can be crossed by thermal fluctuation with periods lower that the equilibration time (the time-constant of the filter).
This means that the diurnal and seasonal fluctuations are smoothed and the cave temperature becomes the average of inflowing fluids, but means also that the cave reacts to long term temperature drifts with some delay.

Corrections for a second-order approximation

The cave temperature is then roughly the average local temperature of atmosphere, but some corrections are needed to include important second-order effects which depend on the cave structure and location.
To understand the importance of these "details" it is sufficient to remember that the "cave temperature" does not depend only on the altitude inside the mountain, but also in the gallery, because the thermal sedimentations can be very significant.
The main corrective terms come from these effects:
1) the average temperature of precipitations is a little lower than the local average temperature;
2) the geothermal flux can play an important role;
3) the entering fluids are selected in dependence of their temperature (Appendix A, Eq. A.3). For instance, below 0 °C the water cannot enter underground and stays outside, the air flow is stronger when the temperature difference is larger, and it is not seasonally symmetrical;
4) the fluids enter at various altitudes, thus with many different temperatures;
5) if the cave is very large, the temperature difference inside it, created by internal processes (fluid heating due to vertical movements and friction inside narrow conduits) can be very important.

Here we simply give this list, leaving the complex discussion to a future work.
We go now to the main aim of this work, the discussion of another similar correction: the underground effect of external long term variations of climate.

Global climatic change

We have implicitly assumed a constant "boundary conditions", that is a constant temperature and flux of inflowing fluids. This makes sense for short times, but if we deal with times longer than Δt_{eq} the general climate behaviour has to be included in the calculation.

It is well known that the climate changes in centuries time-scales (Global Climatic Chance, “GCC” from hereafter).
The deep drilling in Antarctica and Greenland have allowed to reconstruct the Earth climate in the last half-million years, showing fluctuations of almost 10 °C in periods in the order of 10^4 y.
Also direct temperature measures show climatic drift: for instance, the average temperature in the city of Turin has changed from 12.0 to 13.5 °C in the last 150 years (Mercalli, 1997).
For relatively short periods we can assume a linear change of temperature in time

$$T_L = T_{L0} + r_{gc}\, t \qquad (4)$$

Where r_{gc} is the rate of temperature change due to GCC, that can be positive or negative. The experimental data show that in the last two centuries the warming rate is roughly (IPCC, 2001)

$$r_{gc} = +3 \times 10^{-10}\ \mathrm{Ks^{-1}}$$

which is slow, but significant.

Underground effects: the temperature

It is not difficult to solve the problem of the temperature T of a water reservoir supplied by a water flux if temperature increases (or decreases) as $T_L=T_{L0}+r_{gc}t$ (Appendix C).
Assuming that at t=0 the temperatures of the system and the inflowing water are equal $T_{L0}=T_0$ (an equilibrium condition, which means to switch on the process after a long period of stable climate) the solution becomes (Eq. C.3)

$$T - \left(T_{L0} + r_{gc}\, t\right) = r_{gc}\, \Delta t_{eq} \left[\exp\left(-\frac{t}{\Delta t_{eq}}\right) - 1\right] = \Delta T_{IE}(t, H) \qquad (5)$$

The left term is the temperature difference between the average temperature of the internal part of the mountain (our cave…) and the “instantaneous” average external temperature T_L. This term ΔT_{IE} is then due to the GCC and it is a function of time and mountain size. Its value is negative in case of a global warming and positive in case of cooling, because the mountain thermal inertia forces the system to hold at old equilibrium values. In the right term of the same equation appears $r_{gc}\Delta t_{eq}$, which is the main disequilibrium term

$$\Delta T_{gc} = r_{gc}\Delta t_{eq} \qquad (6)$$

It describes the difference between the temperature of the source (the inflowing water) and the water reservoir, after a time equal to the equilibration time scale Δt_{eq}, in the case there would be no mixing at all.
In physics it is usual to interpret similar terms as the processes scale amplitudes, in this case the amplitude of thermal unbalance between the atmosphere and the cave system. In practice, it is the typical temperature difference between the cave and the inflowing fluids.
In the equation this term is multiplied by a usual exponential-negative term, that fades

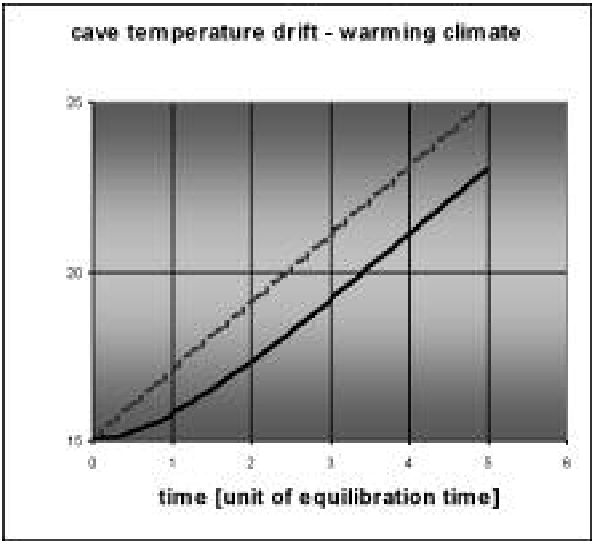

Figure 1 - Dashed line: hypothetical behaviour of external average temperature. Full line: cave temperature. The cave follows the external climate with a delay Δt_{eq} and temperature shift of ΔT_{gcc}.

the temperature unbalance with a time constant Δt_{eq}, as reasonably expected.
This main disequilibrium term ΔT_{gc} can be written in dependence on H (mountain thickness) applying Eq. B.4

$$\frac{\rho_g C_g}{C_w + P_a C_a} H = G_{gc} H \quad (7)$$

This equation shows that the temperature difference between a karstic mountain and the atmosphere, according to the GCC, it is proportional to the thickness of the mountain. We have introduced a new parameter G_{gc}, which is function of local water and air fluxes, and GCC rate.

$$G_{gc} = \frac{\rho_g C_g}{I C_w + P_a C_a} r_{gc}$$

In fact, the air fluxes depend on the cave structure, but represent a small terms in comparison with the water ones ($P_a C_a << I C_w$), then G_{gc} depends essentially on the GCC rate and infiltration intensity.
The Table 2 shows its value for three I values.

Table 2 - Local temperature unbalance cave-atmosphere per mountain size unit

Infiltration I* (mm y^{-1})	Infiltration I (kg $s^{-1} m^{-2}$)	G_{gc} (K km^{-1})
300	1×10^{-5}	15
1000	3×10^{-5}	5
3000	1×10^{-4}	1.5

The parameter has the dimensions of a temperature gradient ($K\cdot m^{-1}$); obviously it is not a temperature variation inside a mountain, but the temperature unbalance cave-atmosphere per unit of mountain thickness.
If we assume, as usual, that the inflowing fluids go quickly in thermal equilibrium with the rock, we can read the above equation as an anomalous temperature increase or decrease (respectively in case of climate cooling or warming) between the entrance and the water springs due to GCC.

The solution behaviour
It is easy to estimate the time dependence of ΔT_{IE} in extreme cases. Remembering that for small x it is possible to assume that $\exp(x)\approx 1+x$, the ΔT_{IE} term for $t \ll \Delta t_{eq}$ becomes

$$\Delta T_{IE} \approx r_{gc}\,\Delta t_{eq}\left(-\frac{t}{\Delta t_{eq}}\right) = -r_{gc}\,t$$

It says that after a long stable period, the thermal unbalance increases linearly.
It is also interesting to estimate the long term behaviour. The temperature difference tends asymptotically to $-\Delta T_{gcc}$ for $t \gg \Delta t_{eq}$, which means that the mountain follows the GCC temperature drift, but with a temperature drop $-\Delta T_{gcc}$, that corresponds to a time-shift of Δt_{eq} between the internal and external climates.

Energy flow
Where does this temperature increase or decrease appears? The inflowing fluids reach a thermal equilibrium with the rock in few metres, and then the thermal anomaly are concentrated in the first parts of absorbing conduits but they are then thermally diffused in the whole mountain with its typical time scales Δt_{eq}. Then in the same more epidermal regions is concentrated the corresponding net energy deposition or extraction, in case respectively of global warming or cooling.
It is easy to estimate the energy flux released by the atmosphere to the cave system due to GCC. The energy flux is due to the thermal unbalance ΔT_{IE} carried by the fluids, then the applied power per square metre is given simply by

$$W = \left(IC_w + P_a C_a\right)\Delta T_{IE} \qquad (8)$$

We have seen that ΔT_{IE} can assume values between 0 and ΔT_{gcc}. Using this last value to estimate the maximum possible power released to the mountain per surface unit we have

$$W_{max} = \left(IC_w + P_a C_a\right)\Delta T_{gcc} = \left(\rho_R C_R H\right) r_{gc} \qquad (9)$$

Note that the fluid flow rate has disappeared because the equilibrium time (and by consequence the ΔT_{gc}) is proportional to the inverse of discharge, but the energy deposition is proportional to the discharge and then the two contributions compensate each other: the power supplied to the mountain due to GCC does not depend on the constancy of precipitations. Including the numerical values we have

$$W_{max} \approx 5\times 10^{-4} H \;\left[\mathrm{Wm^{-2}}\right]$$

It is a specific power release of 0.05 and 1 Wm^{-2} respectively for H=100 and H=1000 m, with the actual r_{gc} value. It is good to note that a value averaged on the last 15 years gives a figure ten times larger (Mercalli, 1997), which means that all our estimation like the energy deposition and so on have to be increased with a factor 10... Obviously, the shorter the average interval, the smaller is the involved rock volume, but the estimation remains impressive and it suggests more detailed studies involving the kinetics of thermal energy transfer.

Nevertheless these values are surprisingly high if compared with the geothermal flux, 0.06 Wm^{-2} (Veerhogen, 1956). The climatic change releases to the cave more energy than the geothermal flux…

This fact suggests a more careful comparison between the two contributions. The geothermal flux can play a role in deep karst due to focusing effects given by deep conduits that modify the rock temperature field (Badino, unpublished), but it is in general completely intercepted by the aquifer. As a rule it has the only effect to heat the water during the travel between the cave bottoms and the springs. The amplitude of heating depends on the water flux that shield it. It is possible to show (Badino, 1995) that this temperature increase is

$$\Delta T_w = \frac{F_w}{C_w I} \approx \frac{500}{I^*}[\mathrm{K}]$$

It does not depend on the mountain size and it is always positive. In the Table 3 its values are compared with the maximum unbalance amplitudes due to the GCC, that can be positive or negative.

Table 3 – Temperature drops due to geothermal flux and GCC vs infiltration

Infiltration I* (mm y^{-1})	ΔT_{gt}, (K)	ΔT_{gc}, H=100 m, (K)	ΔT_{gc}, H=1000 m, (K)
300	1.5	±1.5	±15
1000	0.5	±0.5	±5
3000	0.15	±0.15	±1.5

It is interesting to note that the smaller is the net precipitation, the larger are the two unbalance terms, that compete in the case of global warming, as nowadays, or they add in case of global cooling. But it is necessary to emphasize that the two energy fluxes are applied at different regions, the aquifer the geothermal term and the mountain upper layer the other. This can obviously create strong thermal unbalance inside the mountain.

All these processes can affect the karst chemistries underground and thus, reasonably, leave a track in the cave genesis and in its structural details, mainly in the more epidermal parts of mountain.

Conclusions

The caves allow the thermal inclusion of karstic mountains into the atmosphere and, as a consequence, the whole mountain temperatures attain essentially the local average temperature of the external atmosphere.

Their enormous thermal capacities act as low-pass filters with the crossing fluids and cre-

ate a delay in the processes of equilibration atmosphere-mountain.
These delays prevent the mountain to follow synchronously the global climatic fluctuations, maintaining a disequilibrium that causes strong energy fluxes between the atmosphere and the caves, essentially concentrated in the cave parts more exposed to the external fluids.

Appendix A

Let us estimate the equilibrium temperature T of a system S with thermal capacity C crossed by fluids. Each entering volume V of fluid (thermal capacity C_a, density r_a) at T_a exchanges with S a thermal energy E given by

$$E = \rho_a C_a V (T_a - T)$$

Let us call F(t) the entering fluids flux (cubic metre per second) at temperature $T_a(t)$. In general F(t) is very complex and depends on meteorology.
In a sufficiently long time period t_0 the energy released to S is

$$E_{tot} = \int_0^{t_0} \rho_a C_a F(t) [T_a(t) - T(t)] dt \qquad \text{(A.1)}$$

And the temperature change of S is

$$T_{fin} - T_{in} = \frac{E_{tot}}{C}$$

It is obvious that S warms when the fluids release energy to it, and cools when they subtract it; this trivial observation allows to say that the asymptotic temperature value of S is the value able to zero the algebraic sum of fluids energy fluxes, that is

$$\frac{E_{tot}}{t_0} = \frac{1}{t_0} \int_0^{t_0} \rho_a C_a F [T_a(t) - \overline{T}_C] dt = 0$$

If we assume that the average daily temperature during the year oscillates with amplitude ΔT around a mean T_m

$$T_a(t) = T_m + \Delta T \sin(\omega t) \qquad \text{(A.2)}$$

with Δt phase of the year, we obtain

$$\frac{E_{tot}}{t_0} = \frac{1}{t_0} \int_0^{t_0} \rho_a C_a F [T_m - \overline{T}_C] dt + \frac{1}{t_0} \int_0^{t_0} \rho_a C_a F \Delta T \sin(\omega t) dt = 0 \qquad \text{(A.3)}$$

The second integral is zero if F(t) is able to preserve the symmetry of the sinusoidal function.
In fact this is a very strong assumption, and it is possible to show that in natural conditions it may occur that F(t) does not have this fundamental property (in other words, water and air fluxes are "selected" depending upon their temperature) and a corrective term to the first integral appears. In general, anyway, these are small corrections and only the first integral holds.

It is easy to see that it is zero if the temperature T_C of system S, the cave, it is equal to the average temperature of entering fluids, that in general can be assumed to be T_L, average local yearly temperature.

Appendix B

Let us consider a system S with thermal capacity C and temperature T_0.
At t=0, through S it begin to flow a fluid at rate F_{T1} (in thermal capacity rate (J $K^{-1}s^{-1}$)) with temperature T_1: we want here to estimate the time evolution of system temperature T(t).
In a time step dt the fluid releases thermal energy which causes an increase of system temperature

$$CdT = F_{T1}(T_1 - T)dt \ .$$

Solution of the equation gives the time dependence of T

$$T(t) = T_1 - (T_1 - T_0)\exp\left(-\frac{F_{T1}}{C}t\right) \quad \text{(B.1)}$$

The behaviour is shown in Fig. 2 for two different T_0. The system temperature asymptotically approaches the temperature of the fluid with a time-scale

$$\Delta t_{eq} = \frac{C}{F_{T1}} \qquad \text{(B.2)}$$

which we define as the “equilibration time”.
If we have many entering fluids at different temperatures T_j the solution can be generalized, with obvious meaning of symbols

$$T = \frac{\sum_j F_{Tj} T_j}{\sum_j F_{Tj}} - \frac{\sum_j F_{Tj}(T_j - T_0)}{\sum_j F_{Tj}}\exp\left(-\frac{\sum_j F_{Tj}}{C}t\right)$$

$$\Delta t_{eq} = \frac{C}{\sum_j F_{Tj}} \qquad \text{(B.3)}$$

If we assume that the whole mountain participates to the thermal exchanges (H is the average mountain altitude above the aquifer), and call P the net precipitation intensity, Pa the air flux intensity and C_R, C_w and C_a respectively the thermal capacity per mass unit

$$\Delta t_{eq} = \frac{C}{F_{T1}} = \frac{\rho_R C_R H}{PC_w + P_a C_a} \qquad \text{(B.4)}$$

A cave acts as a low-pass filter with a time constant Δt_{eq}. This fundamental parameter can be read also as

$$C = \Delta t_{eq} \sum_j F_{Tj} \qquad \text{(B.5)}$$

The equilibration time of a cave is the *time during which the total thermal capacity of flowing fluids equals the total thermal capacity of cave*.

Appendix C

Let us consider a system S with thermal capacity C and temperature T(t), with $T(0)=T_{s0}$. System is fed by the water with thermal capacity rate $F=IC_w$, with a temperature that linearly increases

$$T_s = T_{s0} + r\,t \qquad \text{(C.1)}$$

Let us assume a complete mixing between S and the entering flux.
The energy conservation at time t states that the energy released by the water, that enters at T_s and goes out at T is responsible for the temperature increase dT of the system S

$$F\,dt\left(T_{s0} + r\,t\right) - F\,dt\,T = C\,dT$$

The differential equation is then

$$dT = \frac{F}{C}\left(T_{s0} + r\,t - T\right)dt \qquad \text{(C.2)}$$

We recall that the ratio F/C is the inverse of equilibration time scale Δt_{eq} (Eq. B.4). This differential equation can be solved, quite laboriously, with the substitution

$$z = \frac{F}{C}\left(rt - T\right)$$

The solution is

$$T = \left(T_{s0} - r\,\Delta t_{eq}\right) + r\,t + \left[T_0 - \left(T_{s0} - r\,\Delta t_{eq}\right)\right]\exp\left(-\frac{t}{\Delta t_{eq}}\right) \qquad \text{(C.3)}$$

If we assume that at t=0 the inflowing fluid is at the thermal equilibrium with S (that is $T_{s0}=T_0$), we obtain the S temperature at t

$$T(t) = \left(T_0 - r\,\Delta t_{eq}\right) + r\,t + r\,\Delta t_{eq}\exp\left(-\frac{t}{\Delta t_{eq}}\right)$$

and its difference with the temperature of entering fluid is

$$T - \left(T_0 - r\,t\right) = -r\Delta t_{eq} + r\,\Delta t_{eq}\exp\left(-\frac{t}{\Delta t_{eq}}\right) \qquad \text{(C.4)}$$

We can call

$$\Delta T_{dis} = r\,\Delta t_{eq}$$

It is the magnitude of difference between the temperature of the inflowing water and the system after equilibration time in the case of no mixing.

Symbols

a rock thermal diffusivity ($m^2\ s^{-1}$)
C_a air thermal capacity per mass unit ($J\ K^{-1}kg^{-1}$)
C_R rock thermal capacity per mass unit ($J\ K^{-1}kg^{-1}$)
C_w water thermal capacity per mass unit ($J\ K^{-1}kg^{-1}$)
Δt_{eq} thermal equilibration time cave-atmosphere (s)
ΔT_{gc} thermal unbalance scale value cave-atmosphere
DT_{IE} instantaneous temperature difference cave-atmosphere
F_{gt} specific geothermal power (Wm^{-2})
F_w thermal capacity flow rate of inflowing water ($W\ K^{-1}$)
F_{Tj} thermal capacity flow rate of j-inflowing fluid at temperature T_j ($W\ K^{-1}$)
G_{gc} local temperature unbalance cave-atmosphere per mountain thickness unit ($K\ m^{-1}$)
H average mountain altitude above aquifer (m)
I water infiltration underground ($kg\ s^{-1}m^{-2}$)
I* water infiltration underground ($mm\ y^{-1}$)
P_a specific air flux through the cave system ($kg\ s^{-1}m^{-2}$)
r_{gc} climate temperature change rate ($K\ s^{-1}$)
r_a air density
r_R rock density ($kg\ m^{-3}$)
r_w water density
T Instantaneous cave temperature (°C)
T_L local average yearly temperature (°C)
T_{L0} cave temperature at starting time (°C)
x_{sin} penetration length of sinusoidal thermal disturb in limestone (m)
x_{step} penetration length of step thermal disturb in limestone (m)
W_{max} maximum thermal power flux atmosphere-cave (Wm^{-2})

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