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Thomas J. Pfaff
Ithaca College, tpfaff@ithaca.edu

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Review of *Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving* by Sanjoy Mahajan

Abstract

Mahajan, Sanjoy. *Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving* (The MIT Press, Cambridge, Massachusetts, 2010). 152 pp. ISBN 978-0-262-51429-3

Street-Fighting Mathematics is an engaging collection of problem-solving techniques. The book is not for a general audience, as it requires a significant level of mathematical and scientific background knowledge. In particular, most of the book requires knowledge of Calculus I and there are examples that will require knowledge of Physics. At the same time, there are parts of the book that don't require this much background. While the title of the book may be misleading, as it is really street-fighting mathematics for people with a fair amount of training in the subject, there is a lot to be gained from reading this book, and calculus teachers may find it to be a useful resource.

Keywords

Calculus, Problem Solving, Estimation, Approximation

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Cover Page Footnote

Tom Pfaff is a professor of mathematics and Honors Program Director at Ithaca College in New York. He has an interest in applying mathematics to issues of sustainability, producing online resources for use in calculus, statistics, and other mathematics courses available at <http://www.sustainabilitymath.org/>. He has also developed multidisciplinary sustainability modules resulting from NSF grant DUE-0837721 available at <http://www.ithaca.edu/mse/>.

Introduction

Street-Fighting Mathematics, by Sanjoy Mahajan (2010), is an interesting collection of problem-solving techniques. In the preface of the book, the author suggests that the book is a complement to works of Polya, but this book expects a much greater knowledge of mathematics and science and it isn't for a general readership. The author says that the book grew out of a short course taught at MIT, and in the end, it isn't entirely clear who this book is for. Is it for future scientists and mathematicians? Is it for users of mathematics no matter the setting? Maybe it is for teachers of mathematics? There is something in this book for all of these groups, although they may not find the whole book useful. With that said, this book does have value to those that teach Calculus.

Mahajan is an Associate Professor of Applied Science and Engineering at Olin College of Engineering. He is also a visiting Associate Professor of Electrical Engineering and Computer Science at MIT. He was once the Associate Director at MIT's Teaching and Learning Laboratory. Mahajan has a Ph.D. in Physics as well as undergraduate degrees in physics and mathematics. He clearly brings his interest in understanding how we teach STEM subjects to the writing of his book.

The book has six chapters with each focusing on a particular problem-solving technique. Each chapter has several examples as well as some practice problems to reinforce the technique. Before looking at each chapter, we will pause and comment on the foreword by Carver Mead and the book's preface.

Many mathematicians who pick up this book will likely put it down before getting past the first line of the forward as it says, "Most of us took mathematics courses from a mathematician—Bad Idea!" If one manages to get past that, then there is the first sentence of the preface that states, "Too much mathematical rigor teaches *rigor mortis*: the fear of making an unjustified leap even when it lands on a correct result." The general derogatory attitude towards an entire discipline is unfortunate and off putting. All disciplines have their shortfalls and areas that need improvement in their teaching, and comments like these shut down conversation instead of opening up dialogue. Given these comments, it is ironic how much this is a mathematics book. It is not clear what the author means by mathematical rigor because there is plenty of rigor in this book.

The Book

The book gets off to a great start by discussing units and dimensions, with a great news excerpt that compares a rate, GDP, with a net worth. The issue is classic quantitative literacy, and the treatment and is well done. The book really takes the discussion to the next level though by looking at standard free-fall equations showing how a dimensional analysis can be used to estimate answers such as impact speed without much effort. Sure, the result is only accurate up to a constant, but the technique is a great way to check answers to problems. This is something calculus teachers could and should do.

The book goes further in applying dimensional analysis to estimate integrals by using $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx$ as an example. It does take a few pages to get to the estimate, but it is worth it. What is surprising is that there is no discussion about the lack of an algebraic antiderivative for this integral, which makes the estimate more useful. In fact, it is surprising there is no guess and check method in the book as it is a key tool in teaching students, for example, how to find antiderivatives. In this case, a bit of guess and check will lead one to recognize the difficulty in finding an antiderivative for $\int e^{x^2} dx$. Since the integrand has an e^{x^2} we know the antiderivative must have an e^{x^2} , but if we start with this as our guess, then our check requires the use of the chain rule. At this point we can easily see the challenges in finding an antiderivative. Still, this may be the best chapter in the book in terms of usefulness to the large numbers of calculus teachers.

The second chapter of the book examines the tool “the method of easy cases.” Certainly, checking solutions with easy choices of the variable, such as $x = 0$, is a technique that is worthy of being explicitly stated. The first section starts off with the $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx$ example from the previous chapter by asking the question if the integral is equal to $\sqrt{\pi\alpha}$ or $\sqrt{\pi/\alpha}$. If one didn’t read the first chapter, then you’ll be asking where these two possibilities came from. The author suggests checking endpoints for α of 0 and ∞ , excellent, but then goes to check ∞ first. The natural first choice here is $\alpha = 0$, which gives an integrand of 1 and easily rules out $\sqrt{\pi\alpha}$ as a solution.

The second section of this chapter looks at the area of an ellipse, with axis a and b , by considering the five possibilities of ab^2 , $a^2 + b^2$, a^3/b , $2ab$ and πab . Again, the first question should be where did these possibilities come from. The checking of endpoints starts with $a = 0$ and $b = 0$ before getting to $a = b$. For a book that has an entire chapter on the value of pictures, a basic math tool, this section doesn’t make sense. When looking at the picture of an ellipse in the book, one should ask about the object we have when $a = 0$ or $b = 0$.

If one were to try and guess the area of an ellipse, the starting point should be with a circle. In fact, if one is introducing an ellipse for the first time, generating possible expressions for its area starting from a circle is a great exercise. Any possible result needs to reduce to that of a circle when $a = b$, which makes sense with the picture. It would seem that any street-fighting mathematics would use as a main tool building off of previous knowledge. Oh, and problem 2.4 of this section says to use integration to derive the formula for the area of an ellipse. Certainly a good problem, but one might question how it fits into the theme of the book.

Section 2.4 looks at the drag when dropping two paper cones of different sizes. The example here is a nice one, but you need to be familiar with the Navier-Stokes equations. Yet, one wonders if this is really “educated guessing and opportunistic problem solving” or rigorous examination of the problem.

The chapter on lumping has material that would be particularly useful to calculus instructors. There is a nice section in this chapter on estimating integrals by strategically creating one rectangle, which is worth using in any calculus course. The method is then used to estimate the integral representation for $n!$ which is $\int_0^{\infty} t^n e^{-t} dt$. There are two nice applications of Taylor

series in this section, which again makes this a section one that could be used in a calculus class. But again we note that the street fighting going on here requires some significant mathematical sophistication. It is as if our street fighter has a black belt.

This chapter also takes a look at approximating tangent lines with secant lines. The discussion is excellent and useful for calculus teachers. On the other hand, the examples are with functions that are generally easy to differentiate and there could have been a nod to how the limit definition of the derivative effectively deals with the estimation issues. By this point in the book, the role of technology is a glaring absence. For example, if one needs to estimate the slope of a tangent line because a derivative is problematic, there are ways to do this with technology that may be easier and more effective than the estimation techniques. At this point in time, Wolfram alpha is easily accessible. This does not mean that estimation techniques should not be taught, but that the role of technology should not be ignored.

There are occasions in this book where an estimation is used without any justification. In the “Lumping” chapter it says, “Because the cosine changes by 2 (from -1 to 1), call $1/2$ a significant change in $f(x)$.” Why use $1/2$? Will $1/4$ of the range always work or is it just appropriate for this example? It’s hard to see this choice as a technique when it seems more like random guessing, which of course it is not.

Anyone who is a fan of the proof without words compilations by Roger B. Nelsen, properly mentioned at the end of the chapter, will certainly enjoy chapter 4. While fans will find nothing particularly new here, those who are not familiar with the many uses of a good picture will benefit from reading this chapter. There is a nice discussion of the arithmetic and geometric mean which ends with the application of approximating π . Approximating π has a long history in mathematics, and there is a certain irony in finding an example in this particular book.

The chapter concludes with an example of estimating $n!$ and Stirling’s approximation. An integral and a few well-chosen pictures gets $n! \approx n^n \times e^{-n} \times e \times \sqrt{n}$, which becomes equality if the e is replaced by $\sqrt{2\pi}$. This is another example that could be used in a calculus class. The only frustrating thing is stating the error is “only 8%” in comparison to the actual result. This comment raises the question of how useful is an estimation technique if we don’t know how good our estimation is; comparing it to an already derived result seems like cheating a bit.

“Taking Out the Big Part,” chapter 5, is a tale of estimation by various means, with the main idea being to take out the “big part” and then consider a correction factor. Learning to estimate efficiently and effectively is a useful quantitative skill. The chapter is certainly valuable to that end with all but the last section being generally accessible. The lead example looks at the storage capacity of a CD-ROM (an already outdated example?) and boils down to the product $3600 \times 4.4 \times 10^4 \times 32$. Step 1 is the logical factoring out of powers of ten to get $3600 \times 4.4 \times 10^4 \times 32 = 10^8 \times 3.6 \times 4.4 \times 3.2$. We then look at the correction $3.6 \times 4.4 \times 3.2$ and here Mahajan recommends rounding each factor to either 1, few, or 10, where $(\text{few})^2 = 10$. This gives $3.6 \times 4.4 \times 3.2 = (\text{few})^3$ or roughly 30.

The final answer yields 3×10^9 . The introduction of the few term is interesting, but why not go with regular rounding to get $3.6 \times 4.4 \times 3.2 \approx 4 \times 4 \times 3 = 48$ or roughly 50? This would give a final result of $50 \times 10^8 = 5 \times 10^9$, which is more accurate for seemingly little extra work.

Again, it is interesting to see mathematical rigor being applied to justifying estimation techniques, for example, in equation (5.7) where we see

$$\frac{(x + \delta x)(y + \delta y)}{xy} = \frac{x + \delta x}{x} \frac{y + \delta y}{y} = \left(1 + \frac{\delta x}{x}\right) \left(1 + \frac{\delta y}{y}\right).$$

There are plenty of nice nuggets in this chapter, and it ends with estimating the integral $\int_{-\pi/2}^{\pi/2} (\cos t)^{100} dt$. This integral is certainly daunting and the mathematics applied to the estimation is excellent and rigorous.

The last chapter entitled “Analogy” explicitly states, with examples, what mathematicians are taught to do. For example, when asked a question about something in three dimensions, bond angles in methane, first simplify it and consider two dimensions. Section 6.2 asks the question into how many regions do five planes divide three-dimensional space? The technique developed tells us to start with 1 plane, then 2 planes and to look for a pattern. We also look at the easier problem of lines in a plane and asking the related question. These are not so much street-fighting techniques as standard mathematical practice. Later in the chapter, in working to understand the differential operator, the author explores the analogy to functions and numbers. The rest of the section is filled with some excellent mathematics as you’ll see Taylor series, yet again, as well as sums, integrals, and trigonometric functions. The mathematics in this section is somewhat sophisticated, although still at an undergraduate level.

Conclusion

The book is certainly valuable for many practitioners of mathematics including those that teach mathematics, and the introductory comments in the forward and preface raise some interesting questions. For example, what exactly is mathematical rigor and when does “educated guessing and opportunistic problem solving” become a mathematical technique? The author seems to suggest that mathematical rigor is boring and something to be avoided, yet he has turned what he sees as guessing into formal techniques. For example, the title of section 1.3 is “Guessing Integrals,” but he really is not guessing as he is applying an explicit technique, dimensional analysis, to estimate the integral. In fact, isn’t this the way mathematical techniques become techniques? The good idea of using dimensional analysis to estimate an integral has now been written down in a book to be used by others as a technique. If one now uses this book to teach this technique to others, they may now view this technique as boring mathematical rigor. It is ironic that in a book that suggests learning mathematics from a mathematician is a bad idea, the author is behaving very much like a mathematician.

In the end, the book doesn’t really live up to its title. For one, the title “The Art of Mathematical Problem Solving” leads one to believe that this book is going to

be more of a quantitative literacy text. The book is definitely not that, as there is an expectation of significant mathematical background to use this book. This expectation doesn't match the title of *Street-Fighting Mathematics*, which implies much less formality and less background knowledge. The subtitle suggests a more informal book, but really the book is formalizing techniques of estimation, and there is nothing wrong with that.

Many of the problem-solving techniques in the book and the general ideas are covered in mathematics classes, but students don't often fully grasp these concepts until later. Mahajan's class was likely very successful at least in part due to reaching students at a proper time of mathematical maturity. Be warned that many of the problems in the book are challenging, especially if you aren't used to this type of estimation or way of thinking. In the end the book is worth it, not just because you can get it for free¹ and a hard copy isn't expensive either, ([\\$27 at MIT Press](#)) but because there are a lot of good mathematics, ideas, examples, and things to think about.

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¹([pdf version here](#))