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Quantitative Literacy and the Common Core State Standards in Mathematics

Abstract
How supportive of quantitative literacy (QL) are the Common Core State Standards in Mathematics (CCSSM)? The answer is tentative and conditional. There are some QL-supportive features including a strong probability and statistics strand in grade 6 through high school; a measurements and data strand in K-5; ratio and proportional reasoning standards in grades 6 and 7; and a comprehensive and coherent approach to algebraic reasoning and logical argument. However, the standards are weak in supporting reasoning and interpretation, and there are indications that the applications in CCSSM – mostly unspecified – will not include many QL contextual situations. Early indicators of assessment items follow a similar path. Except for statistics, most of the high school standards are aimed at development of algebra and precalculus topics, and there will likely be little room for more sophisticated applications of the QL-friendly mathematics of grades 6-8. The experience with CCSSM is limited at this point, leaving several crucial results uncertain, including assessments, emphases on statistics, and kinds of modeling and other applications.

Keywords
Common Core Standards, quantitative literacy

Cover Page Footnote
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Introduction

Will the implementation of the Common Core State Standards in Mathematics (CCSSM)[1] change education for quantitative literacy (QL), and, if so, how? My answer to this question has several conditions and unknowns, but, in sum, at this time, I conclude that there will be minimal effect. This does not distinguish CCSSM from other K-12 mathematics standards – articulated or not – that I have known over the years.

In "Two Mathematics: Ever the Twain Shall Meet?" (Madison 2004), I echoed Alan Schoenfeld's "Reflections on and Impoverished Education" (Schoenfeld 2001). Similar to Alan's, my education in mathematics was totally devoid of the kinds of problem situations I have learned are central in QL, or, for that matter, any real applications and the whole of probability and statistics. My "Two Mathematics" article in *Peer Review* focused on the two different mathematics of U.S. society, namely school mathematics and QL mathematics, and the divide has existed since pre-colonial days. In several ways, CCSSM (2010) is more supportive of QL than was my mathematics education or some of the state standards being replaced, in part because awareness and importance of QL is worlds above what it was when I was in school or when many of the state standards were developed. The increasing need for QL, however, has outstripped those increases, at least cancelling the relative gain.

Just as the jury is still out on the general success of CCSSM (not our main goal here), it is too soon to know its potential to support QL. As noted above, there are unknowns. Some of them are:

- How will CCSSM be implemented in the steady state, i.e. after the assessments are given and validated? Will the goals of more coherence and more depth be achieved?
- What kind of assessments will be used? Will these become the de facto standards, and if so, will the tests be worth teaching to?
- What kinds of "real world" or "real life" problems will be solved? In numerous places in CCSSM (2010) there are unspecified applications. Will modeling situations be substantial, be realistic, and include QL-friendly applications?

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[1] Throughout, the references to CCSSM are to the standards at http://www.corestandards.org/Math/. Accessed August 19, 2014.

- Will the probability and statistics strand, one that is critical for QL, be a substantial component, or will it be relegated to the "cover if time permits" status?
- Is it realistic to believe that all students can achieve competence in the understandings and skills in CCSSM by grade 11?
- Will the political objections to CCSSM that have surfaced undermine its success?

In what follows, I will elaborate on some of these items, mainly in relation to QL but occasionally on the general effect of CCSSM. First, in the interest of full disclosure, I should elaborate that my experience with CCSSM has been fairly extensive.

The Author and CCSSM

Approximately five years ago, October 2009, my involvement in CCSSM began. I was one of approximately 25 people convened in Washington DC to evaluate CCSSM at that point and advise on its continued development under the auspices of the Conference Board of the Mathematical Sciences (CBMS) and the American Council on Education (ACE). Subsequently I was part of a three-person team (along with Jason Zimba and Pat Thompson) to write for CCSSM on quantitative reasoning. And, later, I was one of three members, chaired by Alan Tucker of State University of New York at Stony Brook, of the Mathematical Association of America (MAA) to evaluate CCSSM for MAA. Later still, I was the higher education mathematics faculty member from Arkansas who advised one of the assessment consortiums, Partnership for Assessment of Readiness for College and Careers (PARCC). Finally, I wrote the first draft of the standards progression (something like a learning progression) for modeling in CCSSM high school[2].

Aside from the progressions draft, I can detect none of my fingerprints on CCSSM, so pride of ownership is not at issue. For the past five years, as a part of an NSF-funded mathematics and physics partnership,[3] I have led several professional development workshops every summer for middle and high school mathematics and science teachers with CCSSM as a guiding framework. The shifting and new responsibilities for teachers brought on by the implementation of CCSSM have been very apparent, and many weaknesses in teachers' preparations to teach using CCSSM have become evident to me. Not that these issues are new; rather, they temper my optimism that CCSSM will significantly change the

[2] Available at http://ime.math.arizona.edu/progressions/. Accessed August 19, 2014.

[3] College Ready Mathematics and Physics Partnership, NSF DUE – 0832091.

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outcome of school mathematics. The most-popular workshops we conducted for teachers in the partnership were the ones on QL. Teachers were hungry for everyday, contemporary applications to use in their classrooms, and the QL workshops provided that because we used the *Casebook of Media Articles* (Madison et al. 2012) that we use in our college course in quantitative reasoning. No such applications are specified in CCSSM.

In October 2009, about the only parts of CCSSM that were available were the eight Standards for Mathematical Practice, which had been distilled from the five strands of mathematical proficiency from *Adding It Up* (Kilpatrick et al. 2001) and the process standards from the NCTM Standards for School Mathematics (NCTM 2000). The distillation and explanation producing the eight practice standards in the form, "Mathematically proficient students do …," was very effective and impressive as a beginning. The practice standards set a high bar. I recall Denny Gulick, University of Maryland, remarking at the CBMS-ACE forum in 2009 that he would be delighted if all faculty colleagues understood and performed as the practice standards indicated. As would be expected, adding detailed content standards to these elegant practice standards was difficult and often troublesome. Many constituencies with varying interests had to be satisfied, so no single perspective prevailed. In 2009 there were some beginnings of example problems that explained the standards and set goals for understanding. Evidently, development of these problem examples did not continue as they are not part of CCSSM at present. Undoubtedly they will be part of the assessments in the guise of sample items that have begun to unfold. One of the dangers of sample items or example problems is that they become the standards.

What Do I Mean by QL?

What I mean by QL will help explain my opinions on how CCSSM does support QL and how it could be more supportive. Over the years there have been several published meanings or frameworks for QL, and CCSSM is more supportive of some than others. For example, the MAA report on quantitative reasoning (MAA 1994) gives a goal for QL that is reasonably well supported by CCSSM, namely applying simple mathematical methods to the solution of real-world problems. This conception of QL is similar to the two key characteristics of a numerate person as stated in the Cockroft (1982) report. The two characteristics were the ability to use mathematics in everyday life and to understand and appreciate information presented in mathematical terms. According to Maguire and O'Donoghue (2002), the Cockroft report's conception of numeracy (called QL in the U.S.) ushered in the mathematical phase of numeracy that gave way to the integrative phase around 2000. Yet, numeracy surveys and literacy testing in the

U.S. and Europe have been and are still dominated by this mathematical context. As Lynn Steen and I (2008) wrote:

> Further arguments for focusing mathematics education in a cross-disciplinary and functional direction emerged in a U.S. report on "what work requires of schools" that stressed practical competencies (in, e.g., resources, information, systems, and technology) built on a broad foundation that included basic skills, decision making, and problem solving (SCANS 1991). The emphasis on numeracy as a functional skill—now giving rise to the term "functional mathematics" (Murnane and Levy 1996; Forman and Steen 1999)—dominates QL assessments and has influenced many state mathematics standards.

It is likely that this view of QL as functional mathematics influenced CCSSM if consideration of supporting QL was ever an issue. A framework for QL by Dossey (1997) moved more toward the integrative version of numeracy by having chance, data interpretation, and measurement as three of the six major aspects. Gal (1997: 41) went further, including affective aspects in his conception of numeracy tasks that "require adults to integrate seamlessly both numeracy and literacy skills." CCSSM would be more supportive of Dossey's version than Gal's, however. Finally, Wilkins (2000) gives a framework for QL that contains affective and motivational aspects of QL, e.g., recognition of societal impact of mathematics, understanding the nature and historical development of mathematics, and having a positive disposition toward mathematics. CCSSM, not surprisingly, is silent on these affective aspects.

My meaning for QL is based on the six core competencies for QL as developed in the AAC&U VALUE rubric for QL as modified by Boersma, et al. (2011). The six core competencies are interpretation, representation, calculation, analysis/synthesis, assumption, and communication. Most, if not all QL situations can be resolved by applying the six competencies. In sum, I find CCSSM supportive of the calculation competency, somewhat supportive of the representation competency (via modeling) and the analysis/synthesis competency, and not very supportive of interpretation and communication competencies. The support for assumptions is unclear.

Overview of my Conclusions

Here I list my conclusions on how supportive CCSSM is of QL. I will elaborate on some reasons in the remainder of this perspective.

Strengths of CCSSM in support of QL

- The practice standards are elegant, challenging and very supportive of QL.

- The mathematics and statistics content is sufficient for QL, especially the strands (called domains in CCSSM) on measurement and data, ratios and proportional relationships, and statistics and probability.
- The development of algebraic thinking and logical reasoning from Kindergarten through grade 11 is coherent and systematic.

Support for QL that is yet undetermined

- The assessments are yet to be administered and validated.
- The applications and mathematical models in CCSSM are mostly unspecified, leading possibly to applications within the assessments becoming dominant in the classrooms.
- The pressure to "cover" the standards by grade 11 may reduce the attention to substantial and challenging applications.

Some evident weaknesses in CCSSM's support of QL

- Beyond the practice standards, the emphasis on interpretation and conceptualization is weak, as is the emphasis on communication and reflection on results of computations.
- There are very few suggestions of applications or mathematical models that deal with critical citizenship issues such as political arguments, government economics, and health risks.
- The language used in CCSSM does not encourage conceptualization and interpretation, especially in regard to quantitative reasoning and conceptualization of functions.

Are Goals of "All Students" and "College and Career Readiness" Realistic?

On a panel that I chaired on CCSSM and college placement examinations at the Joint Mathematics Meetings in 2013, Zalman Usiskin, looked to history to make a point about CCSSM. Usiskin (2013: 17) noted that

> … in 1892, Charles Eliot, the President of Harvard, chaired a committee which came to be called the Committee of Ten, whose purpose was to standardize the high school curriculum in the United States for students intending to go to college. The Committee of Ten (1893) recommended that all students follow a college-preparatory curriculum, that "every subject which is taught at all in a secondary school should be taught in the same way and to the same extent to every pupil so long as he pursues it, no matter what the probable destination of the pupil may be, or at what point his education is to cease.…" The report recommended that all students take one year of algebra and one year of

> geometry, and that is how those courses became standard in the U.S. curriculum in grades 9 and 10. The Common Core committee, charged with coming up a curriculum appropriate for both college and the world of work, thought the same way as the Committee of Ten, and they created a framework for a secondary school curriculum in which appropriateness of mathematics for later college study was viewed as the best preparation for all students, whether college-bound or not."

Usiskin's point was that for a long time colleges have had difficulty dealing with the diverse spectrum of high school graduates and that CCSSM is not likely to change that. It is indeed an ambitious challenge to prepare all students for either college or work. That is not happening now, and CCSSM is not likely to make it so. Along those lines, at the 2009 ACE forum, I remarked that it seemed that an additional c-word should have been added to the Common Core State Standards for College and Career Readiness, namely that readiness for citizenship should have been added. Had that been the case, CCSSM would have had a broader but more QL-friendly goal. In 1892, the Committee of Ten recognized the need for citizenship readiness (Mackenzie 1894: 149):

> … only 3 per cent. of our high school pupils enter our colleges. It follows, therefore, that the best possible provision for secondary education, particularly in our high schools should be made, if we would send into the world with fullest equipment for citizenship the 97 per cent of high school pupils who do not enter college.

The Practice Standards and QL

As I have written elsewhere (Madison 2014), the first four practice standards are central to QL: making sense of problems; modeling with mathematics or statistics; reasoning quantitatively; and drawing, supporting and communicating conclusions. Critiquing the reasoning of others is often the entry point into a QL situation. Less central to QL are practice standards 5 – 8: use appropriate tools strategically, attend to precision, look for and make use of structure, and look for and make use of regularity in repeated reasoning. Tools for QL include calculators or spreadsheets and quantitative benchmarks for detecting reasonableness of answer. There is attention to precision but most numerical precision focuses on the precision needed, realistic or possible, in resolving the QL situation. Precise definitions and correct units are important when available in a context or by assumption. The final two standards – looking for and using structure and looking for and expressing regularity – are more applicable to mathematics development.

Better than the Standards Being Replaced?

Are the CCSSM standards better or more supportive of QL than the ones they are replacing? This question is difficult to answer as the standards being replaced vary from state to state. I can say with confidence that the Common Core standards are stronger and better aligned with QL than were the Arkansas Frameworks as they eventually became practiced. I do not have direct experience with standards from other states. One has to wait to see how CCSSM is implemented and assessed. If the assessments are "tests worth teaching to" and, as is likely, the assessments become the standards, then we will have moved forward.

So far, what I have seen of the sample items released by the two assessment consortia, PARCC[4] and the Smarter Balanced Assessment Consortium,[5] the assessments will not be very supportive of QL. As an example, the PARCC sample assessments in grades 6-high school had 192 items, and only 18 of these were on probability, data analysis and statistics. Since that statistics strand is the most QL-friendly of all the strands, having more items from that strand would be better for QL.

CCSSM Mathematics Content and QL

There are standards at every grade level that are very supportive of QL. Aside from the fundamentals of arithmetic, the parts of CCSSM that are omnipresent in QL are the Measurement and Data strand in K-5, the Statistics and Probability strand in grades 6-high school, and the Ratios and Proportional Reasoning strand in grades 6-7. The latter strand has only six standards, with two of these having four problem types. These standards are important in QL. For example, one of them reads "Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error." One aspect of CCSSM that could very well be very supportive of QL is the development of algebraic thinking from K on. This development would be more supportive were it rooted in quantitative reasoning as I will note below. Nevertheless, this broader view of algebraic thinking in CCSSM – as well as the broader view of logical reasoning – could produce significantly stronger QL reasoning among high school graduates. How well that works depends heavily on implementation and emphases.

Applications and CCSSM

The extent and kinds of applications will be critical in determining how CCSSM supports QL or how the impoverishment that Alan Schoenfeld and I experienced

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4 See http://practice.parcc.testnav.com/#. Accessed August 19, 2014.

5 See http://www.smarterbalanced.org/practice-test/. Accessed August 19, 2014.

in our mathematics education is remedied. Some applied mathematicians have criticized CCSSM because of the lack of applications. Specifically, I recall that Alan Tucker was open in this kind of criticism of CCSSM when we were evaluating CCSSM for the MAA. As noted above, the kinds of "real world" problems that are to be solved by students using various content designations in CCSSM are not specified. There are a few examples that help clarify what is meant by various standards. One of these, on exponential functions, is fairly supportive of QL: "… identify percent rate of change of functions such as $y = (1.02)^t$, $y = (0.97)^t$, … , and classify them as representing growth or decay."

One difficulty of inserting QL-friendly applications in K-12 is that many are sophisticated uses of elementary mathematics.[6] This is difficult because the students may not be knowledgeable about the contexts of the sophisticated uses such as economics, political science, and personal finance. This is especially so in grades 6 and 7 where the standards on ratios and proportional reasoning occur. It is seen as impractical by teachers because they have more complex mathematics to teach and the sophisticated contexts are unlikely to occur on assessments.

Modeling, CCSSM, and QL

Modeling, once viewed as a likely strand, is not a separate strand in CCSSM, and modeling is very much a part of QL. Representing with mathematics or statistics models is a first step after interpreting in resolving many QL situations. Instead of a separate strand, various standards in the existing strands are starred as an indication that they are a modeling standard. As stated in CCSSM,

> Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard of Mathematical Practice and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

From the progression draft[7]:

> … About one in four of the standards in Number and Quantity, Algebra, Functions, and Geometry have a star, but the entire conceptual category of Statistics and Probability has a star. In statistics, students use statistical and probability models—whose data and variables are often embodied in graphs, tables, and diagrams—to understand reality. Statistical problem solving is an investigative process designed to understand variability and uncertainty in real life situations. Students formulate a question (anticipating variability), collect data (acknowledging variability), analyze data (accounting for variability), and interpret results (allowing for variability).

6 As one of the reviewers of this article pointed out, there are many QL applications that do not involve sophisticated contexts. Health, sports, popular culture and shopping provide innumerable settings for proportional reasoning, orders of magnitude, formulas and other mathematics.

7 Available at http://ime.math.arizona.edu/progressions/. Accessed August 19, 2014.

Readiness for College Mathematics

Readiness for college mathematics has different meanings. The most common meaning is readiness for success in a degree credit-bearing college mathematics course, often college algebra. That appears to be the meaning of college readiness assumed in CCSSM. Since college algebra courses generally are not supportive of QL (See, for examples (Gaze 2014) and (Gillman 2010)), CCSSM's targeting college algebra could be the major reason for not being supportive of QL. Not all the CCSSM standards are proposed as necessary for career and college readiness. Standards that are not proposed as necessary are marked with a (+) indicating that these are for preparation for more-advanced courses such as calculus, advanced statistics or discrete mathematics. There are various measures used for determining readiness for college mathematics. The ACT assessment has one (Allen and Sconing 2005), namely an ACT mathematics score of at least 23. One of the stated intents of the PARCC assessment of CCSSM was to have higher education institutions recognize the assessment results as indicating readiness for college mathematics. Whether or not that happens, or becomes operational, is to be determined. It certainly should be an indicator, but likely will not be definitive. Indeed, if the CCSSM summative assessment is first administered in 2014-15, then it will be a few years before the results can be evaluated as to whether they measure readiness for college mathematics. Use as an indicator of college readiness was also the same intent of a predecessor of CCSSM, the Algebra II end-of-course examination, the development of which was led by Achieve,[8] which was also the leader of PARRC. The PARCC CCSSM assessment is still unknown, but the Algebra II end-of-course examination was not friendly to QL. In any event, mathematical sciences faculty are likely to continue placement testing to determine where entering students begin in mathematics. Such tests contain little information about QL. Consequently, no assessments of QL are likely very soon for college admissions or college readiness.

Algebra, Quantitative Reasoning and Functions

The definitions and assumptions in CCSSM tend to be directed toward numbers and procedures rather than conceptualization and interpretation. For example, CCSSM defines quantity as a number with a unit. This definition moves right past the conceptualization of a quantity and avoids student participation in the dialectic among object, attribute and quantification as advocated by Thompson (1993). In

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[8] Achieve is an independent, nonprofit education reform organization created in 1996 by a bipartisan group of governors and business leaders. See http://www.achieve.org/,

fact, Smith and Thompson (2007), arguing for developing algebraic reasoning by way of quantitative reasoning (as opposed to a generalization of arithmetic), state the following:

> For too many students and teachers, mathematics bears little useful relationship to their world. It is first a world of numbers and numerical procedures (arithmetic), and later a world of symbols and symbolic procedures (algebra). What is often missing is any linkage between numbers and symbols and the situations, problems, and ideas that they help us think about.

In a similar vein, Thompson and Carlson (in press) note that CCSSM does not promote student thinking about variation and covariation as the historical and cognitive roots of the concept of functions in mathematics. As they state:

> … the words "covary" and "variation" do not appear in CCSSM's 93 pages. The word "variation" appears just four times – three times in the context of statistics and once about variation in assumptions. The word "vary" also occurs only four times – once about opportunities, twice about changing assumptions, and once in the context of statistical variation.

Both these examples are at the core of potential strengths of CCSSM – successful development of algebraic thinking and correct meanings by students for the fundamental notion of function. Consequently this goes beyond a weak support for conceptualization and interpretation to a way of developing major domains of CCSSM. Since CCSSM should be a formative document, these fundamental issues should be monitored, studied and researched carefully.

Final Thoughts

How supportive of QL will CCSSM be? Will the implementation of CCSSM change education for QL? Probably not, because, as with previous standards, education for QL is not a primary aim – college and career readiness are. Since much of school mathematics now is mired in recall and apply, significant change would require a strong position by CCSSM. That does not seem to be present, and the absence of specific applications means that assessments will likely determine what kinds of applications and models are emphasized, a crucial circumstance. There are QL-supportive features of CCSSM – e.g. the ratios and proportional-reasoning standards, the probability and statistics standards, and the aim of coherence of the development of algebraic thinking and logical reasoning, but assessments are likely to become the de facto standards. There are signs already that the assessments of CCSSM will not be QL-supportive—that, rather, they will focus more on college readiness where QL and statistical analysis are, at this time, not major issues, whereas college algebra is. Complete implementation is still unknown, and implementation issues have generated substantial political opposition. If colleges and universities make QL a more-substantive issue in admission and in graduation, then college readiness, which is dominant in CCSSM, will bend CCSSM toward the Committee of Ten's goal of sending graduates – high school or college – "into the world with fullest equipment for citizenship" (Mackenzie 1894: 149).

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