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Looking at the Multiple Meanings of Numeracy, Quantitative Literacy, and Quantitative Reasoning

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Abstract

The subject of this journal goes by a variety of names: numeracy, quantitative literacy, and quantitative reasoning. Some authors use the terms interchangeably. Others see distinctions between them. Study of psycholinguistic and ontological concepts laid out in the literature of WordNet and familiarity with the papers in this journal suggests a vocabulary matrix consisting of four rows (word senses) and three columns (word forms, namely numeracy, QL, and QR). The four word senses correspond to four sets of synonyms: {numeracy}, {numeracy, QL}, {QL, QR}, and {numeracy, QL, QR}. Each of the word forms is polysemous: "numeracy" points to the first, second and fourth senses; "QL" points to the second, third and fourth; "QR" points to the third and fourth. The four synsets (senses) are on three different branches diverging from the WordNet synset identifying the concept of cognition and knowledge. {QL, QR} is on the cognitive process branch; {numeracy} and {numeracy, QL} are on the cognitive skill and ability branch; and {numeracy, QL, QR} is on the mental attitude branch. For comparison, WordNet places the synsets for mathematics, statistics, and other hyponyms of "disciplines of study" on another branch, the cognitive content branch.

Keywords

polysemy, WordNet, quantitative literacy, quantitative reasoning

Cover Page Footnote

Len Vacher is a professor of geology in the new School of Geosciences at the University of South Florida. He is a fellow of the Geological Society of America, a lifetime member of the National Association of Geoscience Teachers (Neil Miner award, 2004), and a member of the Mathematics Association of America and the American Statistical Association. He was a charter member of the Board of Directors of the National Numeracy Network and is founding co-editor of this journal.

{numeracy, quantitative literacy, quantitative reasoning}

In my experience, the vast majority of proponents of quantitative literacy consider numeracy, quantitative literacy, and quantitative reasoning to be synonymous at least in some contexts. Exhibit A is the masthead of this journal. The title is *Numeracy*. The subtitle is *Advancing Education in Quantitative Literacy*, thereby suggesting synonymy of numeracy and QL. Exhibit B is the National Numeracy Network and its website:

> Some call it **Numeracy**…. Others call it **Quantitative Literacy (QL)**. Still others refer to it as **Quantitative Reasoning (QR)**….[1] (emphasis in original).

Exhibit C is the Association of American Colleges and Universities (AAC&U). It lists quantitative literacy (QL) as one of its six Intellectual and Practical Skills,[2] amongst a total of 12 Essential Learning Outcomes[3] identified in its Liberal Education & America's Promise (LEAP) program (AAC&U 2007). The learning outcomes are now supported by a total of 16 Valid Assessment of Learning in Undergraduate Education (VALUE) rubrics.[4] The VALUE rubric for QL[5] identifies the construct as "Quantitative Literacy, also known as Numeracy or Quantitative Reasoning (QR)." The rubric goes to define and characterize the construct as follows:

> A "habit of mind," competency, and comfort in working with numerical data. Individuals with strong QL skills possess the ability to reason and solve quantitative problems from a wide array of authentic contexts and everyday life situations. They understand and can create sophisticated arguments supported by quantitative evidence and they can clearly communicate those arguments in a variety of formats (using words, tables, graphs, mathematical equations, etc., as appropriate).

If numeracy, QL, and QR are synonymous, they can be considered to be elements of a set of synonyms, i.e., {`numeracy, quantitative literacy, quantitative reasoning`}. Such sets are known as *synsets* in the language of the online lexical database for English, WordNet.[6]

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[1] http://serc.carleton.edu/nnn/index.html (this and all other links in this editorial were accessed June 16, 2014).
[2] The other five are inquiry and analysis; critical and creative thinking; written and oral communication; information literacy; and teamwork and problem solving.
[3] http://www.aacu.org/leap/documents/EssentialOutcomes_Chart.pdf
[4] http://www.aacu.org/value/rubrics/index.cfm. See also, AAC&U 2009.
[5] For an application to a QR course, see Boersma et al. 2011.
[6] http://wordnet.princeton.edu/. For additional information, see Miller (1986), Miller and Fellbuam (1991), Miller (1995), Fellbaum (1998).

The existence of the synset {`numeracy, quantitative literacy, quantitative reasoning`} is confirmed by remarks in the main articles in this issue of *Numeracy*. In framing his paper on the history of numeracy education of journalists in England, Steve Harrison cites Gillman's book for how his own paper "understands 'numeracy' in the broad sense of the term 'quantitative literacy'":

> the ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in individuals' private, civic and work lives (Gillman 2006: vii).

Harrison continues:

> That is, "numeracy" and "quantitative literary" are taken to be ontological terms designating a stance towards the world, rather than epistemological terms delimiting a sphere of knowledge or competency. Hence any training which aims to bolster the ability to reason with number is considered in this paper, not merely training which is explicitly labelled "numeracy" or "mathematical."

Similarly, Bernard Madison, in his paper reverse-engineering his QR course to find principles by which others can design or evaluate their own courses in QR, disposes of the matter of synonymy quickly. He does so in a footnote to the first sentence in the introduction:

> In the remainder of this paper QR will be used for either QL or QR except when referring to existing literature that uses QL.

Eric Gaze and colleagues are even more direct in their paper. They combine QL and QR in the title, "Towards Developing a Quantitative Literacy/Reasoning Assessment Instrument," and then go on to abbreviate it in the running head as "Towards Developing a QLRA instrument." In the introductory section "Purpose, Goals and the QLR Construct," they give a working definition QLR:

> the skill set necessary to process quantitative information and the capacity to critique, reflect upon, and apply quantitative information in making decisions.

They go on to note that cognitive psychologists define numeracy in a similar way:

> A well-established and highly studied construct, numeracy encompasses not just mathematical ability but also a disposition to engage quantitative information in a reflective and systematic way and use it to support valid inferences. (Kahan et al. 2013)

The fact that Harrison, Madison, and Gaze et al. all explicitly state and adopt a working synonymy of numeracy, quantitative literacy, and quantitative reasoning means that they do not take the synonymy for granted: there must be an alternative point of view. Indeed Harrison alludes to it when he contrasts "a stance towards the world" to "merely training which is explicitly labelled 'numeracy' or 'mathematical.'" In fact, Robert Mayes and colleagues fully take the other point of view—that numeracy, QL, and QR are not synonymous—in their paper detailing their ongoing project to develop a Quantitative Reasoning Learning Progression, which is part of a large project to develop learning progressions to promote environmental literacy. Their paper in this issue builds heavily on a previous paper (Mayes et al. 2013) which laid out the framework for the learning progression. There they defined Quantitative Reasoning in Context (QRC) as:

> mathematics and statistics applied in real-life, authentic situations that impact an individual's life as a constructive, concerned, and reflective citizen. QRC problems are context-dependent, interdisciplinary, open-ended tasks that require critical thinking and the capacity to communicate a course of action. (Mayes et al. 2013)

Moreover, Mayes et al. (2013) broke QRC into four progress variables: Quantification Act (QA), Quantitative Literacy (QL), Quantitative Interpretation (QI), and Quantitative Modeling (QM). For each of these four progress variables, they identified a number of fundamental elements. For QL, the elements were four in number, namely numeracy, measurement, proportional reasoning, and basic probability and statistics. In the paper in this issue, Mayes et al. set up a matrix of achievement levels (rows) vs. progress variables (columns). In this work, which involved student interviews and assessments, they reduced the number of progress variables from four to three, moved QL to be an element of the progress variable of QA, and positioned numeracy in one of the achievement levels (the third). The relevance to this editorial is that if numeracy is an element of a quantitative literacy progress variable, and the QL progress variable is part of a quantitative reasoning learning progression, then numeracy, quantitative literacy, and quantitative reasoning can hardly be considered synonymous in this application of the terms.

In short, the usage of the terms numeracy, quantitative literacy, and quantitative reasoning in the four main articles of this issue show that the terms are polysemous; they have multiple meanings. On the one hand, the three terms are synonymous; on the other hand, the three have different meanings from each other. At least two of them must be polysemes.

Polysemy and WordNet

WordNet was developed at the Cognitive Science Laboratory of Princeton University beginning in the mid-1980s and is maintained there. George A. Miller,[7] the original

[7] Miller died two years ago at age 92 (Vitello 2012). He was awarded the National Medal of Science in 1992.

director of the project, is widely regarded as one of the founders of cognitive psychology and, in particular, psycholinguistics. WordNet was "initially conceived as a test bed for a particular model of lexical organization that had never before been implemented on a large scale" (Fellbaum, 1998a: 4). It was an "experiment" (Fellbuam 1998a: 4) to see whether relational lexical semantics could be scaled up from the "toy illustrations of the day" (Miller 1998: xvi). It "was designed as a network, partly because representing words and concepts as an interrelated system seems to be consistent with evidence for the way speakers organize their mental lexicons" (Fellbaum 1998a: 7). Its iterative development included testing by the creation of two semantic concordances[8] − comprehensively tagging 103 passages from the Standard Corpus of Present-Day Edited American English (the Brown Corpus[9]) and the complete 45,600-word text of *The Red Badge of Courage* with word senses contained in WordNet (Landes et al. 1998). The aim was to capture the structure of the vocabulary of "everyday speakers" (Fellbaum 1998a: 6). The development of WordNet has come to be foundational for computational linguistics and natural language processing.

Figure 1 illustrates a small extraction of information from WordNet concerning the word *polysemy* and shaped into a PowerPoint slide. The word occurs once in WordNet, as a member of the synset {`lexical ambiguity, polysemy`}. The sysnset carries the *gloss* (brief definition by which the searched-for word can be identified):

> the ambiguity of an individual word or phrase that can be used (in different contexts) to express two or more different meanings.

Polysemes, in concept, differ from homonyms (e.g., Klepousniotou 2002). Homonyms are different words that are spelled (or sound) the same; a classic example is *bank* (the edge of a river) vs. *bank* (the financial institution). A polyseme is a word (or collocation such as a recurring adjective-noun pair, e.g., *lexical ambiguity*) that is used in different ways in accordance with different *related* meanings; i.e., a polyseme has multiple *senses*. As an example, consider the noun, *noise*. WordNet lists six noun senses for noise; only one of them, {$\text{noise}_{5/6}$}, is shown in Figure 1. The subscript indicates that this sense of noise is the fifth of the six listed in WordNet. The full list of senses is: (1) {`noise`} (sound of any kind (especially unintelligible or dissonant sound)); (2) {`noise, dissonance, racket`}, (3) {`noise, interference, disturbance`} (electrical or acoustic activity that can disturb communication), (4) {`noise`} (a loud outcry

[8] "a textual corpus and a lexicon so combined that every substantive word in the text is linked to its appropriate sense in the lexicon" (Miller et al. 1993: 3, in Landes et al. 1998: 199)

[9] http://clu.uni.no/icame/brown/bcm.html. Brown Corpus Manual, by W. N. Francis and Henry Kucera, Brown University, July 1979.

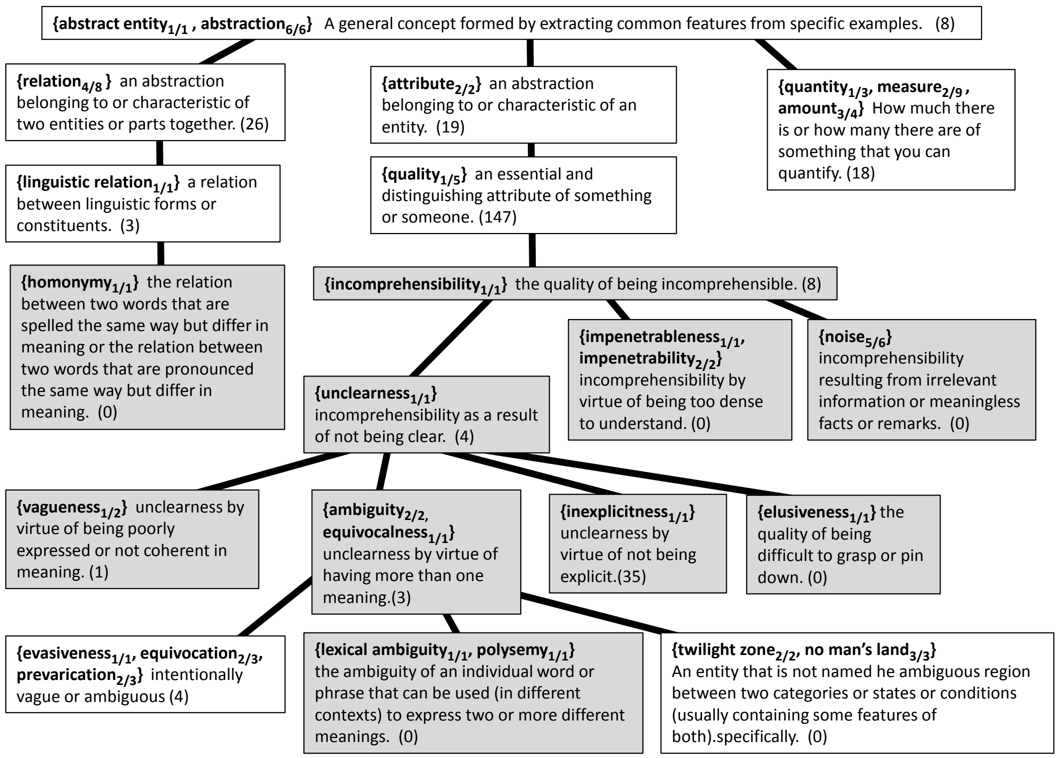

Figure 1. Part of a family tree for *polysemy*. Information extracted from WordNet ("Use WordNet online"). Shaded boxes indicate word senses referred to in the text.

of protest or complaint), (5) {noise} (see gloss in Fig. 1), and (6) {randomness, haphazardness, stochasticity, noise}. Note the set notation (braces): of the six meanings, three are represented by a synset consisting of three words, one of which is noise, and three are represented by the single word, noise. Thus "noise" is polysemous with six meanings; one of them makes the word synonymous with stochasticity, and another makes it synonymous with dissonance, neither of which is the sense shown in Figure 1. How many of those meanings are related to each other? WordNet, which does not get into etymological issues, doesn't distinguish between homonyms and polysemes; its purpose is to recognize a word's various meanings that accord with everyday speakers' various usages.

The boxes of nouns in Figure 1 show a total of 18 word senses, six of which are represented by multiple words, and 27 words, 15 of which are polysemous (although, in each case, only one sense is shown in the figure). According to the WordNet Statistics (wnstats) web page,[10] the noun database now contains 82,115 synsets (senses) and 117,798 strings (words). Think of it as a "vocabulary matrix" (Miller 1986): 82,115 rows and 117,798 columns. According to the wnstats page,

[10] http://wordnet.princeton.edu/wordnet/man/wnstats.7WN.html

there are 146,312 word-sense pairs, meaning that that many of the nearly 10 billion cells are occupied. Of those occupied sites, 101,863 are like the boxes in Figure 1 for {$\text{incomprehensibility}_{1/1}$} and {$\text{unclearness}_{1/1}$}, a synset consisting of a single word with a single meaning – only one occupied cell in that row and column of the vocabulary matrix. On the other hand, 44,449 of the senses (rows) map to multiple words (146,312 – 101,863), and 15,935 of the words (columns) map to multiple senses(117,798 – 101,863). Thus according to the wnstats page, for the nouns in WordNet, the average polysemy including monosemous words is 1.24 (i.e., 146,312/117,798), and the average polysemy excluding the monosemous words is 2.79 (i.e., 44,449/15,935).

WordNet is much more than a matrix displaying many-to-many mappings (Miller 1986) from word senses to words, and words to senses. As conspicuously shown in Figure 1, WordNet organizes the word senses (synsets) into a hierarchical arrangement. In that regard, WordNet can considered a lexical ontology:[11]

> **ontology** (computer science) a rigorous and exhaustive organization of some knowledge domain that is usually hierarchical and contains all the relevant entities and their relations. (WordNet)

For the nouns, the relation that organizes the concepts represented by the synsets is most often hypernymy, which is what comes into play in Figure 1.[12] The hyponym-hypernym relation is a subordinate-superordinate, subset-set relation; it is commonly expressed as ISA, as in *falcon* ISA (kind of) *hawk*, where "hawk" is a hypernym of "falcon" and "falcon" is a hyponym of "hawk." Thus in Figure 1, the word sense represented by {`lexical ambiguity, polysemy`} is a kind of {`equivocalness, ambiguity`}, which is a kind of {`unclearness`}, which is a kind of {`incomprehensibility`}, and so on. The transitive nature of hypernymy assures an inheritance effect as one goes down a branch (Miller, 1990). For example, polysemy is a kind of incomprehensibility, and it inherits that superordinate's characteristics.

The numbers after the gloss in the boxes in Figure 1 indicate the number of hyponyms in WordNet for the word sense. For example, {`incomprehensibility`} has eight hyponyms (only three of which are shown in the figure). Thus one can be incomprehensible in at least eight different ways, including (1) by not being clear, (2) by being too dense to be understood, and (3) by babbling meaninglessly. Meanwhile, there are at least four different ways you can be unclear: you can be vague, ambiguous, inexplicit, or elusive. One of the ways you can be ambiguous is to use words that have multiple meanings. One of the ways you can be vague—from {`haziness`}, the unshown hyponym of {`vagueness`} in Figure 1—is to use words that are not clearly defined.

[11] "So we stumbled into something that others have taught us to call an ontology" (Miller 1998: xix).
[12] The other relation for nouns is meronymy (part of, or member of); for example, "bridge" is a meronym of "nose," which is a meronym of "face."

"numeracy," "literacy," "reasoning," and "mathematics"

The nouns "numeracy," "literacy," "reasoning," and mathematics" are all monosemous in WordNet. As shown in Figure 2, their senses occur on three different branches from {`cognition, knowledge, noesis`}.

The word senses {`numeracy`} and {`literacy`} are sister leaves at the end of the *skills and abilities branch*. Other sisters (not shown on Fig. 2) include {`seamanship`}, {`swordsmanship`}, {`marksmanship`}, {`horsemanship`}, {`showmanship`}, {`mixology`}, and {`craft, craftsmanship, workmanship`}.

The sense for "reasoning" is the synset {`abstract thought, logical thinking, reasoning`}, which is a direct hyponym of {`thought process, thinking, thought, mentation, intellection, cerebration`} in the *cognitive process branch*. The 13 sisters of {`abstract thought, logical thinking, reasoning`} include {`consideration`}, {`free association`}, {`explanation`}, {`mysticism`}, and {`problem solving`}, and its nine hyponyms include {`analysis,`

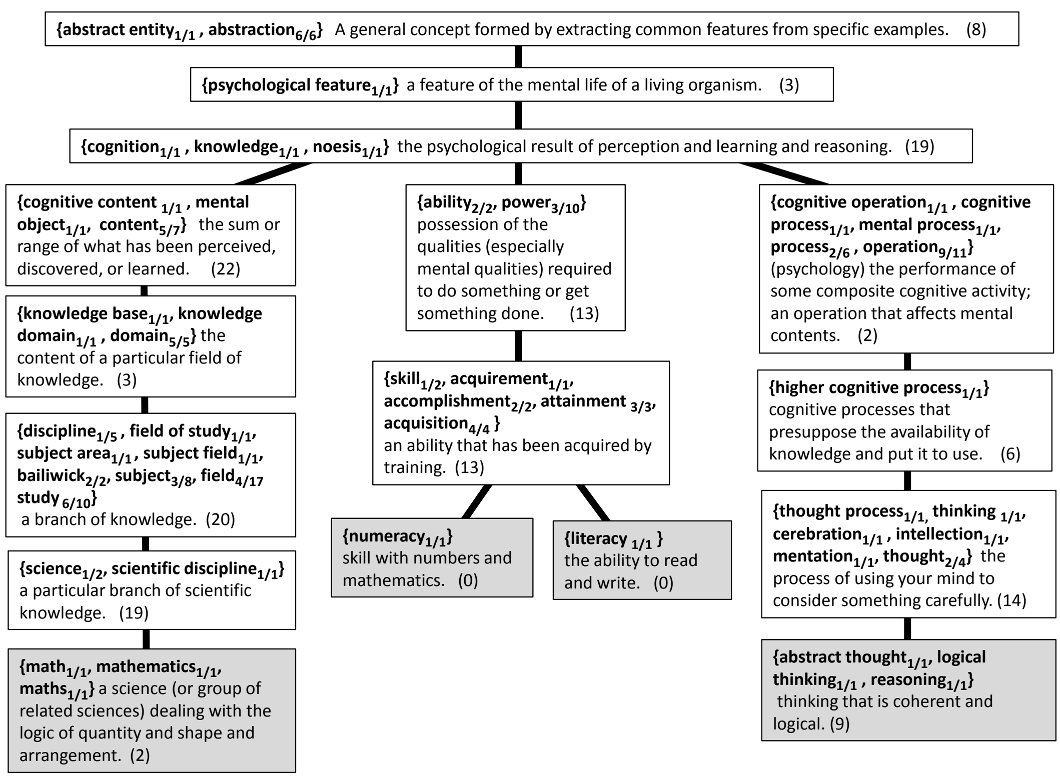

Figure 2. Part of the family tree of numeracy, literacy, and reasoning. Information fromWordNet.

analytic thinking}, {argumentation, logical argument, argument, line of reasoning, line}, {synthesis, synthetic thinking}, {deduction, deductive reasoning, synthesis}, and {conjecture}.

The word sense for "mathematics," {math, mathematics, maths}, is on the *cognitive content* branch as a direct hyponym of {science, scientific discipline}. Its 18 sisters include {natural history}, {natural science}, {cognitive science}, {psychology, psychological science}, {linguistics}, and 13 other twigs and leaves.

Without meaning to go off point, I need to say that I suspect this concept of mathematics will be at odds with the thinking of many colleagues in mathematics, especially those interested in curricula. For example, I have heard: "Mathematics ISA kind of thinking that is coherent and logical." "Mathematics ISA kind of abstract thought." "Mathematics ISA kind of logical thinking." "Mathematics ISA kind of reasoning." "Mathematics ISA kind of problem solving" (see Fig. 2). Disapproval of positioning mathematics in the cognitive content branch is in keeping with the following from a much-cited paper by a distinguished researcher in mathematics education:

> … Goals for mathematics instruction depend on one's conceptualization of what mathematics is, and what it means to understand mathematics. Such conceptualizations vary widely. At one end of the spectrum, mathematical knowledge is seen as a body of facts and procedures dealing with quantities, magnitudes, and forms, and the relationships among them; knowing mathematics is seen as having mastered these facts and procedures. At the other end of the spectrum, mathematics is conceptualized as the 'science of patterns,' an (almost) empirical discipline closely akin to the sciences in its emphasis on pattern-seeking on the basis of empirical evidences.
>
> The author's view is that the former perspective trivializes mathematics; that a curriculum based on mastering a corpus of mathematical facts and procedures is severely impoverished – in much the same way that an English curriculum would be considered impoverished if it focused largely, if not exclusively, on issues of grammar…. (Schoenfeld 1992, p. 334-335)

Repositioning mathematics in the cognitive process branch would be consistent, for example, with the National Council of Teachers' Focus in High School Mathematics: Reasoning and Sense Making[13] (Martin et al. 2009, Graham et al., 2010, King et al. 2010, Strutchens and Quander 2011, Dick and Hollenbrands 2011).

If it is true that mathematicians and mathematics educators would not agree with WordNet's placement of a monosemous "mathematics" in the cognitive content branch rather than the cognitive process branch, then I am reminded of the distinction between word knowledge and world knowledge:

> People often draw the distinction between word (or lexical) knowledge and

[13] http://www.nctm.org/standards/content.aspx?id=23749

> world (or encyclopedic) knowledge. Two kinds of books reflect this distinction: dictionaries are generally the repository of word knowledge, and encyclopedias the repository for world knowledge…. WordNet does not attempt to include encyclopedic knowledge, although the definitions that accompany the synonym sets (synsets) provide information about the concepts that is not strictly part of their lexical structure…. G. A. Miller points out in the foreward that, although WordNet's sysnsets were initially intended to contain no information other than pointers to other synsets, it was found that definitions and illustrative sentences were needed to distinguish closely related synsets whose members were polysemous. And in the case of many technical concepts, such as uncommon plants and animals, lexical and encyclopedic knowledge are merged in the definitions, which are likely to constitute all the knowledge everyday speakers need to access. (Fellbaum 1998a: 6)

Thus I would not be at all surprised if the language of professional mathematicians and mathematics educators (world knowledge) would differ from that of "everyday speakers" (word knowledge) with respect to what hypernym comes to mind when one hears "mathematics." We could ask our students ("everyday speakers"), for example, to complete the following multiple-choice item: "I identify mathematics as: (a) a branch of knowledge distinguished by its content; (b) a set of skills and abilities; (c) a way of thinking." We could ask them to rank the options. We could ask them to assign percentages of the options. We could ask them before and after they took a course that we teach or require.

"numeracy," "quantitative literacy," and "quantitative reasoning"

"Quantitative literacy" and "quantitative reasoning" are not listed in WordNet; however, the adjective "quantitative" is. Adjectives in WordNet are of two main types (Gross and Miller 1990): ascriptive and nonascriptive; the latter are called "pertainyms." An ascriptive adjective ascribes a value (e.g., "dense") to the noun it modifies (e.g., "rock"). A pertainym is relational in that it points to a noun that the modified noun pertains to or is associated with. WordNet has both types under "quantitative." The gloss for the ascriptive type of {`quantitative`} is "expressible or relating to or susceptible of measurement," which does not apply here for "quantitative literacy" or "quantitative reasoning," because we are not considering measuring how quantitative the literacy or reasoning is. The gloss for the pertainym type of {`quantitative`} is "relating to the measurement of quantity," which would suggest that quantitative literacy is *literacy relating to quantities* and quantitative reasoning is *reasoning involving quantities*. Examples of similar pertainym-noun pairs that form collocations in WordNet include "linguistic relation" and "lexical ambiguity" in Figure 1 and "psychological feature," "cognitive process," and "scientific discipline" in Figure 2.

Figure 3 is my attempt to amend Figure 2 to include the collocations "quantitative literacy" and "quantitative reasoning" using the concepts of WordNet and my experience editing papers in this journal. Figure 4 shows the corresponding vocabulary matrix and the arrow diagram for the many-to-many sense-to-word mapping it implies for our three word forms of interest: "numeracy," "quantitative literacy," and "quantitative reasoning." As shown in the figures, the four senses (WS1, WS2, WS3, and WS4) are {`numeracy`}, {`numeracy, QL`}, {`QL, QR`}, and {`numeracy, QL, QR`}, respectively. The columns reveal the polysemes. All three word forms are polysemous: numeracy has three senses (WS1, WS2, WS3); QL has three (WS2, WS3, WS4); and QR has two (WS3, WS4).

Figure 3 shows the hypernyms for the four synsets representing the four word senses. They are on three different branches from {`cognition, knowledge, noesis`}. The word sense {`numeracy, QL, QR`} is on *the mental attitudes branch*, which did not come up in Figure 2. This branch houses the "stance toward the

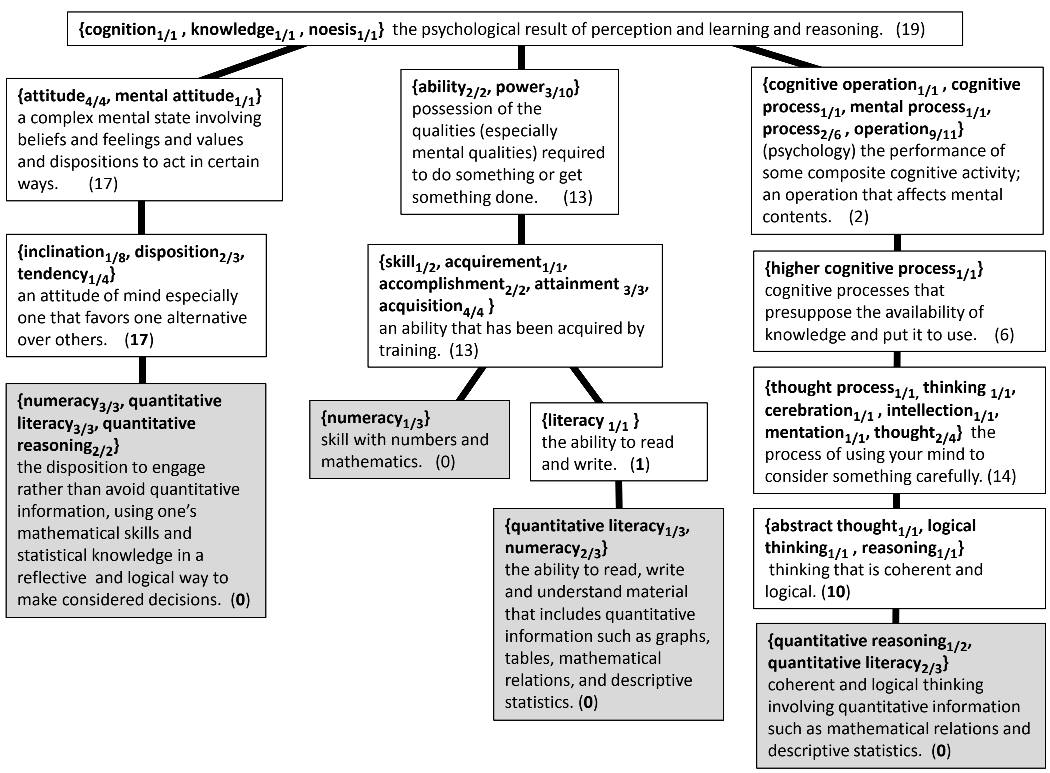

Figure 3. Part of the family tree of numeracy, quantitative literacy, and quantitative reasoning. Information from WordNet with the addition of three boxes containing "quantitative."

world" noted by Harrison (quoted at the beginning of this editorial) and the "habit of mind" mentioned in so many papers in our seven years. The two word senses {numeracy} and {quantitative literacy, numeracy} are on *the skills and abilities branch*. The first focuses the skill on numbers detached from context; the other inherits the properties of literacy and couples them with information about quantities (numbers with units). The word sense {quantitative reasoning, quantitative literacy} is firmly on the *cognitive processes branch*.

Mathematics and statistics are on yet a different branch, *the cognitive content branch*, partly shown in Figure 2, not shown in Figure 3. Following WordNet, the two direct hyponyms of mathematics are pure and applied mathematics, and statistics is one of three hyponyms of applied mathematics.

The lessons I think I have learned from this excursion into the semantic world of word senses and word forms are, first, it is not surprising that *Numeracy* authors are careful to clarify what they mean when they use these words, because they are polysemous. Second, we need to be careful about the distinction between the

	Numeracy	QL	QR
WS1. Skill with numbers and mathematics.	X		
WS2. Ability to read, write and understand material that includes quantitative information such as graphs, tables, mathematical relations, and descriptive statistics.	X	X	
WS3. Coherent and logical thinking involving quantitative information such as mathematical relations and descriptive statistics.		X	X
WS4. Disposition to engage rather than avoid quantitative information, using one's mathematical skills and statistical knowledge in a reflective and logical way to make considered decisions.	X	X	X

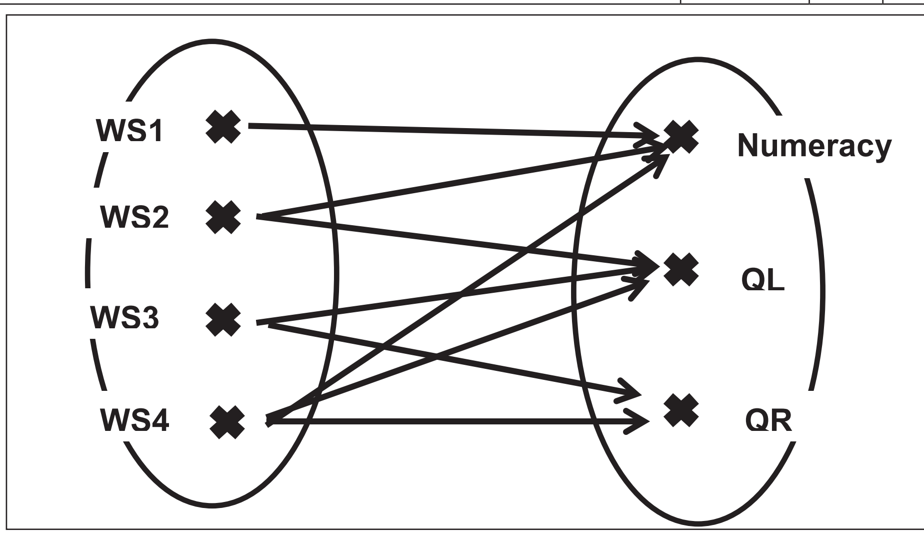

Figure 4. Proposed vocabulary matrix and corresponding arrow diagram for "numeracy," quantitative literacy," and "quantitative reasoning."

language of word knowledge and the language of world knowledge; our students may hear one, as indicated by the dictionary, when we speak the other, reflecting our technical field. Third, leaving the words themselves aside, it certainly seems helpful to distinguish between cognitive content, cognitive ability, cognitive process, and cognitive attitude.

I am less sure that there are only four senses of our three words, "numeracy," "quantitative literacy," and, "quantitative reasoning." I am even less sure that my glosses for the eight word-sense–word-form pairs of Figures 3 and 4 will pass muster with the readers of this journal.

Finally, as repeatedly pointed out by my reviewer, "It remains to be seen what meanings 'everyday speakers' will attach to quantitative reasoning, quantitative literacy, and numeracy – as you do not draw on examples from such sources, only expert sources." It would be interesting to find out.

Acknowledgment

I thank my co-editor Dorothy Wallace for reviewing this editorial. As always, it was a challenging and rewarding experience.

References

AAC&U. See Association of American Colleges and Universities.

Association of American Colleges and Universities. 2007. *College learning for the new global century: A report from the National Leadership Council for Liberal Education & America's Promise*. Washington DC: Association of American Colleges and Universities, 62 pp.

———. 2009. Assessing learning outcomes: Lessons from AAC&U's VALUE project. *Peer Review*, Winter 2009, 11 (1). http://www.aacu.org/peerreview/pr-wi09/pr-wi09_index.cfm (accessed June 7, 2011)

Boersma, S., C. Diefenderfer, S.W. Dingman, and B. L. Madison. 2011. Quantitative Reasoning in the Contemporary World, 3: Assessing student learning. *Numeracy*, 4 (2): Article 8. http://dx.doi.org/10.5038/1936-4660.4.2.8

Dick, T., and K. Hollebrands. 2011. Focus in high school mathematics: Technology to support reasoning and sense making. National Council of Teachers of Mathematics.

Fellbaum. C. (ed.). 1998. *WordNet: An electronic lexical database*. Cambridge

MA: MIT Press.
———. 1998a. Introduction. In Fellbaum 1998, 1-19.
Gillman, R. (Ed.). 2006. *Current Practices in Quantitative Literacy*. Washington DC: Mathematical Association of America.
Graham, K. A. Cuoco, and G. Zimmermann, 2010. *Focus in high school mathematics: Reasoning and sense making in algebra*. National Council of Teachers of Mathematics.
Gross, D., and K.J. Miller. 1990. Adjectives in WordNet. *International Journal of Lexicography* 3 (4): 265-277. http://dx.doi.org/10.1093/ijl/3.4.265
Kahan, D., E. Dawson, E. Peters, and P. Slovic, P. 2013. *Motivated numeracy and enlightened self-government*. The Cultural Cognition Project. Working Paper No. 116.
Klepousniotou 2002. The processing of lexical ambiguity: Homonymy and polysemy in the mental lexicon. *Brain and Language* 81: 205–223. http://dx.doi.org/10.1006/brln.2001.2518
King, J, Y. Orihuela, and E. Robinson, 2010. *Focus in high school mathematics: Reasoning and sense making in geometry*. National Council of Teachers of Mathematics.
Landes, S., C. Leacock, and R. I. Tengi. 1998. Building semantic concordances. In Fellbaum 1998, 199-216.
Martin, W.G., J. Carter, S. Forster, R. Howe , G. Kader, H. Kepner, J. R. Quander, W. McCallum, E. Robinson, V. Snipes, and P. Valdez. 2009. *Focus in high school mathematics: Reasoning and sense making*. National Council of Teachers of Mathematics.
Mayes, R., F.I. Peterson, and R. Bonilla. 2013. Quantitative reasoning learning progressions for environmental science: Developing a framework. Numeracy 6(1): Article 4. http://dx.doi.org/10.5038/1936-4660.6.1.4
Miller, G. A., 1986. Dictionaries in the mind. *Language and Cognitive Processes* 1 (3): 171-185. http://dx.doi.org/10.1080/01690968608407059
———. 1998. Foreward. In Fellbaum 1998, xv-xxii.
———. 1990. Nouns in WordNet: A lexical inheritance system. *International Journal Lexicography* 3(4): 245-264. http://dx.doi.org/10.1093/ijl/3.4.245
———. 1995. WordNet: A lexical database for English. *Communications of the ACM* 38 (11): 39-41. http://dx.doi.org/10.1145/219717.219748
———, and C. Fellbaum. 1991. Semantic networks of English. *Cognition* 41 (1991): 197-229. http://dx.doi.org/10.1016/0010-0277(91)90036-4
Miller. G.A., C. Leacock, R. Tangi, and R. T. Bunker. 1993. A semantic concordance. In *Proceedings of the ARPA Human Language Technology Workshop*, 303-308. San Francisco: Morgan Kaufmann. http://dx.doi.org/10.3115/1075671.1075742
Schoenfeld, A.H. 1992. Learning to think mathematically: Problem solving,

metacognitions, and sense making in mathematics. In *Handbook for research on mathematics teaching and learning*, ed. D. Grouws, 334-369. New York: Macmillan

Strutchens, M.E., and J. R. Quander. 2011. *Focus in high school mathematics: Fostering reasoning and sense making for all students*. National Council of Teachers of Mathematics.

Vitello, P. 2012. George A. Miller, a pioneer in cognitive psychology, is dead at 92. *New York Times*. August 1, 2012. http://www.nytimes.com/2012/08/02/us/george-a-miller-cognitive-psychology-pioneer-dies-at-92.html?pagewanted=all