Effect of Variable Bus Speeds on Bus Network Design

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Abstract

This article provides a methodology for solving the bus network design problem, covering network design and frequency setting and taking into consideration that commercial speeds of buses vary depending on the aggregated frequency of buses on each corridor. This methodology, referred to as Variable Speed Methodology, uses a variation of an algorithm proposed by Baaj and Mahmassani that assumes speeds remain constant (denoted Fixed Speed Methodology). Both methodologies were applied to the street network of Barcelona. Outputs were compared, and it was found that the Variable Speed Methodology produces a bus network with faster average travel speeds, shorter travel times, smaller fleet size, less route kilometer, and fewer buses per link while still serving the same level of demand. These results demonstrate that taking variability of bus speeds into consideration when performing route generation and frequency setting can significantly improve the performance of the bus network produced.

Introduction

The focus of this article is the analysis of the bus network design problem. Planning an efficient transit network is a complex process and usually is divided into three main components: strategic planning (network design), tactical planning (frequency setting) and operational planning (allocation of resources to each route). Through re-evaluation of both network layout and route frequency setting, great

improvements in the efficiency of a transit network are possible, and both user and operator costs can be reduced. This is particularly important as public transportation systems have become an integral and essential tool for cities to tackle the problems of escalating vehicle emissions and congestion.

It is notable that in several cities, such as Barcelona, a common practice of public transportation planners has been the concentration of several key bus routes along the main corridors of the city. This measure strengthens the ease and number of transfers at shared stops along the corridor, although as number of buses increases, the aggregated bus flow may approach the theoretical capacity value of the lane or bus stop. The key consequences of this effect are queues of vehicles at stops and a significant drop in the commercial speed of buses. It is important to take such factors into consideration when creating or expanding a transit network.

Previous research on route generation and frequency setting has not taken congestion into consideration. The main objective of this article is to provide a methodology for the network design problem that covers both strategic planning and tactical planning, taking into consideration that commercial speeds of buses vary depending on the aggregated flow of buses on each corridor. Its main application is in areas with high bus frequency, as is the case in many cities in South America and Europe. The aim of the methodology is to generate a set of routes and frequencies that minimize both user and operator costs.

This article is organized as follows. The following section summarizes past research on this topic, and then the methodology is described. Next, the methodology is applied to Barcelona's street network, and experimental results are detailed. The most important conclusions of the work are summarized, information for applying the model is provided, and steps for future research are briefly discussed.

Background

Much attention has been paid to the bus network design problem and the setting of efficient frequencies to cover demand. The problem is considered NP-complete and, therefore, a way to find an optimal solution can take a considerable amount of time, especially for large problems (Van Ness 2002). For this reason, most research related to this topic has included adding constraints to the problem or reducing the search space in order to shorten calculation time within reasonable limits. However, the resulting solutions may not be optimal.

In past research, two approaches generally have been used for transit route generation: a continuous approach and a discrete approach. The continuous approach formulates a problem on a solution space with certain completeness. In general, this approach provides a global optimal solution, but the solution might not be realistic. For example, the solution might contain stop spacing or line spacing that is not applicable on the actual network (Van Ness and Bovy 2000). This approach works well for small problems, but as the size of the problem increases, solution time quickly reaches unreasonable values.

The discrete approach formulates the problem directly on possible solution subspaces defined based on domain specified heuristic guidelines. This approach will provide a feasible solution, but often not a global or even local optimal solution. However, the discrete solution generally requires much less computing time, demonstrating the tradeoff between solution optimality and computational time.

Recently, the development of algorithms based on local search and metaheuristics have been implemented in the bus network design problem in order to further optimize the network produced (Chien et al. 2001; Ngamchai and Lovell 2003; Verma and Dhingra 2005). Finally, other metaheuristics such as taboo search or simulated annealing have been used to search the optimal set of routes in the solution domain (Fan and Machemehl 2006).

Table 1 provides a summary of past research on this topic, including both the continuous and discrete approaches. The objective of each of these models is to produce a set of bus routes and route frequencies.

Table 1. Overview of Past Research on Transit Network Design

Author	Description
Lampkin and Saalmans (1967)	Minimize travel time given fleet size and vehicle size.
Hasselstrom (1981)	Maximize number of passengers (demand) given budget and minimum frequency constraints.
Ceder and Wilson (1986)	Minimize travel time, transfer time and fleet size given constraints on route length, number of routes and frequency.
Janarthanan and Schneider (1988)	Includes manual network design, assignment and feedback.
Van Nes et al. (1988)	Maximize the number of passengers with no transfer given a budget constraint.
Baaj and Mahmassani (1995)	Includes computer aided network design, assignment and line improvement.
Ceder and Israeli (1998)	Minimize travel time, empty seat hours and fleet size.
Shih et al. (1998)	Includes computer aided network design, transfer nodes, assignment and line improvement.
Chien et al. (2001)	Minimize user and supplier costs subject to constraints, applies genetic algorithm.
Saka (2001)	Reduce operating costs by finding optimal spacing of buses.
Ngamchai and Lovell (2003)	Includes frequency setting and headway coordination. Applies a genetic algorithm to help optimize bus transit route design.
Verma and Dhingra (2005)	Routes are generated based on shortest paths, also considers transfers to rail stations. Applies a genetic algorithm.
Fan and Machemehl (2006)	Includes computer aided network design and assignment. Applies a simulated annealing procedure to select an optimal set of routes.

Methodology

The aim of the methodology proposed in this article is to solve the bus network design problem and frequency determination. This methodology includes a modification of the solution approach proposed in Baaj and Mahmassani (1995). While the contribution of Baaj and Mahmassani assumes that the shortest path between nodes remains fixed, our methodology recalculates the shortest path during the network building process. This information subsequently is used to determine frequencies and develop the assignment of passengers to the network. The overall methodology process is outlined in Figure 1. On the right is the methodology proposed in Baaj and Mahmassani (1995), referred to here as the Fixed Speed Methodology (FSM), and on the left is the new methodology proposed in this article,

referred to as the Variable Speed Methodology (VSM). Both consist of two principal components: (1) the route generation algorithm (RGA) which designs routes, and (2) the assignment process, which assigns demand to the network, determines frequencies, and evaluates network performance. In the following subsections, the Variable Speed Methodology is described as are the differences between it and the Fixed Speed Methodology.

Figure 1. Model Methodology

Methodology Inputs and Formulation

Both methodologies require information about the underlying street configuration on which the bus network will be created, the demand distribution in the network and the minimum performance quality required. We represent the street configuration by a directed graph $G=(N,L)$, with node set N representing transit stops and intersections and link set L representing links between nodes. We denote d as an asymmetrical bus demand matrix, where the element d_{ij} represents the demand between node i and node j . Regarding quality of service, a number of parameters are set by the user, including the minimum percentage of demand that must be satisfied by the bus network.

The solution of these methodologies is the bus network and its frequencies. The bus network can be described as a set of routes, $R=\{r_1, r_2, r_3, \dots, r_s\}$, where each route, r_z , is defined by a sequence of nodes: $r_z=\{(i,j),(j,k), \dots, (u,v)\} / i,j,k,u,v \in N$. Each route, r_z , has a scheduled frequency, f_z (bus/h).

Route Generation

The route generation algorithm (RGA) is a heuristic algorithm for route design. Its three main features are (1) it is heavily guided by the demand matrix, (2) it allows the designer's knowledge to be implemented so as to reduce the search space, and (3) it generates different sets of routes corresponding to different trade-offs among conflicting objectives. The algorithm starts by creating a number of initial skeletons (M) for the routes, which are expanded and complemented as demand is assigned to existing or new route segments. At the end of the process, a minimum percentage of total demand must be satisfied directly with zero transfers (D_0), and a minimum percentage of total demand with one or fewer transfers (D_1).

Modified Route Generation Algorithm. The RGA described in Baaj and Mahmassani (1995) was modified to include additional features. The main difference is that the Modified RGA (MRGA) recalculates travel times on links in order to account for reduced speeds due to bus congestion (particularly related to interference at bus stops). In addition, the MRGA accommodates networks with one-way streets by designing both an initial route and a return route. The structure of the MRGA consists of the following steps.

1. Select the node pair (i,j) with the highest demand not yet satisfied and search for the shortest path between the nodes.
2. Expand the path generating a new route.
3. Design the return route.

4. Filter the set of routes: check if any routes are overlapped, if there is an overlap delete the smallest route.
5. Re-calculate new travel times on the links with congestion.
6. Add new route to set of routes and compute the directly served demand. If it is greater than D_0 go to the next step; otherwise, go to step 1.
7. Compute the demand served with zero or one transfer. If it is greater than D_1 , stop and return the set of routes; otherwise, go to step 1.

Travel Time Calculation. A key variation between the FSM and VSM is the consideration that bus travel speeds will vary based on the flow of buses in the lane. This variation will affect the travel times calculated on each link. The FSM assumes that travel speeds are constant no matter how many buses are using the link. However, the *Transit Capacity and Quality of Service Manual (TCQSM)* by Kittleson & Associates et al. (2003) presents an analysis of bus speeds, providing a compact formula to estimate this operational variable. Equation 1 is taken from this report and evaluates bus commercial speeds as a function of a basic travel time, time spent at intersections, the effect of skip-stop operations, and the effect of interference from other vehicles.

$$S_t = \left(\frac{60}{t_r + t_l} \right) f_s f_b \quad (1)$$

Where:

S_t : Speed on link t

t_r : Basic travel time considering dwell time

t_l : Time spent at intersections considering effects of other vehicles

f_s : Parameter measuring the effect of skip-stop operations

f_b : Parameter measuring the effect of interference from other buses

The basic travel time, t_r , is determined by taking an estimate of bus running times as a function of stop spacing and average dwell time per stop. Running time losses, t_l , are estimated considering effects of traffic signals, intersections, and other vehicles sharing the lane. TCQSM contains estimated values for these variables (based on field measurements) considering five different lane configurations: (1) bus lane, (2) bus lane with no right turns, (3) bus lane with right turn delays, (4) bus lane blocked by traffic, and (5) mixed traffic flow.

Our modification does not consider the effect of skip-stop patterns (f_s is equal to one). However, the parameter f_b is especially important because it is ruled by the relationship between bus flow and lane capacity. As the flow of buses in the lane increases, the probability of buses having to wait for other buses at bus stops or buses needing to pass other buses increases, thus reducing the overall speed of buses. Table 2 displays the values used for f_b , as evaluated in TCQSM. This parameter is the key behind our measurement of the variation of bus speeds.

Table 2. Values for Bus-Bus Interface Factor

Lane Volume/Capacity Ratio	Bus-Bus Interface Factor (f_b)
<0.5	1.00
0.5	0.97
0.6	0.94
0.7	0.89
0.8	0.81
0.9	0.69
1.0	0.52
1.1	0.35

The travel time on each link is calculated by taking the length of the link divided by the speed on the link (S_t). In the VSM, link travel times are recalculated after each route is generated using the speed of Equation 1. If the new route shares a link with an existing route, the parameter f_b could be affected, thus changing the values of the link speed and link travel time. The link travel times are used to calculate the shortest path between each node pair. Therefore, the adjusted link travel times are used when determining each subsequent route.

One important consideration is that the travel times on each route generated by the FSM will appear lower than would be evaluated in a real network with high transit frequencies. Therefore, after the routes have been generated for the FSM, the speed on each link is recalculated, using Equation 1, to account for the effect of multiple buses on a link.

Network Assignment

Network assignment is the process of assigning demand to the bus network. Once routes have been generated by RGA and MRGA, the network assignment process is applied to the routes to determine frequencies and generate a set of performance indicators. The network assignment process used in the models is a program called TRUST, described in Baaj and Mahmasani (1990). This assignment method uses a transit path choice logic to apply the demand to the network, considering number of transfers as the most important criteria.

Analysis

The FSM and VSM were programmed using JAVA. The two models were applied to the street network of Barcelona, which is composed of 5,928 street nodes and 8,783 links. Of the street nodes, 198 are potential bus stops with a total of 12,254 non-zero origin-destination pairs. The average daily demand matrix during the peak hour (8:00 AM - 9:00 AM), with an associated demand of 51,689 passengers per hour, was obtained from data from a mobility survey of the metropolitan area of Barcelona (IDESCAT 2001).

Speed Calculation Validation

An analysis was performed to assure that the speed calculation shown in Equation 1, defined in TCQSM, was appropriate for the case of Barcelona. The values of t_r , t_l , and f_b were analyzed using real data from the city of Barcelona. The real values were found to be similar to the values detailed in TCQSM in all cases (CENIT 2006).

In particular, data for the bus-bus interface factor (f_b) collected for the city of Barcelona produced the following best-fit curve (Equation 2). These values match very closely with the values from TCQSM, listed in Table 2.

$$f_b = 1 - 0.45 \left(\frac{v}{C} \right)^{3.86} \quad (2)$$

Where:

f_b : Parameter measuring the effect of interference from other buses

v : Lane volume (vehicles/hour)

C : Lane capacity (vehicles/hour)

Subsequently, the TCQSM model for calculating velocity was used to estimate velocity in Barcelona during different periods of the day. These values were compared with the actual velocity values of buses in Barcelona during those periods, and the differences were very small, with a maximum error of 15 percent. This analysis validates the TCQSM model as an appropriate model for estimating the velocity of buses in the city of Barcelona. Therefore, values of t_r , t_l and f_b were taken from the corresponding tables in TCQSM.

Parameter Inputs

As mentioned, many parameter values must be defined by the user. The parameters in Table 3 were selected specifically for the case of Barcelona. In addition,

the values from Table 2 were used to define f_b , which varies based on the flow of buses in the lane.

Table 3. Parameter Values for Barcelona Network

Variable	Description	Value
M	Initial number of skeleton routes	7
D_0	% of demand that must be satisfied with 0 transfers	20% - 80%
D_1	% of demand that must be satisfied with 1 or fewer transfers	50% - 100%
t_r	Base bus running time (min/km)	3.49
t_l	Base bus running time losses (min/km)	1.2
T_{tran}	Transfer time penalty (min)	5
Cap	Seated bus passenger capacity (passengers/bus)	90
LF_{max}	Maximum allowable load factor	1.25

Several scenarios have been considered regarding various combinations of D_0 and D_1 . These variables greatly influence the composition of the bus network created because the algorithms will continue adding routes to the network until these minimum demand values are satisfied. The value of D_0 governs the directness of service, while D_1 influences network coverage. Table 4 summarizes the various combinations of D_0 and D_1 used.

Table 4. Input Values for D_0 and D_1

D_0 (%)	D_1 (%)
20	50
20	60
20	80
20	90
20	100
40	60
40	80
40	90
40	100
60	80
60	90
60	100
80	90
80	100

Sensitivity Analysis

The inputs listed in Table 3 and Table 4 were used for both the VSM and FSM. Figure 2 displays the total travel time in terms of passenger-minutes for each model run versus the percentage of the total demand satisfied by one or fewer transfers (each point represents a different combination of D_0 and D_1 for each of the models). The slightly larger markers represent the case where $D_0 = 40\%$ and $D_1 = 60\%$. It should be noted that the demand satisfied by the network might be higher than the minimum demand required by D_1 , since as routes are added to the network, demand satisfied increases in discrete increments. These results show that as demand satisfied increases, total passenger travel time also increases. This is intuitive since as the number of passengers served increases, the total passenger-minutes also increase. The networks created by the VSM satisfy the demand using fewer overall passenger minutes than the networks created by the FSM.

Figure 2. Total Travel Time vs. Satisfied Demand

Figure 3 shows the mean passenger travel time versus satisfied demand. The VSM model provides shorter travel times than the FSM model. This is a very attractive quality for users.

Figure 3. Mean Travel Time vs. Satisfied Demand

Figure 4 shows the number of buses versus satisfied demand. This plot shows that as demand satisfied increases, the number of buses required to serve that demand also increases, which is expected. The VSM generally requires fewer buses to serve the same level of demand as the FSM. This suggests that the buses in the VSM networks are used more efficiently.

Figure 5 depicts the percentage of total routes that have low ridership versus the minimum demand satisfied directly. Low ridership routes are defined as those requiring fewer than one bus per hour. As demand satisfied directly increases, percent of low ridership routes also increases. Furthermore, when $D_1 = 100\%$, meaning all demand must be satisfied with one or fewer transfers, the percent of low ridership routes increases significantly. These results demonstrate that forcing a higher percentage of demand to be satisfied directly or requiring that all demand be satisfied will lead to a higher occurrence of low ridership routes. This reduces the efficiency of the network because more resources are required to provide services on routes that are used by fewer passengers, thus requiring more resources per passenger served. These passengers might be more efficiently served by demand-responsive transit services such as paratransit.

Figure 4. Fleet Size vs. Satisfied Demand

Figure 5. Percent Low Ridership Routes vs. Demand Satisfied Directly

Figure 6 shows the mean passenger travel speed versus satisfied demand. The mean passenger travel speed does not include waiting time or time stopped at bus stops, but includes only the average time all passengers spend traveling on links in the system. As Figure 6 demonstrates, networks created by VSM have higher passenger travel speeds in networks with less than 95 percent of demand satisfied.

However, when demand satisfied reaches 100 percent, the networks designed by both VSM and FSM seem to have similar travel speeds. This could be because, to satisfy a higher percentage of the demand, the network must be extended to cover more nodes, thus creating routes on outlying links and reducing frequencies, hence reducing congestion in the network, which, in turn, increases travel speeds. However, it should be noted that this increase in travel speed comes at the cost of reduced efficiency in the network and higher operating costs.

Figure 6. Mean Passenger Travel Speed vs. Satisfied Demand

Figure 7 shows mean route speed versus mean route frequency. As frequency increases for FSM, route speed decreases dramatically. This is likely a result of several bus routes running on the same streets, causing bus speeds to decrease as bus congestion and bus interference at stops increases. In the VSM, however, route speed decreases at a slower rate as frequency increases. This suggests that the bus networks generated by VSM are more spread out across the street network and, as a result, encounter less congestion. Even as bus frequencies increase overall, it has a much lower impact on bus speeds.

As discussed, a variety of potential bus networks can be produced depending on the model used and inputs selected. The above analysis shows that VSM generally provides a more efficient network than FSM.

Figure 7. Mean Passenger Travel Speed vs. Mean Route Frequency

Model Results Using Same Inputs

Table 5 displays the most important parameters for measuring the performance of the bus networks produced by each of the models when the same set of inputs are entered into each. The values $D_0 = 40\%$ and $D_1 = 60\%$ were selected for this analysis because the networks produced have reasonable values for each of the indicators analyzed; for example, the percent of demand served is high while the number of low ridership routes and fleet size are relatively low. These networks are denoted in the previous figures by slightly larger markers. The network produced by the VSM serves about the same amount of demand as the FSM. Both serve the minimum amount required plus additional demand due to the discrete addition of routes to the network. The VSM network requires more routes but fewer buses than the FSM network and fewer route kilometers, which would result in lower expenses for purchasing buses and providing bus maintenance.

The mean passenger travel speed is higher for VSM than for FSM. This reflects the importance of variable speed consideration in the network design process. In addition, the VSM network has a lower mean passenger travel time than the FSM network. This signifies that the VSM network is more attractive to passengers than the FSM network.

The maximum number of buses operating on a single link is lower for VSM; furthermore, the network produced by VSM has the highest percentage of links in

the street network covered by routes. This demonstrates that VSM produces a network that is more spread out, covering more links, and the links in the network are less congested than the network produced by the FSM.

These results are specific to this set of inputs and results can vary depending on the inputs used, but based on this analysis, VSM tends to produce a more efficient network and one that is more attractive to both users and operators than FSM.

Table 5. Comparison of Networks

	NETWORK COMPARISON	FSM	VSM	% Difference
	Total demand (pax/hr)	51,689	51,689	
	Demand assigned, D1 (%)	60	60	
	Demand with no transfer, D0 (%)	40	40	
INPUTS	Demand served with 0 transfers (pax/hr)	19,858	19,805	-0.3%
	Demand served with 1 transfer (pax/hr)	22,582	21,978	-2.7%
	Demand satisfied (%)	85	84	-1.2%
	Number of routes	112	115	2.7%
	Percent of low ridership routes (%)	12	11	-8.3%
	Fleet (vehicles)	861	803	-6.7%
	Max buses per link	110	98	-10.9%
	Links covered by routes (%)	51	53	3.9%
	Mean frequency of the routes (buses/hr)	3.5	3.5	0.0%
	Mean passenger travel time (min)	57	55	-3.5%
OUTPUTS	Mean person speed (km/hr)	11.8	12.4	5.1%
	Total travel time (pax·min)	2,669,388	2,585,779	-3.1%
	Total route kilometer	2,805	2,798	-0.2%
	Total seat kilometer offered (pax·km), SKO	908,290	896,633	-1.3%
	Total person kilometer transported (pax·km), PKT	462,438	455,045	-1.6%
	Load factor (PKT/SKO)	0.51	0.51	0.0%

Bus Network Layout Comparison

The bus network layout and vehicle flow distribution associated with these two networks are represented in Figures 8 and 9. At first glance, both networks share the corridors with more demand (transversal arterials). Nevertheless, the network

proposed by the VSM covers more streets, providing a more extensive network, and, generally, each street has less frequency, therefore reducing congestion. On the other hand, the network proposed by the FSM presents a consolidation of routes on fewer streets, which worsens the bus congestion phenomenon.

Figure 8. Network Layout for FSM

Figure 9. Network Layout for VSM

Conclusions

In this article, we have proposed a bus network design methodology that takes into consideration the reduced speed caused by multiple buses using the same link in generating routes and setting frequencies for a bus network. We have developed a model using this methodology and have shown that this model produces different results than a model that does not include this consideration. Not only does the Variable Speed Methodology more accurately simulate the actual practice of buses than the Fixed Speed Methodology, it also produces a more efficient and more attractive bus network.

The FSM and VSM models were applied to the street network of Barcelona. This, itself, is a valuable contribution as past research generally has applied transit network design models to small networks. This is an example of a network design model applied to a large network of an actual city. Overall, the VSM was found to produce bus networks with faster travel speeds, lower travel times, fewer buses, less route kilometers, and fewer buses per link than the networks produced by the FSM. This demonstrates that the VSM model was able to create bus networks that were more spread out, less congested, faster, and able to use resources more efficiently than the networks created by the FSM model.

These results show that the variable speed consideration is an important improvement to network design models that have been created in the past. Therefore, this adjustment is a valuable contribution to research attempting to solve the bus network design problem.

Application

The Variable Speed Model offers a flexible modeling tool that transit operators can use to create and evaluate various sets of bus networks. The model can be adapted to any street network using inputs specific to that network.

The sensitivity analysis performed in this article also gives some insight to operators on how to use the model. When selecting values for D_0 (the percent of demand that must be served directly) and D_1 (the percent of demand that must be served with one or fewer transfers), the operator should consider the trade-off between these values and travel time, fleet size, and percent of low ridership routes. The values chosen should be high enough to ensure that a sufficient level of demand is served, but low enough to maintain reasonable user and operator costs.

Further Research

Future work on this model would consider a multi-modal approach. Information from the local metro and regional train system would be included to determine how these systems would affect efficient bus network design.

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