How Does One Design or Evaluate a Course in Quantitative Reasoning?

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Abstract

In the absence of generally accepted content standards and with little evidence on the learning for long-term retrieval and transfer, how does one design or evaluate a course in quantitative reasoning (QR)? This is a report on one way to do so. The subject QR course, which has college algebra as a prerequisite and has been taught for 8 years, is being modified slightly to be offered as an alternative to college algebra. One modification is adding a significant formal writing component. As the modification occurs, the current course and the modified one are judged according to six sets of criteria: the six core competencies of the Association of American Colleges and Universities rubric on quantitative literacy; the five mathematical competencies from the National Research Council (NRC) study report, *Adding It Up*; the eight practice standards from the Common Core State Standards for Mathematics; the five elements of effective thinking as articulated by Edward Burger and Michael Starbird, the summary research findings on human cognition from the NRC study report, *How People Learn*; and the ten principles gleaned from applying the science of learning to university teaching. The QR course, as described by ten design principles, is determined to be generally well aligned with most of the overlapping criteria of the six sets, providing cogent evidence of high educational value.

Cover Page Footnote
Bernard L. Madison is professor and former Chair of the Department of Mathematical Sciences, University of Arkansas, and former Dean of its Fulbright College of Arts and Sciences. He was founding president of the National Numeracy Network and is a frequent contributor to this journal.

Introduction[1]

Over the past decade or so, education for quantitative literacy (QL) or quantitative reasoning (QR) in the US has gained limited recognition as a critical and perhaps distinct component of school and college curricula, but effective educational methods for QR are tentative and unproven.[2] Focused around the publication of *Mathematics and Democracy* in 2001, several authors (e.g., Steen 1997 and 2001; Madison and Steen 2003 and 2008a), have made the case forcefully for QR education. Various post-secondary professional societies, notably the Mathematical Association of America (MAA 2004), Association of American Colleges and Universities (AAC&U 2004), the American Association of Two Year Colleges (AMATYC) (Blair 2006), and the National Numeracy Network (NNN),[3] have initiated policies and structures supporting QR education. Courses are being offered or are under development at individual colleges and universities, and consortia of institutions are working in concert to produce effective college level courses in QR, some in conjunction with developmental mathematics and statistics. Two of the efforts by consortia are centered at the Charles A. Dana Center[4] at the University of Texas in Austin and at the Carnegie Foundation for the Advancement of Teaching[5] in Palo Alto, California.

QR education in post-secondary institutions has two major resource hurdles to overcome. First, it has no academic home in either K-12 or post-secondary education (Madison 2001; Steen 2001). In K-12 QR education is highly dependent on the mathematics and statistics curricular strand, and less so on the sciences. Most post-secondary courses and quantitative learning centers (Madison and Steen 2008b; Gillman 2006) have evolved from mathematics or statistics units, but QR units and courses remain largely marginalized in college and university mathematics curricula. In contrast to most mainline collegiate disciplines, collegiate mathematics has long used its standard content-designated courses as general education courses – algebra, geometry, and calculus. Most collegiate mathematics courses have titles derived from the mathematical content of the course – e.g., calculus, differential equations, linear algebra. College and university mathematics faculty members, not unlike many of their colleagues in other STEM (science, technology, engineering, and mathematics) disciplines, have limited and varying interests in the role of their courses in service of general education. Various attempts at general education mathematics courses over the past century have met with limited acceptance, so mathematics faculty are strongly influenced by this in considering

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[1] This paper is a significant revision of a manuscript that was published as "Reverse Engineering a Course in Quantitative Reasoning" in *Quantitative Reasoning in Mathematics and Science Education: Papers from an International STEM Research Symposium (*R. L. Mayes and L. L. Hatfield, eds.), p. 43-64, vol. 3, WISDOM Monograph, 2013, Laramie, WY: College of Education, University of Wyoming.

[2] In the remainder of this paper QR will be used for either QL or QR except when referring to existing literature that uses QL.

[3] http://serc.carleton.edu/nnn/index.html. (all links in the footnotes were accessed 1 May 2014).

[4] http://www.utdanacenter.org/amdm/index.php

[5] http://www.carnegiefoundation.org/quantway

and supporting courses such as QR. The titles of such courses do not describe the mathematical content, so faculty are justifiably puzzled by what they are and how effective they would be in promoting learning in mathematics. (One of the author's colleagues characterized the content of the QR course as "fluff.") Mathematics in grades 9-16, from high school through the early years of college, is very linear, equaled only by that of a foreign language, and general education courses have no established place in this linearity (Madison 2003).

The second major resource hurdle for QR education is connected to the first. There are no clear guidelines for courses and no generally accepted measures of success. Consequently there are no widely accepted curricular materials.

Both of these hurdles were obvious when mathematical sciences faculty at the University of Arkansas considered[6] whether or not to establish a QR course as an alternative to college algebra for students who would not study further mathematics that needed many of the methods of college algebra, i.e. many of the students who were majoring in non-STEM disciplines. Essentially the same QR course as the one proposed had been offered for several years, but with college algebra as a pre-requisite. The new version of the course was more visible in that it was being proposed as a course in the (Arkansas) state minimum core as a substitute for college algebra, and, as such, had attracted critiques in the public media (*Arkansas Democrat-Gazette* 2012; Brawner 2012). The effectiveness of the current mathematics curriculum, including algebra, had been questioned in two highly visible op-ed pieces in the *New York Times*, one by David Mumford and Sol Garfunkel (2011) and one by Andrew Hacker (2012). As was verified by many people who commented on the Hacker article, arguments in favor of QR courses as alternatives to college algebra fall victim to being interpreted as finding an easier route for algebra-phobic students. Because QR is neither well established nor well understood, and because QR courses often do not develop any specific mathematical content, the standards for acceptance within the academic community are higher than those for a course such as statistical methods that indicates some generally acceptable (now, but less so a few decades ago) mathematical content. This backdrop prompted an articulation of the analysis of the methods and content of the QR course that is reported here, the results of which provide a framework for designing QR courses.

Developing the Design Principles

The design principles that are presented below evolved over the past eight years of teaching QR courses to college students and are rooted in the author's work with Robert Orrill and Lynn Steen in the QR initiative that Orrill led during 2000-2004 (Madison and Steen 2008b). Almost all of the principles have been described in three research reports on the QR course (Dingman and Madison 2010; Madison and Dingman 2010; Boersma et al. 2011). These principles were articulated more thoroughly in light of five research-based and one experience-based sets of criteria on student learning in the analysis of the QR course at the University of Arkansas in a reverse engineering process prompted by an increased need to evaluate, justify and improve the course and its outcomes.

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[6] The course was recommended narrowly but was questioned as to both its mathematical content and the ways that the success of the course would be measured. To be fair, many courses could be questioned on the latter issue, especially college algebra.

The process of taking something apart and revealing the way it works is often an effective way to learn how to build a device or make improvements to it; this is an aspect of reverse engineering. In order to reverse engineer the QR course I identified six collections of content and process standards and research findings on how students learn in college classrooms and used them as criteria for improvements and evaluation. In brief, these collections are:

- The six core competencies for QL as articulated in the Association of American Colleges and Universities (AAC&U) QL rubric (AAC&U 2009; Boersma et al. 2011).

- The five strands of mathematical proficiency from *Adding It Up* (Kilpatrick et al. 2001). This will be referred to as *Adding It Up*.

- The eight Standards for Mathematical Practice of the Common Core State Standards for Mathematics (CCSSM 2010).

- *The 5 Elements of Effective Thinking* as articulated by Edward Burger and Michael Starbird (2012).

- Three principles from *How People Learn* (Bransford et al. 2000) as applied to successful classroom practice. This will be referred to as *How People Learn*.

- Ten principles from *Applying the Science of Learning to University Teaching and Beyond* (Halpern and Hakel 2003).

Five of these six collections are based on research on student learning, and the sixth, by Burger and Starbird, is based on years of highly successful college classroom teaching. There are other possibilities for criteria, especially if one focuses more on immediate outcomes of QR courses rather than long-term retention and instructional design, the primary issues here. Examples are QR assessments such as the Quantitative Literacy & Reasoning Assessment (2012) and the Critical Thinking Assessment Test (Stein, Haynes and Redding 2007). Another possibility is a 1994 MAA committee report that gave a list of mathematical outcomes of QR in college (MAA 1994).

Of course, the real measure of the effectiveness of a course is student learning, especially the learning for long-term retrieval and transfer. Such measures are elusive for single college courses, to say the least, and other reasons why any measures of student learning are both difficult and of limited value will become apparent as we discuss the characteristics of the QR course in question and compare those characteristics to characteristics specified or implied by the six collections of standards and research findings. In the absence of traditional content for a QR course and reliable measures of desired learning outcomes, the six collections of criteria seem a reasonable approach to developing design specifications for or evaluating a QR course. Throughout, two rather startling conclusions from a report (Halpern and Hakel 2003) of the research findings on learning for long-term retrieval and transfer should serve as motivating beacons of a QR course design, and I present them here verbatim, for emphasis:

- "But, ironically (and embarrassingly), it would be difficult to design an educational model that is more at odds with the findings of current research about human cognition than the one being used today at most colleges and universities." (p. 38)

- "There is a large amount of well-intentioned feel-good psychobabble about teaching out there that falls apart upon investigation of the validity of the supporting evidence." (p. 41)

Evolution of the QR Course

As of this writing there are two QR courses, one with college algebra as a prerequisite and one without that prerequisite. The resolution of the relationship between these courses will take a few semesters, but both have the characteristics and philosophies discussed here. Consequently, "the QR course" will refer to either. First offered experimentally in fall 2004, the QR course has been offered each semester since. At present, the enrollment is approximately 600 students per year, mostly majors in the arts, humanities and social sciences. The course is taught in sections of 20-30 students in interactive classroom environments with tables for four, a document projector, and Internet access. The only textbook is the third edition of *Case Studies for Quantitative Reasoning* (referred to as the *Casebook*) (Madison et al. 2012) that evolved from duplicated notes and two earlier editions. The course was expanded and enhanced through the support of the National Science Foundation (DUE-0715039) from 2007 to 2012. Typically, the class meets twice weekly for 75-80 minutes throughout a semester. The *Casebook* has 30 case studies of media articles, consisting of an article, warm-up exercises, and study questions on the article. The topics of the case studies are sorted into six sections: 1) using numbers and quantities; 2) percent and percent change; 3) measurement and indices; 4) linear and exponential growth; 5) graphical interpretation and production; and 6) counting, probability, odds, and risk. A typical class meeting begins with students presenting or discussing at tables media articles they have found and brought to class that contain quantitative information. This feature has been referred to as News-of-the-Day, and students are sometimes awarded credits for presenting articles. There is usually a homework assignment of warm-up exercises, but the core activity is addressing the study questions, which probe the quantitative content of the article being discussed. Often, students address the study questions in groups of 3-4 at a single table. Quizzes and tests consist of exercises similar to the warm-up exercises and study questions on one or two articles new to the students. Mathematics is developed or reviewed as needed, when needed. For example, the sum of a geometric series is developed when needed for compounding interest or exploring installment savings or purchasing.

The success rate (grade of A, B, or C) for the course is over 80%, significantly higher than other introductory mathematics courses. The higher success rate is partly due to the prominent role of daily homework in the course but likely also due to the students' heightened interest in the subject matter. Student evaluations of the course have been favorable, and it has received high marks from faculty advisors in departments whose students enroll in the course. Pre- and post-tests were used in 2007-2008 to compare learning in this course with that in two other similar courses (see Table 1 for some summary results), and pre- and post-tests for an attitude survey were administered to the same populations. Although the results were not dramatic, learning gains as measured by the test were larger in the QR course and attitude shifts were all in the desired direction. Former students were surveyed by email after 2-3 years to see if they continued to

practice QR in looking at media articles[7]. The response rate was very low (42/300), but about half reported that they continued to practice QR; about 2/3 responded that their confidence in their QR ability had increased; and about ¾ reported that they now believed QR to be more important to them. Various effects of the course, e.g., on productive disposition, are currently under investigation.

Table 1
Comparative pre-test and post-test results

Fall 2007 – 15 multiple-choice items. Three different courses			
Course	Number of Students	Number of Items with significant increase in mean scores ($p<0.05$)	Number of items with significant increase in mean scores ($p<0.1$)
Survey of Calculus	106	6	9
For All Practical Purposes	77	6	7
QR	96	9	10

Spring 2008 – 17 multiple-choice items. Two different courses			
Course	Number of Students	Number of Items with significant increase in mean scores ($p<0.05$)	Number of items with significant increase in mean scores ($p<0.1$)
For All Practical Purposes	83	5	6
QR	95	5	9

Writing and critical reading have been important all along in responding to study questions. In fact, 26 of the 30 case studies have questions that require communication, including writing, and all 30 require interpretation, usually interpreting quantitative information given in words so it can be represented in another form, usually a function or an equation. See Table 2 below for the competency requirements of the case studies in the *Casebook* that are given in full on the website[8] that supports the *Casebook*.

Table 2.
Prevalence of competencies in questions and cases

Competency	Percent of Study Questions	Percent of Case Studies
Interpretation	67	100
Representation	30	73
Calculation	48	90
Analysis/synthesis	35	90
Assumption	7	40
Communication	38	87

Over the past three years a significant writing component has been added to a few sections of the course. The results of that and the belief that writing is important to

[7] An important outcome of QR courses is development of a QR habit of mind that would be expected to continue beyond the course and beyond school. Assessment of such a habit of mind remains to be developed and demonstrated. See Boersma and Klyve (2013).

[8] http://www.cwu.edu/~boersmas/QRCW/mappingtesting/index.html

improved QR prompted adding a significant formal writing requirement. The writing requirement, added in consultation with English composition program,[9] adds to the rigor of the course as well as to the difficulty of teaching and assessment. Part of the reason for the writing requirement is to maintain the level of rigor and to protect against the course degenerating to the methods contained therein. The more important reason for adding writing is that writing strengthens quantitative reasoning (Madison 2012; Grawe and Rutz 2009) and increases the metacognitive skills of the students. Because the course requires instructors not only familiar with using case studies in a collaborative learning classroom but also with instruction and assessment in writing, the preparation of instructors for the QR course will be expanded to include writing.

Two challenges that have not been solved are:

- What contextual examples should be generalized and abstracted? The power of mathematics is in abstraction and generalization, and students should not only see this power when it is needed but should combine results of contextual examples with abstractions to increase the long-term retrieval and transfer (Halpern and Hakel 2003; *How People Learn* 2000).

- One of the research findings (*How People Learn* 2000: 16) about developing competence in an area of inquiry is to "understand facts and ideas in the context of a conceptual framework." What are the conceptual frameworks for a QR course, or, more generally, for QR?

Guidance and Boundaries for this Paper

The multiple, complex, and interrelated lists of criteria and principles needed here prompt me to offer some guidance to the reader and place some boundaries on the following discussion. First, the six lists of criteria measures will be articulated more fully than the abbreviated list above. Second, the design principles of the QR course will be listed and discussed both from the point of view of how they influence and are reflected in the QR course and how some of the criteria measures support the principles. Third, the criteria measures will each be discussed in light of how they are reflected in the QR course. Obviously, the six lists of criteria measures overlap and have numerous connections. Comparisons among the six sets of criteria will be minimal here to avoid distractions from our primary purpose of supporting the design principles by noting their alignment with the sets of criteria.

Criteria Measures

How the QR course fares with respect to the six sets of criteria described briefly above is discussed below. First, the criteria are given in more detail. The first set of criteria, the core proficiencies for QR as developed by AAC&U and adapted by Boersma et al.

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[9] Paired sections of the QR course and a composition course were tried, but students did not find an English and mathematics course for six semester hours very attractive. Having a composition instructor alongside a mathematics instructor in a writing intensive section of QR was far more appealing to students, but too labor intensive for staffing.

(2011), is the only set of the six criteria measures that was developed with QR in mind, so this set is listed first.

AAC&U QL Rubric and an Adaptation

In 2009, AAC&U published fifteen rubrics as products of its Valid Assessment of Learning in Undergraduate Education (VALUE) project. One of those fifteen was the Quantitative Literacy rubric.[10] According to AAC&U, "the rubrics are intended for institutional-level use in evaluating and discussing student learning, not for grading." The author and colleagues (Boersma et al. 2011) adapted the AAC&U VALUE QL rubric to one to assess individual student work. The result was the Quantitative Literacy Assessment Rubric (QLAR).[11] Like the VALUE rubric, QLAR has six core competencies that are required for responses to QR prompts: interpretation, representation, calculation, analysis/synthesis,[12] assumption, and communication. These are described as follows:

1. *Interpretation*: Ability to glean and explain mathematical information presented in various forms (e.g., equations, graphs, diagrams, tables, words).

2. *Representation*: Ability to convert information from one mathematical form (e.g., equations, graphs, diagrams, tables, words) into another.

3. *Calculation*: Ability to perform arithmetical and mathematical calculations.

4. *Analysis/Synthesis*: Ability to make and draw conclusions based on quantitative analysis.

5. *Assumptions*: Ability to make and evaluate important assumptions in estimation, modeling, and data analysis.

6. *Communication*: Ability to explain thoughts and processes in terms of what evidence is used, how it is organized, presented, and contextualized.

Two of the six – interpretation and communication – involve critical reading and writing (or speaking). In fact, all but calculation can involve non-quantitative communication.

The next two sets of criteria are descriptions of mathematical proficiency (for K-12, but clearly more broadly applicable) that were developed by groups of mathematicians and mathematical educators and have bases in research on teaching and learning mathematics.

Mathematical Proficiency from Adding It Up

Adding It Up is a 2001 report of the Mathematics Learning Study Committee of the National Research Council that summarizes research results on mathematics learning from pre-kindergarten through grade 8. The model of mathematical proficiency articulated in *Adding It Up* consists of five intertwined strands that are described as follows.

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[10] See http://www.aacu.org/value/rubrics/index_p.cfm,

[11] See http://www.cwu.edu/~boersmas/QRCW/Casebook/QLAR.pdf.

[12] This was application/analysis in the QL VALUE rubric.

1. *Conceptual understanding*: Comprehension of mathematical concepts, operations and relations.

2. *Procedural fluency*: Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

3. *Strategic competence*: Ability to formulate, represent, and solve mathematical problems.

4. *Adaptive reasoning*: Capacity for logical thought, reflection, explanation, and justification.

5. *Productive disposition*: Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence in one's own efficacy.

The Standards of Mathematical Practice of the Common Core State Standards

The Common Core State Standards' Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students (CCSSM 2010). These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM 2000) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second consists of the strands of mathematical proficiency from *Adding It Up* as described above. The eight practice standards are below, each with a one-sentence description. The full descriptions of the standards are at the Common Core State Standards for Mathematics website.[13] The eight practice standards will be referred to as CCSSM # where # is 1-8.

1. *Make sense of problems and persevere in solving them:* Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

2. *Reason abstractly and quantitatively:* Mathematically proficient students make sense of quantities and their relationships in problem situations.

3. *Construct viable arguments and critique the reasoning of others:* Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.

4. *Model with mathematics:* Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

5. *Use appropriate tools strategically:* Mathematically proficient students consider the available tools when solving a mathematical problem.

6. *Attend to precision:* Mathematically proficient students try to communicate precisely to others.

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[13] http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

7. *Look for and make use of structure:* Mathematically proficient students look closely to discern a pattern or structure.

8. *Look for and express regularity in repeated reasoning:* Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.

The 5 Elements of Effective Thinking

The fourth set of criteria is documented largely with anecdotes from the classrooms of the book's two authors, Edward Burger and Michael Starbird, both notable award-winning collegiate mathematics faculty members. Although the title of the book (Burger and Starbird 2012) is *5 Elements*, there are four core building blocks of effective thinking; the fifth element, change, is an expected outcome of applying the first four. The authors use the five classical elements that were once believed to be the essential parts of nature and matter – earth, fire, air, and water, plus the quintessential heavenly element aether. Contrary to what was believed about aether (that it was incapable of change), Burger and Starbird have change as their fifth and quintessential element. Briefly, these four building blocks of effective thinking are (p. 6):

- *Earth* – Understand deeply. Don't face complex issues head-on; first understand simple ideas deeply. Clear the clutter and expose what is really important.

- *Fire* – Ignite insights by making mistakes. Fail to succeed. Intentionally get it wrong to inevitably get it more right. Mistakes are great teachers – they highlight unforeseen opportunities and holes in your thinking.

- *Air* – Raise questions. Constantly create questions to clarify and extend your understanding. What's the real question? Working on the wrong question can waste a lifetime. Be your own Socrates.

- *Water* – Follow the flow of ideas. Look back to see where ideas came from and then look ahead to see where the ideas may lead. A new idea is a beginning, not an end.

Research Findings from How People Learn

Quantitative reasoning has become an indispensable skill for 21st century US residents. In *How People Learn* (2000: 4-5), the situation is summarized as follows:

> In the early part of the twentieth century education focused on the acquisition of literacy skills: simple reading, writing, and calculating. It was not the general rule for educational system to train people to think and read critically, to express themselves clearly and persuasively, to solve complex problems in science and mathematics. Now, at the end of the century, these aspects of high literacy are required of almost everyone in order to successfully negotiate the complexities of contemporary life. The skill demands for work have increased dramatically, as has the need for organization and workers to change in response to competitive workplace pressures. Thoughtful participation in the democratic process has also become increasingly complicated, as the focus of attention has shifted from local to national and global concerns.

The expanded edition of *How People Learn* represents reports on the work of two National Research Council committees, both published in 1999, one that summarized research developments in the science of learning, and one that summarized research findings on linking learning research to classroom practices. The expanded volume,

published in 2000, begins with three key findings on how students learn. These findings have strong implications for teaching and are connected to our practices in the QR course as listed below (numerals indicating findings; *T* indicating implication for teaching).

1. Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that they are taught, or they may learn them for purposes of a test but revert to their preconceptions outside the classroom.

    1T. Teachers must draw out and work with preexisting understandings that their students bring to them.

2. To develop competence in an area of inquiry, students must: (a) have a deep foundation of factual knowledge, (b) understand facts and ideas in the context of a conceptual framework, and (c) organize knowledge in ways that facilitate retrieval and application.

    2T. Teachers must teach some subject matter in depth, providing many examples in which the same concept is at work and providing a firm foundation of factual knowledge.

3. A "metacognitive" approach to instruction can help students learn to take control of their own learning by defining learning goals and monitoring their progress in achieving them.

    3T. The teaching of metacognitive skills should be integrated into the curriculum in a variety of subject areas.

Principles from Applying the Science of Learning to the University and Beyond

What can research on human learning tell us about how to best conduct classes in college (or in any adult education setting) to teach for long-term retention and transfer? About a dozen years ago 30 experts from different areas of the learning sciences met to answer this question. As reported by Halpern and Hakel (2003), these experts identified ten "basic laboratory-tested" principles drawn from what is known about human learning. They follow below and will be referred to as Halpern and Hakel # with # being 1-10.

1. The single most important variable in promoting long-term retention and transfer is "practice at retrieval."

2. Varying the conditions under which learning takes place makes learning harder for learners but results in better learning.

3. Learning is generally enhanced when learners are required to take information that is presented in one format and "re-represent" it in an alternate format.

4. What and how much is learned in any situation depends heavily on prior knowledge and experience.

5. Learning is influenced by both our students' and our own epistemologies.

6. Experience alone is a poor teacher. Too few examples can situate learning. Many learners don't know the quality of their comprehension and need systematic and corrective feedback.

7. Lectures work well for learning assessed with recognition tests, but work badly for understanding.

8. The act of remembering itself influences what learners will and will not remember in the future. Asking learners to recall particular pieces of information (as on a test) that have been taught often leads to "selective forgetting" of related information that they were not asked to recall.

9. Less is more, especially when we think about long-term retention and transfer. Restricted content is better.

10. What learners do determines what and how much is learned, how well it will be remembered, and the conditions under which it will be recalled.

Design Principles for the QR Course

As the QR course was refined and expanded over the past eight years, some principles have evolved and been articulated in composing curricular materials and in conducting the QR classes. Some of these are strongly influenced by the circumstances of having a one-semester QR course with no continuing formal education in QR. These ten principles are articulated and discussed in the following with references to the six sets of criteria.

1. Provide a venue for continued practice beyond the course (and beyond school). Quantitative reasoning is a habit of mind, and habits are developed by practice. Especially because the QR course is for only one semester, extending the practice of QR beyond the course is critical for long-term recall and transfer. As noted in Halpern and Hakel 8, the act of remembering influences what learners will and will not remember in the future. The venue for continued practice for the QR course is media articles with quantitative content. The course utilizes case studies of media articles as the focus of study; the *Casebook* for the course consists of 30 such case studies. Media articles similar to the ones discussed in the course are now and will continue to be part of the everyday world of the students. There are several examples of successful application in professional education in the US of using problem-based case studies that prepare one for professional practice, even using the word, practice. Among these are case studies in education, medicine, law, architecture, social work, and business. Quantitative reasoning is analogous to a lifelong profession, as effective quantitative reasoning will be needed for informed performance as citizens and for personal prosperity. The QR course moves students toward developing their own habits of analysis of media articles, taking charge of their learning as promoted by principles from *How People Learn* and by Burger and Starbird (2012). In one of the activities in the QR course, students develop study questions about articles they bring to class, possibly as a News-of-the-Day contribution. Study questions, like those in the *Casebook*, can aim at, for examples, clarification of quantitative content, checking of quantitative assertions, or extending the quantitative conclusions. The variety is so extensive as to defy definition. This encourages the use of questioning to increase understanding, the element of air by Burger and Starbird (2012).

2. Keep the material relevant to students' everyday contemporary world. According to John Dewey, "School should be less about preparation for life and more about life itself." Connecting classroom learning to the everyday contemporary world not only can enhance learning at the time of study in the classroom but can lead students to adapt their classroom learning to the changing environment of everyday life. As noted in *How People Learn* (2000: 73), "The ultimate goal of schooling is to help students transfer what they have learned in school to everyday settings of home, community, and workplace." The variety of media articles and contexts in non-school environments in the QR course regularly requires adapting thinking in one context to another context. Again following *How People Learn* (2000: 73), "Since these environments change rapidly, it is also important to explore ways to help students develop the characteristics of adaptive expertise." Adaptive reasoning is one of the five strands of mathematical proficiency from *Adding It Up*.

Another of the strands of mathematical proficiency is productive disposition, a double edged proficiency depending on students seeing that mathematics (or QR) is sensible, useful, and worthwhile, coupled with a belief in their ability to understand and use it. Keeping the material relevant to the students' lives aims directly at half of this proficiency.

For various reasons, subject matter should be fresh and authentic. Even older[14] articles can be related to the present, as in Burger and Starbird's looking forward and backward. For example, a 2003 article on a political debate about how to measure the budget deficit (nominal dollars, constant dollars, or percent of GDP) easily relates to the current continuing discussion of deficits and national debt. Or a 2001 opinion piece about the economics of increasing the fuel efficiency of automobiles is analogous to the economics of choosing between a hybrid version and a gasoline version of a type of automobile.

Over the decade of developing the QR course, paper copies of newspapers and magazines have continued to give way to online sources, and online sources are available via numerous personal technologies. The shifting of sources and methods of delivery have changed the way students access media articles and has increased the variety (and uncertain reliability) of articles, adding importance to the question of evaluation of the information reported.

There are potential problems with learning in contexts. As stated in *How People Learn* (2000: 77), "Simply learning to perform procedures and learning in a single context, does not promote flexible transfer," leading to design principle 3.

3. Use multiple contexts to practice quantitative reasoning. According to Halpern and Hakel (2002; 2003), "The purpose of formal education is transfer" (p. 38 in 2003). Halpern and Hakel go on to identify retrieval in multiple contexts as one of the most basic principles to enhancing long-term retention and transfer of learning, and that spaced, not massed, practice at retrieval is best. In a one-semester QR course, significant spacing of retrieval is not possible. Consequently, there is more need for continued practice at retrieval beyond the course. With multiple contexts, students are more likely to abstract the relevant features of concepts and develop a more-flexible representation of knowledge, whereas instruction based on single contexts may lead to situated learning.

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[14] a few years to an 18-year-old!

Contextual situations need to be abstracted and generalized, which is closely related to principle 4.

4. Promote appreciation of arithmetical precision and the power of mathematical concepts and processes. This fourth principle is difficult to apply in a QR course that is based on analyzing contextual situations, especially so when contextual circumstances dictate degrees of reasonable accuracy. The CCSSM practice standard, attend to precision, has to be interpreted appropriately here because attending to precision is influenced by context.

Developing mathematical formulas and models when they are needed points to reasons why the work is worthwhile. As stated in *How People Learn* (2000: 139), "An alternative to simply progressing through a series of exercises that derive from a scope and sequence chart is to expose students to the major features of a subject domain as they arise naturally in problem situations. … Ideas are best introduced when students see a need or a reason for their use – this helps them see relevant uses of knowledge to make sense of what they have learned." In the QR course, an example of this just-in-time-as-needed development is summing of a geometric series when the length of the sum has exceeded calculator capability.

Much of the power in mathematics is in abstraction and generalization, and this is a motivation for the eight CCSSM practice standards. In fact, it is stressed in CCSSM 7, look for and make use of structure, and CCSSM 8, look for and express regularity in repeated reasoning. Abstraction and generalization trouble many students, especially those who are somewhat math-phobic. By seeing uses of and reasons for abstraction and generalization, their difficulties can be reduced. However, multiple uses of similar processes in different contexts give rise to the need for abstraction and generalization, which can organize information to facilitate retrieval.

5. Help students to structure their quantitative reasoning in resolving problematic situations, including ample doses of critical reading and writing. One way to help students structure their quantitative reasoning is to use the core competencies of interpretation, representation, calculation, analysis/synthesis, assumptions, and communication (AAC&U 2010; Boersma et al. 2011). If students understand that they need some or all of these six competencies to address a QR situation, then they can organize their responses accordingly and produce a full response. Curricular materials and questioning prompts should be composed in consideration of which competencies are needed for the proper responses. For example, if the student should communicate a response in writing, the prompt should so indicate. Requiring students to write responses promotes clearer thinking and deeper understanding, and writing requirements should progress from sentences to paragraphs to multi-page reports. Students in one of the sections of the QR course in Spring 2012 commented about combining writing and quantitative reasoning (called math by many students). One wrote, "… instead of just working a problem and moving on, I had to evaluate the process and determine how to explain the process in words." Another was more explicit, showing some negativism toward mathematics, "Math is virtually useless without proper communication of its meaning." College faculty who were participants at a 2012 Conference on Interdisciplinary Teaching and Learning at Michigan State University discussed why writing was an effective vehicle for assessing interdisciplinary learning. As one participant stated, "writing manifests thinking." Students need to get writing structure

down in order to progress intellectually and communicate that progress to others. Reflective writing can reveal how well students are integrating ideas from different sources or disciplines. One participant quoted from Richard Guindon's 1989 *San Francisco Chronicle* cartoon: "Writing is nature's way of showing you how sloppy your thinking is."

The QR course now has multiple (currently, four) significant writing assignments (200-500 words) with peer review of the first draft by 2-3 classmates. This moves students toward taking charge of their learning by not only having them judge their own writing but also judge each other's writing. Of course, writing prompts are aimed at having quantitative reasoning as a significant part of an appropriate response.

6. Encourage on-the-fly calculations and estimations. If students are able to quickly assess the validity of a quantitative assertion or mentally compute a numerical result, then they will be more able to practice QR in many aspects of their daily lives. This increased practice will strengthen their analysis and calculation, thereby building formidable QR skills. This practice is one of the places where one can develop automaticity of skills. Facility with mental arithmetic and estimation allows one to "function effectively without being overwhelmed by attentional requirements" (*How People Learn* 2000: 139). This skill is part of the *Adding It Up* strand of procedural fluency, i.e. ability to carry out procedures flexibly, accurately, efficiently, and appropriately. This practice is also part of CCSSM 6, attend to precision**.** Knowing the degree of accuracy needed to understand a quantitative situation allows for simplification that promotes mental calculations. Further, knowing the constraints that contexts place on precision not only allows simplification but also reflection on the contextual circumstances.

7. Increase students' supplies of quantitative benchmarks. Personal quantitative benchmarks are critical for understanding quantities and being able to determine reasonableness of quantitative assertions or numerical answers to questions. Having known benchmarks to measure results of reasoning can help learners know the quality of their comprehension. Comprehending quantities, especially very large or very small ones, can be aided by expressing them in personally understandable units. One's personally understandable units depend heavily on one's supply of personal quantitative benchmarks. Joel Best (2008: 7) points to the importance of statistical benchmarks in spotting dubious data. "Having a small store of factual knowledge prepares us to think critically about statistics. Just a little bit of knowledge – a few basic numbers and one important rule of thumb – offers a framework, enough basic information to let us begin to spot questionable figures." Best gives four benchmarks that go a long way in understanding US social statistics. These are the US population (approx. 300 million), the annual birth rate (approx. 4 million), the annual death rate (approx. 2.4 million), and the approximate fractions of the population of major ethnic or racial groups.

At the 2012 Quantitative Reasoning Symposium in Mathematics in Savannah, GA, Gail Jones (North Carolina State University) began a presentation by showing a highly magnified image of part of a familiar biological entity and began showing successive images with less magnification (Jones 2012). She asked audience members to take note of the point at which they were able to identify the entity. Namely, at what magnification was the entity understandable—i.e., when could you recognize what it was? (In my case, it was at either the penultimate image or the final image that I was able to see that the entity was a common ant.) Understanding the whole better than parts of the whole is an

inversion of the problem given students in a think-aloud session, namely, express $1.2 trillion in terms that make it understandable to you. One reasonable solution was to note that $1.2 trillion is enough to purchase every person in the states of Arkansas and Kentucky a house costing approximately $150,000 each. Note that in the ant visualization example, one understands by seeing the whole, or nearly whole, animal as opposed to small pieces magnified. In the $1.2-trillion example one understands by breaking the large entity into smaller pieces. Of course, experts on ants might recognize the ant at higher magnifications of its parts, and managers of large money accounts might not need to re-express the $1.2 trillion.

As students use quantitative benchmarks, their supply grows, as does their understanding of quantities. Broadening the possibilities of comprehending quantities is consistent with Burger and Starbird's understanding deeply, clearing the clutter of meaningless measurements.

8. Encourage students to use technology to enhance and expedite understanding. Technology, including personal devices, is omnipresent in the everyday lives of QR students, so it is leveraged in service of understanding. As examples, students are encouraged to use technology for calculations exceeding on-the-fly abilities, to graph functions on graphing calculators, and to use spreadsheets for repetitive calculations. In QR class sections, a statistic or another piece of information is often needed. Students use smart phones or sometimes rely on one designated student as "Googler of the Day." How personal technologies affect learning is not clear; research projects to determine answers will have difficulty keeping pace with the changing technologies. However, since these technologies are certain to be a part of students' future everyday lives, they are a part of the QR classes. As stated in the CCSSM 5, use appropriate tools strategically.

9. Allow student interests to emerge. As reported in *How People Learn* (2000: 77)**,** "Students are motivated to spend time needed to learn complex problems that they find interesting. Opportunities to use knowledge to create products and benefits for others are particularly motivating for students." The QR class addresses student interests by way of students finding media articles with quantitative content, bringing them to class and explaining them to the class or formulating questions (like the study questions in the *Casebook*) that they can or cannot answer. Students who are interested in baseball may bring a comparison of the statistics of Albert Pujols and Henry Aaron. Students who are interested in the military may bring a statistical analysis of military budgets of different countries. Increasing student interest encourages student-generated questioning, one of the four elements of effective thinking.

10. Provide interactive classroom environment. Inquiry-based learning is emphasized in the QR classes, and students often work in groups of 3-4. Social interaction is important as a motivation and as a vehicle for developing understanding. According to Halpern and Hakel 10, "What learners do determines what and how much is learned, how well it will be remembered, and the conditions under which it will be recalled" (p. 41). Inquiry-based learning and interactive classrooms are fundamental in the elements of effective thinking by Burger and Starbird. Understanding deeply, making mistakes, asking questions, and looking forward and backward are common components of interactive classrooms.

How Criteria Are Reflected in the QR Course

The discussion above indicates how the QR course design principles are supported by some of the criteria in the six sets. Below, each of the six sets is discussed as to how it is reflected in and influences the QR course.

Core Competencies and the QR Course

The QR core competencies – interpretation, representation, calculation, analysis/ synthesis, assumption, and communication – serve multiple purposes. They provide the basis for rubrics to assess student work; they offer ways to structure students' understandings; they are reminders of what we are seeking to develop in curricular materials and assessments. There are 268 study questions in the 30 case studies in the 3rd edition of the *Casebook*. Although most (1st and 2nd editions) of the *Casebook* was written before the QL core competencies were articulated, the changes for the 3rd edition focused on incorporating what was learned from adapting the AAC&U rubric to assess student work (Boersma et al. 2011). The competencies to assess with study questions were classified, and the rubric for scoring student work was incorporated in the introduction of the *Casebook*. The proportions of the 30 case studies and the 268 study questions that require each of the six competencies are shown in Table 2.

An example of a case study (Boersma et al. 2011, p. 7) where the study questions require all six competencies is an op-ed article that argues that forcing fuel efficiency on consumers does not work. The argument is based on the economics of buyers, namely making assertions that the $1466 extra for a more fuel-efficient pick-up truck is a bad investment. Study questions focus on testing the economic assertions made in the article. Interpretation, representation, calculation, analysis/synthesis, and communication are required for answers to several of the questions, and assumptions need to be made about the cost of gasoline and the number of miles driven annually.

Strands from Adding It Up *and the QR Course*

Although the five strands – conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive disposition - were part of the basis for the CCSSM standards for mathematical practice, the articulation of these five as above is more succinct and identifies what appears to be a critical proficiency for many of our students – productive disposition.

The core competencies in QLAR are manifestations of these and related proficiencies. In work with QR students, productive disposition seems to be critically important for practicing QR in contemporary society, and all six core competencies seem to depend on productive disposition. As reported in describing the experience in developing the QRCW course (Dingman and Madison 2010), the students are initially (on average) negative about their view of and experiences in mathematics, both in its utility to them and their abilities to use it. Improving this productive disposition is paramount in efforts to help the students toward stronger QR.

Interpretation in QLAR depends more on conceptual understanding; representation depends more on both conceptual understanding and strategic competence; calculation is strongly related to procedural fluency; analysis/synthesis depends on strategic competence and adaptive reasoning as does assumptions; and communication is closest to

adaptive reasoning. Reflection, explanation, and justification in adaptive reasoning play major roles in resolving contemporary QR situations.

CCSSM Practice Standards and the QR Course

Practice standards 1, 2, 3, and 4 are dominant in contemporary QR as addressed in the QR course. Making sense of problems; modeling with mathematics or statistics; reasoning quantitatively; and drawing, supporting and communicating conclusions are integral parts of QR. Critiquing the reasoning of others is often the entry point into a QR situations as they appear in public media articles. Practice standards 5–8 are less central to QR. There is attention to precision (CCSSM 6), but most attention focuses on the precision needed or possible in resolving the QR situation. Certainly the use of appropriate units is crucial in QR and somewhat unusual as noted above in QR course design principle 7. Tools (CCSSM 5) for our QR students include calculators (and sometimes, spreadsheets) and quantitative benchmarks for detecting reasonableness of answers. CCSSM standards 7 and 8 are less obvious in resolving QR situations.

5 Elements *and the QR Course*

Burger and Starbird's five elements are aimed at students (and others) taking control of their own learning, as in *How People Learn* #3. Although there are anecdotes from their classrooms that illustrate the five elements in action, the real message is to the learner-thinker.

Earth. Burger and Starbird (2012) get at teaching in depth of *How People Learn* #2 in several ways. While giving advice on how to understand deeply, they say, "Sweat the small stuff" (p. 25). They note that when studying some complex issue, instead of attacking it in its entirety, find one small element of it and solve that part completely.

Deep understanding at first blush seems like something that one cannot achieve in a one-semester QR course. In fact, as mathematics faculty tend to judge mathematics courses, they are likely to consider a QR course such as the one discussed here as not promoting or requiring deep understanding. They likely are judging on the depth of understanding of the mathematical concepts and not on the sophisticated and habitual use of rather elementary mathematical concepts to understand quantitative situations. Deep understanding of ratios, proportions, rates of change, and graphical representations are not the aim of most college mathematics courses, but they are among the aims of the QR course. Clearing the clutter in analyzing a quantitative argument in a media article and getting to the gist is a critical first step in understanding. This requirement of depth in understanding contextual situations is one of the major distinctions of a quality QR course.

Fire. Mistakes can be great teachers, but QR students initially are not inclined to venture opinions or propose solutions. In the QR class every mistake is a learning opportunity. This is a major issue in the student presentations of News-of-the-Day articles. Many students are reluctant to stand up in front of a class (and the teacher) and demonstrate their quantitative reasoning, which often contains errors. Reluctance can be defused by handling mistakes carefully and straightforwardly because everyone makes mistakes, and everyone can learn from them. One of the most common mistakes occurs in backing up a percentage change. Canonically, one knows the value of a quantity now and a percent change from some point in the past and wants to find the value at the point in the past.

About ¾ of the students entering the QR course answer this incorrectly, and these same mistakes persist throughout the semester. This canonical mistake in a News-of-the-Day presentation provides an opportunity to point out how common this is and urge remembering the correct way. By semester's end, about half the students still make this mistake.

Air. Raising questions by QR students is initially stymied by the same attitudes that keep them from venturing solutions or opinions. Their experiences in traditional mathematics and statistics courses point them toward responding to questions that have definite and often unique answers. The core material in the *Casebook* consists of study questions on media articles that serve as examples of questioning that they should employ in QR in everyday life beyond the course. Many of these questions do not have clearly defined answers, which can be frustrating to students not accustomed to such situations. However, the vague nature of some situations invites student questioning, and QR instructors model such questioning, especially in regard to News-of-the-Day articles being presented by students.

Water. News media articles invite looking backward at the origins of the information and forward to where it might lead. Further, the ideas developed in exploring and understanding one media article are often applicable to other articles. So the flow of ideas has two channels, one regarding a particular context of one article and one that takes the understanding of one article and utilizes it in understanding other articles, perhaps even in very different contexts. As an example, one of the QR case studies aims at understanding inflation by way of looking at the cost of a product (in this case, the Chuck Taylor All Star canvas shoe) that has remained essentially the same over the past half century. This is a very real situation as it is often the case that some student in a QR class may be wearing the All Star shoe. One has the chance to think backward to the 1950s and forward to see what the shoe might cost in 20 years. And the ideas here easily extend to more complex situations, say, considering arguments about how to measure federal revenues, spending, and deficits or surpluses.

How People Learn *and the QR Course*

The three principles, in brief, are: (1 and 1T) engaging preexisting understandings; (2 and 2T) factual knowledge, conceptual framework, and facilitating retrieval; and (3 and 3T) metacognition. How does the QR course respond to these principles?

1 and 1T. Some of the preconceptions that students bring to the QR course are molded by their experiences in previous mathematics classes (Dingman and Madison 2010). They are accustomed to courses with structured lectures, template problems, textbooks with numerous example exercises, and homework that utilizes the method of the day to solve problems that have one and only one solution. Because this is very different from the everyday QR challenges these students will face, the QR course and "textbook" are different. The absence of multiple template problems frustrates some students, illustrating that varying conditions of learning makes it more difficult for students but results in more learning. Students are also not accustomed to seeing mathematics, especially algebra, as a tool for understanding media articles, and this is the central purpose of the QR course. Students usually are not prepared to make the connections between the QR circumstances and their previous learning in arithmetic and algebra. They do not see the utility of their

arithmetic and algebra in resolving the QR issues, and so these connections are made within the QR class often serving to review the algebra, in particular.

2 and 2T. Presentation of an organized set of facts is not specified in the QR course. The knowledge that students are to apply consists of mathematics and statistics learned in school or early college. Beyond that, they need to understand or learn the basics of various contexts – political, social, economic, etc. – of the media articles in the case studies and articles brought to class by students. One of the weaknesses (noted above) of the QR course is in developing conceptual frameworks for QR, and the absence of conceptual frameworks takes away a powerful retrieval and transfer mechanism.

3 and 3T. The one-semester QR course functions like a prelude to continuing practice beyond the course. Having students take charge of their learning is a major goal. Much of what is done is aimed at that: creating a venue for continued practice, contexts from contemporary student life, increasing the supply of personal quantitative benchmarks, asking good questions, reflective writing on answers, and being able to judge ones comprehension.

Applying the Science of Learning to the University and Beyond *and the QR Course*

What can research on human learning tell us about how to best conduct classes in college (or in any adult education setting) to teach for long-term retention and transfer? About a dozen years ago 30 experts from different areas of the learning sciences met to answer this question. As reported by Halpern and Hakel (2003), these experts identified ten "basic laboratory-tested" principles drawn from what we know about human learning. They follow below, and after each principle, connections to the QR course are given.

1. **The single most important variable in promoting long-term retention and transfer is "practice at retrieval."** Practice at retrieval within the QR course can take place with questioning in class, collaborative learning situations where one student explains to another, and responding to assessment items or homework assignments. Spaced practice is better than massed practice, so spreading concepts such as relative change versus absolute change over an entire course, in different contexts, facilitates learning for long-term transfer.

2. **Varying the conditions under which learning takes place makes learning harder for learners but results in better learning.** The absence of template problems, as noted above, is the main adherence of our QR course to this principle. Each case study is different, but there are conceptual strands that run through multiple cases. Identifying and emphasizing these strands remains one of the challenges of the course.

3. **Learning is generally enhanced when learners are required to take information that is presented in one format and "re-represent" it in an alternate format.** As noted earlier, all of the case studies and 2/3 of the study questions require interpretation (i.e., gleaning and explaining information presented in various forms) and ¾ of the cases require representation, i.e., converting information from one mathematical form to another.

4. **What and how much is learned in any situation depends heavily on prior knowledge and experience.** This principle is basically the same issue as 1 and 1T above, so the discussion there applies. In addition, all of the QR students have demonstrated modest facility with algebra, and almost all have been successful in two English composition courses.

5. **Learning is influenced by both our students' and our own epistemologies.** One of our findings about the QR students is that they are weak on productive disposition, i.e., the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence in one's own efficacy. A principal aim of the QR course is to convince students that QR is important to them and that they are capable of making use of it in their daily lives.

6. **Experience alone is a poor teacher.** Too few examples can situate learning and deter transfer. Many learners do not know the quality of their comprehension and need systematic and corrective feedback. The use of a variety of authentic cases can point out to students the consequences of various conclusions in real-life situations. The feedback can convince students that their experiences are not conclusive and push them to consider other alternatives.

7. **Lectures work well for learning assessed with recognition tests, but work badly for understanding.** Learning via case studies as in the QR course does not rely on extensive lecturing, relying more on just-in-time mini-lectures to address a needed concept or method. Assessments are not recognition tests and the QR habits rely more on questioning, relationships, and elementary arithmetic than on algorithms and formulas.

8. **The act of remembering itself influences what learners will and will not remember in the future.** Asking learners to recall particular pieces of information (as on a test) that have been taught often leads to "selective forgetting" of related information that they were not asked to recall. As noted above, the QR course does not emphasize facts and processes that students need to remember. Identifying a few conceptual frameworks that have broad application would alleviate the possibility of promoting "selective forgetting."

9. **Less is more, especially when we think about long-term retention and transfer. Restricted content is better.** The mathematical and statistical methods in the QR course are quite restricted but broadly applicable. Mathematical formulas or concepts are developed only if there is an immediate reason, and most of those developed have broad applications to QR.

10. **What learners do determines what and how much is learned, how well it will be remembered, and the conditions under which it will be recalled.** We keep the admonition that the mind remembers what it does in front of all our instruction. Collaborative inquiry-based learning is a major theme of the course.

Final Thoughts

The QR course was not designed with the principles listed above explicitly stated. Nor was it designed in overt consideration of any of the six sets of criteria, except perhaps the

research results on human cognition, which were reasonably well known to the author as the course was initiated and refined by the author and colleagues S. Dingman, S. Boersma, and C. Diefenderfer over the past eight years. And looking at the result in light of the six sets of criteria has no doubt influenced forming the now-recognized ten design principles. The qualitative evidence that the design principles of the course align reasonably well with most of the principles in the six sets of criteria is a good starting point for a more rigorous evaluation of the course. The alignment is far from perfect. As noted earlier there are two unresolved alignment issues:

1. What contextual examples should be generalized and abstracted to take advantage of the power of mathematics?

2. What are the conceptual frameworks for QR?

The alignment with the QL core competencies is understandably strong since these are competencies for QR. The alignment with the five strands of mathematical proficiency is stronger than that with the practice standards of CCSSM, which are attuned more to traditional mathematics proficiency. Alignment with the *5 Elements* of Burger and Starbird (2012) seems reasonably strong, but the explication of these in their book by the authors points clearly to the personal pedagogies of the authors, so alignment here likely depends more on the implemented course. Alignment with the principles from *How People Learn* and those articulated by Halpern and Hakel (2003) is probably the strongest of all, and this might be surprising except for the fact noted above that I knew of these principles before I began designing and teaching the QR course. There are sprinkles of other evaluative evidence, some of it quantitative – surveys of faculty advisors, student evaluations, some pre- and post-test data, and some follow up survey data of former students. Most of the evidence appears to support the conclusion that the design of the course supports strong learning by QR students.

However there are uncertainties. One is the uncertainty of how well aligned the implemented course is with the designed course. With most of the instructors inexperienced in leading this kind of course, implementation can vary from design. The design has been reviewed rather thoroughly during the past year, and professional development programs for QR instructors are being formulated.

Until there are assessment instruments that are reliable measures of long-term retention and transfer or QR habits of mind, qualitative evidence of alignment with research-based principles that apply to QR learning will continue to be useful. These principles constitute a fairly high standard as indicated by the six sets of criteria here, and not the "well-intentioned feel-good psychobabble about teaching out there that falls apart upon investigation of the validity of the supporting evidence," as quoted from Halpern and Hakel (2003) in the introduction. Such alignment with accepted principles adds some concurrent validity to the face validity of the QR course at the University of Arkansas.

Acknowledgments

References

AAC&U. See Association of American Colleges and Universities.

Arkansas Democrat-Gazette. 2012. Down with algebra! Editorial, May 5, p. 16.

Association of American Colleges and Universities. 2004. Quantitative literacy. *Peer Review*: 6(4). List of contents available at https://www.aacu.org/peerreview/pr-su04/pr-su04contents.cfm (accessed 1 May 2014).

———— . 2009. Assessing learning outcomes: Lessons from AAC&U's VALUE project. *Peer Review*: 11(1). Available at https://www.aacu.org/peerreview/pr-wi09/pr-wi09_index.cfm (accessed 1 May 2014).

Best, J. 2008. *Stat-spotting: A field guide to identifying dubious data*. Berkeley and Los Angeles, CA: University of California Press.

Blair, R. (ed.). 2006. *Beyond crossroads: Implementing mathematics standards in the first two years of college*. Memphis, TN: American Mathematics Association of Two-Year Colleges. Available at
http://beyondcrossroads.matyc.org/doc/PDFs/BCAll.pdf (accessed 1 May 2014).

Boersma, S., C. L. Diefenderfer, S. W. Dingman, and B. L. Madison. 2011. Quantitative reasoning in the contemporary world, 3: Assessing student learning. *Numeracy* 4(2): Article 8. http://dx.doi.org/10.5038/1936-4660.4.2.8

Boersma. S. and D. Klyve. 2013. Measuring habits of mind: Toward a prompt-less instrument for assessing quantitative literacy. *Numeracy* 6(1): Article 6.
http://dx.doi.org/10.5038/1936-4660.6.1.6

Bransford, J. D., A. L. Brown, and R. R. Cocking. (eds.). 2000. *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academies Press. Available at http://www.nap.edu/openbook.php?isbn=0309070368 (accessed 1 May, 2014).

Brawner, S. 2012. The new math equation: no algebra. *Log Cabin Democrat*, 8 September. Available at http://arkansasnews.com/sections/columns/news/steve-brawner/new-math-equation-no-algebra.html (accessed 1 May 2014).

Burger, E., and M. Starbird. 2012. *The 5 elements of effective thinking*. Princeton, NJ: Princeton University Press.

CCSSM. 2010. Standards for mathematical practice. In Common Core State Standards for Mathematics 2010, 6-8.

Common Core State Standards for Mathematics. 2010. Washington, DC: National Governors Association and The Council of Chief State School Officers.
http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf (accessed 1 May, 2014).

Dingman, S. W., and B. L. Madison. 2010. Quantitative reasoning in the contemporary world, 1: The course and its challenges. *Numeracy* 3(2): Article 4.
http://dx.doi.org/10.5038/1936-4660.3.2.4

Gillman, R. (2006). *Current practices in quantitative literacy*. Washington, DC: Mathematical *Association* of America.

Grawe, N. D. and C. A. Rutz. 2009. Integration with writing programs: A strategy for quantitative reasoning program development. *Numeracy* 2(2): Article 2.
http://dx.doi.org/10.5038/1936-4660.2.2.2

Hacker, A. 2012. Is algebra necessary? *New York Times*, 29 July. Available at:

http://www.nytimes.com/2012/07/29/opinion/sunday/is-algebra-necessary.html?pagewanted=all&_r=0 (accessed 1 May 2014)

Halpern, D. F., and M. D. Hakel. (eds.) 2002. *Applying the science of learning to the university and beyond.* New York, NY: Jossey-Bass.

———. 2003. Applying the science of learning to the university and beyond. *Change: The Magazine of Higher Learning* (July/August): 36-41. http://dx.doi.org/10.1080/00091380309604109

How People Learn. 2000. See Bransford et al. 2000.

Jones, M. G. 2012. Conceptualizing size and scale. Expert Reading, An International STEM Research Symposium: Quantitative Reasoning & Science Education, Savannah, GA, May 31- June 2.

Kilpatrick, J., J. Swafford, and B. Findell. (eds.). 2001. *Adding it up: Helping children learn mathematics*. Mathematics Learning Study Committee, National Research Council. Washington DC: The National Academies Press. Available at http://www.nap.edu/catalog.php?record_id=9822 (accessed 14 May 2014).

MAA. See Mathematical Association of America.

Madison, B. L. 2001. Quantitative literacy: Everybody's orphan. *MAA FOCUS* 6: 10-11.

———. 2003. Articulation and quantitative literacy: A view from inside mathematics. In Madison and Steen 2003, 153-164.

———. 2012. If only math majors could write... *Numeracy* 5(1): Article 6. http://dx.doi.org/10.5038/1936-4660.5.1.6

———, and S. W. Dingman, S. W. 2010. Quantitative reasoning in the contemporary world, 2: Focus questions for the numeracy community. *Numeracy* 3(2): Article 5. http://dx.doi.org/10.5038/1936-4660.3.2.5

Madison, B. L., and L. A. Steen. (eds.). 2003. *Quantitative literacy: Why numeracy matters for schools and colleges.* Princeton, NJ: National Council on Education and the Disciplines. http://www.maa.org/sites/default/files/pdf/QL/WhyNumeracyMatters.pdf (accessed 13 May 2014)

——— . (eds.) 2008a. *Calculation vs. context: Quantitative literacy and its implications for teacher education*. Washington, DC: Mathematical Association of America. http://www.maa.org/sites/default/files/pdf/QL/cvc/CalcVsContext.pdf (accessed 13 May 2014)

———. 2008b. Evolution of numeracy and the National Numeracy Network. *Numeracy* 1(1): Article 2. http://dx.doi.org/10.5038/1936-4660.1.1.2

Madison, B. L., S. Boersma, C. L. Diefenderfer, and S. W. Dingman. 2012. *Case studies for quantitative reasoning: A casebook of media articles*, 3rd ed. New York, NY: Pearson.

Mathematical Association of America. 1994. *Quantitative reasoning for college graduates: A complement to the standards*. http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/quantitative-literacy/quantitative-reasoning-college-graduates (accessed 13 May 2014)

Mathematical Association of America. 2004. *Undergraduate programs and courses in the mathematical sciences: CUPM curriculum guide 2004*. Washington D.C.: author. Available at http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/cupm/cupm-guide-2004 (accessed 13 May

2014)

Mumford, D., and S. Garfunkel. 2011. How to fix our math education. *New York Times,* 24 Aug. Available at: http://www.nytimes.com/2011/08/25/opinion/how-to-fix-our-math-education.html (accessed 1 May 2014).

National Council of Teachers of Mathematics. 2000. *Principles and standards for school mathematics.* http://www.nctm.org/standards/content.aspx?id=16909 (accessed 1 May, 2014).

NCTM. See National Council of Teachers of Mathematics.

Quantitative Literacy & Reasoning Assessment . 2012. http://serc.carleton.edu/QLRA (accessed 1 May 2014)

Steen, L. A. (ed.). 1997. *Why numbers count: Quantitative literacy for tomorrow's America.* New York, NY: The College Board.

———. (ed.). 2001. *Mathematics and democracy.* Princeton, NJ: National Council on Education and the Disciplines.

Stein, B., A. Haynes, and M. Redding. 2007 Project CAT: Assessing critical thinking skills (CAT), In D. Deeds and B. Callen, Eds., *Proceedings of the National STEM Assessment Conference,* pp. 290-299, Washington, DC: National Science Foundation. http://www.tntech.edu/files/cat/reports/ProjectCat_NSF_NationalSTEMAssessmentConference.pdf (accessed 1 May, 2014).