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Reorganizing School Mathematics for Quantitative Literacy

Abstract

This paper offers an alternative curriculum for high school mathematics. It proposes replacing the Algebra-Geometry-Algebra rush to calculus model with one which focuses on improving student problem-solving skills and general quantitative literacy skills while reinforcing basic manipulative skills. Most of these goals are gained by expanding the current single-year algebra-one course into two years. The model proposes moving “learning to write proofs” from the traditional geometry course into a separate discrete mathematics course. It requires statistics for every student, and requires a senior-level modeling course for every college-going student. In addition, the proposed model creates opportunities for students to move at their own pace through the program by organizing courses in semester units rather than year-long units.

Keywords

curriculum, high school, mathematics, standards

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Cover Page Footnote

Rick Gillman is Professor and Chair of the Department of Mathematics and Computer Science and formerly Assistant Dean for Research and Teaching Support of the College of Arts and Sciences at Valparaiso University. His teaching has included courses in graph theory, finite mathematics, and a graduate seminar in game theory for sociologists, as well the gamut of the precalculus-calculus-analysis train. He has served as Chair of the SIGMAA-QL of the Mathematics Association of America (2003). His most recent book is *Models of Conflict and Cooperation* (with David Housman), published by the American Mathematical Society (2009).

Introduction

How and where does school mathematics teach quantitative literacy (QL)? This question was the topic of an extended conversation on the SIGMAA-QL list-serve. The assumption in the conversation was that school mathematics is still being taught using a model that attempts to strongly emphasize mathematical techniques, in particular techniques necessary to succeed in science, technology, engineering, or mathematics (STEM) disciplines in college, rather than more generally useful QL skills.

In another independent, but concurrent, conversation I am aware of, a conference panel found consensus with the statement that “pushing kids to take algebra in 7th grade, pushing them to accelerate through the curriculum, and teaching them nothing is not helpful when they appear on college campuses.” Motivated by those conversations, I suggest an alternative curriculum.

Background

The SIGMAA-QL is one of several special interest groups (SIGs) within the Mathematical Association of America (MAA). The SIGMAA-QL has over 200 members. As suggested by its name, this group of mathematicians is interested in promoting and enhancing the quantitative abilities of college graduates—specifically, the ability and habit of mind to use elementary mathematical tools to model and resolve problems that they face in their daily lives (Steen 2001; Gillman 2006) Most of the SIGMAA-QL’s work is focused on establishing standards for QL, identifying entering college students’ level of QL, and promoting appropriate general education courses and programs to enhance QL. While the work of the members is focused on college-level issues, it is natural that they would have a conversation about school mathematics.

Both conversations demonstrate concerns that secondary students may be frequently pushed too fast into, or through, material that they may not need and may not be prepared to take. Taken together, the two conversations reflect a view that preparing students better for life and for future mathematical study implies more than simply pushing the traditional Algebra-Geometry-Algebra-Precalculus-Calculus curriculum earlier into the students’ program of study.

Table 1 illustrates how this traditional curriculum is usually implemented in the State of Indiana, my home state. It is my understanding that this is a reasonable model of the implementation of the curriculum across most of the country. More importantly, as the country and individual states have established mathematics standards, testing regimes, and “reformed” curricula, they have routinely maintained this model consisting of five year-long courses with these

names. Thus, the Algebra 1 course may in fact contain elements of statistics, geometry, and other topics without identifying them in the course title.

Table 1
Indiana's Curriculum

Grade	Goal	Traditional
		Often
8	Algebra 1	Pre-Algebra
9	Geometry 1	Algebra 1
10	Algebra 2	Geometry 1
11	Precalculus with trig	Algebra 2
12	AP Calculus	Pre-calc without trig (maybe) General statistics (maybe)

In Table 1, the Traditional Goal column represents the ideal path of a student preparing for a STEM major in college. Unfortunately, although many students complete this path, most are not continuing on to STEM disciplines in college. For students who cannot keep up with the pace of the Traditional Goal path, the Traditional Often column represents a typical high school experience

Purpose and Scope

This paper will attempt to sketch an alternative model curriculum for secondary mathematics that I believe balances the general need for QL with a need to prepare students for a STEM major in college. It is necessarily a sketch because, as a single author writing a short paper, I cannot address the location of each learning objective, the pacing of the individual courses, and the general availability of resources for them. I do plan to demonstrate that it would be possible to do all of those things quickly if a school district, state education board, or community of educators decided to do so.

The model proposed here is consistent with multiple sets of learning standards, both at the national level (e.g., NCTM 2000; National Governor's Association 2009; National Mathematics Advisory Panel 2008) and at the individual state level (e.g., Indiana State Department of Education 2000). Any re-organization of the present curriculum must be consistent with the expectations of these standards. It must also be responsive to the natural resistance to change that is inherent in state testing structures, parent expectations, and teacher skill sets. Within those constraints, I propose a curriculum model that:

- Responds to the fundamental need of algebraic preparation for all students.
- Provides sufficient time for instructors to focus on using algebra for problem solving.
- Provides all students the quantitative skills necessary for their citizenship roles and career aspirations.
- Provides a large number of students the fundamental mathematical skills required to succeed in college, both generally and in STEM disciplines.

In addition to these fairly obvious goals, the proposed model also provides a distinguished secondary mathematics education that will be valued on its own merits and is responsive to the varying learning needs of students.

The model described in the following section is largely a re-organization of current topics in the secondary curriculum. It organizes the curriculum in such a way that it puts the highest priorities—algebra skills and problem solving—first and revisits those priorities multiple times. Because this model is primarily a re-organization of the existing curriculum, it will meet the various standards described above and may lower the resistance to shifting to it.

Proposed Standard Sequence

In this section, I describe a different five-year (grades 8–12) plan of study. Table 2 summarizes what I see as the “standard” sequencing of the material. I will describe typical deviations from the standard sequence in the next section.

Table 2
Proposed Curriculum

Grade	Semester	Standard Sequence
8	1 st	Algebra, part 1
	2 nd	Algebra, part 2
9	1 st	Algebra, part 3
	2 nd	Algebra, part 4
10	1 st	Discrete Mathematics
	2 nd	Fundamentals of Geometry
11	1 st	AP Statistics
	2 nd	Analytic Geometry
12	1 st	Functions with trigonometry
	2 nd	Mathematical Modeling

The material in grades 8–11 should be required of all students for graduation. The material in grade 12 is additional college preparatory work. My assumption is that most students will complete all five years of the curriculum.

The modeling aspects of QL can be comfortably introduced in the initial two-year algebra course, revisited multiple times in the next two years (most obviously in the AP statistics course), and enhanced in the capstone mathematical modeling course.

Grade 8

The study of algebra begins in eighth grade and extends through ninth grade. Using current textbooks that are intended for a single year, this two-year model for algebra provides room in the curriculum to begin to focus on QL through extensive discussion of mathematical problem solving and algebra as a modeling tool, while still helping students to master the fundamental algebraic manipulative skills. The sequencing given here is based on Holliday et. al. (2004), but it is also very similar to that of Larson et. al. (2004a), which is also approved by the State of Indiana.

By extending the study of algebra through the second year, teachers will have sufficient time to provide instruction and to assess student mastery of each skill. There will be plenty of time for students to engage in both manipulative drill and problem-solving practice; the teacher will not be forced to choose between them. Sufficient time is available for digression into significant real-world modeling projects that involve data collection, data manipulation, simulation, and interpretation, while still preparing students for a standardized graduation exam, such as Indiana's End of Course Assessment in Algebra (Indiana State Department of Education 2009).

Algebra, part 1. The first course in the algebra sequence begins a discussion of the language of algebra. Variables, expressions, and order of operations are reviewed. The key properties—distributive, associative, and commutative—are introduced in an algebraic setting. The course continues with a review of the arithmetic of rational numbers.

The majority of this first course is spent developing and solving systems of equations, including ratio and proportion problems. It is at this point that problem solving and mathematical modeling are first emphasized. Students begin learning to implement the four-step modeling process: understanding the real-world problem; conversion to a mathematical model; application of solution techniques; and interpretation in the real-world context (Polya 1945; Giordano and Weir 1985; Holiday et. al. 2004; Larson et. al. 2004a; Gillman and Housman 2009). They learn that writing the statement “Let x represent the number of ...” is an early crucial step in the mathematical modeling process. It is at this point that students should aggressively be taught that they should not be submitting answers to problems, but rather written solutions that completely explain the solution process.

A simple illustration shows the connection between the mathematics, the writing, and the understanding. Problem: An American alligator hatchling is about 8 inches long and grows approximately 12 inches per year. How old would an alligator that is 10 feet 4 inches be? Solution: Let x denote the age of the alligator. Then we have $8 + 12x = 124$ because 10 feet 4 inches is 124 inches. Solving, $x = 9.66$. This means that the alligator is about 9 years and 8 months old.

Algebra, part 2. The second semester continues to focus on linear relationships as it explores the properties of linear functions, the various ways to construct and represent them (point-slope, slope-intercept, and standard forms). It extends the work with linear relationships in two directions: it introduces linear inequalities, and it introduces the solution of (two-variable) systems of equations and inequalities.

In both this and the first course, there will be time to explore connections between the geometric and algebraic properties of linear relationships. Students should have time, for example, to discover the relationship between the line $y = x$ and the line $y = x + 1$.

One sophisticated, but approachable, application of these topics is in simple linear programming models. These types of multi-step problems provide students the opportunity to review the formulating of simple mathematical models, while they also practice the associated manipulative skills. Over the years, hundreds, if not thousands, of these problems have been published in Finite Mathematics textbooks. For example, in Goldstein et. al. (2007), the following problem is an example: A manufacturer makes chairs and sofas. The production process uses three operations: carpentry, finishing, and upholstery. Manufacturing a chair requires 6 hours of carpentry, 1 hour of finishing, and 2 hours of upholstery. Manufacturing a sofa requires 3 hours of carpentry, 1 hour of finishing, and 6 hours of upholstery. For a variety of reasons, there are 96 labor hours available for carpentry, 18 labor hours for finishing, and 72 labor hours for upholstery. The profit per chair is \$80 and the profit per sofa is \$70. How many chairs and how many sofas should be manufactured to maximize the profit?

Grade 9

The second year continues the algebra studies begun in the previous year, using the same textbook. This continuity will also assist students in the transition between schools/learning environments that usually occurs at this point in their lives.

Algebra, part 3. The third course in algebra focuses on polynomial expressions and equations. The distributive, associative, and commutative properties are reviewed as the critical tools needed to manipulate these more general expressions. Factoring skills are introduced as the key method for solving

polynomial equations. Quadratic equations and functions can be studied here as both a special case of polynomial equations and as a link to the geometric thinking in other courses.

Algebra, part 4. The final course in the algebra sequence extends students' applications of algebraic manipulations to expressions involving radicals and rational expressions. Problem-solving applications of the former might center on various uses of the Pythagorean Theorem. Again, a simple illustration demonstrates this possibility. Margy has a large corner lot and wants to keep a triangular flower bed in the corner. She decides to run the bed 6 foot along one side of the corner and 8 foot along the adjoining side. How much fencing will she need to encompass the bed? (Sons et. al. 1994)

Grade 10

My most radical change in the curriculum is in Grade 10. The traditional geometry course is reduced to a single semester because a new one-semester course, Discrete Mathematics, will introduce students to deductive reasoning, proofs, and formal logic. Discrete Mathematics also provides another natural opportunity to connect mathematical problem solving and writing skills as students write solutions to these types of problems.

Discrete Mathematics. Students begin to develop formal reasoning skills in this curriculum through the study of discrete mathematics topics. I do not know of a high school-level book organized for such a course, but an excellent model at the college level is *Discrete Mathematics with Applications* (Epp 2004). In the first two chapters of this book, students study topics such as logical form and equivalence, conditional statements, valid and invalid arguments, quantifiers and quantified statements. This study is done both in a verbal context and in a quantitative context. The third chapter uses topics from elementary number theory to introduce various methods of proof.

As suggested by Wanko (2009), students can solve puzzles and games to hone their skills at mathematical reasoning and mathematical manipulation. For example, to play and win the game POISON, students need to be familiar with divisibility by three. Similarly, to win at the game NIM, students need to understand the binary number system and be nimble in binary computations and conversions.

The use of models such as those found in the Tarski's World program (Barker-Plummer et. al. 2009) can introduce students to simple axiomatic systems and the development of proofs in this system.

It would be reasonable to include a short unit on financial mathematics—in particular a unit on the computation of compound interest—in this course. This is

the age that many students are first entering the work community and are beginning to make independent financial decisions. This topic also allows the class to explore finite arithmetic and geometric sums, and possibly to introduce the idea of mathematical induction.

Fundamentals of Geometry. By shifting the primary instruction of deductive reasoning to the Discrete Mathematics course, the basic geometry course can be reduced to a single semester. Again referencing approved Indiana geometry textbooks (Larson et. al. 2004b; Boyd et. al. 2004), the content topics of the course are: lines and angles, triangles and relationships among them, circles, and notions of area and volume.

Connections can be made to the first algebra course by considering transformations of lines and the geometric interpretation of proportion.

Grade 11

The fourth year again consists of two one-semester courses which could be taken in either order, or even concurrently. We will see later that this flexibility adds strength to the proposed model.

Introduction to Statistics. Consistent with various standards documents, all students should be introduced to the fundamental ideas of probability and statistics. The optimal course for this experience is the AP Statistics course (College Board 2009a), which covers all of the essential topics with an appropriate focus on data analysis and manipulation.

I believe that the AP statistics course is well known, and so I will not include a description of the course. There are two issues, however, that I believe should be made clear about the role of this course in this proposal.

1. A statistics course should be a requirement for ALL students. Choosing to take a statistics course is not simply a “bailout” option for students unprepared for pre-calculus topics.
2. The AP Statistics course should be the default choice for a statistics course for ALL students. The material is not heavily algebraically driven, so many students will have an opportunity to find success in a course that could reward them with college credit.

Analytic Geometry. Building on and reviewing the material from Algebra, the Analytic Geometry course emphasizes the quadratic function and other conic functions. It also introduces matrix arithmetic, real and complex numbers, and sequences and series. Again, this material is contained in standard textbooks such as Larson et. al. (2004c). It should also be noted that there are many opportunities in a course such as this for students to practice formal reasoning and proof-writing skills as they verify identities and make conjectures about relationships among

mathematical objects. Students might be asked, for example, to determine whether matrix multiplication has the associative and commutative properties.

Grade 12

As mentioned earlier, the two 12th-year courses transition students into college-level mathematics as well as into college-level study in many non-STEM disciplines. The two courses also lay a strong foundation for study in STEM disciplines. The first is clearly the pre-calculus course offered at most colleges and universities around the country. The second course, if constructed properly, might satisfy the quantitative analysis requirement of non-STEM majors at many of these same institutions.

Functions and Trigonometry. The pre-calculus course introduces a wide range of functions—polynomial, rational, exponential, logarithmic, and trigonometric—and their properties. This course will provide students another opportunity to make conjectures and to prove results. For example, students in this course should be able to explore examples, develop a conjecture, and eventually prove that $f(x+a)$ is a translation to the left of $f(x)$ for any function. This course could be taught using the same text as the Analytic Geometry course (e.g., Larson et. al., 2004c).

Mathematical Modeling. The capstone course of the curriculum will reinforce the modeling, reasoning, and technical skills introduced throughout the curriculum in a mathematical modeling context. The course will again emphasize the four-step modeling process (understanding the real-world problem; conversion to a mathematical model; application of solution techniques; and interpretation in the real-world context) that will have been introduced in the Algebra sequence.

There are many different ways that this course could be implemented based on the needs of a particular school. The course could be a project-based course with students working on individually identified problems. In this version, as students identify a problem and attempt to model it, they will have the opportunity to see both the power and the limitations of mathematical modeling. Alternatively, the course might be centered on the many modeling problems described in articles published in the *Mathematics Teacher* (e.g., Pendleton, 2009; Miller Mariner and Miller 2009). In this version, there would again be no textbook, but the risks of a project failing would be minimized because the instructor, at least, would have some sense of how it should be completed. Finally, the course could be a theme-based course using a single text. For example it might be a course on game theory using either Gillman and Housman (2009) or Straffin (1993) in which students investigate the use of quantitative approaches for dealing with social conflict and conflict resolution.

Alternative Sequences

What about AP Calculus (College Board, 2009b)? I can almost hear the responses that this model does not provide students an opportunity to take a calculus course while still in high school. But it does and, I believe, it provides this opportunity in a more rational way that reduces the potential of students being pushed along too fast, while not restricting access to students who mature a bit later. Table 3 shows two ways that the proposed curriculum provides the opportunity for students to take a calculus course in high school.

Table 3.
Alternative Sequences

Grade	Semester	Alternative 1	Alternative 2
8	1 st	Algebra, parts 1 and 2	Algebra, part 1
	2 nd	Algebra, parts 3 and 4	Algebra, part 2
9	1 st	Discrete Math	Algebra, part 3
	2 nd	Geometry	Algebra, part 4
10	1 st	AP Statistics	Discrete Math and Geometry
	2 nd	Analytic Geometry	Analytic Geometry and AP Statistics
11	1 st	Functions with/Trigonometry	Functions with Trigonometry
	2 nd	Mathematical Modeling	Mathematical Modeling
12	1 st	AP Calculus, part 1	AP Calculus, part 1
	2 nd	AP Calculus, part 2	AP Calculus, part 2

Alternative 1 (Table 3) allows students to complete the Algebra sequence in a single year (8th grade) in order to reach a calculus course in grade 12. Clearly, with significant variations across school districts, some percentage of students can be identified as gifted entering the eighth grade. These students can be given the opportunity to complete the two-year Algebra course in a single year and move more rapidly through the program. The expectations of student performance for this type of eighth grade course can be kept high, since a student who cannot keep up will be able to finish the second year of Algebra with other students.

The advantage of the semester structure of the curriculum becomes apparent at this point. If students in the two-year algebra course demonstrate high ability and motivation in mathematics, the independent semester organization allows the students to easily accelerate their study by taking extra courses over the summer or by doubling up courses during the academic year. One example of this is shown as Alternative 2 (Table 3).

The semester structure also responds in several key ways to concerns for students who struggle through the mathematics curriculum. Some students may be identified to be at risk by the end of the first year of their study of algebra.

Alternative 3 (Table 4) would offer these students the second year of algebra in a course intentionally designed to remediate issues from the first year while continuing to pursue the essential ideas of the second year of the course. These students would then be on track to continue with the semester courses in the subsequent years.

Table 4
More Alternative Sequences

Grade	Semester	Alternative 3	Alternative 4
8	1 st	Algebra, part 1	Algebra, part 1
	2 nd	Algebra, part 2	Algebra, part 2
9	1 st	Algebra, part 3 (modified)	Algebra, part 3
	2 nd	Algebra, part 4 (modified)	Algebra, part 4
10	1 st	Discrete Math	Discrete Math (failed)
	2 nd	Geometry	Geometry and Discrete Math (again)
11	1 st	AP Statistics	AP Statistics
	2 nd	Analytic Geometry	Analytic Geometry
12	1 st	Functions with Trigonometry	Functions with Trigonometry
	2 nd	Mathematical Modeling	Mathematical Modeling

Other students may fail one of the single semester courses in grades 10 or 11. Again, because of the independent, single semester structure of the curriculum, a second attempt at successfully completing the course is possible by doubling up courses the following semester, taking a course over the summer, or by extending the minimal four-year plan into the fifth year. Alternative 4 (Table 4) illustrates an example.

Discussion

There is a consensus that understanding and being able to utilize algebra skills is essential to success in modern life. Unfortunately, in the current one-year model for teaching algebra, an artificial conflict is created between the need of students to master the technical skills of algebra and the need of the same students to internalize the ability to utilize algebra as a modeling system. By distributing the algebra-1 curriculum over two years, I believe that both of these goals can be achieved by providing opportunities for instruction that focuses student learning on the fundamental skills of algebra while also grounding students in the problem-solving context necessary for QL.

The proposed re-organization shifts the focus of the curriculum to QL and away from calculus preparation. By doing this, the curriculum can serve the needs of a much larger percentage of its students (i.e., those not intending to enter STEM disciplines) and add value back into a high school education. In particular, it changes the answer to the question “Why are we studying this?” from “So you can get to college-level mathematics,” to “Because it will help you solve

problems now and in the future.” With this curriculum, students can know that they’ve graduated from high school with valuable knowledge and skills.

Although the focus of the proposed curriculum model is on QL, it develops these abilities in students in ways that will not prohibit them from pursuing STEM careers, or even from taking calculus in high school. In fact, because the curriculum uses a semester-based system, it makes the opportunity to study calculus available to a larger group of students—not just to the early bloomers, but also to students who discover their talents and interests a year or two into high school. In addition, it addresses many of the concerns raised by Bressoud (2009, 2010) about teaching calculus in high school (e.g., accessibility, appropriateness, dual credit, quality control).

Further, the intentional early focus on problem solving, analytical reasoning, and statistics allows these tools and skills to be utilized in other courses in the curriculum. Junior- and senior-level science courses, for example, can build on the fact that students will have stronger interpretive skills and basic statistical skills. Students will be able to understand, process, and interpret the data that they collect in science labs in more meaningful ways.

Similarly, junior- and senior-level social science courses will be able to explore contemporary issues using careful arguments, real data, and critical reading of popular journalism. Improved algebra skills in modeling and interpretation, for example, will allow students in economics courses to use quantitative methods to study cost and revenue functions and the associated marginal cost and revenues at the micro-economic level and to study the role of slope in the consumption function at the macro-economic level.

Finally, we need to ask if the model curriculum proposed here can significantly improve student performance. The model curriculum is consistent with the goals of the NCTM and Indiana standards, and it addresses the concerns of those who compare the performance of U.S. students to their international peers. The proposed curriculum attends to both the technical skills that students need to acquire to succeed in STEM disciplines in college and the habits of mind that they need to develop to be quantitatively literate citizens.

I also believe that by clearly identifying skills-based problem solving as the central theme of the curriculum, we can encourage students to continue to pursue quantitative fields of study after graduation from high school.

Conclusion

As noted in the introduction, this curriculum proposal is offered in the context of existing standards and assessment regimens. As such it is a modest proposal; implementing it does not require any changes in the current standards or assessment practices. Implementation does require that a school district and its

teachers think outside of the traditional curriculum box to design two new courses (Discrete Mathematics and Mathematical Modeling) and re-package existing courses. I believe that there are sufficient resources available to quickly develop the courses. Thus, I believe that the proposed curriculum is easily within the reach of ambitious and innovative school districts.

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