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## Arsenic and Growth of *Amphistegina gibbosa*

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## Arsenic and Growth of *Amphistegina gibbosa*

### Abstract

A laboratory tested various concentrations of arsenic on the growth of foraminifera and recorded their findings. Upon examination, the plotted probability density function for each of these trials resembled a similar shape. The plots were then characterized in a general model composed of linear segments. Using calculus, statistics such as the expected value, variance and standard deviation were calculated to interpret the collected data. The statistics revealed that arsenic limits the growth of ocean life.

### Keywords

Arsenic, Amphistengina Gibbosa, Foraminifera

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## PROBLEM STATEMENT

The purpose of this project is to explore the relationship between the growth of foraminifers and the concentration of arsenic found their surrounding environment.

## MOTIVATION

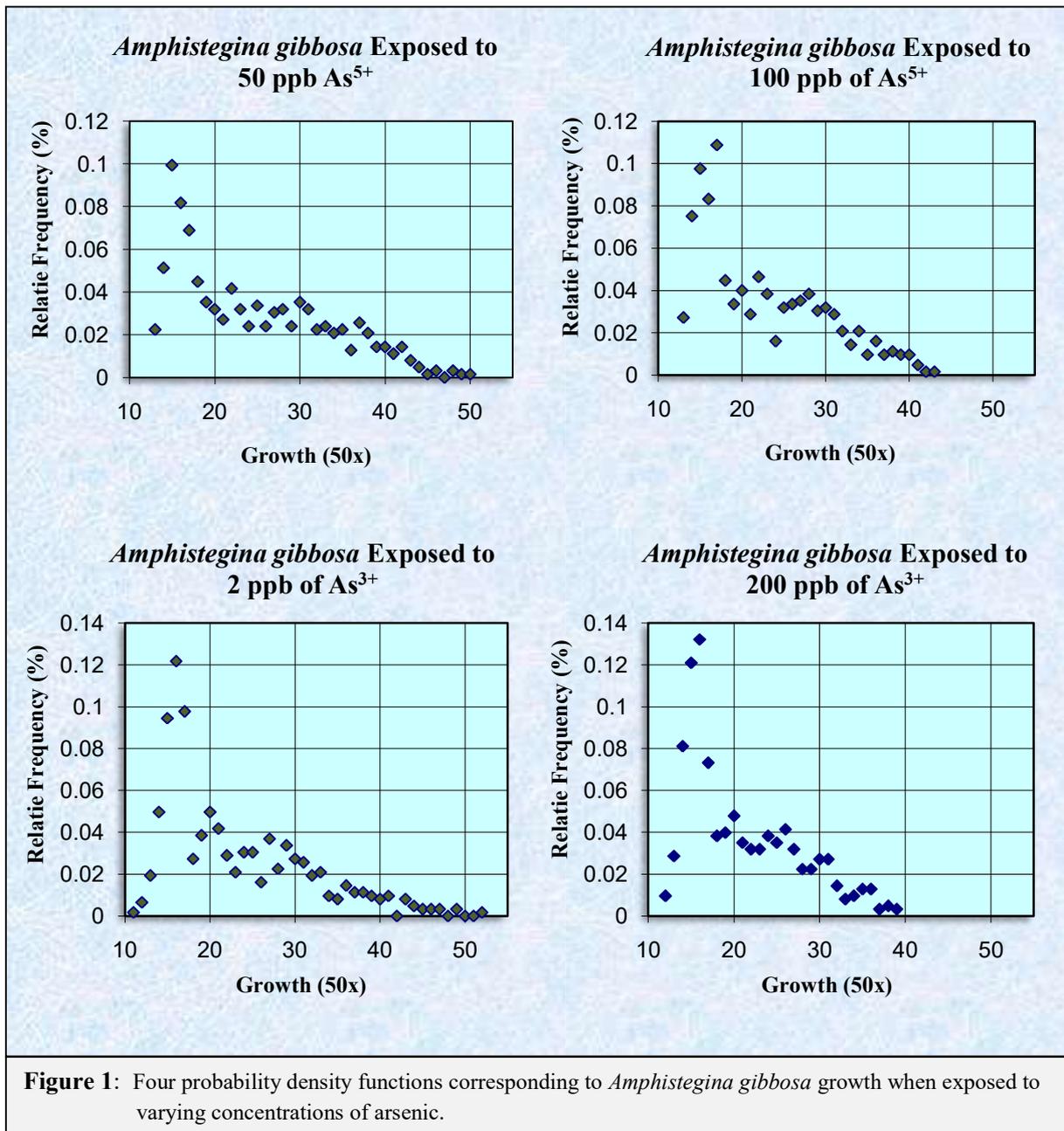
For the past few decades, foraminifera in the world's oceans have been used as a bio-indicator measuring the declining health of coral reefs. In particular, the growth of the common Caribbean reef foraminifer *Amphistegina gibbosa* d'Orbigny is ideally suited for such measurements (Hallock et al, 2003). Foraminifers are exposed to arsenic (As) both naturally and through manmade pollutants, making this heavy metal preferable in experimentation (McCloskey, 2009).

The data utilized by this project was taken from a study that looked at the impact of arsenic on the growth of foraminifera. Unlike previous experiments conducted in the field, this study was conducted in a laboratory allowing the researchers to control the environment and vary the only the arsenic concentration levels (McCloskey, 2009). Specifically, *Amphistegina gibbosa* d'Orbigny was exposed to  $As^{5+}$  and  $As^{3+}$  at concentrations ranging from 2 to 1,000 micrograms/kilogram (McCloskey, 2009) to determine the most harmful level.

Determining the level of arsenic which begins to threaten the life of coral is necessary to protect the ocean's natural wildlife. If dangerously high levels of arsenic are recorded in an area, the public can be alerted and further steps taken to decrease the anthropological and environmental input.

## MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

From the eleven trials conducted, four were selected for analysis. The plotted charts in Figure 1 represent the *Amphistegina gibbosa* growth size probability for each of the four trials. Note that the probability densities for each of the charts have a similar shape which can be



approximated as a composition of linear functions as in Figure 2.

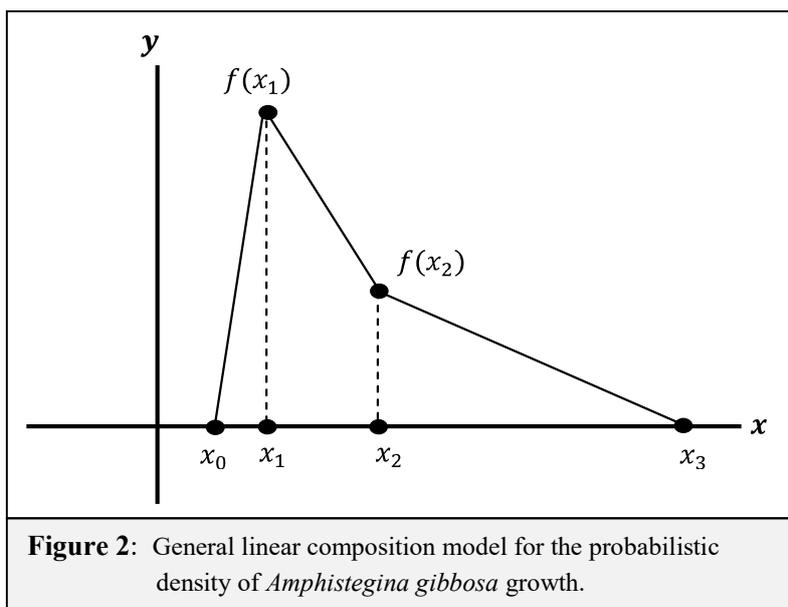
By first considering the probability density when the foraminifers were exposed to 200 parts per billion (ppb) of arsenic, the following estimate was linearly modeled after the shape in Figure 2:

$$y = f(x) = \begin{cases} 0, & x < 12, \\ 0.0331(x - 12), & 12 \leq x < 16, \\ 0.1322 - 0.0470(x - 16), & 16 \leq x < 19, \\ 0.0382 - 0.0017(x - 19), & 19 \leq x < 39, \\ 0, & x \geq 39. \end{cases}$$

This model was achieved by first associating the values of  $x_0 = 12$ ,  $x_1 = 16$ ,  $x_2 = 19$ , and  $x_3 = 39$  according to the corresponding features of the measured probabilities. Next, the equation for each segment is calculated algebraically using the two endpoints in the following manner. If  $(x_a, h_a)$  and  $(x_b, h_b)$  are two points then

$$y = \left( \frac{h_b - h_a}{x_b - x_a} \right) (x - x_a) + h_a$$

describes the equation of the segment between them. The equation for Figure 2 can be expressed



more generally as:

$$y = f(x) = \begin{cases} 0, & x < x_0, \\ a(x - x_0), & x_0 \leq x < x_1, \\ h_1 + b(x - x_1), & x_1 \leq x < x_2, \\ h_2 + c(x - x_2), & x_2 \leq x < x_3, \\ 0, & x_3 \leq x. \end{cases}$$

where  $a = \frac{h_1}{x_1 - x_0}$ ,  $b = \frac{h_2 - h_1}{x_2 - x_1}$ , and  $c = \frac{h_2}{x_2 - x_3}$  such that  $h_1 = f(x_1)$ ,  $h_2 = f(x_2)$ , and  $h_3 = f(x_3)$ .

For this model to be a well-defined continuous distribution, the sum of the probabilities must total to one. To ensure this is so we scale the function by a constant  $d$  and equate to one, i.e.

solve  $1 = \int_{-\infty}^{\infty} d f(t) dt$  for  $d$ . For the general model,

$$d = \frac{2}{h_1(x_2 - x_0) + h_2(x_3 - x_1)}.$$

The cumulative distribution function,  $F(x) = \int_{-\infty}^x d f(t) dt$ , for the linear composition approximation is given by

$$F(x) = \begin{cases} 0, & x < x_0, \\ \frac{d}{2} a(x - x_0)^2, & x_0 \leq x < x_1, \\ \frac{d}{2} (b(x - x_1)^2 + 2h_1(x - x_1) + h_1(x_1 - x_0)), & x_1 \leq x < x_2, \\ \frac{d}{2} (c(x - x_2)^2 + h_2(2x - x_2 - x_1) + h_1(x_2 - x_0)), & x_2 \leq x < x_3, \\ 1, & x_3 \leq x. \end{cases}$$

and the trial for *Amphistegina gibbosa* exposed to a 200 ppb concentration of  $\text{As}^{3+}$  may be modeled as

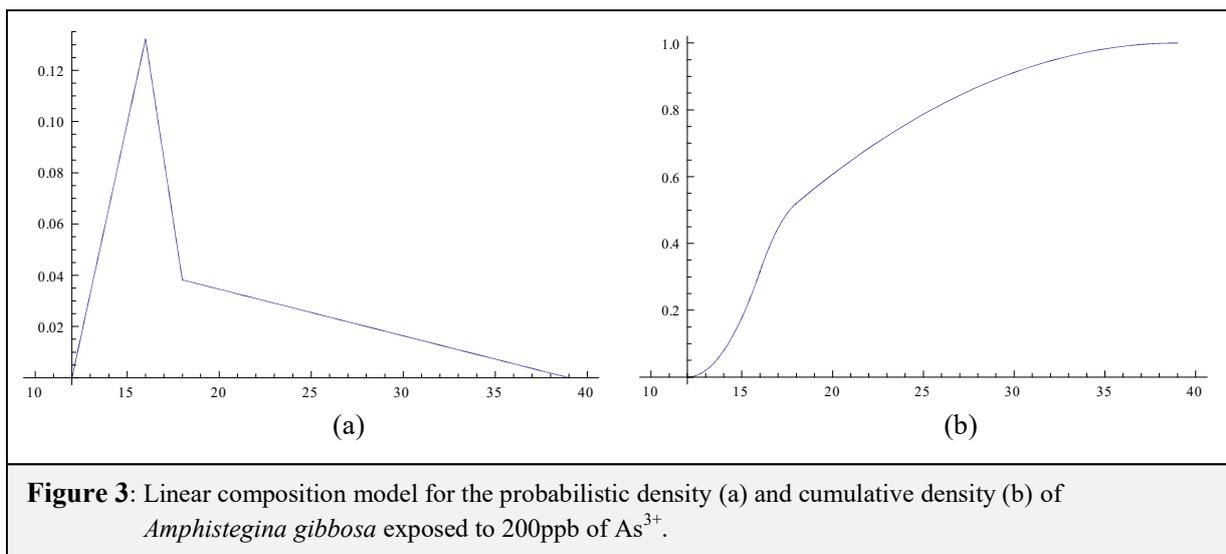
$$F(x) = \begin{cases} 0, & x < 12, \\ 2.8468 - 0.4745x + 0.0198x^2, & 12 \leq x < 16, \\ -9.4112 + 1.0578x - 0.0281x^2, & 16 \leq x < 18, \\ -0.6550 + 0.0849x - 0.0011x^2, & 18 \leq x < 39, \\ 1, & 39 \leq x. \end{cases}$$

To see  $F(x)$  graphically, see Figure 3(b). From this, the probability that a tested *Amphistegina gibbosa* falls between a predefined growth range may be computed directly. For instance if a sample is randomly selected from this trial, the probability that the growth rate falls between 15 and 21 is computed as follows:

$$\Pr(15 \leq x \leq 21) = F(21) - F(15) = 0.6475 - 0.1779 = 0.4695.$$

Therefore, 47% of all the samples in the *Amphistegina gibbosa* trial test with 200ppb  $\text{As}^{3+}$  recorded growth rates between 15 and 21. This is a fairly large portion of all the samples and Figure 3 further supports this conclusion.

The expected value is a form of averaging and a useful descriptive statistic. The expected value of a continuous random variable is defined as  $E[X] = \int_{-\infty}^{\infty} x f(x) dx$ . For the 200ppb trial, the expected growth rate is 20.0632. Other statistics may be expressed in terms of the expected



**Figure 3:** Linear composition model for the probabilistic density (a) and cumulative density (b) of *Amphistegina gibbosa* exposed to 200ppb of  $\text{As}^{3+}$ .

value such as variance and standard deviation. While the expected value will determine the statistical mean of the probabilistic density function, the variance will determine how far the data varies from the mean on average. Mathematically, variance is defined as

$$\text{Var}(X) = E[X^2] - E[X]^2.$$

For our example, the variance of the linear composition model for the 200 ppb of  $\text{As}^{3+}$  trial is 35.1540 and the standard deviation is  $\sqrt{\text{Var}(X)} = 5.9291$ . These statistics allow for a clear insight to the characteristics of the arsenic trials. Most of the *Amphistegina gibbosa* had a growth rate of  $20.0632 \pm 5.9291$  after being exposed to an arsenic concentration of 200 ppb. Similarly, the other trials may be characterized by their linear composition models, expected values and standard deviations. The results for the remaining trials using the composition of linear segments model are summarized in Table 4 below.

## DISCUSSION

From Table 4, it is clear that a higher concentration of arsenic leads to a lower expected growth rate of the *Amphistegina gibbosa* foraminifer. The lower standard deviation associated

	$\text{As}^{5+}$		$\text{As}^{3+}$	
	50 ppb	100 ppb	2 ppb	200 ppb
<b>Expected Value</b>	23.914	22.4895	23.6595	20.0632
<b>Variance</b>	77.1593	47.6613	82.7030	35.1540
<b>Standard Deviation</b>	8.7840	6.9037	9.0941	5.9291

**Table 4:** *Amphistegina gibbosa* growth rates in arsenic exposure trials using the linear composition model.

with higher arsenic concentrations signify a more uniform growth rate centered on the lower expected value. The probability densities from Figure 1 indicate that higher arsenic concentrations reduce the number of foraminifers with large growth rates leaving only the coral with smaller growth rates. This interpretation accounts for the decreased expected values and smaller standard deviations.

## CONCLUSION AND RECOMMENDATIONS

Foraminifer growth is a biological indicator for the overall health of coral reef.

*Amphistegina gibbosa* d'Orbigny is ideally suited for heavy metal environmental experimentation. McCloskey's experiment showed that higher concentrations of arsenic ( $\text{As}^{3+}$  and  $\text{As}^{5+}$ ) yields lower growth rates in *Amphistegina gibbosa*.

Further experimentation should determine the arsenic concentration levels which make the reef environment unsustainable. Once threatening levels have been identified, public policy can be enacted to prevent irreparable contamination in oceanographic wildlife.



**Figure 5:** Photograph of *Amphistegina gibbosa* exposed to 200 ppb  $\text{As}^{3+}$  for 12 weeks

## NOMENCLATURE

Abbreviation	Description
As	Arsenic
Ppb	Parts Per Billion

## REFERENCES

- Hallock, P., Lidz, B. H., *et al.* 2003. Foraminifera as bioindicators in coral reef assessment and monitoring: The FORAM Index. *Environmental Monitoring and Assessment* 81 (1) : 221-238.
- McCloskey, B. 2009. Foraminiferal Responses to Arsenic in a Shallow-Water Hydrothermal System in Papua New Guinea and in the Laboratory. University of South Florida Marine Science Dissertation.