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Detecting Edges

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Detecting Edges

Abstract
In human vision the first level of processing is edge detection. Edges are determined by the transitions from dark points to bright points in an image. For this paper, we consider an edge profile model representing a boundary or edge in an image. From this model we can determine the strength of the edge, the width of the edge, and either the transition from dark to bright to dark or the transition from bright to dark to bright. Our first step was to take the given edge profile and determine the type of edge that is represented and the characteristics of the edge, such as that of the varying width of the edge as the variable a is either increased or decreased. In the next step, we calculated the derivate of the edge profile model. The final step involved utilizing the properties of the function defined by the derivative. Finding the second derivate of the edge profile model allowed us to determine the maximum and minimum values of x for the derivative of the edge profile model.

Keywords
Age profile Model, Intensity, Differentiation proces

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PROBLEM STATEMENT

Human vision is an intriguing ability. The fact that we can effortlessly recognize objects, people, and read this document belies the complexity of the problem that a significant portion of our brain is devoted to solving. It is amazing to learn that this almost photographic perception of the world that we have is not what our eyes send to our brain. What we perceive is a reconstruction from the noisy, shaky output of elementary “features” detected in the retina. There is strong evidence that the first level of processing done in the retina involves detecting something called “edges” or positions of transitions from dark to bright or bright to dark points in the images. These points usually coincide with boundaries.

Figure 1: On left is an image, with the corresponding edge image shown on the right.

In **Figure 1** above we see an example of an edge image. The process of detecting edges in an image is called edge detection and can be modeled as a differentiation process. In this module we will look into this differentiation process.

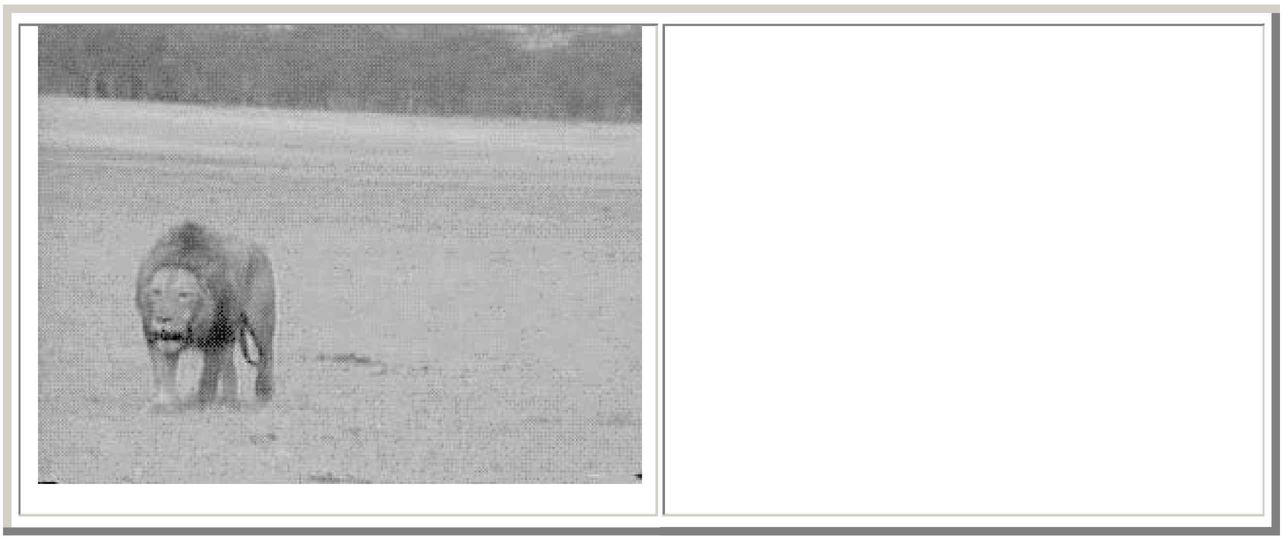

Figure 2: Edges corresponds to points in the image where there are intensity changes.

In the above figure we see the profile of the intensity value along a line (shown in red) cutting an edge in the image. We use small values to represent dark points in the image and greater values to detect bright regions in the image. The value increases and saturates to the value of the bright region. The edges can be marked at point of the maximum change in intensity, or where the *derivative is a maximum*. We will study the behavior of this derivative for a class of models of the edges.

We will be answering the following questions:

1. A) What kind of "edge" is represented by the following edge profile model:

   $$e(x) = \exp(-ax^2)$$

   B) What happens to the width of the "edge" as *a* varies?

2. Compute the first derivative of the above function.

3. A) Where, i.e. for what values of *x*, does this derivative reach a maximum or a minimum values?

   B) What happens to the distance between the maximum and minimum value locations as *a* is increased?

4. Where is the value of the derivative zero?

MOTIVATION

Technology has become an increasingly integral part of the daily lives of human beings in the form of necessity, convenience, and entertainment. Its impact can be seen in our daily lives from cars that are able to park themselves, everyday electronics and face recognition cameras. One of the goals of engineers is to continuously create and develop smarter and more sophisticated technologies. To accomplish this, engineers often devise ways to mimic the abilities of human beings in thinking, understanding, and seeing. Sight is a complex ability often taken for granted without the understanding of its inner workings. In order for engineers to mimic the abilities of human sight they first must understand how it works. When electronics view our world they see a flat image, nothing like our three-dimensional world made up of real objects. The task, then, is to find a way for those electronic devices to differentiate between the separate objects in the flat image. To differentiate between objects, engineers need a way to find edges in the image. In this paper, this is done by finding the derivative across a line within the image, allowing the edges of a picture to become defined by the contrasting of dark and light between colors that occurs at an object's boundary. This process is repeated everywhere throughout the picture.

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In this paper I will explore the method engineers have found to find edges and differentiate between the objects in an image. For the stated problem the edge profile model is given as an exponential function. This edge model determines the type and characteristics of the edge. Using the derivate of this function, we can find the “intensity” of the edge boundary.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

The edge profile model for this problem is given in the form of an exponential function, where the edge is denoted by $e(x)$.

$$e(x) = e^{f(x)}$$

The function $e^{f(x)}$ is a composition of functions and so this equation can be rewritten as

$$h(x) = e^{f(x)} = g[f(x)], \text{ where } g(x) = e^{x}$$

This problem requires finding the derivate of the edge profile model $e^{f(x)}$. By the chain rule,

$$h'(x) = g'[f(x)]\, f'(x)$$

A characteristic of the natural exponential function is that it is its own derivative, so $g'(x) = e^{x}$ and the derivate of $e^{f(x)}$ can be written as

$$h'(x) = e^{f(x)}\, f'(x)$$

Once the derivative of the exponential function has been found, the remaining tasks associated with this paper involve studying the characteristics and properties of the exponential function, such as the location of the maximum and the minimum value of the function as a is increased. This involves finding the derivate of the new function and finding where the value of the function is zero, yielding the critical points for the function. Also, we must find where the derivative of the edge profile model equals zero, this time finding the location where the actual function (derivate of the edge profile model) is equal to zero.

DISCUSSION

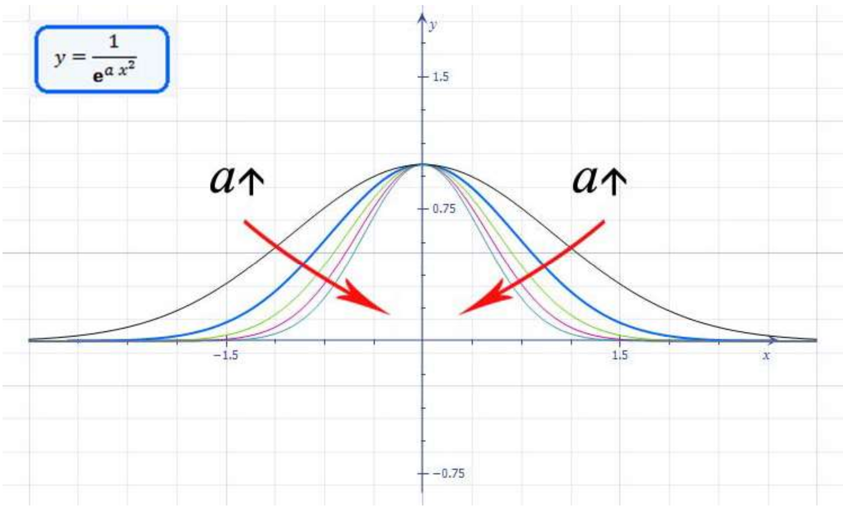
I. SOLUTIONS

Question 1)

A) What kind of "edge" is represented by the following edge profile model? $\mathrm{e(x)} = \left(e^{-ax^2}\right)$

The edge profile model $\mathrm{e(x)} = e^{-ax^2}$ represents a curve. This curve would appear white in a dark background, evident in the shape and properties of the graph below.

B) What happens to the width of the "edge" as you change *a*?



As the variable *a* is increased the value of function expressed deceases for every value of x except when x=0. At the point where x=0 the value of the function is 1 regardless of the value of *a.* While the value of the function decreases as *a* is increased it also increases as *a* is decreased until the value of the function is 1 for every x. From the graph it is clear that as *a* is increased the width of the "edge" decreases and inversely as *a* is decreased the width of the "edge" increases.

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Question 2)

A) Compute the first derivative of the above function.

$$e(x) = e^{-ax^2} \Rightarrow e'(x) = \frac{d}{dx}(e^y)\,\frac{d}{dx}(-ax^2) \quad ,$$

where y is equal to $-ax^2$

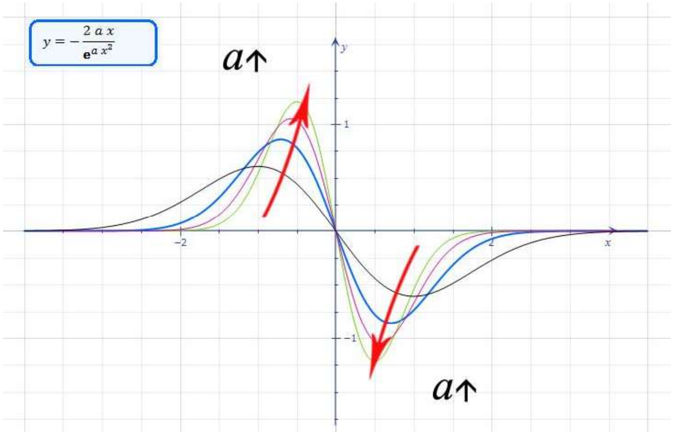
III) $e'(x) = -2ax\left(e^{-ax^2}\right)$

Question 3)

A) For what values of *x* does this derivative reach a maximum or a minimum values?

To find the maximum or minimum of any function we need to find the derivate of the function and the critical points, the values of *x* at which the derivative is equal to zero. The derivate of $-2x(e^{-x^2})$ is $2(\mathrm{x}^2 - 1)\mathrm{e}^{-\mathrm{x}^2}$. Once the derivate is calculated the next step is to find the critical points where the derivate is equal to zero. The critical points can be found when $(\mathrm{x}^2 - 1) = 0$, therefore we need all values of x that cause $x^2 = 1$. These values are $x = 1$ and $x = -1$. When plugged back into the derivate, it is found that the derivate of the edge profile model has a maximum at $\mathrm{x} = -1$ and a minimum at $\mathrm{x} = 1$.

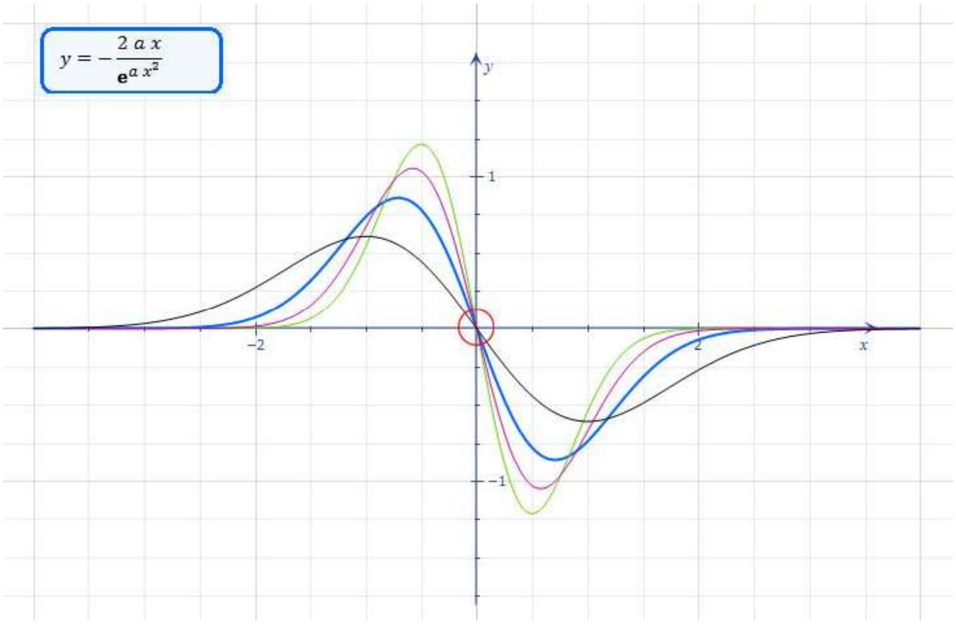
B) What happens to the distance between the maximum and minimum value locations as *a* is increased?



From the graph of the derivative $y = -\frac{2ax}{e^{ax^2}}$ it can be seen that as the variable *a* is increased, the maximum and minimum values of the derivate increase while also approaching closer to the y-axis. Therefore, as *a* is increased, the maximum and minimum values of the derivate become closer with respect to the x-axis and further apart with respect to the y-axis.

Question 4)

A) Where is the value of the derivative equal to zero?



If we look at the equation of the derivate $y = -\frac{2ax}{e^{ax^2}}$ it can be seen that the equation will equal zero when the numerator $2ax$ is equal to zero. The denominator can be disregarded because regardless of the values of *a* or *x*, e^{ax^2} will never be equal to zero as $e^0 = 1$. Looking at the numerator $2ax$, the value of the derivate will be equal to zero when either *a* is equal to zero or *x* is equal to zero.

II. OUTCOME

The aim of this paper was the computation and understanding of an edge profile model. These models mimic a function of human vision, namely that of detecting the boundaries between objects. In this paper, the utility and the inner works of an edge profile model came to light. The achieved results were as expected and matched the background information provided in the problem statement.

This technology will allow a new level of computerized intelligence and ability. The revolution of electronics and their integration into our world is becoming increasingly solidified. The innate capabilities of humans that distinguish us can now be "given" to an object. Our curiosity in ourselves is leading technology into grander and more unimaginable realms than most would think possible.

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CONCLUSIONS AND RECOMMENDATIONS

From any given edge profile model the characteristics and properties that define an edge or boundary in an image can be determined. From the edge profile model itself the transition type—from bright to dark to bright, or from dark to bright to dark—can be determined by the graph of the function and by the variable *a*. While simply looking at the graph provided in the solution to the first question, the shape yields a form that the edge could posses. The next problem involved calculating the derivate of the edge profile model. This was done by using the fact that an exponential function is the derivative of itself. With the derivative calculated, the final problems were solved. When viewing the graph of the derivative, the maximum, minimum and the location where the derivate is equal to zero can clearly be observed. The focus of this paper was to define and understand an edge profile model. Through the plotting of the model, calculating the derivative, and solving for specific properties of the derivative, a thorough understanding of how edges in an image are formed can be obtained.

I feel that this problem provided a comprehensive understanding how edge detection in an image works. For a more in-depth project, one could include reconstructing a portion of an image through given edge profile models at a specific locations. Inversely, after solving the initial problems another task could involve trying to formulate an edge profile model from a graphed location of an image.

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