

2009

2009 Ford Mustang Performance Test

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Recommended Citation

Fernandes, Daniel (2009) "2009 Ford Mustang Performance Test," *Undergraduate Journal of Mathematical Modeling: One + Two*: Vol. 1: Iss. 2, Article 4.

DOI: <http://dx.doi.org/10.5038/2326-3652.1.2.4>

Available at: <https://scholarcommons.usf.edu/ujmm/vol1/iss2/4>

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Abstract

Our goal is to find the time required for a 2009 Ford Mustang to accelerate from rest to 88 feet per second. We begin with three equations involving force, velocity, and force inverse, which is a value derived from Newton's Law, $F=ma$. The Mustang has three gears with three different gear ratios that must be used as the car accelerates. We found results from 2000 to 6000 RPMs for all three gears. Once the force inverse was found, we plotted the force inverse vs. velocity graph. The area beneath this curve from 0 to 88 feet per second is the time it takes to reach that speed. Using a Riemann sum approximation, we found that the time to be 4.109 seconds. This is a very brief time and is comparable to 5.2 seconds, the time found in a real test by RSPORTSCARS.com.

Keywords

Ford Mustang, Newton's Law, Force Inverse

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Problem Statement

Using a Torque vs. RPM curve of a vehicle of your choice, find the time it takes for that vehicle to travel from 0 to 88 feet per second. To find this time, plot a “Force Inverse” vs. Velocity graph and find a Riemann sum approximation of the curve from 0 to 88 feet per second. This value will be the time. The equations below must be used to find “Force Inverse” and Velocity. For all gears, the Torque is a specific value at certain RPMs. Gear ratios, tire radius, mass, drive ratio, and the Torque vs. RPM curve are all needed for the chosen vehicle to find Force, Velocity, and “Force Inverse”.

Derived Force Inverse Equation

$$\text{Force} = \frac{\text{mass}}{g_c} \cdot \frac{dV}{dt}$$

$$dt = \frac{m}{g_c} \cdot \frac{dV}{\text{Force}}$$

$$t = \int \frac{m}{g_c} \cdot \frac{dV}{\text{Force}}$$

Equations

$$\text{Force} = \frac{\text{Torque} \cdot \text{Drive Ratio} \cdot \text{Ratio for that Gear}}{\text{Tire Radius}}$$

$$\text{Velocity} = \frac{\text{RPM} \cdot \text{Tire Radius} \cdot 2\pi}{\text{Drive Ratio} \cdot \text{Ratio for that Gear} \cdot 60}$$

$$\text{"Force Inverse"} = \frac{\text{Mass}}{g_c} \cdot \frac{1}{\text{Force}}$$

$$g_c = 32.2 \frac{\text{ft}}{\text{s}^2}$$

(All values except for g_c are specific to the particular car.)

Motivation

Today, every consumer vehicle has among its specifications the time it takes to accelerate from 0 to 60 miles per hour, or 0 to 88 feet per second. Car companies are constantly trying to improve this time in order to make their sports cars more appealing. This problem is very pertinent to automobile engineers, who want to know the time it takes for the car to accelerate from 0 to 88 feet per second without having to actually time the car. Instead of developing the car and adjusting it once it is built, the engineers can make adjustments on paper. Mechanical engineers can use this method for all vehicles: airplanes, motorcycles and even robots. Figuring out these values on paper avoids wasting time and money on trial and error of the vehicle.

Mathematical Description and Solution Approach

Equations with Fixed Numbers Included

(The "Ratio for that Gear" is fixed, but different for each gear.)

(Gear 1: 3.75, Gear 2: 2.19, Gear 3: 1.41)

$$\text{Force} = \frac{\text{Torque} \cdot (3.55) \cdot \text{Ratio for that Gear}}{15/12 \text{ (ft)}}$$

$$\text{Velocity} = \frac{\text{RPM} \cdot (15/12) \cdot 2\pi}{3.55 \cdot \text{Ratio for that Gear} \cdot 60(\text{s/min})}$$

$$\text{Force Inverse} = \frac{3356 \text{ lbs.}}{32.2 \text{ ft/s}^2} \cdot \frac{1}{\text{Force}}$$

(See appendix for all values found using these equations. Torque is a function of RPM. (See figure 1.))

Mustang Specifications:

Mass: 3356 lbs.

Drive Ratio: 3.55

Tire Radius: (15/12) feet

I found the Force, Velocity, and “Force Inverse” for nine different RPMs for the Mustang (see Appendix). Then, I plotted the “Force Inverse” vs. Velocity curve (see Figure 2). The area beneath the curve from 0 to 88 ft/s is the desired time. This is found using a Riemann Sum approximation. The outlined portion of the curve in Figure 3 is the portion of the graph under which we are approximating the area. The line from (0,0) to the leftmost point of Gear 1 is an approximation due to the fact that the vehicle cannot be in gear until it reaches a certain speed. This region is a triangle, so its area, $\frac{1}{2} \cdot \text{base} \cdot \text{height}$, will be added to the Riemann sum. The jump at 59 (ft/s) is due to a shift in gears.

The Riemann sum is given by

$$\sum_{i=1}^n f(c_i)\Delta x.$$

For this Riemann sum there are 17 subintervals, each of length 4 ft/s from 20-88 ft/s. The time is given by integrating the “Force inverse” with respect to velocity:

$$\sum_{20}^{88} (\text{“Force Inverse” at each Velocity}) \cdot (4\text{ft/s}) + (0.5 \cdot 20 \cdot 0.0439)\text{s} = 4.109 \text{ s.}$$

The second term on the right hand side gives the area under the line from (0,0) to the leftmost point of Gear 1, as depicted in Figure 2.

Discussion

The result of the Riemann sum approximation was 4.109 seconds. This is a great number because, for most sports cars, the time required for acceleration from 0-88 (ft/s) varies between 4 (ft/s) and 6 (ft/s). The number is slightly lower than the

value recorded by SPORTSCARS.com, but this is due to the approximation used. With more subintervals or a computer program that calculates area, the results would be even more accurate. This problem proves that Mechanical Engineers can calculate important values using the vehicle's basic specifications.

Conclusions and Recommendations

A large number of calculations was needed to find the trend of "Force Inverse" for each gear. The time of 4.109 was exactly the type of number I was hoping I would find. With the many equations and slight rounding there was plenty of room for error but the results were accurate. The Ford Mustang is a high performance vehicle that is capable of attaining this fast 0-88(ft/s) time. Its high torque at certain RPMs greatly contributes to its fast acceleration. Because my results were accurate, I don't have a recommendation for further development of this problem except to find a program that can calculate the area more accurately than a Riemann sum.

References

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<http://www.hypertech-inc.com/media/attachments/aa/42004%2084009%202005-2009%20mustang%20gt%204%206l.pdf>
- "2009 Ford Mustang GT Specifications" RSportsCars.com. 26 April 2009.
<http://www.rsportscars.com/ford/2009-ford-mustang-gt/>

Appendix**Gear 1 (Gear Ratio=3.75:1)**

RPM	Torque (lbs-ft)	Force (lbf)	Velocity (ft/s)	"Force Inverse" (s ² /ft)
2000	223	2375.0	19.67	0.0439
2500	236	2513.4	24.58	0.0415
3000	242	2577.3	29.50	0.0404
3500	244	2598.6	34.41	0.0401
4000	255	2715.8	39.33	0.0384
4500	263	2801.0	44.25	0.0372
5000	262	2790.3	49.16	0.0374
5500	246	2619.9	54.08	0.0398
6000	219	2332.4	59.00	0.0447

Gear 2 (Gear Ratio=2.19:1)

RPM	Torque (lbs-ft)	Force (lbf)	Velocity (ft/s)	"Force Inverse" (s ² /ft)
2000	223	1387.0	33.67	0.0751
2500	236	1467.8	42.09	0.0710
3000	242	1505.1	50.51	0.0692
3500	244	1517.6	58.93	0.0687
4000	255	1586.0	67.35	0.0657
4500	263	1635.8	75.77	0.0637
5000	262	1629.5	84.19	0.0640
5500	246	1530.0	92.60	0.0681
6000	219	1362.1	101.0	0.0765

Gear 3 (Gear Ratio=1.41:1)

RPM	Torque (lbs-ft)	Force (lbf)	Velocity (ft/s)	"Force Inverse" (s ² /ft)
2000	223	893.00	52.30	0.1167
2500	236	945.00	65.38	0.1103
3000	242	969.10	78.45	0.1075
3500	244	977.10	91.53	0.1067
4000	255	1021.1	104.6	0.1021
4500	263	1053.2	117.7	0.0990
5000	262	1049.2	130.8	0.0993
5500	246	985.10	143.8	0.1058
6000	219	877.00	156.9	0.1188

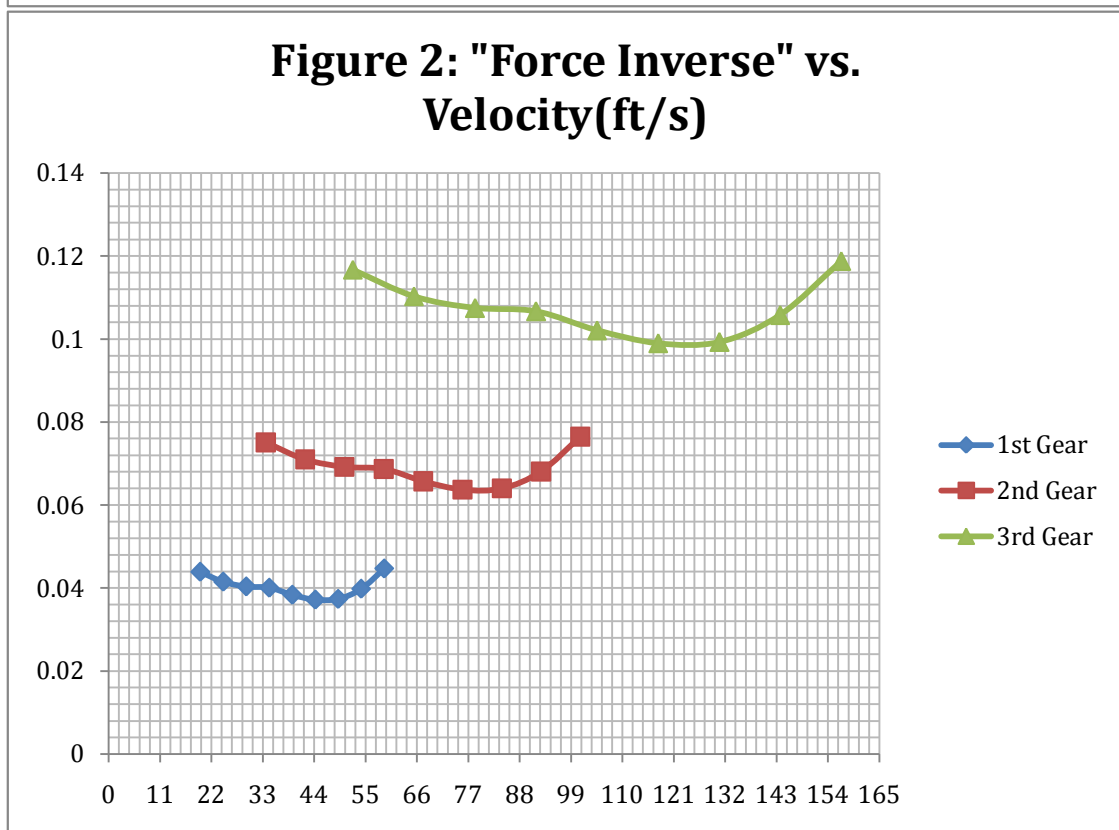
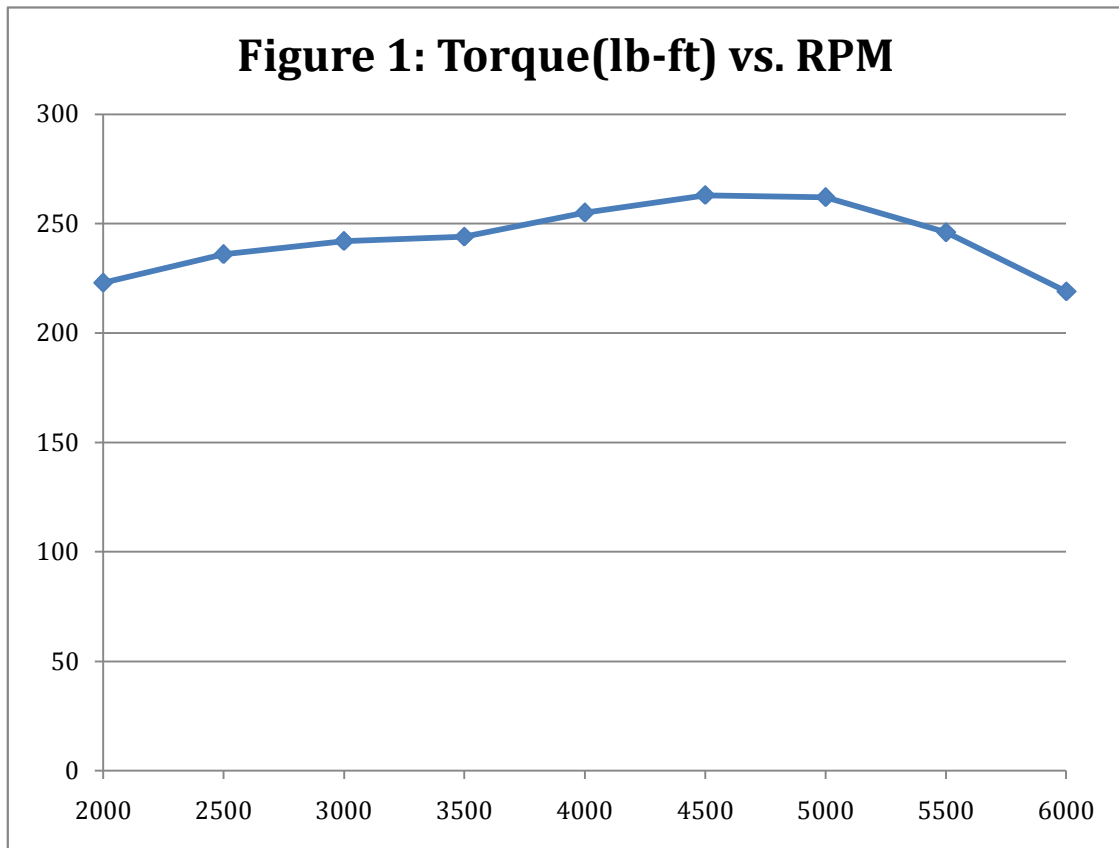
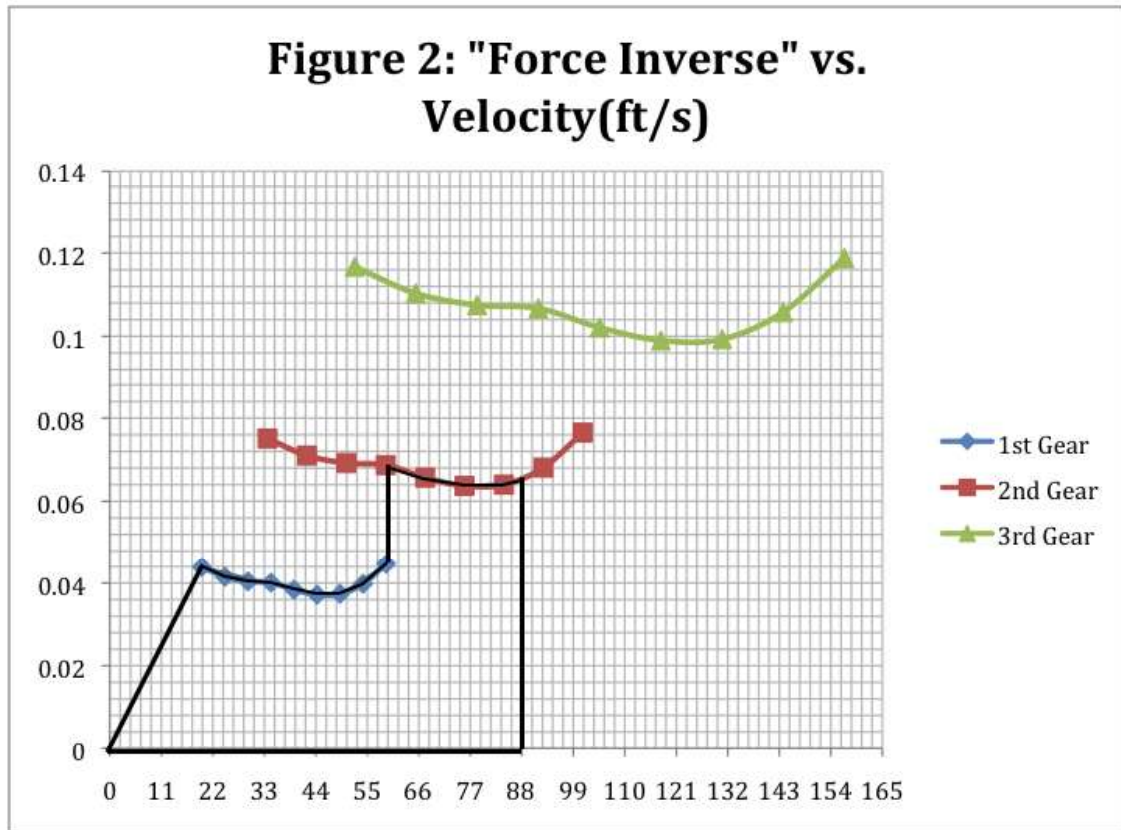


Figure 3: Black Line Highlights Area from 0 to 88 (ft/s)



Sample Calculation:

Gear 1

$$\text{Force} = \frac{223 \cdot 3.55 \cdot 3.75}{15/12} = 2375.0$$

$$\text{Velocity} = \frac{2000 \cdot 15/12 \cdot 2\pi}{3.55 \cdot 3.75 \cdot 60} = 19.67 \text{ (ft/s)}$$

$$\text{Force Inverse} = \frac{3356}{32.2} \cdot \frac{1}{2375.0} = 0.0439 \text{ (s}^2\text{/ft)}$$