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Algae Bloom in a Lake

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Algae Bloom in a Lake

Abstract
The objective of this paper is to determine the likelihood of an algae bloom in a particular lake located in upstate New York. The growth of algae in this lake is caused by a high concentration of phosphorous that diffuses to the surface of the lake. Our calculations, based on Fick's Law, are used to create a mathematical model of the driving force of diffusion for phosphorous. Empirical observations are also used to predict whether the concentration of phosphorous will diffuse to the surface of this lake within a specified time and under specified conditions.

Keywords
Algae, Phosphorous, Fick's Law

Motivation

Florida is known for its wetlands, which contain many lakes, rivers, ponds, and fountains; whether man-made or not, algae can grow in all of them. In lakes, it is known that the growth of algae is stimulated by concentrations of phosphorous. The phosphorous, initially present with dead vegetation at the bottom of the lake, rises to the top of the lake over time. Another way that phosphorous gets into lakes is through fertilizers that are washed into the lake by the rain. This causes algae to bloom. There are some debates over how much damage algae causes. One of the most noticeable problems with algae growth is its unappealing visual effect on a body of water. Most people would like to control the rate of algae growth. This paper attempts to calculate the likelihood of an algae bloom. This could be used to ensure that Florida's waters are cleaner and more presentable for its inhabitants and tourists.

Mathematical Description and Solution Approach

In this paper, we consider a specific lake in upstate New York that has an average depth of 5.7 m. In the spring, the lake mixes and forms a uniform phosphorous concentration of $\rho_0 = 0.46$ mg/L. All year round, there is a high concentration of phosphorous at the sediment-water interface ($\rho^* = 0.36$ mg/L). This creates a driving force for diffusion, which is modeled mathematically by Fick's Law. This equation gives the phosphorous concentration ρ as a function of time *t* and the depth of the lake, *y*:

$$\frac{\rho-\rho_0}{\rho^*-\rho_0} = 1 - \frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-x^2}\,dx, \qquad (1)$$

where $\eta = \frac{y}{\sqrt{4Dt}}$ and $D = 0.085$ m^2/day is the diffusivity of phosphorous.

It is known by empirical observation that algae blooms are probable if the surface water concentration of phosphorous reaches 0.08 mg/L within 180 days of the spring mixing (caused by the rise in temperatures). Our objective is to determine whether this particular lake will have a fall bloom of algae and to make a plot of depth, y, versus time, t, in order to determine if the lake will bloom.

To solve our first problem we must know the phosphorous concentration, ρ, on the surface of the lake after 180 days of diffusivity. In other words we must use Fick's Law to solve for ρ. If $\rho < 0.08$ mg/L then we may conclude that there will not be an algae bloom, but if $\rho \geq 0.08$mg/L then there will be an algae bloom.

Before solving for ρ, we rewrite the integral as an infinite power series:

$$\begin{aligned} 1 - \frac{2}{\sqrt{\pi}}\int_0^{\eta}\sum_{n=0}^{\infty}\frac{(-1)^n\, x^{2n}}{n!}\,dx &= 1 - \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n\, \eta^{2n+1}}{n!\,(2n+1)} \\ &\cong 1 - \frac{2}{\sqrt{\pi}}\left(\eta - \frac{\eta^3}{3} + \frac{\eta^5}{10} - \frac{\eta^7}{42} + \frac{\eta^9}{216}\right). \end{aligned} \qquad (2)$$

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Now we plug in the values of y = 5.7 m, D = 0.085 m^2/day and t = 180 days into the equation $\eta = \frac{y}{\sqrt{4Dt}}$ and calculate η. We obtain

$$\eta \cong 0.7286.$$

Inserting this value into formula (2) gives

$$1 - \frac{2}{\sqrt{\pi}}\left(\eta - \frac{\eta^3}{3} + \frac{\eta^5}{10} - \frac{\eta^7}{42} + \frac{\eta^9}{216}\right) = 0.3028.$$

Now, from equation (1),

$$0.3028 = 1 - \frac{2}{\sqrt{\pi}}\left(\eta - \frac{\eta^3}{3} + \frac{\eta^5}{10} - \frac{\eta^7}{42} + \frac{\eta^9}{216}\right) \cong \frac{\rho - \rho_0}{\rho^* - \rho_0} = \frac{\rho - 0.046}{0.36 - 0.046}.$$

Solving for ρ gives ρ = 0.1411 mg/L. Therefore, since ρ > 0.08 mg/L, we may conclude that there will be a fall bloom of algae in this particular lake.

The second part of this paper concerns the relationship between the depth of the lake and the time required for the necessary concentration of phosphorous to develop. More specifically, there needs to be a 0.08mg/L-concentration of phosphorous in order to get an algae bloom. Here, ρ = 0.08 mg/L is held constant. We need to find the corresponding value of η. We can rewrite equation (1) as

$$\frac{2}{\sqrt{\pi}}\int_0^{\eta} e^{-x^2}dx = 1 - \frac{\rho - \rho_0}{\rho^* - \rho_0} = 1 - \frac{0.08 - 0.046}{0.36 - 0.046} = 0.892.$$

The term on the left side is called the "error function"; it is denoted erf(η). The following table lists values for this function:

x	erf(x)	x	erf(x)
0.00	0.0000000	0.9340079	0.0659921
0.05	0.0563720	0.9522851	0.0477149
0.10	0.1124629	0.9661051	0.0338949
0.15	0.1679960	0.9763484	0.0236516
0.20	0.2227026	0.9837905	0.0162095
0.25	0.2763264	0.9890905	0.0109095
0.30	0.3286268	0.9927904	0.0072096
0.35	0.3793821	0.9953223	0.0046777
0.40	0.4283924	0.9970205	0.0029795
0.45	0.4754817	0.9981372	0.0018628
0.50	0.5204999	0.9988568	0.0011432
0.55	0.5633234	0.9993115	0.0006885
0.60	0.6038561	0.9995930	0.0004070
0.65	0.6420293	0.9997640	0.0002360

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0.70	0.6778012	0.9998657	0.0001343
0.75	0.7111556	0.9999250	0.0000750
0.80	0.7421010	0.9999589	0.0000411
0.85	0.7706681	0.9999779	0.0000221
0.90	0.7969082	0.9999884	0.0000116
0.95	0.8208908	0.9999940	0.0000060
1.00	0.8427008	0.9999969	0.0000031
1.10	0.8802051	0.9999985	0.0000015
1.20	0.9103140	0.9999993	0.0000007

http://en.wikipedia.org/wiki/Error_function

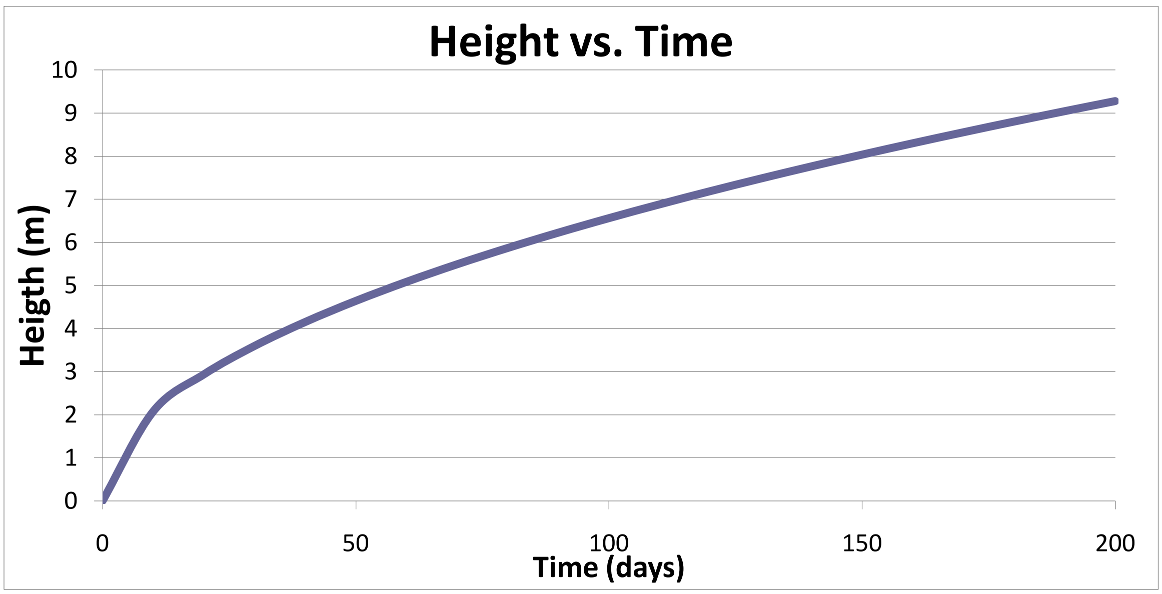
Notice that erf(1.10) = 0.8802051 < erf(η) = 0.892 < erf(1.20) = 0.910. Using the approximation (2), we calculated erf(1.125) ≅ 0.892. Thus,

$$\eta = \frac{y}{\sqrt{4Dt}} = 1.125.$$

We can rewrite this equation as

$$y = \left(\sqrt{4Dt}\right)\eta = 1.125\sqrt{4(0.085)t}.$$

Using Microsoft Excel, we graph this function.



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Discussion

Now we may look at the graph or plug in t = 180 days into the formula $y = \left(\sqrt{4Dt}\right)\eta$ to see that after 180 days the concentration of phosphorous will reach $\rho = 0.08\ \mathrm{mg/L}$ for a lake at least as deep as 8.80 m. Therefore, since the lake in New York is only 5.7 m deep, the surface water will attain a concentration of 0.08 mg/L prior to 180 days. This further proves our prior conclusion that there would be a algae bloom in the lake.

Conclusions and Recommendations

The results of this paper provide a lot of information about the concentration of phosphorous and how it mixes in lakes. The first calculation allowed us to conclude that there would be a fall bloom of algae because the concentration of phosphorous in the surface water was greater than 0.08 mg/L. This conclusion was reinforced by the second calculation and the graph that showed the time it takes for lakes of varying heights to reach phosphorus concentrations of 0.08mg/L. The graph shows that at 180 days a lake of height 8.80 m would reach a phosphorus concentration of 0.08 mg/L. Since our lake is only 5.7 m deep, it is reasonable to conclude that a larger concentration of phosphorous would have built up over the span of 180 days.

With the aid of our graph or its equation, we can calculate how many days it would take for there to be an exact phosphorous concentration of 0.08 mg/L in the surface water of our lake. To do this, we could use the formula $y = 1.125\sqrt{4(0.085)t}$, plug in 5.7m for y and solve for t or we could look at the graph. Either way, we see that it would take a little more than 75 days for a phosphorous concentration of 0.08 mg/L to reach 5.7 m, which is the average depth of the lake in upstate New York. So, in a span of a little more than 75 days, there would already be a sufficient phosphorous concentration (0.08mg/L) for an algae bloom. These results would allow us to better understand the growth of algae and learn how to prevent algae bloom in other lakes.

With this information, we now have a way to calculate the likelihood of an algae bloom. This helps engineers to come up with ways to prevent algae, not only in lakes, but also in beaches where coral reefs are affected. This paper allowed us to predict a bloom of algae using known concentrations of phosphorous and a mathematical model of the diffusivity of phosphorous. (VENERA-PONTON)

References

"Error function." Wikipedia. Web.29 Nov 2008. http://en.wikipedia.org/wiki/Error_function.

VENERA-PONTON, Dagoberto. "Contrasting Effects of Benthic Algae on Coral Recruits in an Upwelling Reef from the Colombian Caribbean ." NOVA SouthEastern University. Instituto de Investigaciones Tropicales-INTROPIC. Web.17 Jul 2009. <www.nova.edu/ncri/11icrs/abstract_files/icrs2008-002441.pd>.

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