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Light Attenuation at Molasses Reef

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Abstract

Estimating the amount of light available at different depths of the ocean is important for gaining a better understanding of coral reefs. It is especially useful to be able to get such estimates without having to perform direct measurements. Although accurate, the direct measurements are costly, time consuming, and usually limited to relatively small areas of interest.

One statistic that could be useful for estimating the amount of available light is the rate at which it declines with depth (the rate of light attenuation). In this project, this rate is calculated for four different wavelengths using data for 30m water column profile collected in upper Florida Keys. The rate of light attenuation is calculated for the entire profile as well as the intervals corresponding to 5m increments.

Keywords

Available Light, Rate of Light Decline, Coral Reef

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Erratum

This article was previously called Article 20.

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PROBLEM STATEMENT

Estimate the rate of light decline for each wavelength for the following intervals:

- a) Surface (0m) to 30 m
- b) Surface (0m) to 5 m (downward)
- c) 5 m to 10 m
- d) 10 m to 15 m
- e) 15m to 20 m
- f) 20 m to 25 m
- g) 25 m to 30 m

Repeat b) through g) for the upward profile. Discuss any differences you see in the rates that you calculated.

MOTIVATION

The amount of light available at certain depths has a significant effect on the organisms in the area (Hallock-Muller). Photosynthetic organisms, such as various reef inhabitants and benthic algae need a specific amount of light to survive. Furthermore, the amount of available light could affect survival rates or success rates in photosynthetic mutualisms (Hallock and Schlager).

The objective of this project is to estimate the rate of light attenuation over different depth intervals. This is achieved using the data quantifying the amount of available light for four different wavelengths at depths ranging from the surface (0m) to the benthic zone (30m) of Molasses Reef located near the Florida Keys.

MATHEMATICAL DESCRIPTION AND SOLUTION APPROACH

One possible way to calculate the rate of light attenuation is to first fit a function to the data and then find the derivative of this function with respect to the depth. Because the plot of the amount of available light with respect to height suggests exponential decay (see Fig. 1), the data is fitted to the function

$$f(x) = Ce^{-kx}$$

where $f(x)$ is the amount of available light, C is a constant, k is the rate of decline, and x is the depth. The rate of decline of $f(x) = Ce^{-kx}$ is given by its derivative $f'(x) = -Cke^{-kx}$. Thus instead of explicitly stating $f(x)$ and $f'(x)$ for each interval and each wavelength it is convenient to simply give the corresponding values of C and k .

DISCUSSION

Appendix-Tables contains a summary of all our calculations. As pointed out in the previous section, instead of giving a function representing the amount of light and the rate of light attenuation (derivative) on a given interval, we simply state the corresponding values of C and k . The graphs of these functions can be found in Appendix-Figures.

The amount of light with 305nm and 330nm wavelengths seems to decline faster than the 380nm and PAR light (see Fig. 1). This suggests that most of the light at these wavelengths is absorbed at relatively small depths. The spike in the amount of available light between 15m and 20m in the downward profile (see Fig. 5) seems abnormal. Because it is not present on the upward profile, it is probably a sensor malfunction. The other discrepancies in the amount of available light between upward and downward profiles can be contributed to (1) differences in coverage of each interval and (2) changes in the sunlight intensity.

CONCLUSION AND RECOMMENDATIONS

Using the surface to 30m profile, we have calculated the rate of attenuation by taking the derivative of the function of the best fit:

- $g_1(x) = 1.03e^{-0.17x} \mu\text{W}/\text{cm}^2/\text{nm}/\text{m}$ for the light with 305nm wavelength,
- $g_2(x) = 3.37e^{-0.08x} \mu\text{W}/\text{cm}^2/\text{nm}/\text{m}$ for the light with 330nm wavelength,
- $g_3(x) = 1.77e^{-0.03x} \mu\text{W}/\text{cm}^2/\text{nm}/\text{m}$ for the light with 380nm wavelength,
- $g_4(x) = 58.91e^{-0.05x} \mu\text{E}/\text{m}^2/\text{s}/\text{m}$ for the light with PAR wavelength,

where x is the depth.

It is likely that these estimates are more reliable than those for many 5m intervals. This is because a cluster of outliers in a 5m interval affects its estimated rate of attenuation significantly more than the rate of attenuation for the entire profile.

The rates of light attenuation observed on 15m-20m (downward profile), surface-5m (upward profile) intervals for the light with wavelength 380nm and on 15m-20m (upward profile) interval for PAR wavelength are probably due to the sensor malfunction. The unusual spike in the amount of available light on 15m-20m (downward profile) interval has distorted the rates of light attenuation for all four wavelengths.

It may be possible to avoid these problems by preprocessing the data after its collection by removing the values having a high probability of being erroneous.

NOMENCLATURE

Symbol	Description	Units
x	Depth	Meters
$f(x)$	Amount of available light at depth x	$\mu\text{E}/\text{m}^2/\text{s}$ PAR wavelength $\mu\text{W}/\text{cm}^2/\text{nm}$ for other wavelengths
$g(x)$	Rate of light attenuation at depth x	$\mu\text{E}/\text{m}^2/\text{s}/\text{m}$ for PAR wavelength $\mu\text{W}/\text{cm}^2/\text{nm}/\text{m}$ for other wavelengths:
k	Rate of decay	n/a
C	Constant	n/a

REFERENCES

Hallock, Pamela and Schlager, Wolfgang. "Nutrient Excess and the Demise of Coral Reefs and Carbonate Plateforms." PALAIOS (1986): 389-398.

Hallock-Muller, Pamela. "Light Measurements at Molasses Reef." 2005.

APPENDIX I - FIGURES

Figure 1: The amount of available light in surface to 30m interval with the curve of the best fit depicted in red. The (C, k) values are $(6.08, 0.17)$ for 305nm, $(42.16, 0.08)$ for 330nm, $(58.87, 0.03)$ for 380nm, and $(1178.11, 0.05)$ for PAR.

Figure 2: The amount of available light in surface to 5m interval (downward profile) with the curve of the best fit depicted in red. The (C, k) values are (8.34, 0.32) for 305nm, (57.62, 0.23) for 330nm, (82.58, 0.19) for 380nm, and (2174.78, 0.34) for PAR.

Figure 3: The amount of available light in 5m to 10m interval (downward profile) with the curve of the best fit depicted in red. The (C, k) values are (4.56, 0.16) for 305nm, (31.00, 0.07) for 330nm, (40.44, 0.02) for 380nm, and (611.70, 0.04) for PAR.

Figure 4: The amount of available light in 10m to 15m interval (downward profile) with the curve of the best fit depicted in red. The (C, k) values are (4.49, 0.17) for 305nm, (30.68, 0.07) for 330nm, (40.19, 0.02) for 380nm, and (526.11, 0.03) for PAR.

Figure 5: The amount of available light in 15m to 20m interval (downward profile) with the curve of the best fit depicted in red. The (C, k) values are (2.07, 0.12) for 305nm, (11.51, 0.02) for 330nm, (10.47, 0.06) for 380nm, and (55.37, 0.11) for PAR.

Figure 6: The amount of available light in 20m to 25m interval (downward profile) with the curve of the best fit depicted in red. The (C, k) values are (12.50, 0.21) for 305nm, (56.60, 0.10) for 330nm, (80.67, 0.04) for 380nm, and (1119.07, 0.04) for PAR.

Figure 7: The amount of available light in 25m to 30m interval (downward profile) with the curve of the best fit depicted in red. The (C, k) values are (27.57, 0.24) for 305nm, (37.43, 0.08) for 330nm, (46.91, 0.02) for 380nm, and (229.54, 0.01) for PAR.

Figure 8: The amount of available light in surface to 5m interval (upward profile) with the curve of the best fit depicted in red. The (C, k) values are (5.21, 0.1) for 305nm, (31.95, 0.01) for 330nm, (48.52, 0.06) for 380nm, and (1085.56, 0.004) for PAR.

Figure 9: The amount of available light in 5m to 10m interval (upward profile) with the curve of the best fit depicted in red. The (C, k) values are (7.08, 0.18) for 305nm, (50.05, 0.1) for 330nm, (73.44, 0.04) for 380nm, and (1378.18, 0.07) for PAR.

Figure 10: The amount of available light in 10m to 15m interval (upward profile) with the curve of the best fit depicted in red. The (C, k) values are (6.40, 0.17) for 305nm, (42.80, 0.08) for 330nm, (45.78, 0.02) for 380nm, and (841.42, 0.02) for PAR.

Figure 11: The amount of available light in 15m to 20m interval (upward profile) with the curve of the best fit depicted in red. The (C, k) values are (8.75, 0.19) for 305nm, (54.96, 0.09) for 330nm, (76.64, 0.04) for 380nm, and (1051.44, 0.04) for PAR.

Figure 12: The amount of available light in 20m to 25m interval (upward profile) with the curve of the best fit depicted in red. The (C, k) values are (11.34, 0.20) for 305nm, (54.91, 0.09) for 330nm, (77.07, 0.04) for 380nm, and (1007.05, 0.03) for PAR.

Figure 13: The amount of available light in 25m to 30m interval (upward profile) with the curve of the best fit depicted in red. The (C, k) values are $(40.40, 0.25)$ for 305nm, $(57.79, 0.09)$ for 330nm, $(79.87, 0.04)$ for 380nm, and $(972.91, 0.03)$ for PAR.

Appendix II - Tables

Downward Profile								
Depth	305nm		330nm		380nm		PAR	
	C	k	C	k	C	k	C	k
0m-5m	8.34	0.32	57.62	0.23	87.58	0.19	2174.78	0.34
5m-10m	4.56	0.16	31.00	0.07	40.44	0.02	611.70	0.04
10m-15m	4.49	0.17	30.68	0.07	40.19	0.02	526.11	0.03
15m-20m	7.07	0.12	11.51	0.02	10.47	-0.06	55.37	-0.11
20m-25m	12.50	0.21	56.60	0.10	80.67	0.09	1119.07	0.04
25m-30m	27.57	0.24	37.43	0.08	46.90	0.02	229.54	0.01

Table 1: The values of the constant (C) and the rate of decay (k) for the function of the best fit for a given depth interval of the downward profile.

Upward Profile								
Depth	305nm		330nm		380nm		PAR	
	C	k	C	k	C	k	C	k
0m-5m	5.21	0.1	31.95	0.01	48.52	-0.06	1085.56	0.004
5m-10m	7.08	0.18	50.05	0.1	73.44	0.04	1378.18	0.07
10m-15m	6.40	0.17	42.80	0.08	45.78	0.02	841.42	0.02
15m-20m	8.75	0.19	54.96	0.09	76.64	0.04	1051.44	0.04
20m-25m	11.34	0.20	54.91	0.09	77.07	0.04	1007.05	0.03
25m-30m	40.40	0.25	57.79	0.09	79.87	0.04	972.91	0.03

Table 2: The values of the constant (C) and the rate of decay (k) for the function of the best fit for a given depth interval of the upward profile.