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# Geometric Transformations in Middle School Mathematics Textbooks

Barbara Zorin
*University of South Florida*, DrBZorin@gmail.com

# Geometric Transformations in Middle School Mathematics Textbooks

by

Barbara Zorin

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Major Professor: Denisse R. Thompson, Ph.D.
Richard Austin, Ph.D.
Catherine A. Beneteau, Ph.D.
Helen Gerretson, Ph.D.
Gladis Kersaint, Ph.D.

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## Dedication

This dissertation is dedicated to my father, Arthur Robert, who provided me with love and support, and who taught me, from a very early age, that I could accomplish anything that I put my mind to. To my son, Rick, for always believing that I was the smartest person he had ever met and showing me that he knew that I would complete this endeavor. To my departed husband, Glenn, for all of the love, faith, and understanding that he brought into my life. And to my partner, Michael, who has provided me with love, respect, understanding, and the support that I needed to finish what I started.

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**Abstract**

This study analyzed treatment of geometric transformations in presently available middle grades (6, 7, 8) student mathematics textbooks. Fourteen textbooks from four widely used textbook series were evaluated: two mainline publisher series, Pearson (Prentice Hall) and Glencoe (Math Connects); one National Science Foundation (NSF) funded curriculum project textbook series, Connected Mathematics 2; and one non-NSF funded curriculum project, the University of Chicago School Mathematics Project (UCSMP).

A framework was developed to distinguish the characteristics in the treatment of geometric transformations and to determine the potential *opportunity to learn* transformation concepts as measured by textbook physical characteristics, lesson narratives, and analysis of student exercises with level of cognitive demand. Results indicated no consistency found in order, frequency, or location of transformation topics within textbooks by publisher or grade level.

The structure of transformation lessons in three series (Prentice Hall, Glencoe, and UCSMP) was similar, with transformation lesson content at a simplified level and student low level of cognitive demand in transformation tasks. The types of exercises found predominately focused on students applying content studied in the narrative of lessons. The typical problems and issues experienced by students when working with transformations, as identified in the literature, received little support or attention in the lessons. The types of tasks that seem to embody the ideals in the process standards, such

as working a problem backwards, were found on few occurrences across all textbooks examined. The level of cognitive demand required for student exercises predominately occurred in the *Lower-Level*, and *Lower-Middle* categories.

Research indicates approximately the last fourth of textbook pages are not likely to be studied during a school year; hence topics located in the final fourth of textbook pages might not provide students the opportunity to experience geometric transformations in that year. This was found to be the case in some of the textbooks examined, therefore students might not have the opportunity to study geometric transformations during some middle grades, as was the case for the Glencoe (6, 7), and the UCSMP (6) textbooks, or possibly during their entire middle grades career as was found with the Prentice Hall (6, 7, Prealgebra) textbook series.

## Chapter 1: Introduction and Rationale for the Study

The branch of mathematics that has the closest relationship to the world around us, as well as the space in which we live is geometry (Clements & Samara, 2007; Leitzel, 1991; National Council of Teachers of Mathematics (NCTM), 1989). Furthermore, geometry is a vehicle by which we develop an understanding of space that is necessary for comprehending, interpreting, and appreciating our inherently geometric world (NCTM, 1989). Spatial geometry provides us with the knowledge to understand (Leitzel, 1991) and interpret our physical environment (Clements, 1998; NCTM, 1992); this knowledge provides us with intellectual instruments to sort, classify, draw (NCTM, 1992), use measurements, read maps, plan routes (NCTM, 2000), create works of art (Clements, Battista, Sarama & Swaminathan, 1997; NCTM, 2000), design plans, and build models (NCTM, 1992). Spatial geometry also provides us with the knowledge necessary for engineering (NCTM, 2000) and building (Clements, Battista, Sarama & Swaminathan, 1997), in addition to the aptitude to develop logical thinking abilities, creatively solve problems (NCTM, 1992), and design advanced technological settings and computer animations (Clements et al, 1997; Yates, 1988). Additionally, spatial geometry helps us understand and strengthen other areas of mathematics as well as provides us with the tools necessary for the study of other subjects (Boulter & Kirby, 1994).

Spatial geometry includes the contemporary study of form, shape, size, pattern, and design. Spatial reasoning concentrates on the mental representation and manipulation of spatial objects. Geometry is described by Clements and Battista (1992) and Usiskin (1987) as having four conceptual aspects. The first conceptual aspect is visualization, depiction, and construction; this conception focuses on visualization, sequence of patterns, and physical drawings. The second aspect is the study of the physical situations presented in the real world that direct the learner to geometric concepts, as a carpenter squaring a framing wall with the use of the Pythagorean Theorem. The third aspect provides representations for the non-physical or non-visual, as with the use of the number line to represent real numbers. The fourth aspect is a representation of the mathematical system with its logical organization, justifications, and proofs. The first three conceptual aspects of geometry necessitate the use of spatial sense, which can be learned and reinforced during the study of geometric transformations.

The study of transformations supports the interpretation and description of our physical environment as well as provides us with a valuable tool in problem solving in many areas of mathematics and in real world situations (NCTM, 2000). The study of geometric transformations begins with the student’s journey into the understanding of visualization, mental manipulation, and spatial orientation with regard to figures and objects. Through the study of transformations, Clements and Battista (1992) and Leitzel (1991) assert that students develop spatial visualization and the ability to mentally transform two dimensional images. Two dimensional transformations are an important topic for all students to study and the recommendation is that all middle grades students study transformations (NCTM, 1989, 2000, 2006).

The study of geometry with transformations has enhanced geometry to a dynamic level by providing the student with a powerful problem-solving tool (NCTM, 1989).

Spatial reasoning and spatial visualization through transformations help us build and manipulate mental representations of two dimensional objects (NCTM, 2000). Students need to investigate shapes, including their components, attributes, and transformations. Additionally, students need to have the opportunity to engage in systematic explorations with two dimensional figures including representations of their physical motion (Clements, Battista, Sarama, & Swaminathan, 1997). Geometric transformations, for middle school students, are composed of five basic concepts: translations (slides), reflections (flips or mirror images), rotations (turns), dilations (size changes), and the composite transformation of two or more of the first three (Wesslen & Fernandez, 2005).

Transformation concepts provide background knowledge to develop new perspectives in visualization skills to illuminate the concepts of congruence and similarity in the development of spatial sense (NCTM, 1989). Spatial reasoning, including spatial orientation and spatial visualization, is an aptitude that directly relates to an individual’s mathematical ability (Brown & Wheatley, 1989; Clements & Sarama, 2007). It also directly influences success in subsequent geometry coursework and general mathematics achievement, which, in turn, directly affects the student’s future career options (Ma & Wilkins, 2007; NCTM, 1989).

Research suggests that students should have a functioning knowledge of geometric transformations by the end of eighth grade in order to be successful in higher level mathematics studies (Carraher & Schlieman, 2007; Flanders, 1987; Ina-Wilkins, 2007; Ladson-Billings, 1998; Knuth, Stephens, McNeil, & Alibali, 2006; National Assessment of Educational Progress (NAEP), 2004; NCTM, 2000; National Research Council (NRC), 1998). However, the academic performance of United States students in geometry, and more specifically in spatial reasoning, is particularly low (Battista, 2007; Silver, 1998; Sowder, Wearne, Martin, & Strutchens, 2004).

Because of long standing concerns about student achievement, recommendations by major national mathematics and professional educational organizations, such as the NCTM, the National Commission on Excellence in Education, and the NRC, call for essential alterations in school mathematics curricula, instruction, teaching, and assessment (NCTM, 1989, 1991, 1992, 1995, 2000, 2006; NRC, 1998). In particular, the NCTM published three milestone documents which developed mathematics curriculum standards for grades K - 12 that focused on school mathematics reform. The *Curriculum and Evaluations Standards* (NCTM, 1989) includes a vision for the teaching and learning of school mathematics, including a vision of mathematical literacy. This document also includes recommendations for the study of transformations of geometric figures to enhance the development of spatial sense for all students. The document's recommendations suggest that students should have an opportunity to study two dimensional figures through visualization and exploration of transformations.

NCTM revised and updated the Standards with its publication of the *Principles and Standards for School Mathematics* (PSSM) (NCTM, 2000). This document extends the previous recommendations by providing clarification and elaboration on the curricula described, as well as specifically identifying expectations for each grade band: preK-2, 3-5, 6-8 and 9-12. PSSM offers specific content guidelines for all students, and examples for teaching, as well as specific principles and features to assist students in attaining high quality mathematics understanding. The expectations for students are delineated in each of the mathematical strands. For example, in the PK - 2 grade band, PSSM recommends that students should be able to recognize symmetry and geometric transformations of figures with the use of manipulatives; in grades 3-5, students should be able to predict and describe the results of geometric transformations and recognize line and rotational symmetry. In the 6 - 8 grade band, PSSM recommends that students should apply transformations; describe size, positions and orientations of geometric shapes under slides, flips, turns, and scaling; identify the center of rotation and line of symmetry; and examine similarity and congruence of these figures.

The *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (Focal Points) (NCTM, 2006) further extended the recommended standards and delineated a coherent progression of concepts and expectations for students with descriptions of the most significant content for curriculum focus within each grade level from pre-kindergarten through grade eight. *Focal Points* extends the mathematics ideals set forth in the PSSM by targeting curriculum content and by providing resources that support the development of a coherent curriculum (Fennell, 2006). The Focal Points document reinforces the need for students to discuss their thinking, to use multiple representations that bring out mathematical connections, and to use problem solving in the process of learning.

Of these milestone documents, PSSM (2000) offers the most specific and delineated recommendations for school mathematics content. Sufficient time has passed since the publication of PSSM to expect to observe substantial alignment to the recommended content in published textbooks. The NCTM (1989) stated that they expected the standards to be reflected in textbook content and that the standards should also be used as criteria for analyzing textbook content.

The PSSM (NCTM, 2000) can only be put into practice when its recommendations can be implemented. Hiebert and Grouws (2007) emphasize that the most important factor in student achievement is *opportunity to learn*, and one criterion for student *opportunity to learn* is the expectation that the prescribed curriculum standards be reflected within textbook contents (NCTM, 1989). The textbook is an influential factor on student learning (Begle, 1973; Grouws et al., 2004; Schmidt et al., 2001; Valverde et al., 2002), and it represents a variable that can be easily manipulated. *On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Evaluations* (National Research Council, 2004) suggests that curriculum evaluation should begin with content analyses. Confrey (2006) affirms that content analysis is a critical element in the link between standards and the effectiveness of the curriculum. Textbook content analysis typically focuses on specific characteristics of the textbooks' content. Of the various characteristics analyzed, *opportunity to learn* and levels of cognitive demand are frequently used as measurements of the potential effectiveness of the reviewed materials. Both the characteristics, *opportunity to learn,* and levels of cognitive demand, are discussed in the next section.

**Opportunity to Learn and Levels of Cognitive Demand**

Tornroos (2005) describes the *intended* curriculum as the goals and objectives that are set down in curriculum documents; the curriculum documents most frequently used in the classroom are textbooks. An important contributing factor in learning outcomes is the *opportunity to learn* (OTL) based on textbook content (Tornroos, 2005). Tornroos found a high correlation between an item level analysis and student performance on the Third International Mathematics and Science Study (1999) and suggested that content analysis of textbooks would be valuable when looking for justification for different student achievement in mathematics.

Schmidt (2002) suggested that differences in student *opportunity to learn* did not suddenly appear in the eighth grade level, but rather in earlier grades, and that differences in curriculum diversity, to a large degree, cost student achievement exceedingly. Tarr, Reys, Barker, and Billstein (2006) report that it is crucial to identify and select textbooks that present critical features of mathematics that support student learning and assist teachers in helping students to learn. Tarr et al. describe the critical features of providing support, focus, and direction in the mathematics textbook and they call for the analysis of content emphasis within a textbook and across the span of textbooks within a series.

*Opportunity to learn* can be studied in various ways as indicated above, and OTL can have a variety of meanings. Although Tornroos and Schmidt considered the relationship of OTL to test performance, Floden (2002) determined the *opportunity to learn* by the emphasis a topic receives in the written materials in the form of textbooks since they are the form used by the student. This study takes a somewhat broader view and considers *opportunity to learn* not only by the amount of emphasis a mathematical concept receives in student textbooks but also by the nature of lesson presentations, types of tasks presented for student activity, and the level of cognitive demand required by students to complete tasks.

The NCTM set forth ideals for mathematics with recommendations for the teaching and learning of worthwhile tasks, including expectations that students will develop problem solving skills and critical thinking abilities. The PSSM (NCTM, 2000) document describes the necessity for learning mathematics content through meaningful activities that focus on the *Process Standards*: problem solving, reasoning and proof, communications, connections, and representations.

The mathematical tasks that students experience are central to learning because "tasks convey messages about what mathematics is and what doing mathematics entails" (NCTM, 1991, p. 24). Tasks need to provide an opportunity for the student to be active (Henningsen & Stein, 1997) and provoke thought and reasoning in complex and meaningful ways as categorized by Stein and Smith (1998). The results reported in Stein and Lane (1996) suggest, that in order for students to develop the capacity to think, reason, and problem solve in mathematics, it is important to start with high-level, cognitively complex tasks. Some of the *high-level* cognitive demand tasks include:

- exploring patterns (Henningsen & Stein, 1997)
- thinking and reasoning in flexible ways (Henningsen & Stein, 1997; Silver & Stein, 1996)
- communicating and explaining mathematical ideas (Henningsen & Stein, 1997; Silver & Stein, 1996)
- conjecturing, generalizing, and justifying strategies while making conclusions (Henningsen & Stein, 1997, Silver & Stein, 1996)
- interpreting and framing mathematical problems (Silver & Stein, 1996)
- making connections to construct and develop understanding (Silver & Stein, 1996; Stein & Smith 1998).

A major finding of Stein and Lane (1996) and Smith and Stein (1998) was that the largest learning gains on mathematics assessments were from students who were engaged in tasks with high levels of cognitive demand. Thus, the key to improving the performance of students was to engage them in more cognitively demanding activities (Boston & Smith, 2009) and hence provide the foundation for mathematical learning (Henningsen & Stein, 1997; Stein & Smith, 1998). Different types of tasks require higher levels of cognitive demands through active reasoning processes and the higher level demand tasks require students to think conceptually while providing a different set of opportunities for student cognition (Stein & Smith, 1998). Hence, students need to have the *opportunity to learn* worthwhile mathematical concepts, and be immersed in their mathematical studies with cognitively demanding tasks.

NCTM (1989) stated “let it be understood that we hold no illusions of immediate reform” (p. 255), but they held the vision of having classroom materials, such as textbooks, produced so that standards would be aligned and in-depth learning take place. Yet, since the initial publication of the *Standards*, little has been done to analyze textbook contents. Because students do not learn what they are not taught (Tornroos, 2005), it is essential to examine the extent to which mathematical topics are presented in textbooks. Clements (1998) indicates it is essential to examine the extent to which middle school mathematics textbooks attend to the development of the concept of transformations in available instruction and in mathematics research. If there is a barrier to students in “*opportunity to learn*” which prevents them from attaining the full benefits from the Standards, educators need to address what can be done to eliminate the barriers; one way to know if a problem exists due to the lack of included content is to analyze the content of textbooks.

With the inception of this study a pilot investigation was enacted to analyze the extent and treatment of geometric transformations lessons in two middle grades textbooks to discern if sufficient differences in the curricula were present (Appendix A). The results suggested that an analysis of a larger variety of textbooks was a worthwhile endeavor, and hence this study was implemented.

**Statement of the Problem**

Research indicates that students have difficulties in understanding the concepts and variations in performing transformations (Clements & Battista, 1998; Clements, Battista, & Sarama, 1998; Clements & Burns, 2000; Clements, Battista, Sarama, & Swaminathan, 1996; Kieran, 1986; Magina & Hoyles, 1997; Mitchelmore, 1998; Olson, Zenigami & Okzaki, 2008; Rollick, 2009; Soon, 1989). Given recommendations from the mathematics education community about the inclusion of transformations in the middle grades curriculum, we might expect to observe the concepts in published textbooks; hence, there is a need to analyze contents. However, few examinations of the contents within textbooks have been found with respect to the alignment or development of mathematics concepts with current recommendations (Mesa, 2004), and none have been found to focus on the analysis of presentations and *opportunity to learn* for the study of geometric transformations.

Because textbooks are the prime source of curriculum materials on which the student can depend for written instruction (Begle, 1973; Grouws et al., 2004; Schmidt et al., 2001; Valverde et al., 2002), the nature of the treatment of these concepts needs to be examined to insure that students are provided appropriate opportunities to learn. As a result there emerges a need to analyze the treatment of geometric transformations in middle school mathematics textbooks. This study examined the nature and treatment of geometric transformations through the analysis of published middle grades textbooks in use in the United States. The textbooks chosen included publisher generated textbooks, curriculum project-developed textbooks, and National Science Foundation (NSF) funded curriculum materials; it was assumed that these textbook types would likely present the concepts differently. The lesson concepts were analyzed in terms of content of the narrative, examples offered for student study, number and types of student exercises, and the level of cognitive demand expected by student exercises. Additionally, this investigation addressed the possible changes of focus in the progression of content from grade six through grade eight.

**The Purpose of the Study**

This study had three foci: 1) to analyze the characteristics and nature of geometric transformation lessons in middle grades textbooks to determine the extent to which these textbooks provide students the potential *opportunity to learn* transformations as recommended in the curriculum standards; 2) to describe the content of geometric transformation lessons to identify the components of those lessons, including how they are sequenced within a series of textbooks from grades 6 through grade 8 and across different publishers; 3) to determine if student exercises included with the transformation lessons facilitate student achievement by the inclusion of processes that encourage conceptual understanding with performance expectations.

Four types of middle school transformations were examined: the three rigid transformations and their composites (translations, reflections, and rotations), where rigid refers to the preimage figure and resulting image figure being congruent; and dilation where figures are either enlarged or shrunk. The sections of student exercises that follow the lesson presentations were investigated for the level of cognitive demand required for completion because problems of higher levels of cognitive demand increase students' conceptual understanding (Boston & Smith, 2009; Smith & Stein, 1998; Stein, Smith, Henningsen, & Silver, 2000).

**Research Questions**

This study investigated the nature and treatment of geometric transformations in student editions of middle grades mathematics textbooks in use in the United States. In doing so, the following research questions were addressed.

1. What are the physical characteristics of the sample textbooks? Where within the textbooks are the geometric transformation lessons located, and to what extent are the transformation topics presented in mathematics student textbooks from sixth grade through eighth grade, within a published textbook series, and across different publishers?
2. What is the nature of the lessons on geometric transformation concepts in student mathematics textbooks from sixth grade through eighth grade, within a published textbook series?
3. To what extent do the geometric transformation lessons' student exercises incorporate the learning expectation in textbooks from sixth grade through eighth grade within a published textbook series, and across textbooks from different publishers?
4. What level of cognitive demand is expected by student exercises and activities related to geometric transformation topics in middle grades textbooks? The level of cognitive demand is identified using the

parameters and framework established by Stein, Smith, Henningsen, and Silver (2000).

Together, the answers to these four questions give insight into potential *opportunity to learn* that students have to study geometric transformations in the middle grades textbooks.

**Significance of the Study**

The mathematics curriculum in the United States has been defined as being in need of vast improvement (Dorsey, Halvorsen, & McCrone, 2008; Grouws & Smith, 2000; Kilpatrick, 1992, 2003; Kilpatrick, Swafford, & Findell, 2001; Kulm, Morris, & Grier, 1999;  McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers, & Cooney, 1987; National Center for Education Statistics (NCES), 2001, 2004; National Commission on Mathematics and Science Teaching for the 21st Century, 2000; NCTM, 1980, 1989, 2000; NRC, 2001, 2004; Schmidt, McKnight, & Raizen, 1996; U. S. Department of Education, 1996, 1997, 2000) and professional organizations have recommended changes (National Commission on Mathematics and Science Teaching for the 21st Century, 2000; NCTM, 1980, 1989, 2000; NRC, 2001; 2004; U. S. Department of Education, 1996, 1997, 2000). Three of the most influential documents since the late 1980s were published by the NCTM (1989, 2000, 2006) and these documents set forth recommendations for the teaching and learning of worthwhile mathematical tasks in which students are expected to think critically.

Analysis of literature from both national (AAAS, 1999a; Braswell, Lutkus, Grigg, Santapau, Tay-Lim, & Johnson, 2001; Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Dorsey, Halvorsen, & McCrone, 2008; Fey & Graeber, 2003; Flanders, 1994b;

Kouba, 1988; NCES, 2000, 2001b, 2004, 2005; U. S. Department of Education, 1999, 2000) and international (Adams, Tung, Warfield, Knaub, Mudavanhu, & Yong, 2000; Ginsburg, Cook, Leinward, Noell, & Pollock, 2005; Husen, 1967; McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers, & Cooney, 1987; NAEP, 1998; NCES,2001a; Robitaille & Travers, 1992; U. S. Department of Education, 1998) reports indicate that achievement of U. S. students lags behind those of other countries; one specific area is spatial reasoning which is the foundation for understanding our three-dimensional world. Spatial reasoning, taught through transformations, has been neglected as an area for study by students in the middle grades (AAAS, 1999b; Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996; Clements & Battista, 1992; Clements & Sarama, 2007; Clopton, McKeown, McKeown, & Clopton, 1999; Gonzales, 2000; McKnight, Travers, Crosswhite, & Swafford, 1985; Sowder, Wearne, Martin, & Strutchens, 2004) and has been recognized as a mathematical topic in need of development within the world of learning (Battista, 2001a, 2007; 2009; Clements & Battista, 1992; Hoffer, 1981).

Many educators report that textbooks are common elements in mathematics classrooms and that textbook content influences instructional decisions on a daily basis (Brasurell et al., 2001, Grouws & Smith, 2000; NRC, 2004; Weiss, Banilower, McMahon, & Smith, 2001). Because approximately three fourths of textbook content is typically covered each year in middle school mathematics classrooms (Weiss et al., 2001), the textbook directly affects students' *opportunity to learn*. Because the textbook is an influential factor on student learning (Begle, 1973; Grouws, Smith & Sztajn, 2004; Schmidt et al., 2001; Valverde et al., 2002), it becomes important to document the opportunities presented in textbooks for students to gain competency in important mathematical concepts beyond the level of procedural skills. If the content is not present in the textbook, or placed where it is easily omitted, then students most likely will not learn it.

School mathematics curriculum is generally delivered by use of the textbook in the classroom (Herbel-Eisenmann, 2007; Jones, 2004; Jones & Tarr, 2008; Lee, 2006; Pehkonen, 2004; Tarr, Reys, Barker, & Billstein, 2006). Clements, Battista, and Sarama (2001) and Battista (2009) state geometric topics in middle school textbooks tend to be a jumble of unrelated topics without a focus on concept development or problem solving. The insufficient development of spatial sense prior to the study of formal geometry in high school places students at a disadvantage for achievement and success in future mathematics courses (Clements, 1998). Flanders (1994a) and Tarr, Chavez, Reys and Reys (2006) indicate that textbooks in grades K-8 tend to be uniform in giving arithmetic topics preferential treatment over geometry, and that the topics in geometry are the least covered and are usually found at the end of the textbooks. Topics that appear near the end in textbooks can easily be eliminated from the material that is covered by the teacher in the classroom to conserve time for various other mandatory curriculum requirements.

**Conceptual Issues and Definitions**

Composite Transformation - A complex transformation achieved by composing a sequence of two or more rigid transformations to a figure (http://www.cs.bham.ac.uk). The transformations that are combined in composite transformations are translations, reflections, and rotations. Any two rigid transformations can be combined to form a composite transformation, and the resulting image can be redefined as one of the original transformations (Wesslen & Fernandez, 2005).

Congruent Figures - Two dimensional figures are congruent when they are the same shape and size; all points coincide when one figure is superimposed over the other.

Curriculum - Herein is defined as the written (textbook) curriculum. The curriculum is described as the vehicle by which the course content is dispersed. The written materials are designed to include all of the components of the course curriculum and contain the course topics, both scope and sequence.

Dilation - Dilation is a transformation that either reduces or enlarges a figure. Dilation stretches or shrinks the original figure and alters the size of the preimage; hence, it is not rigid because it does not satisfy the condition that the image is congruent to the preimage. Dilation is a similarity transformation in which a two-dimensional figure is reduced or enlarged using a scale factor ($\neq 0$), without altering the center of dilation.

Glide Reflection - A glide reflection is a reflection followed by a translation along the direction of the line of reflection. In order to perform a glide transformation, information about the line of reflection and the distance of the translation is needed; in the glide all points of the preimage figure are affected by the movement (Wesslen & Fernandez, 2005).

Image - The name given to the figure resulting from performing a transformation is called an image. The letters marking the image points are the same letters as used on the preimage but often marked with a prime symbol (′).

Line of Symmetry - A line that can be drawn through a figure on a plane so that the figure on one side of the line is the mirror image of the figure on the opposite side.

Middle Grades / Middle School - for this study consists of grades 6, 7, and 8.

Mira® - A geometric manipulative device that has reflective and transparent qualities is a Mira®.

Opportunity to Learn - For the purpose of this study, *opportunity to learn* is defined as how a concept is addressed in the curriculum, including the amount of emphasis a mathematical concept receives in the written curricula, the nature of the presentations, the types of tasks that are presented for student study, and the level of cognitive demand required by students to complete provided tasks.

Preimage - The name given to the original figure to which a transformation is applied is called a preimage. The original figure is called the preimage, and the resulting figure, after a transformation is applied, is called the image. The preimage figure's points, or vertices, are usually labeled with letters.

Reflection - A type of rigid transformation where the figure appears to be flipped over an axis or line on a plane is called a reflection; the line may be the x- or y-axis, or a line other than one of the axes. This line is called the line of reflection. The object and its reflection are congruent but the position and alignment of the figures is reversed. A mental picture of the reflection motion would be described as lifting the shape out of its plane and flipping it over an indicated line and then putting it back down on the plane. When a reflection figure is viewed in a mirror, the mirror edge becomes the line of reflection, or the line over which the preimage is reflected. The terms "*flip* or *flipping*" are often used to describe this type of transformation.

Rigid Transformation - A transformation whereby the pre-image figure and the resulting image are congruent is called a rigid transformation. Three types of transformations are rigid motion transformations - translations (slides), reflections (flips), and rotations (turns) - because the original figure is not distorted in the process of being transformed (Yanik & Flores, 2009).

Rotation - A rotation is a type of rigid transformation where a two-dimensional figure is turned a specified angle and direction about a fixed point called the center of rotation. A rotation is also called *turn*. The rotation turns the figure and all of the points on the figure through a specific angle measurement where the vertex of the angle is called the center of rotation. For a description of rotation, two pieces of information are needed: the center and angle of rotation, and the direction of the rotation; the center of rotation is the only point that is not affected by the rotation (Wesslen & Fernandez, 2005).

Scale Factor - The size change of the figure in dilation is called the scale factor. The change in size of the length of a side of the image to the corresponding side length on the preimage is given by a comparison of the size of the image over the size of the preimage; this is represented as a ratio which represents the scale factor for the dilation. For example, a preimage of 3 (units), and an image of 12 (units), would be written as 12 over 3 in simplest form, i.e. 12/3 = 4/1, hence the scale factor is 4. The scale factor is always expressed with the image units first, or in the numerator of the fraction. If the scale factor is between zero and one, the dilation is a reduction; if the scale factor is greater than one the dilation is an enlargement. If the scale factor is 1, the preimage and the image are the same size.

Similar Figures - Two polygons are similar if the measures of their corresponding sides are proportional and their corresponding angle measures are congruent. The result of a dilation transformation produces similar figures.

Size Change - Size Change is another term for dilation of a figure.

Student Performance Expectations - Performance Expectations are defined as the type of responses elicited by the work required in the tasks, activities, and exercises presented for student experience.

Symmetry - Symmetry is the correspondence in size, form, and arrangement of parts of a figure on opposite sides of a line. In rigid motion transformations, congruent (symmetric) figures are produced, hence there is symmetry in the pair of figures constructed by translations, reflections, and rotations. A pattern is said to be symmetric if it has at least one line of symmetry. In symmetric figures, the angle measures, sizes, and shapes of the figures are preserved (http://www.math. csusb.edu). A figure is said to have rotational symmetry if the figure can be rotated less than 360 degrees about its center point and the resulting figure is congruent to the preimage.

Transformation - The process by which a two-dimensional figure is moved on a plane by mapping the preimage set of points to a second set of points called the image. A transformation involves a physical or mental manipulation of a figure to a new position or orientation on a plane (Boulter & Kirby, 1994).

Translation - A geometric translation consists of moving a point, line, or figure to a new position on a two dimensional surface. The definition of translation specifies that each point of the object is moved the same distance and in the same direction.

Usiskin et al. (2003, p. 302) calls a geometric translation “the sliding of an object from one to another place without changing its orientation.” The simplest of the transformations is the translation, sometimes called a *slide,* or a *shift.* The symbolism for a point translation may be labeled as A → A′. The arrow indicates that the point A is being moved to a new position labeled A prime (A′). An arrow may be illustrated on a graph to indicate the direction for movement of the object, and the shaft of the arrow indicates the intended distance of the movement.

Two-dimensional - A term used to represent figures in which only the length and width are measured on a plane, there is no thickness.

Vector - An arrow symbol representing the distance and direction for the translation of a figure is called a vector. The arrow symbol, when illustrated on the graph, is called a “translation vector” because it shows the direction and magnitude of the translation. The direction and distance that the preimage is to be moved can also be represented by an ordered pair, (±x, ±y), where the ±x represents the amount of movement right or left along the x-axis, and the ±y represents the amount of movement up or down along the y-axis of a coordinate graph. The intended movement values are relative to the original position of the point or object, not to the origin.

## Chapter 2: Literature Review

The purpose of this literature review is to present relevant findings and investigations to establish the foundation on which this study was developed, as well as delineate the concepts and content on which the conceptual framework for analysis was constructed. This review is divided into three major sections. The first presents discussion on different types of curriculum, the influence that textbooks bring to bear on determining classroom curriculum, as well as criticisms of the curriculum and the need for content analysis. The second section reviews findings from existing content analysis studies, and identifies foci of content analysis studies. The third section presents findings on the issues raised in research relating to misconceptions and difficulties that students experience with learning geometric transformation concepts to determine the areas and specific concepts that should be delineated for investigation.

**Literature selection.** Articles, research reports and studies, dissertations, and conference reports were located using Dissertation Abstracts International, Education Full Text, Education Resources Information Center (ERIC), Google Search, JSTOR Education, and H. W. Wilson Omnifile, as well as University Library services. An exploration of related research was conducted starting with appropriate chapters from the National Council of Teachers of Mathematics (NCTM) *Handbook of Research on Mathematics Teaching and Learning* (1992)*, Second Handbook of Research on Mathematics Teaching and Learning* (2007), NCTM Standards documents (1989, 2000, 2006), the NCTM's journals, including the *Journal for Research in Mathematics*

*Education* and *Mathematics Teaching in the Middle School*, and the NCTM Yearbooks (1971, 1987, 1995, 2009) that focused specifically on geometry. The reference lists in the documents provided additional resources for locating related studies and publications from additional educational sources.

**The Curriculum and the Textbook**

This section of the literature review presents discussion on different types of curriculum, textbook use in the classroom, and the influence that textbooks have on course content. The information presented here further illustrates the need for content analysis.

**Types of curriculum.** Many educators have written about different types of curriculum and the specific characteristics that delineate each (Jones, 2004; Klein, Tye, & Wright, 1979; Porter, 2002, 2006; Reys, Reys, Lapan, Holliday, & Wasman, 2003; Stein, Remillard, & Smith, 2007; Usiskin, 1999; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002; Venezky, 1992). In general, the term curriculum has different meanings specific to the context in which it is used (Stein, Remillard, & Smith, 2007).

Stein, Remillard, and Smith (2007) used the terms curriculum materials and textbooks interchangeably, and called this type of curriculum “formal”, “institutional”, or “intended”. Usiskin (1999) and Klein et al. (1979) also used the term “formal” while Jones (2004) labeled this type of curriculum “prescribed”, as did Porter (2006); however, Porter used the term “intended” synonymously for this curriculum. The enacted curriculum refers to how the written curriculum is delivered in the classroom (Porter, 2004, 2006; Stein, Remillard, Smith, 2007), the assessed curriculum is the content being tested (Porter, 2004, 2006; Jones, 2004), and the attained or received curriculum is the knowledge obtained by the student (Jones, 2004; Venezky, 1992).

The textbook with instructional resources and guides prepared for use by students and teachers is the vehicle by which the written curriculum of a course is dispersed. The written materials are designed to include all of the components of the course and contain the course topics, both scope and sequence. Because the student normally has direct access to the mathematics textbook, it is the student textbook that represents the written curriculum in the classroom. Thus, it is important to reflect on the role of the textbook because the textbook represents the scope and sequence of concepts as they are generally presented to students.

In summary, the term curriculum can have different meanings depending on the focus and topic being discussed, examined, or investigated. The textbook serves as the obvious link between the content prescribed for a course and the scope and sequence of what is actually taught in the classroom (i.e., “enacted curriculum”).

**The mathematics textbook and the curriculum.** Senk and Thompson (2003) offer a detailed observation of mathematics in the nineteenth century, and explain that textbooks were structured so that topics were typically introduced by stating a rule, showing an example and then offering numerous exercises for student practice. Commercially published textbooks were primarily used as instructional guides (Clements, 2007; Richaudeau, 1979; Senk & Thompson, 2003). Throughout the 20$^{th}$ century and even into the first part of the 21$^{st}$ century, the most prevalent type of textbook presentation was still the style offering exposition, examples, and exercises (Kang & Kilpatrick, 1992; Love & Pimm, 1996; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002), hence the present type of textbook lesson presentation and relevant emphasis placed on specific mathematics topics needs to be examined to determine the alignment with the Standards recommendations.

Educators suggest that the textbook has a marked influence on what is taught and presented in the classroom (Begle, 1973; Driscoll, 1980; Haggarty, & Pepin, 2002: Porter, 1989; Reys, Reys, Lapan, Holliday, & Wasman 2003; Robitalle & Travers, 1992; Schmidt, McKnight, & Raizen, 1997; Schmidt et al., 2001; Schmidt, 2002; Tornroos, 2005). Students typically do not learn what is not in the textbook (Begle, 1973; Jones, 2004; Porter, 1995; Reys, Reys, & Lapan, 2003; Schmidt, 2002) and teachers are unlikely to present material that is not there (Reys, Reys, & Lapan, 2003). Begle (1973) noted that the textbook is a powerful influence on learning so that learning seems to be directed by the textbook rather than by the teacher. Haggarty and Pepin (2002), on their evaluation of learners, indicate that presentation of different mathematics offers students different opportunities to learn prescribed mathematics. Similarly, Lenoir (1991, 1992) and Pellerin and Lenoir (1995) indicate that the textbook exerts a large degree of control over the curriculum and teaching practices in general. Therefore textbook content analyses are needed.

The textbook continues to be a determining factor in the curriculum in many mathematics classrooms in this nation, particularly at the elementary and middle school levels (Howson, 1995; Venezky, 1992; Woodward, Elliott, & Nagel, 1988). Teachers rely heavily on the textbook for curriculum design, scope, and sequence (Stein, Remillard, and Smith, 2007) as well as for guidance on pedagogical issues. Thus, the textbook is the most common channel through which teachers are exposed to the communications from professional organizations in reference to mathematics standards and to recommendations from the research community (Collopy, 2003); both standards and recommendations translate into immediate determinants for teaching practices (Ginsbury, Klein, & Starkey, 1998). Grouws and Smith (2000), Peak (1996), and Tarr, Reys, Barker, and Billstein (2006) report that throughout mathematics classrooms in the United States, the textbook holds a prominent position and represents the expression of the implicit curriculum requirements.

These various educators suggest that the mathematics textbook is regarded as the authoritative voice that directs the specified mathematics curriculum content in the classroom (Haggarty & Pepin, 2002; Olson, 1989). The influence that the textbook maintains is related to most of the teaching and learning activities that take place in the mathematics classroom (Howson, 1995).

**The textbook and its use in the classroom.** Although professional organizations (NCTM, 1989, 2000), individual states, and local educational governing departments have designed frameworks to guide mathematics curriculum, the development of the structure and content of the written curriculum in publisher generated textbooks is done by textbook authors and publishing staff. However, because "publishers attempt to meet the criteria of all such frameworks, including scope and sequence requirements, the educational vision of any one state framework is, at best, diluted" (Clements, 2008, p. 599). The effect is often poor performance by students (Ginsburg, Cook, Leinwand, Noell, & Pollock, 2005; Kouba, 1988; McKnight et al., 1987; McKnight, Travers, Crosswhite, & Swafford, 1985; Mullis et al., 1997) and a U. S. school mathematics curriculum that is labeled a "mile wide, inch deep" (NCES, 1996; Schmidt et al., 1997, p122). The problem with the written curriculum exists in

- the large quantity of topics presented (Clements & Battista, 1992; Ginsburg, Cook, Leinward, Anstrom, & Pollock, 2005; Jones, 2004; Porter, 1989; Snider, 2004; Valverde et al., 2002),
- the lack of depth of study for specific topics (Jones, 2004; McKnight et al., 1987; Schmidt, McKnight, & Raizem, 1997; Snider, 2004; Tarr, Reys, Barker, & Billstein, 2006; Valverde et al., 2002),
- the superficial nature of the material presented (Fuys, Geddes, & Tischler, 1988; Schmidt, McKnight, & Raizem, 1997; Schmidt et al., 1997; Tarr, Reys, Barker, & Billstein, 2006),
- the highly repetitive nature of topics appearing year after year (Flanders, 1987; McKnight et al., 1987; Schmidt, McKnight, & Raizem, 1996; Senk & Thompson, 2003; Snider, 2004; Tarr, Reys, Barker, & Billstein, 2006; Usiskin, 1987),
- the number of breaks between mathematics topics (Valverde et al., 2002),
- the fragmentation of mathematical topics (Flanders, 1994; Herbst, 1995; McKnight, Crosswhite, Dossey, Keffer, Swafford, Travers, & Cooney, 1987; U. S. Dept. of Ed., 1996, 1997, 1998),
- the contextual features and problem performance requirements (Herbst, 1995; Li, 1999, 2000; Schmidt et al., 1996; Schutter & Spreckelmeyer, 1959; Stevenson & Bartsch, 1992),
- the low level of expectations for student performance (McKnight, Crosswhite, Dossey, Keffer, Swafford, Travers, & Cooney, 1987; Snider, 2004),
- the low level of cognitive demand for student performance (Fuys, Geddes, & Tischler, 1988; Jones, 2004; Li, 2000; Smith & Stein, 1998, Stein & Smith, 1998;

Senk & Thompson, 2003),

- the placement as well as the amount of new material, enrichment activities, and the inclusion of the use of technology and manipulatives (Clements, 2000; Flanders, 1987, 1994; Jones, 2004).

The above provides a partial list of studies that have investigated different aspects of the written curriculum. Any or all of these issues with curriculum might be analyzed related to content analysis.

Dissatisfaction with textbooks in the United States has been reported by many educators (Ball, 1993; Flanders, 1987; Jones, 2004; Heaton, 1992; Ma, 1999; Schifter, 1996). Project 2061, by the American Association for the Advancement of Science (AAAS), and the U. S. Department of Education found that commercially published textbooks were “unacceptable” with regard to content emphasis (p. 1), and that the textbooks provided little sophistication in the presentation of mathematical topics from grade six to grade eight. Inconsistency and weak coverage of mathematical concepts were found in most of the textbooks examined (AAAS, 2000). Valverde et al. (2002) voiced their concern that, with the composition of presently published U. S. textbooks and the classroom time available, the student is severely limited in the number of concepts that would be experienced and the level of importance that the topics receive.

Yet, simultaneously, reports indicate that mathematics textbooks are frequently used in classrooms for teaching practices and student activities. From the 2000 national survey of the National Assessment of Educational Progress, researchers found that more than 90% of teachers in grades 5-8 use commercially published textbooks in their classrooms, and more than 60% of the classrooms use a single textbook during the school year. A large number of 8th grade teachers reported using the textbook "almost every day" (p. 133). More than 95% of teachers reported that they use the textbook more than half of the classroom teaching time, and 60% of the teachers reported using the textbook as the main source for lesson presentations and student exercises (Grouws & Smith, 2000).

Similarly, approximately 75% of eighth grade students worked from their textbooks on a daily basis (Braswell et al., 2001; Grouws & Smith, 2000). More than 90% of students reported doing mathematics problems from their textbooks during almost every class (Linquist, 1997; Tarr, Reys, Barker, & Billstein, 2006). Collectively, these reports suggest that the textbook has come to represent the formal curriculum, and that the textbook determines and dominates what goes on in the classroom (Hummel, 1988) as well as what students have an *opportunity to learn* (Down, 1988). Hence, because the textbook is used to determine classroom curriculum it is important to analyze the content of textbooks used.

**Curriculum analysis.** The curriculum was not recognized as an entity to be developed until the 1950s (Howson, Keitel, & Kilpatrick, 1981; Kilpatrick, 2003) and little attention was given to the design or quality of textbooks prior to the 1970s (Senk & Thompson, 2003; Woodward, Elliott, & Nagel, 1988). So the need for specific formal content analysis did not arise until after the products of the curriculum development projects of the 1970s and 1980s were completed. Kilpatrick (2003) states "the job of curriculum analyzer, like the job of the curriculum developer, is a 20th century invention" (p. 182). Hence, the study of mathematics textbook content analysis has only appeared in the literature during, approximately, the last 30 to 40 years.

Many questions about the characteristics and influences of the textbook still remain to be answered (Chappell, 2003), such as: “to what extent are these curricula similar to or different from each other?” (p. 285) and to what extent are different series different from one another? Reys, Reys, Lapan, Holliday, and Wasman (2003) suggest that “different types of curriculum materials tend to focus on different priorities” (p. 77). Chappell (2003), in summarizing the research on middle school programs developed in the 1990s, states “differences among the three middle school curricula are apparent in their structure and design” (p. 297). Yet, what seems to be missing in a comparison of the reported curricula is an analysis of the contents that provide the students with the *opportunity to learn* the topics that are the focus of mathematical learning.

As seen in the previous section, the textbook plays a prominent role in the mathematics education of students in the United States. Hence, an investigation of the content within these textbooks appears to be needed to determine the level of students’ *opportunity to learn* from the available mathematics presentations (Grouws & Smith, 2000; Herbst, 1995; Julkunen, Selander, & Ahlberg, 1991; Kilpatrick, 2003; Leburn, Lenoir, Laforest, Larosse, Roy, Spallanzani, & Pearson, 2002; Peak, 1996; Venezky, 1992).

**Related Textbook Content Analyses**

Assessment of student achievement normally follows the teaching-learning process. Analysis of student achievement must address multiple variables; one of these variables is to focus on the instructional materials that are used in the educational setting (AAAS, 2000). Textbook content analysis is certainly not new in its appearance in publications, but on closer inspection the title “content analysis” encompasses many different aspects of investigating the written materials. Various types of studies were identified under the general category of content analysis. The general ideas gathered from these content analyses guided the structure of this study.

The first type of content analysis literature reviewed synthesizes content analysis studies that have focused on development of generalized instructions and directions on how to evaluate and select textbooks for specific goals and curriculum for classroom use. These reports offer insight into the development of a coding instrument to analyze textbooks.

The second type of literature reviewed summarizes content analysis that specifically evaluated textbooks in reference to coverage of mathematical content in comparison to items on international tests, as for example, the Second International Mathematics Study (SIMS) and Trends in Mathematics and Science Study (TIMSS). These studies addressed students' *opportunity to learn* the material addressed on international tests in comparison to textbook presentations and student exercises.

The third type of content analysis literature reviewed focuses on the content of mathematical topics and concepts, lesson narrative presentations, examples offered for student study, expected student performance in presentations, and the *levels of cognitive demand* needed for student engagement. This section's reviewed literature was most applicable to the development of the coding instrument used for this study.

**Types of textbook content analyses.** Textbook content analyses have focused on many different aspects of available curriculum resources. It was the aspects identified in these studies that provide insight into the different types of data collected. There have been investigations on

- gender and ethnicity bias (Rivers, 1990)
- page count (Flanders, 1987; Jones, 2004)
- total area of lesson presentation and the weight of textbooks (Shields, 2005)
- topics of mathematics covered at particular grade levels (Flanders, 1987, 1994a; Herbel-Eisenmann, 2007; Jones, 2004; Jones & Tarr, 2007; Li, 2000; Mesa, 2004; Remillard, 1991; Stylianides, 2005, 2007; Sutherland, Winter, & Harris, 2001; Wanatabe, 2003)
- repetition of topics from one year to the next (Flanders, 1987; Jones, 2004)
- teacher edition content (Flanders, 1987; Stylianides, 2007; Watanabe, 2003)
- teachers' use of textbooks (Freeman & Porter, 1989; Leburn, Lenoir, Laforest, Larosse, Roy, Spallanzani, & Pearson, 2002; Tarr, Chavez, Reys, & Reys, 2006; Witzel & Riccomini, 2007)
- comparison of international textbook series (Adams, Tung, Warfield, Knaub, Mudavanhu, & Yong, 2001; Haggarty & Pepin, 2002; Li, 2000; Mesa, 2004; Sutherland, Winter, & Harris, 2001)
- voice of the textbook (Herbel-Eisenman, 2007)
- content of textbook topics in comparison to national or international test questions (Flanders, 1994a; Mullis, 1996; Tornroos, 2005; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002)
- how to analyze content for textbook selection (Confrey, 2006; Kulm, 1999; Lundin, 1987; McNeely, 1997; U. S. Department of Education, Exemplary and Promising Mathematics Programs, 1999)
- analysis of content to align or explain student achievement (Kulm, Morris, Grier,

2000; Kulm, Roseman, & Treistman, 1999)

- analysis of student exercises and performance expectations (Jones & Tarr, 2007; Li, 2000; Tornroos, 2005)
- narrative of specific content over multiple topics in mathematics (AAAS, Project 2061, 2000; Flanders, 1987; Haggarty & Pepin, 2002; Herbel-Eisenman, 2007; Johnson, Thompson, & Senk, 2010; Jones, 2004; Jones & Tarr, 2007; Li, 2000; Martin, Hunt, Lannin, Leonard, Marshall, & Wares, 2001; Mesa. 2004; Porter, 2002, 2004; Remillard, 1991; Rivers, 1990; Shield, 2005; Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998; Sutherland, Winter, & Harris, 2001; Stylianides, 2005, 2007; Tarr, Reys, Barker, & Billstein, 2006, Watanabe, 2003)
- evaluation of experimental and quasi-experimental designs on series and student achievement (NRC, 2004; Senk & Thompson, 2003; What Works Clearinghouse).

These delineated studies have contributed to construction of the coding instrument for this study in the area of physical characteristics of the textbooks, itemization of content in student exercises, and student performance expectations. Also closely related to the research of this dissertation were studies on the following topics:

- content analysis of targeted areas of topics in mathematics (Haggarty & Pepin, 2002; Johnson, Thompson, & Senk, 2010; Jones, 2004; Jones & Tarr, 2007; Mesa, 2004; Rivers, 1990; Soon, 1989)
- textbook lesson narratives (Herbel-Eisenman, 2007; Mesa, 2004; Johnson, Thompson, & Senk, 2010; Shield, 2005; Sutherland, Winter, & Harris, 2001)
- student opportunity to learn content (Floden, 2002; Haggarty & Pepin, 2002; Tornroos, 2005),

- student cognitive demand (Jones, 2004; Jones & Tarr, 2007; Porter, 2006; Stein, Grover, & Henningsen, 1996; Smith & Stein, 1998; Stein & Smith, 1998)

The preceding list delineates the types of analyses that provide the background for the context used in this study as they address targeted content, lesson narratives, cognitive demand required to complete student exercises, and student potential *opportunity to learn*.

**Curriculum content analysis for textbook selection.** In 1987, the California State Board of Education rejected the 14 textbook series that were submitted for adoption (Flanders, 1987). In response, the California State Department of Education published a resource entitled Secondary Textbook Review: General Mathematics, Grades Nine through Twelve (Lundin, 1987). The purpose of this document was to assist in the selection of textbooks that would align with curriculum standards in California. This document is termed "a trailblazer" (pp. iv) because it suggested new procedures and offered an instrument for review of published textbooks. The document contains reviews of 18 textbooks and addresses four major areas:

- publisher description and information on the textbook's intended audience
- emphasis given to each mathematical topic
- extent to which content aligns to the curriculum standards using the number of lessons as the method of analysis
- extent and location of mathematics topics in the textbook's instructional material and teacher resources.

The textbook areas reviewed encompass only the textbook's instructional pages and did not include supplementary pages, appendices, index, etc.

In 1997, the U. S. Department of Education published *Attaining Excellence: TIMSS as a Starting Point to Examine Curricula: Guidebook to Examine School Curricula* (McNeely). This publication extended the process of content analysis to offer five methods for analysis that vary on the resources needed for implementation of the procedures as well as the types of conclusions that can be drawn from the analyses. The five methods are: 1. The TIMSS Curriculum and Textbook Analysis; 2. National Science Foundation (NSF) Instructional Materials and Review Process; 3. California Department of Education Instructional Resources Evaluation; 4. Council of Chief State School Officers (CCSSO) State Curriculum Frameworks and Standards Map; and 5. American Association for the Advancement of Science (AAAS) Project 2061 Curriculum-Analysis Procedure. These five methods were included in the AAAS reports because the methods employed in the evaluation process specifically tied the analysis to mathematics standards.

The American Association for the Advancement of Science Project 2061 (2000) designed procedures to critique published middle school mathematics curriculum materials to assess the degree of alignment of the content to selected benchmarks and mathematics standards. Thirteen NSF and traditional textbooks were evaluated and rated on their core content on number concepts and skills, geometry concepts and skills, and algebra graphing concepts and skills. The analysis procedures included four phases:

- identify a specific set of learning goals and benchmarks for analysis
- execute a preliminary inspection of the content of the textbooks
- perform an in-depth analysis of the curriculum materials for alignment between the content and the benchmarks

- summarize the findings (Kulm, 1999; Kulm, Roseman, & Treistman, 1999).

Further literature reviewed on content analyses has added to the elements included in this research study. As for example, in 2004, the National Research Council (NRC) identified and examined almost 700 evaluative studies on 19 mathematics textbook series curricula from grades K-12. The value of the NRC work was in the development of models for curricular analyses. The NRC report indicates a full comprehensive content analysis should include identification and description of the curriculum theory; scrutiny of program objectives; applicability to local, state or national standards; program comprehensiveness, content accuracy; and support for diversity.

The work of the NRC (2004) was extended by Shield (2005) and Tarr, Reys, Barker, and Billstein (2006) to focus on developing mathematics textbook analysis strategies. Shield's work focused on textbook concepts and presentations with alignment to prescribed standards. The overall initial framework included four stages of evaluation methodology that are similar to those used in Project 2061. Tarr, Reys, Barker, and Billstein developed a general framework for reviewing and selecting mathematics textbooks; their framework is built around three dimensions, namely instructional focus, content emphasis, and teacher support.

**Curriculum content analysis for comparison to international tests.** Flanders' (1994a, 1994b), and Tornroos' (2005) content analysis compared textbook content to the mathematical content on international tests. The results from both studies were similar; they found that student achievement was directly related to the mathematical content presented in the textbook. In addition to evaluating the textbook content, Valverde et al. (2002) evaluated the physical features which included the lesson characteristics of the textbooks. These studies reinforce the relationship between the need for content inclusion as it relates to student achievement through *opportunity to learn* and the need for textbook content analyses. A summary of these reports follow.

Flanders (1994a, 1994b) published two investigations that examined eighth grade textbooks from six commonly used publishers. He compared the content of middle school textbooks with the subject matter found on the *Second International Mathematics Study* (SIMS) test, a total of 180 multiple-choice test questions. Flanders' study focused on the coverage of content in six non-algebra textbooks, and teachers' evaluation of student *opportunity to learn* and level of student performance expectations for achievement. Special attention was given to record topics that were classified as new, in $8^{th}$ grade text only; or reviewed, in both the $7^{th}$ and $8^{th}$ grade text; or not covered in either textbook. His findings showed that the textbooks were lacking in coverage of the topics of algebra and geometry. He found that approximately 50% of the geometry items were not covered in the middle grades textbooks at all, and the newest curriculum topics on algebra and geometry were presented least and latest in the sequence of the curriculum.

Similarly, Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002) examined 192 textbooks from grades 4, 8, and 12 from approximately 50 educational systems that took part in international testing. The focus of their analysis was the content of textbooks as well as the features of the textbooks themselves. Features classified included total number of pages, total text page area, and dimensions of the textbooks. Researchers identified the topics of mathematical content addressed, the number of times that the mathematical content changed in the sequence, the characteristics and nature of the lesson narratives, and student performance expectations. Their framework divided the material into blocks where each part could be analyzed independently. The findings indicated that many mathematics textbooks were mostly composed of exercises and questions posed.

In contrast, Tornroos (2005) was concerned with content validity of international tests and he analyzed student *opportunity to learn* by comparing student performance on the TIMSS 1999 assessment with an item based analysis of textbook content. Tornroos' study addressed the topic of *opportunity to learn* in three different ways. Among these approaches, an item-based analysis of textbook content resulted in fairly high correlations with student performance at the item level in TIMSS 1999. This study compared 162 mathematics items from the 1999 TIMSS test against 9 textbooks from grades 5, 6, and 7. Data were collected on the proportions of the textbooks that were dedicated to different topics, describing the mathematical content, and analyzing the textbook against the test items to see if the textbook contained sufficient material to provide the students with the ability to answer the questions correctly. Results indicated that the use of comparative analysis of international test results with textbook analysis provides a fairly high correlation with overall student performance, and hence yields a good measure of student *opportunity to learn*.

The preceding studies have contributed to elements incorporated into this study and helped to inform the development of the framework which will be discussed later in this chapter. In particular these studies suggest the need to look at the elements and include *"Where"* content is positioned in the textbook including: page count (Flanders, 1987; Jones, 2004), the quantity of content (Jones, 2004; Lundin, 1987) together with the sequence of the topics, and comparison of topics covered by grade level (Flanders, 1987, 1994a; Herbel-Eisenmann, 2007; Jones, 2004; Jones & Tarr, 2007; Li, 2000; Mesa, 2004; Remillard, 1991; Stylianides, 2005, 2007; Sutherland, Winter, & Harris, 2001; Wanatabe, 2003). These studies also suggest the need to look at *"What"* mathematical content is included, as: the nature of the lesson presentations (AAAS, Project 2061, 2000; Flanders, 1987; Haggarty & Pepin, 2002; Herbel-Eisenman, 2007; Johnson, Thompson, & Senk, 2010; Jones, 2004; Jones & Tarr, 2007; Li, 2000; Martin, Hunt, Lannin, Leonard, Marshall, & Wares, 2001; Mesa, 2004; Porter, 2002, 2004; Remillard, 1991; Rivers, 1990; Shield, 2005; Stein, Grover, & Henningsen, 1996; Shields, 2005; Soon, 1989; Stein & Smith, 1998; Sutherland, Winter, & Harris, 2001). Additionally, these studies suggest the need to focus on the *"How"* processes, including analysis of student exercises and performance expectations (Jones & Tarr, 2007; Li, 2000; Tornroos, 2005), and the level of student cognitive demand (Jones, 2004; Jones & Tarr, 2007; Porter, 2006; Stein, Grover, & Henningsen, 1996; Smith & Stein, 1998; Stein & Smith, 1998).

**Content Analysis on Textbook Presentations and Student Expectations**

This section presents studies that focus on textbook presentations, nature of mathematical content, and student performance expectations in textbooks. Even though this section is limited to the studies that concentrated on the written curriculum, there were variations noted among the topics of these studies. The variations delineated illustrate the differences in content analyses that contributed to the structure of the conceptual framework developed for this study.

Rivers (1990) investigated the content of textbooks adopted in 1984, and a second set adopted in 1990 for the inclusion of topics of interest to females or ethnic minorities, motivational factors, and technical aids or manipulatives. Findings indicate that the frequency of topics of interest increased from 1984 to 1990. Remillard (1991) studied how problem solving is presented in one elementary level traditional and three standards type textbooks; and Sutherland, Winter, and Harries (2001) and Haggarty and Pepin (2002) focused on multi-national comparisons of mathematics textbooks. Sutherland, Winter, and Harries examined similarities and differences in ways that images, symbols, tables, and graphs presented for the study of multiplication compared in textbooks from England, France, Hungary, Singapore, and the USA. Additionally, Haggarty and Pepin (2002) examined textbooks from England, France, and Germany for differences in their treatment of measurement of an angle. They found that clear differences exist in the ways that this topic is offered between and within textbooks from different countries, hence providing support for the theory that content analysis is a valuable addition to mathematics education research.

Porter (2006) developed a two-dimensional language to explain the content of the mathematics curriculum to compare intended, enacted, and evaluated curricula. The developed framework used a matrix listing the topics being evaluated and the cognitive demands on students based on the nature of the presentations. Herbel-Eisenmann (2007) also focused on language, which she called the "voice" of the textbook, that is, the interaction between the reader of the textbook and the textbook's authors. The findings suggested that the particular language used in the textbook sets up the student as either "scribbler", taking orders, or a member of the mathematical community in doing mathematics. These findings suggest that written materials can either support or undermine the goals for improving student achievement, and that many different aspects of analysis can be targeted.

Yeping Li (2000) extended the focus of Porter's investigation to include analysis of the required level of cognitive demand of example problems in lesson narratives and student exercises. She published results of cross-national similarities and differences on the content of addition and subtraction of integers in 7th grade textbooks from the United States and China. She analyzed five American and four Chinese textbooks for differences in the textbooks' problems, including the type of response elicited, cognitive demand, and problem features that would influence students' performance. The findings of this investigation indicated that the differences in the problems' performance requirements were larger than the differences in the problem presentation features, and that the American published textbooks had more of a variety of performance requirements than the Chinese textbooks.

Mesa (2004) examined 24 middle school textbooks from 14 countries to assess the practices associated with the notion of function in grades 7 and 8. The textbook sample chosen was based on the Third International Mathematics and Science Study (TIMSS) data, textbooks that were intended for middle school students, and that specifically contained references to linear functions and graphing. Mesa used a framework adapted from the theories of Balacheff and Biehler when she analyzed 1218 tasks identified in the textbooks to do an in-depth analysis of the exercise sections. The specific inquiry addressed the function in each task, and what needed to be done to solve the problem. The findings of the study suggested that few textbooks offered clear suggestions to the students to assist in their performance activities or information on how to solve a problem in different ways.

Johnson, Thompson, and Senk (2010) investigated the character and scope of reasoning and proof in high school mathematics textbooks in the United States to determine the variation in the treatment of reasoning and proof that might be evident in different textbook series. The researchers evaluated the narrative and exercises in 20 student editions of textbooks from four nationally marketed textbook series and two curriculum development projects. The analysis focused on mathematical topics dealing with polynomials, exponents, and logarithms. The framework used in this investigation utilized constructs based on the *Principles and Standards for School Mathematics* (NCTM, 2000). Findings indicated proof and reasoning were evidenced in greater instances in the narrative portion of the lessons than in the exercises, and the amount of reasoning and proof related work varied by mathematical topic and by textbook.

Gabriel Stylianides' (2005) developed and used an analytic framework he developed to investigate the opportunities to engage in reasoning and proof in a reform-based middle-grades mathematics curriculum. Units in algebra, geometry, and number theory in the Connected Mathematics textbooks were analyzed. The framework developed by this researcher distinguished the differences in the textbook authors' design on reasoning and proof opportunities within the textbook context in comparison to the opportunities provided for students to learn other mathematical topics. In contrast, Andreas Stylianides (2007b) investigated proof in the context of an elementary school classroom. Four characteristics or major features were examined in mathematical arguments: foundation, the definitions or axioms available for student use; formulation, the development system in use, as generalizations or logical equivalencies; representation, response expression expected, as appropriate mathematical language or algebraic language; and social dimension, the context of the community in which it is to be constructed. Stylianides (2007b) found that these four characteristics are derived from how proofs or mathematical arguments are conceptualized in the framework of mathematics. The framework he used evaluated mathematical intellectual honesty and continuity over different grade levels to experience proof in a coherent progression. Both Stylianides (2005, 2007b) provided background for the examination of the textbook authors' design and characteristics of transformation lessons.

Jones (2004) and Jones and Tarr (2007) evaluated the nature and extent to which probability content was treated in middle school textbooks. They examined two comprehensive textbook series from four recent eras intended for use in grades 6, 7, and 8. Their research questions (Jones, 2004; Jones & Tarr, 2007) focused on the components and structure of lessons and the extent of the incorporation of probability tasks over four eras. Comparatively speaking, they assessed the introduction or repetition of probability topics in students' tasks and the use and type of manipulatives suggested. The level of cognitive demand required in textbook activities and tasks, as related to probability, were assessed in student exercises using the framework developed by Smith and Stein (1998) and Stein and Smith (1998). The composed framework allowed for the collection of the total number of pages in the textbooks, the number of pages devoted to probability, the location and order of the probability lessons within each textbook, the identification of the probability lesson's topics, the suggestion for the use of manipulative devices, and the level of cognitive demand required by the student in performance expectations to complete the probability tasks. The work of Jones (2004) and Jones and Tarr (2007) illustrated the examination of the components and structure of lessons as well as providing a sample application of the levels of cognitive demand as devised by Stein and

Smith (1998).

**Analyses on Levels of Cognitive Demand Required in Student Exercises**

Stein and Smith (1998) designed and tested a framework to identify the level of cognitive demand needed for students to complete exercises and tasks in textbooks. Their framework document identified the level of cognitive demand in mathematical tasks by providing an evaluation of student thinking and reasoning required by the types of questions posed. This framework was used to evaluate the level of cognitive demand in student textbook lesson exercises in their study.

Smith and Stein classified questions that require memorization or the application of algorithms into categories of tasks that require *lower-level* demands. Questions that required students to use *higher-level* thinking were less structured, often had more than one solution, or were more complex or non-algorithmic. Four categories of level of cognitive demand for middle school students were identified, as indicated in Table 1. The outline suggested by Smith and Stein (1998); Stein, Grover, and Henningsen (1996); Stein, Lane, and Silver (1996); and Stein and Smith (1998) provides suggestions for determining the level of demand of mathematical tasks. This delineation of levels of cognitive demand was used in this study to determine the level of cognitive demand required for student performance expectations in the lesson exercises examined.

**Research on Transformation Tasks and Common Student Errors**

Research on the geometric transformational constructs and typical student misconceptions and errors when dealing with transformational tasks are discussed in this section. The subject matter content is the rigid transformations (translations, reflections, rotations) and dilations. This research and related curriculum recommendations helped to

Table 1

*Levels of Cognitive Demand for Mathematical Tasks*

| *Level of Cognitive Demand* | *Characteristics* |
|---|---|
| *Lower-Level (LL) demands (memorization):* | Memorization, exact reproduction of learned facts, vocabulary, formulas, materials, etc., lack of defined procedures, no connections to mathematical facts, rules |
| *Lower-Level (LM) demands (procedures without connections):* | Procedures lacking mathematical connections, requires use of algorithm, no connection to mathematical concepts, no explanations needed. |
| *Higher-Level (HM) demands (procedures with connections):* | Procedures with connections, procedures for development of mathematical understanding of concepts, some connections to mathematical concepts and ideas, multiple representations with interconnecting meaning, effort and engagement in task required. |
| *Higher-Level (HH) demands (doing mathematics):* | Doing mathematics, requires non-algorithmic procedures, requires exploration of mathematical relationships, requires use of relevant knowledge and analysis of the task requires cognitive effort to achieve solution required. |

*Note:* Based on Stein and Smith (1998) and Smith and Stein (1998).

inform the construction of the conceptual framework in the delineation of specific content that would address common student errors and misconceptions.

***Transformations.*** In this section, studies reviewed provided background information on the types of issues students experienced when dealing with two-dimensional transformation tasks. This section delineates the tasks reviewed and demonstrates the specific issues where students experienced difficulties.

When students perform transformation tasks, Soon (1989) concluded that her students, ages 15 and 16, were most successful with transformations in this order:

reflections, rotations, translations, and dilations. However, Kidder (1976), Moyer (1978), and Shah (1969) report translations were the easiest transformation for students. Soon (1989) and Meleay (1998) both indicated that students did not spontaneously use specific or precise vocabulary when communicating about translations, but rather used finger movements or words like “move” or “opposite” to indicate the direction of change. Thus, Meleay emphasized the importance of stressing vocabulary and the development of drawing skills during instruction about transformations.

Students need concrete opportunities to supplement the words and visuals that are represented in transformational geometry (Martinie & Stramel, 2004; Stein & Bovalino, 2001; Weiss, 2006). Williford (1972) states manipulatives provide students with a concrete avenue for understanding concepts that are abstract (Martinie & Stramel, 2004).

> Transformational geometry topics may be approached quite naturally through the manipulation of concrete objects or figure drawings. . . . Initially, the child performs actions upon objects. But eventually, after the object becomes distinct images, the child is able to perform mental transformations (actions) upon these images. … imagery evolves from an initial level of reproductive images based completely upon past perceptions to a level of true anticipatory images which are imagined to be the results of an unforeseen transformation. (p. 260)

Several common misconceptions were often exhibited by students when studying transformations. Many studies indicate that students focused on the whole figure being moved in the transformation process rather than each point being mapped to a corresponding location (Boulter & Kirby, 1994; Hollebrands, 2003, 2004; Kidder, 1976; Laborde, 1993; Soon, 1989), and students also experienced problems seeing the features or properties of the figures themselves (Kidder, 1976; Laborde, 1993). Kidder noted that students in grades 4, 6, and 8 experienced a specific difficulty with the property of conservation of length. Students focused on the visual features and the movement of the shape under the transformation rather than on properties of the transformation (Soon, 1989; Soon & Flake, 1989). Laborde went on to suggest that higher level reasoning powers were required for understanding preservation of properties of figures. Next, the misconceptions and errors students experience with specific transformations are discussed.

***Issues students experience with transformations concepts.*** In this section issues that students experience with the four principle types of transformations and composite transformations are discussed. The literature identified characteristics and issues with elements of specific performance within the transformation tasks. The issues discussed provided background and reasoning for the collection of specific performance tasks in each of the transformation types as well as the division of tasks into categories of difficulty.

*Translations.* The NCTM (Illuminations Lessons List: Translations), Moyer (1975, 1978), and Shah (1969) state that translations are the easiest transformation for students to understand. In their work with third and fifth grade students, Schultz and Austin (1983) and Shultz (1978) found that the direction of the movement of the translation had a definite impact on the difficulty of the problem; they found that translations to the right, then to the left were easier than diagonal translations, either up and to the right or up and to the left. They also found that as the distance between the initial and final figure increased in the translation, the students experienced increasing difficulty in performing the translation tasks.

Flanagan (2001) indicated that students have problems recognizing that the movement of the figure in a translation is the magnitude of movement and is related to the length of the vector shaft represented on the coordinate graph. Hollebrands (2003) affirms that students should recognize that a figure and its image are parallel and that the distances between the preimage and image points are equal and the same length as the translating vector. Flanagan (2001) and Wesslen and Fernandez (2005) found that students did not realize that translating a figure moves every point on the figure the same distance and in a parallel orientation. The findings above illustrate that it is important to look at the direction of the translation of the figure since certain directional movements are easier for students to perform than others, especially the movement of a figure in a translation that is in a diagonal direction to the horizontal.

*Reflections*. Through interviews, Rollick (2007, 2009) found that pre-service teachers had various problems with reflections. The specific reflection that the participants found the easiest was the movement of a figure from the left to right position over the y-axis or a vertical line. The participants had problems performing the right to left reflection and had a tendency to interpret the movement as being top to bottom instead. Many of the participants identified a reflection as a translation when symmetric shapes were used. Additionally, sometimes they misunderstood reflections and confused them with rotating the figure. Rollick (2009) explains that developing the concept of invariant relationships between the figure and its image is needed to help dismiss these misconceptions.

Yanik and Flores (2009) and Edwards and Zazkis (1993) both indicated that preservice elementary teachers interpreted the line of the mirror as cutting the figure in half, or alternatively interpreting the edge of the figure as the predetermined line of reflection. Hence, if pre-service elementary teachers struggle with reflection so might middle school students. Kuchemann (1980, 1981) found that students had the most difficulties with reflection over a diagonal line, the students were found to often ignore the angle or slope of the reflection line and perform a horizontal or vertical reflection instead; this finding was also evident in the works of Burger and Shaugnessy (1986), Perham, Perham, and Perham, (1976), and Schultz (1978). The most difficult type of reflection for students is reflecting a figure over a line of reflection that intersects the preimage, this type of transformation reflects the image to overlap itself (Edwards & Zazkis, 1993; Soon, 1989; Yanik & Flores, 2009). In this particular case the use of tracing paper (Patty paper) would be useful for assisting with this concept (Serra, 1994). The axes and the preimage would be traced; then, the tracing paper would be flipped over and aligned to show the position of the image.

The findings on reflections indicate that it is important to document the direction of movement of the figure since reflection right to left, over a diagonal line, and of a figure over itself are increasingly difficult. The use of manipulatives was recommended to clarify these problem tasks.

*Rotation.* Clements and Burns (2000) observed that fourth grade above average students first learned about rotation from the experience with physically turning their own bodies; further the concept of turn to the right and left was developed, followed by the amount of turn. Of all of the rigid motion transformations, Moyer (1975, 1978) and Shah (1969) indicate that elementary students, from 7 to 11 years old, had the most difficulty working with rotations.

Kidder (1976) found, in his testing of nine, eleven, and thirteen year old students of average mathematical ability, that students were often unable to imagine the existence of the angle and the rays necessary for a rotation. The students were unable to hold some factors constant while varying others to perform a rotation. Kidder also indicated that students had difficulty holding the distance from the point of rotation to the vertices of the figure constant while performing a rotation. The students were unaware that angle measures of the figure remain unchanged under the turn. Olson, Zenigami, and Okazaki (2008) found that students had a weak understanding that when rays of different lengths rotated the same number of degrees the same angle measure resulted. Students' demonstrated common misconceptions about the measure of an angle being determined by the lengths of the rays that make up the angle (Clements, & Battista, 1989, 1990; Krainer, 1991). Additionally, Clements, Battista and Sarama (1998) found that students had difficulty assigning the number of degrees to the angle of rotation, but they were more comfortable using the measures of 90 and 180 degrees.

Edwards and Zazkis (1993), Yanik and Flores (2009), and Wesslen and Fernandez (2005) concur that students' image of rotation was usually at the center of the figure being rotated, and students had more success with this type of rotation. Wesslen and Fernandez (2005) found that students were not confident with rotating figures where the center of rotation was defined as other than the center of the shape or a vertex of the figure; but students also experienced problems with using the figures' vertices for center of rotation and had difficulty with clockwise and counterclockwise directionality.

Soon (1989) and Soon and Flake (1989) found that students experienced the most difficulty in rotation of a figure with the center of rotation given as a point external to the figure. Students had a tendency to ignore the prescribed center of rotation and instead rotated the figure about the center of the figure or a vertex of the figure; and they frequently disregarded the direction of turn indicated in the transformation (Soon & Flake, 1989). Soon (1989) and Wesslen and Fernandez (2005) found that students did not illustrate knowledge of angle of rotation or center of rotation or both.

Clements and Burns (2000) and Clements and Battista (1992) found that average 4$^{th}$ graders have many misconceptions and have difficulty learning the concepts of angle and rotation; these concepts are central to the understanding of rotation. Clements and Burns suggest that the static definition of angle (An angle is the part of the plane between two rays meeting at a vertex) may be part of the cause for the misconception. Clements et al. (1996) found that students did not give notice to the directionality of right or left of a turn in performing a rotation.

The studies presented above describe numerous problems that students experience with performing rotational tasks. Among the problems that appear most frequently are the concept of angle measure, measure of angle of rotation, and center of rotation. Additionally, the difference between the factors that vary, and those that remain constant during a rotation appear to create supplementary problems for students when completing rotational tasks.

***Dilations.*** Soon (1989) found the geometric transformation of dilation to be the most difficult concept for students as reported by assessment results. Students experienced confusion with the scale factor in enlargements; they believed that a positive scale factor meant an enlargement and a negative factor meant a reduction in size of the figure (Soon, 1989). Students were reluctant to use specific vocabulary for center of dilation or for scale factor and would instead use, for example, "equal angle but sides enlarged two times" (Soon, 1989, p. 173). Also, students consistently expected a change to occur and could not accept a scale factor of 1/1 or 1 as the identity property for dilation (Soon, 1989). Hence, discussion on the topics of scale factor, similarity, and identity, with evidence of terminology use would be expected to be found in the presentations on dilation.

*Composite Transformations.* Wesslen and Fernandez (2005) state "the national curriculum, as it is today in England and Canada for middle grades, does not include glides" (p. 27) or the general topic of composite transformations. The recommendation for the inclusion of composite transformations was added to the standards curriculum documents in the United States (NCTM, 2000). The study of composite transformations increases understanding for the concept of congruence of two dimensional figures and provides meaning and closure to the mathematical system of transformations (Wesslen & Fernandez, 2005), because two transformations can be combined to form a composite transformation, and the resulting image can be redefined as one of the original transformations (Wesslen & Fernandez, 2005).

With the inclusion of composites to the topic of geometric transformations, it becomes possible to define a pattern as simple as a set of footsteps across the sand. Wesslen and Fernandez indicate that adding composite transformations to the curriculum ". . . make[s] interesting mathematics because it is a complete system with plenty of patterns to be discovered. For example, any two transformations combined seem always to be one of the already existing transformations" (p. 27). The need to include composite transformations in the curriculum is reiterated by numerous educators (Burke, Cowen, Fernandez & Wesslen, 2006; Schattschneider, 2009; Wesslen & Fernandez, 2005). The properties and a sampling of composite transformations are presented in Appendix B.

The issues students experience with the concept of composite transformations include the difficulties experienced with each individual type of transformation and difficulties identifying and understanding the combination of composite transformations (Addington, http://www.math.csusb.edu/). Students often do not see congruence of figures when the shapes are placed in different orientations and that using different direction or distance of movement still yields the same resulting shaped figure. Usiskin et al. (2003) indicated that a rotation can be considered a composite of reflections, hence yielding various possible conjectures for students to make. Additionally, problems experienced by students include determining the distance a figure was to be moved for a transformation on a coordinate plane; the students seemed to experience difficulty in determining the distance and direction to move the figure (Usiskin et al., 2003)

**Conceptual Framework for Content Analysis of Geometric Transformations**

Researchers investigating the effects of curriculum on student achievement focus on various issues, for example, how to ensure that students are comparable at the start of an experience, how to randomize students assigned to different treatments, and what measures to use to evaluate effects on student achievement. But the question of the comparability of the content of the curricula used has been less evident in research studies. Stein, Remillard, and Smith (2007) indicate that one approach to analyzing students' *opportunity to learn* includes looking at what is covered in the content of the curriculum and how the content is presented.

**Summary of Literature Review**

This chapter described the curriculum and the textbook, the use of the textbook in the classroom, the impact that the textbook has on classroom curriculum, criticisms of the curriculum and the textbook, and the need for content analysis. Next, the literature was reviewed on various types of textbook analyses as well as on textbook content analyses of specific mathematics topics. Then findings were presented on an in-depth delineation of the geometric transformational constructs related to this study, and the types of difficulties that students experience when learning transformation concepts.

This review of relevant literature has delineated the need for analysis of content on transformations and has provided background for construction of the conceptual framework for this study. The next chapter presents the conceptual framework for content analysis with the methods and the coding instrument utilized.

# Chapter 3: Research Design and Methodology

This study analyzes the nature and treatment of geometric transformations included in middle grades student textbooks published from 2009 to the present. This chapter presents the research design and methods used for this study.

The content of this chapter is divided into five sections. The first section presents the research questions, the second presents the sample of textbooks used for analysis, the third discusses the development of the instrument used for coding the transformation lessons, and the fourth describes data collection. Lastly, this chapter culminates with a summary of the design and methodology.

## Research Questions

This study investigates the nature and treatment of geometric transformations (translations, reflections, rotations, dilations, and composites) in student editions of middle grades textbooks presently in use in the United States. The intent of this study is to investigate the following research questions.

1. What are the physical characteristics of the sample textbooks? Where within the textbooks are the geometric transformation lessons located, and to what extent are the transformation topics presented in mathematics student textbooks from sixth grade through eighth grade, within a published textbook series, and across different publishers?
2. What is the nature of the lessons on geometric transformation concepts in student mathematics textbooks from sixth grade through eighth grade, within a published textbook series?

3. To what extent do the geometric transformation lessons' student exercises incorporate the learning expectations in textbooks from sixth grade through eighth grade within a published textbook series, and across textbooks from different publishers?

4. What *level of cognitive demand* is expected by student exercises and activities related to geometric transformation topics in middle grades textbooks? The *level of cognitive demand* is identified using the parameters and framework established by Stein, Smith, Henningsen, and Silver (2000).

Together, these four questions give insight into potential *opportunity to learn* that students have to study geometric transformations in the middle grades textbooks.

**Sample**

Different types of developed curricula were included for analysis because they are constructed on different philosophies and focus on different goals; it was expected that they would deal with the concepts of geometric transformations differently. Standards-based textbooks, that is, those developed in response to the *Curriculum and Evaluation Standards* (NCTM, 1989) typically place greater emphasis on conceptual understanding through problem solving and topic investigation, hence focusing on mathematical structures (Kilpatrick, 2003; NCTM, 1989, 2000; Senk & Thompson, 2003). The publisher generated textbook has historically emphasized procedural skills and exercises (Begle, 1973; Senk & Thompson, 2003). Although the mainline publishers continue to emphasize procedural skills they are including a balance between procedural skill and conceptual understanding to follow the NCTM recommendations. Therefore, it was important to include both types of curricula in the sample.

The sample included four middle grades textbook series available for classroom use in the United States. Two were from widely used mainline commercial publishers, Pearson (Prentice Hall) and Glencoe; one was a National Science Foundation (NSF) funded curriculum project textbook series, Connected Mathematics 2 (CM2); and one was a non-NSF funded curriculum project textbook series, the University of Chicago School Mathematics Project (UCSMP). The Pearson and Glencoe textbook series contain a 6 to 8 basal set and a pre-algebra textbook for grade 8 to accommodate choice on curriculum content for the study of pre-algebra concepts in grade 8. The CM2 and UCSMP textbook series contain one textbook for each grade 6 to 8; students would be expected to complete all three in the series. With the latter two textbook series, students have completed the equivalent of middle grades algebra by the end of 8$^{th}$ grade.

To ensure a comparison of comparable achievement levels, the pre-algebra textbooks from Pearson and Glencoe were included in the sample to provide a comparable analysis to the Connected Mathematics 2 and the UCSMP series that have pre-algebra and algebra topics embedded within their curricula. The inclusion of textbooks available for the study of beginning algebra provides information to analyze the content for variations in potential *opportunity to learn* depending on the curriculum sequence that may be chosen by individual districts. Thus, for each of the Prentice Hall and Glencoe series the books are grouped into two series, 6-7-8 or 6-7-pre-algebra (-pa) to provide two basis for comparison. The four textbook series included a total of 14 textbooks that were analyzed. The symbols used for the textbooks in this study are presented in Table 2.

Table 2

*Textbooks Selected for Analysis with Labels Used for This Study*

| *Publisher* | *Title* | *Grade* | *Symbol* |
|---|---|---|---|
| Pearson | Prentice Hall Mathematics | | PH |
| | Course 1 | 6 | PH6 |
| | Course 2 | 7 | PH7 |
| | Course 3 | 8 | PH8 |
| | Algebra Readiness | Pre-algebra 8 | PH-pa |
| McGraw Hill | Glencoe Math Connects: Concepts, Skills, and Problem Solving | | G |
| | Course 1 | 6 | G6 |
| | Course 2 | 7 | G7 |
| | Course 3 | 8 | G8 |
| | Glencoe Pre-Algebra | Pre-algebra 8 | G-pa |
| Pearson | Connected Mathematics 2 | | CM2 |
| | Grade 6 | 6 | CM6 |
| | Grade 7 | 7 | CM7 |
| | Grade 8 | 8 | CM8 |
| McGraw Hill, Wright Group | University of Chicago School Mathematics Project | | UCSMP |
| | UCSMP Pre-Transition Mathematics | 6 | U6 |
| | UCSMP Transition Mathematics | 7 | U7 |
| | UCSMP Algebra | 8 | U8 |

One set of mainline publisher generated textbooks was from Pearson Publications: *Prentice Hall Mathematics*, *Course 1* (© 2010), *Course 2* (©2 010), *Course 3* (© 2010) and *Algebra Readiness* (© 2010). The Prentice Hall series provides for differentiated instruction while engaging students in problem-solving skills and procedural understanding. The Prentice Hall series helps students develop problem solving skills, test taking strategies, and conceptualize abstract concepts with activities in a structured approach to mathematics topics. Additionally the use of technology is incorporated in the presentations of lessons (http://www.pearson school.com).

A second mainline publisher generated series was from McGraw Hill

Publications: *Glencoe Math Connects: Concepts, Skills, and Problem Solving, Course 1* (© 2009), *Course 2* (© 2009), *Course 3* (© 2009) and Glencoe *Pre-Algebra* (© 2010). The Glencoe: Math Connects series features thee key areas: mathematics vocabulary building to strengthen mathematics literacy; intervention alternatives to improve achievement levels; and enhanced differentiated instruction to match the needs of individual students. The curriculum provides a balanced program for mathematics understanding, skills practice, and problem solving application with problem solving guidance. The series also contains student feedback after each lesson example, progressive student exercise sets, self assessment options for students, and higher order thinking problems in each lesson (http://www. glencoe.com).

The third set of textbooks was from a widely used National Science Foundation (NSF) funded Standards-based series, from Pearson Publications: *Connected Mathematics 2*, Grade Six (© 2009), Grade Seven (© 2009), and Grade Eight (© 2009). The philosophy of this curriculum is that students can make sense of mathematics concepts when they are embedded within the context of real problems. Student learning is to be achieved in the curriculum by problem-centered investigations of mathematical ideas that include explorations, experience-based intuitions, and reflections that help students grow to reason effectively and to use multiple representations flexibly (http://www.phschool.com).

The Connected Mathematics 2 curriculum presentation is quite different from more familiar curricula formats (http://connectedmath.msu.edu). The Connected Mathematics 2 is a modular series designed to develop mathematical thinking and reasoning by using an investigative approach with engaging real-world situations with students working in small groups (http://www.Pearson school.com). This series, Connected Mathematics 2, was chosen because it is the most widely used NSF funded middle grades series (Dossey, Halvorsen, & McCrone, 2008).

The fourth set of textbooks was from a non-NSF funded curriculum development project considered to be a hybrid of publisher generated and Standards-based textbooks, the University of Chicago School Mathematics Project (UCSMP) *Pre-Transition Mathematics (*© 2009*)*, *Transition Mathematics (© 2008)*, and *Algebra (© 2008)*. This curriculum research and development project began in 1983 in response to recommendations by the government and professional organizations to update mathematics curriculum. The UCSMP curriculum focuses on interconnected mathematical components throughout the kindergarten–grade 12 levels to improve the understanding of mathematics (Senk, 2003; D. R. Thompson, personal communication, March 6, 2010).

Although the UCSMP textbook series was initially developed before the *Standards*, it is specifically perceived to align with the recommendations of the NCTM Standards to use realistic applications, cooperative learning strategies, problem solving with reading and technology in the instructional format (Thompson & Senk, 2001; Usiskin, 1986). These textbooks are specifically designated for use in the middle grades (UCSMP, n.d.).

**Development of the Coding Instrument for Analysis of Transformations**

This section describes the development of the coding instrument used to collect data for the analysis of the nature and treatment of geometric transformations (Appendix C) in middle grades textbooks. The instrument was initially constructed during the pilot study using recommendations for the inclusion of geometric transformation concepts from the *Principles and Standards for School Mathematics* (NCTM, 2000) in conjunction with the properties of geometric transformations, and reviewed literature which suggested collecting data on the physical characteristics of textbooks. The properties provided background for the contents of the geometric constructs that were expected to be present in lesson narratives and explained in student examples.

The coding instrument was tested as part of the pilot study using *Glencoe Mathematics Applications & Concepts Course 3* (© 2004) and *Prentice Hall Course 3 Mathematics* (© 2004). As indicated in the discussion of the pilot study in Appendix A, the coding instrument provided confirmation that differences were found in the presentation and treatment of transformation lessons and in the student exercises in the textbooks analyzed. Hence, the pilot study provided confirmation that an analysis of transformation concepts and student performance expectations could delineate differences in potential *opportunity to learn* transformations. However, some changes were made in the coding instrument as a result of the pilot study and the review of literature. For instance, more space was left for additional totals on page counts and record of what was observed in the lesson narrative. Based on the research literature the coding instrument was later extended to look for concepts to address potential misconceptions. Appendix D presents aspects of transformations that were important to capture because of issues raised in the research on misconceptions or difficulties that students experience with these tasks. In view of the difficulties that students experience with transformations, it seemed logical to document what is available within instructional content to provide students with the opportunity to avoid these difficulties.

The review of student exercises for the pilot study confirmed the need to capture the nature of the tasks that students were expected to complete. Specifically, students were often asked to respond by providing vocabulary terms, applying steps previously given, finding coordinates or angle measures of rotation, graphing an answer, correcting an error in a given problem, or assessing true/false statements about transformations. Additionally, exercises included an expectation that students would engage with the *process standards* (problem solving, communication, connections, reasoning and proof, and representations) from the *Principles and Standards for School Mathematics* (NCTM, 2000). Hence, a decision was made to capture the extent to which students are expected to write about their solutions, work a problem backwards, or give a counterexample. Because of the recommendation for the inclusion of real world relevance in posing questions, a decision was also made to document real world connections. Appendix E illustrates Examples of Student Performance Expectations in Exercise Questions.

**Global Content Analysis Conceptual Framework**

The description of content analysis from the literature review revealed similarities and differences among various types of content analysis investigations. In particular, the body of literature provided background content for the validation in the construction of the Global Content Analysis Conceptual Framework (Figure 1) which aims to delineate the areas of textbook content that need to be examined. The center portion of the Framework contains three segments that encompass the areas of the textbooks that were analyzed. The left segment addresses the question “Where”, where is the content located within the textbook. The middle segment addresses the question “What”, what is the nature of the narrative of the lessons including the content scope, and the opportunities

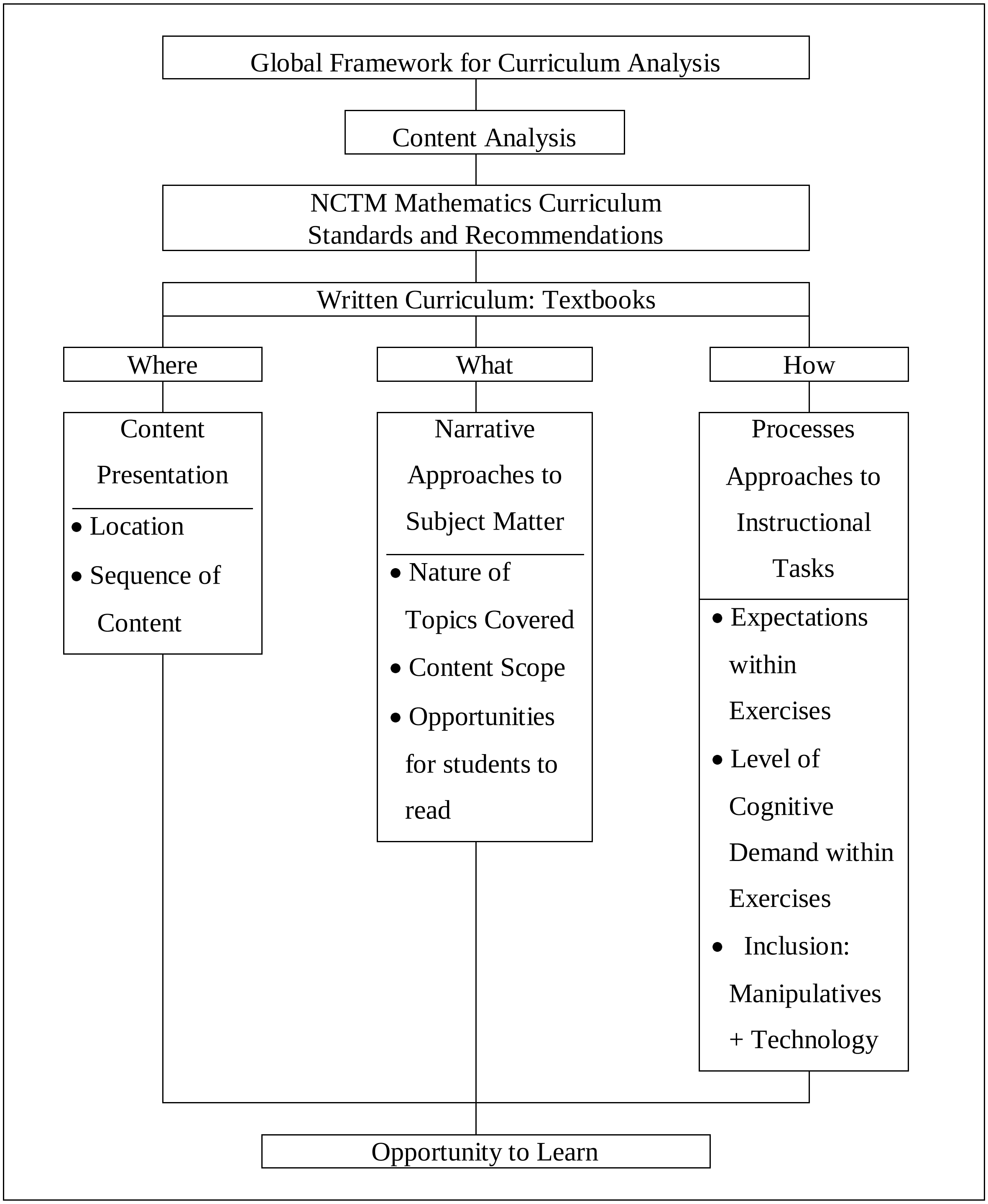


*Figure 1.* Global Content Analysis Conceptual Framework

provided for student study. And, the right segment addresses the question "How", how are the concepts reinforced in the tasks and exercises, including the level of cognitive demand required by the students to complete the exercises and the suggestions for the inclusion of manipulatives and technology use. These collective segments provide insights about the students' potential *opportunity to learn* the mathematical content.

The aforementioned content analyses on specific mathematical concepts and student performance expectations has helped to extend the "What" portion of the framework, including the lesson's narrative content with a focus on components and structure of presentation (Johnson, Thompson, & Senk, 2010; Jones, 2004; Jones & Tarr, 2007; Porter, 2006), delineation of objectives and properties, and inclusion of definitions (Stylianides, 2007b). The coding instruction portion containing the "How" processes was further extended by the content analyses of student performance requirements (Johnson, Thompson, & Senk, 2010; Mesa, 2004), student exercise features (Li, 2000) with analysis of the level of cognitive demand required to complete student exercises (Jones, 2004; Jones & Tarr, 2007; Li, 2000; Porter, 2006; Stein & Smith, 1998), and the recommendation for the inclusion of manipulatives and technology use (Jones, 2004; Jones & Tarr, 2007; Rivers, 1990).

Figure 2 illustrates the Conceptual Framework: Content Analysis of Two Dimensional Geometric Transformations in Middle Grades Textbooks that has been constructed using the literature reviewed herein and the Global Framework previously presented (Figure 1). The left hand segment of the illustration concentrates on '*Where*' the content is located and the sequence of topics presented in the textbook. The center segment focuses on '*What'* content is covered in the curriculum by examining the nature of the topics covered, the scope of the constructs, and the extent to which lesson content

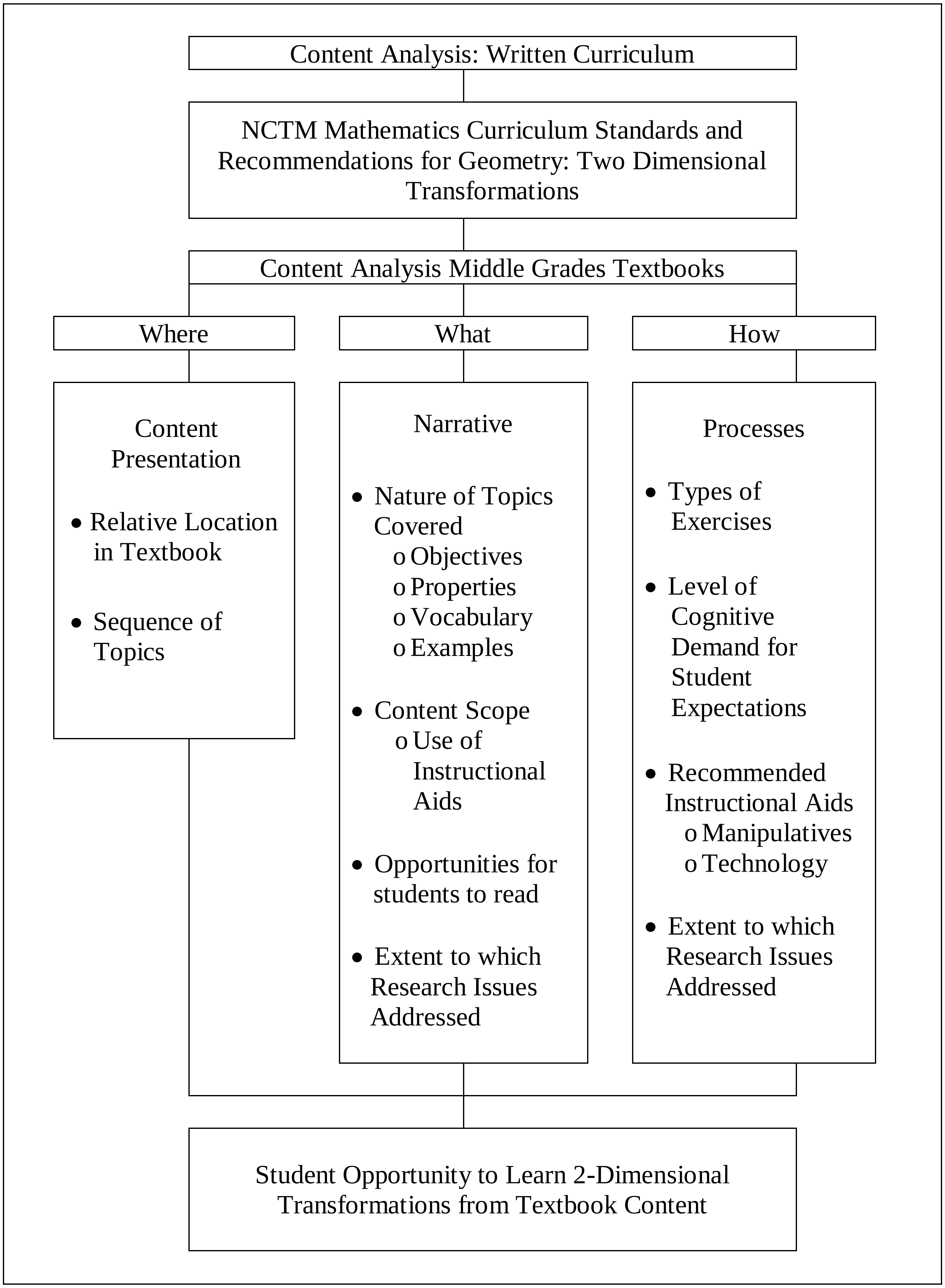


*Figure 2.* Conceptual Framework: Content Analysis of Two Dimensional Geometric

may help lessen the development of student misconceptions. The right hand segment of the illustration focuses on the '*How'* processes to support student learning in exercises, and level of cognitive demand required by the students to accomplish the performance expectations. Together these areas of examination provide a conceptual framework and a scaffold to analyze student *opportunity to learn* geometric transformations in middle grade textbooks sampled in this study.

The coding instrument has three segments, corresponding to the three segments in the Content Analysis Middle Grades Textbooks conceptual framework (Figure 2). Segment 1, "Where", was designed to support data collection on the physical characteristics and content of the textbooks as well as the relative placement of the transformation lessons and sequence of topics. Segment 2, "What", captures the nature of the lesson narratives, including the objectives, properties, and vocabulary. Segment 3, "How", was designed to capture the processes in the exercises, including types of exercises, types of performance expectations, and the required levels of cognitive demand. Table 3 summarizes the data collected in each segment.

The coding instrument Segment 1a provided space to record the physical characteristics of the textbook including: total number of pages, number of chapters, number of student instructional pages, number of chapter sections, and number of pages for chapter review and practice tests as well as additional features, such as example projects or activities. The number of supplemental pages at the end of the book, for prerequisite skills, selected answers, extra practice, word problem examples, index, and glossary was also recorded to provide a basis for reconciliation of lesson pages to the

Table 3

*Three Stages of Data Collection and Coding Procedures*

| *Segment Name* | *Segment Designation and Contents* |
|---|---|
| "Where" -Content | Segment 1a - Textbook contents |
| | Segment 1b - Transformation lesson locations and sequence |
| | Segment 1c - Glossary - vocabulary/terminology check |
| "What" -Narrative | Segment 2 - Lesson Presentation |
| "How" -Processes | Segment 3 - Exercise type, student performance expectation, and level of cognitive demand |

total page count in the textbook. Segment 1b provided space to record all textbook sections/pages that discuss geometric transformation concepts, these pages were determined by a page by page inspection of the textbook. Collection of this data was patterned after the work of Tarr, Reys, Baker, and Billstein (2006) and Jones (2004).

To insure that all geometric transformation content was identified for analysis, this researcher examined the index of the textbook to identify the location of related vocabulary and the page numbers of appearance. Initially, the transformation vocabulary list was amassed during the pilot study from the Glencoe ©2004 and the Prentice Hall ©2004 textbooks and expanded as additional terms were located in lesson narratives and indices (Table 4). Additional space was provided to add relevant terms when found. Segment 1c of the coding document lists the vocabulary with space to record the page(s) on which each term is mentioned. The comparison of identified transformation lesson locations (Segment 1b) with listed page locations where vocabulary and transformation topics were located (Segment 1c) was conducted by this researcher to insure that all transformation lessons throughout the textbook were listed for analysis. Additionally, this

Table 4

*Terminology for Transformation Concepts*

| *Segment 1c: Terminology for Transformations Concepts* | | |
|---|---|---|
| *Term* | *Term* | *Term* |
| Transformation | Composite Transformation | Coordinate Plane |
| Congruence | • Glide | Two dimensional figures |
| Similarity | | |
| Rotation | Symmetry | Dilation |
| • Turn | • Line of Symmetry | • Dilate |
| • Rotary Motion | • Bilateral symmetry | • Reduction |
| • Rotation Motion | • Turn Symmetry | • Stretch |
| • Clockwise | • Rotational Symmetry | • Scale model |
| • Counterclockwise | | • Scaling |
| | Translation | • Scale drawings |
| Reflection | • Vector | • Expand |
| • Flip | • Slide | • Enlarge |

reconciliation provided a verification check that all transformation instruction was identified for further examination. A mathematics education colleague reviewed the method and checked lesson inclusion in the sample textbooks.

Segment 2 “What” (Narrative) of the coding instrument focused on the transformation content from the narrative of the lessons. The lesson objectives were recorded when explicitly presented in the lesson. The vocabulary as defined in the lesson narrative was recorded along with any other pertinent fundamentals observed. When presented, specific transformation properties were recorded. Space was provided to note lesson features, including types of examples offered for student study, references to real world topics, and the suggestions for the use of manipulatives and technology because recommendations to improve student assessment on geometric transformation tasks included general indications to provide various types of manipulatives and technology.

Hence these suggestions for use were incorporated into the coding instrument (Jones, 2004; Jones and Tarr, 2007; Kieran, Hillel, & Erlwanger, 1986; Magina & Hoyles, 1997; Martinie & Stramel, 2004; Mitchelmore, 1998; NCTM, 1989, 2000; Stein & Bovalino, 2001; Weiss, 2006; Williford, 1972).

Segment 3 “How” (Processes) of the coding instrument focused on the student exercises presented following the lesson’s narrative. Each exercise was analyzed for specific transformation topic(s) included in the questions, type of student performance expected, inclusion of real-world or other academic subject relevance, suggestions for the inclusion of manipulatives and/or technology, and level of cognitive demand needed for students to complete the task.

The complete Coding Instrument is presented in Appendix F and the Instrument Codes for Recording Transformations in Appendix G. Note, each transformation type was sub-divided into specific tasks that were identified in the literature as they related to student difficulties or misconceptions. The codes were delineated to capture specific requirements of each exercise. Appendix H provides illustrated sample exercises of each specific characteristic to be coded in the exercises. Appendix I provides sample exercises classified by the level of cognitive demand required for students to complete the work. Finally, Appendix J: Background for Content Analysis and Related Research Studies illustrates the connections of the coding instrument with the ideas based on similar content analyses (Doyle, 1983, 1988; Jones, 2004; Jones, & Tarr, 2007; Senk, Thompson, & Johnson, 2008; Smith & Stein, 1998; Stein & Smith, 1998).

Changes to some of the instrument codes occurred during coding of the first lessons, such as incorporating arrows for direction of movement in reflections and translations to insure that questions arising from the various difficulties of directional movement could be delineated when analyzed. The coding symbol for translation changed from ‘tl’ to ‘tr’ to provide a direct connection of the word to the symbol. All of these influences and decisions were collated to create the coding instrument for analyzing geometric transformations as described above. The next section illustrates the application of the coding instrument with sample questions.

**Sample application of the coding instrument.** The following four examples illustrate the application of the coding instrument. The exercises are from the two textbooks used in the pilot study.

| |
|---|
| Graph each point. Then rotate it the given number of degrees about the origin. Give the coordinates of the image.<br>16. L ( 3 , 3 ); 90º    17. M ( -4 , -2 ); 270º    18. N ( 3 , -5 ); 180º<br>(Prentice Hall, 2004 p. 172) |

*Figure 3.* Example 1 - Sample of Student Exercise for Framework Coding

The three exercises in Figure 3 were each coded as follows:

- rotation about the origin (Ro),
- apply steps given (Y),
- find the coordinates (Y),
- graph the answer (G),
- level of cognitive demand (LM) required to complete this task [i.e., follow algorithmic procedure provided within the narrative of the lesson to produce the correct answer].

| |
|---|
| 23. Error Analysis  A square has rotational symmetry because it can be rotated 180º so that its image matches the original. Your friend says the angle of rotation is 180º / 4 = 45º. What is wrong with this statement?  (Prentice Hall, 2004, p. 172) |

*Figure 4.* Example 2 - Sample of Student Exercise for Framework Coding

Example 2 exercise # 23 was coded as follows:

- rotation about the origin (Ro),
- apply steps given (Y),
- correct the error in the given problem (Y),
- written answer (Y),
- level of cognitive demand (HM) (i.e., some degree of cognitive effort-general procedures with close connections to concepts).

For Exercises 15 and 16, graph each figure on dot paper.

15. a square and its image after a dilation with a scale factor of 4.
16. a right triangle and its image after a dilation with a scale factor of 0.5.

(Glencoe, 2004, p. 196)

*Figure 5.* Example 3 - Sample of Student Exercise for Framework Coding

Example 3, exercise # 15 was coded as follows:

- Dilation (En),
- apply steps given (Y),
- graph answer (G),
- Manipulative (M) (dot paper),
- level of cognitive demand (HM).

Exercise # 16 was coded as follows:

- Dilation (Di),
- apply given steps (Y),
- graph answer (G),
- Manipulative (M) (dot paper),
- level of cognitive demand (HM).

31. Graph the equation Translate the line right 2 units and up 4 units. Find the equation of the image line.

(Prentice Hall, 2004, p. 162)

*Figure 6.* Example 4 - Sample of Student Exercise for Framework Coding

Example 4, exercise # 31 was coded as follows:

- Translation (Tr),
- apply steps given (Y),
- find the coordinates (Y),
- graph answer (G),
- subject related (alg),
- level of cognitive demand (HH).

Each transformation exercise, either individually numbered or each part of a multi-part task was counted as one exercise on the instrument. As in the previous examples each was numbered; exercises labeled with letters a, b, c, etc., instead of numbers, were each counted as one exercise. Exercises requiring two different parts to complete were counted as two exercises except in the case of composite transformations, because the exercise required two steps in the student expectation.

**Reliability Measures**

Reliability is concerned with stability and reproducibility (Krippendorff, 1980). Krippendorff refers to stability as consistent coding at different time intervals, where ambiguities in the text and/or the coding rules and changes in the coder's judgment on specific codes are minimized. Krippendorff also refers to reproducibility, called inter-rater reliability by Gay and Airasian (2000). Inter-rater reliability is concerned with the extent to which the coding for the study is consistent across different coders. Inter-coder reliability, also called inter-rater agreement, is a term used for the measurement of the consistency to which individual coders evaluate characteristics (Budd, Thorp, & Donohew, 1967; Tinsley & Weiss, 1975, 2000).

Inter-coder agreement is necessary in content analysis to provide measures of "the extent to which the different judges tend to assign exactly the same rating to each object" (Tinsley & Weiss, 2000, p. 98). Inter-coder reliability is an important component of content analysis, and although it does not insure validity, if it is not present the interpretation of data cannot be considered valid (Lombard, Snyder-Dutch, & Bracken, 2008). Kolbe and Burnett (1991) state

> Interjudge reliability is often perceived as the standard measure of research quality. High levels of disagreement among judges suggest weaknesses in research methods, including the possibility of poor operational definitions, categories, and judge training. (p. 248)

The data collected for this study was subjected to a reliability measures check with two mathematics education colleagues. The coders were doctoral level mathematics education students and are well versed in mathematics. The coders were provided with information on the topics of geometric transformations that are the focus of this study. Both coders felt very comfortable with the concepts.

The coding procedures started with discussion of the geometric transformation concepts under investigation, the coding symbols, and the coding instrument. The characteristics of tasks being identified on the coding instrument were discussed and symbols reviewed. The coding instrument was reviewed and coding procedures were discussed. The coders felt that the constructed documents were all encompassing. Sample questions were used to identify each type of characteristic identified and the coding symbols were again discussed. The coders agreed the codes *no*, *na*, and an entry left blank meant that the characteristic was not present in the exercise being examined. For example, if the question did not ask for graphing, the response for graphing was left blank on the coding document or the word *no* or *na* would mean the same. The level of cognitive demand required by the student to complete performance expectation in the exercise task was discussed. A copy of the framework developed by Stein and Smith (1998) and Smith and Stein (1998) with explanations of the characteristics of each level of demand was available for use during the coding session.

Next specific questions were identified on each transformation and coded from the textbooks to clarify any further ambiguities in the coding framework and in the coding procedures. During coding of the first lesson, the coders collaborated on the coding of the exercises. In the next phase, a textbook was picked and the coding was done with further collaborative discussion. Coding continued with occasional collaborative discussion when a coder felt the need. One coder noticed that to identify a figure being reflected over/onto itself the coder had to graph the points in order to determine the location of the image with respect to the reflecting line.

The specific lessons to be analyzed were each recorded on index cards prior to the start of coding. The index cards were then used to randomly draw the next lesson to be analyzed. The double coded lessons were highlighted on a master list to determine that all published series and grade level textbooks were being represented in the analysis. Stratification by publisher and grade level was insured in the last round of the card draw by segregating the remaining cards into groups that had been less represented by a second coding. The total number of lessons double coded by the raters totaled slightly more than 44% of the total number of lessons coded by this researcher (14% more than originally planned). Approximately 50% of the total number of transformation lessons in each series was coded by a second rater.

Lombard, Snyder-Dutch, and Bracken (2008) indicate that when coding nominal categories the percent agreement is an inappropriate and misleading liberal measure of inter-coder consistency, and they list the widely used *Holsti*'s method as the proposed indicator. Holsti's (1969, p. 140) method uses the following formula:

$$\text{Reliability} = 2\ M / (Na + Nb)$$

where M is the number of agreed upon coding decisions, and Na and Nb represent the total number of coding decisions made by the raters. Results of these calculations will yield a coefficient value between .00 (no agreement) and 1.00 (total agreement).

Berelson (1952) suggested inter-coder reliability would be acceptable with coefficients of 0.66 to 0.96. Lombard, Snyder-Dutch, and Bracken (2008) list 0.70 as appropriate for some purposes, 0.80 acceptable in most situations, and 0.90 as always acceptable. For the purposes of this study an inter-rater agreement of 0.80 or higher was deemed acceptable, in agreement with Lombard et al. Lombard, Snyder-Dutch. Bracken (2008) indicates that the minimum sample size to assess reliability is 10% of the full sample.

A total of 17 lessons (44%) containing 8,112 coding decisions were coded in the inter-coder reliability process. Of this total exercise number, 7549 represents the number of agreed upon coding decisions. Using Holsti's formula, the overall reliability measure 0.931 was obtained, representing an acceptable level of inter-coder reliability according to Berelson (1952) and Lombard et al. (2008). Additionally the inter-coder reliability level between each of the two mathematics education colleagues and this researcher were 0.915 and 0.940, respectively. A breakdown of the reliability measures by textbook series is presented in Table 5, showing the reliability ranged from 0.921 to 0.952.

Table 5

*Reliability Measures by Textbook Series*

| *Textbook Series* | *Total Questions* | *Total Coding Decisions* | *Total Agreement* | *Reliability Measure per Textbook Series* |
|---|---|---|---|---|
| Prentice Hall | 131 | 2096 | 1932 | 0.922 |
| Glencoe | 164 | 2624 | 2424 | 0.924 |
| Connected Mathematics 2 | 139 | 2224 | 2117 | 0.952 |
| UCSMP | 73 | 1168 | 1076 | 0.921 |
| Overall Total | 507 | 8112 | 7549 | 0.931 |

Stein, Grover, and Henningsen (1996) indicate that coding for cognitive demand of tasks necessitates an evaluation regarding the entire task as presented; this task appraisal requires a comprehensive judgment and makes coding consistency somewhat tentative. Jones (2004) reported that the inter-rater reliability percentage for the category of level of cognitive demand was lower than expected because coding was difficult to reliably assign. Jones reported the level of reliability on the level of cognitive demand and included a secondary report on the percent of tasks that differed by only one level. Using the suggestion by Jones (2004), herein, there were 163 disagreements in level of cognitive demand that differed by one level. By using Jones' method, reliability was increased from 0.931 to a second reliability measure of 0.951.

**Summary of Research Design and Methodology**

The research design and methodology for this study were delineated in this chapter. The research questions were reviewed and the sample of textbooks examined was identified. The next section examined the constructed coding instrument and the procedures for the coding, including locating the transformation lessons and the content to be analyzed. A pilot study (Appendix A) was conducted using two eighth grade textbooks; this study demonstrated the usefulness of the results and reliability of the coding instrument, as well as the differences found in the series providing indications that more could be learned from an in depth study. In Chapter 4 the findings of this study are described; Chapter 5 provides a discussion of the findings, conclusions, and implications for further research.

## Chapter 4: Findings

The purpose of this study was to analyze the nature and extent of the treatment of geometric transformations in middle grades mathematics textbooks in an attempt to gauge students' potential *opportunity to learn* transformations. Four series of textbooks available for classroom use in the United States were examined, each with a textbook for grades 6 to 8; for two of the series, an additional alternate textbook focusing on pre-algebra in grade 8 was also included. Consequently, the sample size consisted of 14 textbooks.

### Research Questions

The following research questions were addressed in this study:

1. What are the physical characteristics of the sample textbooks? Where within the textbooks are the geometric transformation lessons located, and to what extent are the transformation topics presented in mathematics student textbooks from sixth grade through eighth grade, within a published textbook series, and across different publishers?
2. What is the nature of the lessons on geometric transformation concepts in student mathematics textbooks from sixth grade through eighth grade, within a published textbook series?
3. To what extent do the geometric transformation lessons' student exercises incorporate the learning expectations in textbooks from sixth grade through eighth grade within a published textbook series, and across

textbooks from different publishers?

4. What *level of cognitive demand* is expected by student exercises and activities related to geometric transformation topics in middle grades textbooks? The *level of cognitive demand* is identified using the parameters and framework established by Stein, Smith, Henningsen, and Silver (2000).

Together, these four questions give insight into potential *opportunity to learn* that students have to study geometric transformations in the middle grades textbooks.

**Analysis Procedures**

Both descriptive statistics and qualitative methods were employed in the analysis of the collected data. The data analysis utilized percents, graphical displays, and narratives to illustrate the level of opportunities that students have to learn geometric transformations. The data collected was analyzed by comparing the textbooks from the sixth grade through the eighth grade within a published textbook series, and across textbooks from different publishers.

In particular, the data were analyzed within the sampled textbooks in terms of comparison of number of pages devoted to concepts, location of lessons within the textbooks, order in which lessons were presented, kinds of examples offered in the narrative for students study, number and types of student exercises presented, type of work required by students to complete exercises, kinds of manipulatives and technology suggested for student use, and the *level of cognitive demand* required by the student to complete lesson exercises.

Both the Prentice Hall (PH) and Math Connects (G-Glencoe) textbook series offered a choice of two textbooks for use in grade eight (PH8 or PH-pa, and G8 or G-pa) allowing individual school/district choice for the middle school curriculum to include pre-algebra and algebra topics that were included in the three textbook Connected Mathematics 2 and UCSMP series. To allow for comparison depending on the nature of the series used, analysis for PH and G was done using the basal series and again using the 67-pa textbook sequence. It was believed this would provide a fairer comparison with the CMP and UCSMP textbook series. Connected Mathematics 2 was coded using the single bound edition of the textbook, even though it is primarily used in modular form where instructional units could be presented in different sequences depending on district or teacher's choice. The lesson placement was determined using the publisher's suggested order in the single bound edition of the CM textbooks. The descriptive statistics were based on the transformation modules presented third in CM6, second in CM7, and fifth in CM8, as ordered in the single bound editions.

**Organization of the Chapter**

This chapter was organized into four sections to address the four research questions. The first section presents findings on "Where" the content was located within the textbooks, including physical characteristics of the instructional pages, lesson locations and sequence within the textbook layout. The second section presents findings on "What" was included in the Narrative components of the lesson, including structure of the lesson presentations and lesson components with the scope of the concepts. The third section presents data on student exercises, including total number and specific characteristics of the student exercises, expected student performance required to complete the exercise tasks, as well as the types of learning processes utilized in answering the exercise questions, and the suggested use of manipulatives and technology. The fourth section presents findings on the *level of cognitive demand* necessary for student expected performance in the exercises. This chapter ends with a summary of the results. Together these results are used in the discussion chapter to address students' potential *opportunities to learn* geometric transformations from middle grades mathematics textbooks presently available for use in the United States.

**Physical Characteristics of Transformation Lessons in Each Series**

This section presents data addressing the research question: What are the physical characteristics of the sample textbooks? Where within the textbooks are the geometric transformation lessons located, and to what extent are the topics presented in mathematics student textbooks from sixth grade through eighth grade, within a published textbook series, and across different publishers?

**Location of pages related to transformations.** Table 6 displays the physical characteristics of the textbooks and the location "Where" the transformation lessons appear. Presented are the number of instructional pages in each textbook, page number of the first transformation lesson and percent of textbook pages prior to the first transformation lesson in each textbook. The total number of textbook pages related to transformations was calculated using linear measurement of the pages to the closest one quarter of a page and then rounded to the tenths place in the table presentation. The table summary presents the total number of pages of transformation lessons contained in each textbook and the percent of the transformation lesson pages to the total number of instructional pages.

Table 6 also presents the number of chapters and sections contained in each

Table 6

*Pages Containing Geometric Transformations in the Four Textbook Series*

| *Text book* | *Total Page Count* | *Number Instr. Pages* | *Number of Chapters* | *Number Total Lessons* | *Number Transf. Lessons* | *Percent Transf. Lessons* | *Page # First Transf. Lesson* | *% Pages Prior to First Transf. Lesson* | *Number Transf. Lesson Pages* | *Percent Transf. Pages to Total* |
|---|---|---|---|---|---|---|---|---|---|---|
| PH6 | 730 | 603 | 12 | 94 | 2 | 2.1 | 398 | 66.0 | 7.3 | 1.2 |
| PH7 | 622 | 622 | 12 | 94 | 3 | 3.2 | 509 | 81.8 | 11.3 | 1.8 |
| PH8 | 746 | 596 | 12 | 88 | 4 | 4.5 | 136 | 22.8 | 14.5 | 2.4 |
| PH-pa[a] | 808 | 648 | 12 | 100 | 3 | 3.0 | 476 | 73.5 | 12.5 | 1.9 |
| G6 | 853 | 669 | 12 | 100 | 3 | 3.0 | 604 | 90.3 | 15.5 | 2.3 |
| G7 | 857 | 674 | 12 | 100 | 2 | 3.0 | 546 | 81.0 | 9.5 | 1.4 |
| G8 | 856 | 690 | 12 | 100 | 4 | 4.0 | 225 | 32.6 | 19.3 | 2.8 |
| G-pa[a] | 1033 | 806 | 13 | 99 | 3 | 4.0 | 101 | 12.5 | 16.8 | 2.1 |
| CM6[2] | 683 | 596 | 8 | 34 | 1 | 2.9 | 154 | 25.8[2] | 5.5 | 0.9 |
| CM7[2] | 738 | 650 | 8 | 34 | 2 | 5.9 | 87 | 13.4[2] | 24.5 | 3.8 |
| CM8[2] | 717 | 639 | 8 | 35 | 3 | 8.6 | 323 | 50.5[2] | 53.8 | 8.4 |
| U6 | 860 | 765 | 13 | 106 | 4 | 3.8 | 644 | 84.2 | 20.0 | 2.6 |
| U7 | 885 | 791 | 12 | 105 | 5 | 4.8 | 356 | 45.0 | 29.3 | 3.7 |
| U8 | 918 | 835 | 13 | 108 | 0 | 0 | 835[3] | 100.0 | 0.0 | 0.0 |

*Key:* Transf. = Transformation Instr. = Instructional

[a] Textbooks -pa (pre-algebra) are offered by publishers as an alternate textbook for grade eight curriculum.

[2] Textbooks are composed of modules that can be rearranged for instructional choice; calculations are based on the order of modules presented in the single bound edition.

[3] U8 contained no lessons on transformations.

textbook. The transformation lessons included in this table were complete lessons, including narrative, examples, and student exercises. Pages of lesson extensions, additional activities, and projects were not included as an individual lesson. With the exception of the PH6, G7, CM6 and CM7 textbooks, that each contained two or fewer lessons on transformations, the sampled textbooks each contained between three and four lessons on transformation topics.

The number of chapters and sections contained in the textbooks of the Prentice Hall, Glencoe, and UCSMP series appeared to be consistent. The number of chapters (student unit paperbacks) and sections contained in the CM2 series was found to be lower than the other three textbook series, although the total page counts were somewhat similar across all four series. The textbook series CM2 included two textbooks with the highest percentage of lesson pages focused on transformations, followed by UCSMP and PH, even though there were no transformation lessons contained in the U8 textbook.

The number of instructional pages in the fourteen textbooks ranged from 596 to 806, with an average of 685 and standard deviation of 78 pages. The UCSMP Algebra (U8) textbook did not contain any transformation lessons, and was excluded from the page total analysis. The percent of instructional pages of transformation topics ranged from 0.9% to 8.4%, with an average of 2.5% and a standard deviation of 1.9%. Only one textbook (CM8) had more than 5% of total textbook pages devoted to transformation lessons, and only two (CM7 and U7) had more than 3%.

The Prentice Hall and Glencoe textbooks appeared to be similar in the percent of pages devoted to transformations. Notice that CM8 placed a larger amount of emphasis on transformations indicated by 8.4% of pages devoted to transformations, that is, twice the percent of pages as in any other textbook herein examined. The analysis highlights that PH7, G6, G7 and U6 placed transformation lessons in the fourth quartile of the textbook pages.

Table 7 presents the relationship of transformation instructional pages to student exercise pages. The division of instructional pages to the number of exercise pages was approximately equal in the Prentice Hall series. This equality of pages was also true for the Glencoe and the UCSMP series. The Connected Mathematics series provided almost three times more page count devoted to student exercises than to instruction. Two of the Connected Mathematics 2 textbooks, CM7 and CM8, contained 25 and 54 lesson pages on transformations respectively. The majority of transformation lesson pages were dedicated to student exercises, with 70% in the CM6 textbook and 80% in the CM7 textbook. The UCSMP textbooks (U6, U7) contained 20 and 30 lesson pages on transformations respectively, with 40% and 50% of these pages devoted to student exercises.

**Relative position of transformation lessons.** Figure 7 displays the position of each type of transformation lesson within the textbooks with respect to the percentage of pages covered prior to the introduction of each topic. Ten of the textbooks presented the topic of translations and reflections in lessons following one another. One of the textbooks (PH8) presented all four of the transformation topics in lessons in close proximity to one another, whereas three of the textbook groupings did not address one or more of the transformations over the three book sequence.

Analysis of the physical characteristics revealed that transformations were contained in 13 of the 14 textbooks that comprised the sample. The UCSMP Algebra

Table 7

*Geometric Transformations Lessons/Pages in Textbooks*

| *Textbook* | *Total Transformation Pages* | *Number of Tranfs. Instructional Pages* | *Number of Transf. Student Exercises Pages* |
|---|---|---|---|
| PH6 | 7.3 | 3.8 | 3.5 |
| PH7 | 11.3 | 6.0 | 5.3 |
| PH8 | 14.5 | 7.8 | 6.8 |
| PH-pa | 12.5 | 6.0 | 6.5 |
| G6 | 15.5 | 7.3 | 8.3 |
| G7 | 9.5 | 4.5 | 5.0 |
| G8 | 19.3 | 10.8 | 8.5 |
| G-pa | 16.8 | 8.5 | 8.3 |
| CM6 | 5.5 | 1.3 | 4.3 |
| CM7 | 24.5 | 7.0 | 17.5 |
| CM8 | 53.8 | 12.3 | 41.5 |
| U6 | 20.0 | 11.5 | 8.5 |
| U7 | 29.3 | 16.0 | 13.3 |
| U8 | 0.00 | 0.0 | 0.0 |

(U8) textbook was the only textbook that did not contain any transformation lessons. The first transformation lesson occurred not until the first 90% in pages of the Glencoe, G6, textbook but within the first 12.5% of pages in the Glencoe Pre-Algebra textbook.

The first transformation lesson was placed in the first quartile of pages in the Prentice Hall PH8, Glencoe Pre-Algebra, and Connected Mathematics 2, CM7* textbooks. (*The position of the transformation topics in the Connected Mathematics 2 textbooks was determined by the order of units as recommended by the publisher; because the units were stand-alone soft covered workbooks, the order of use could be

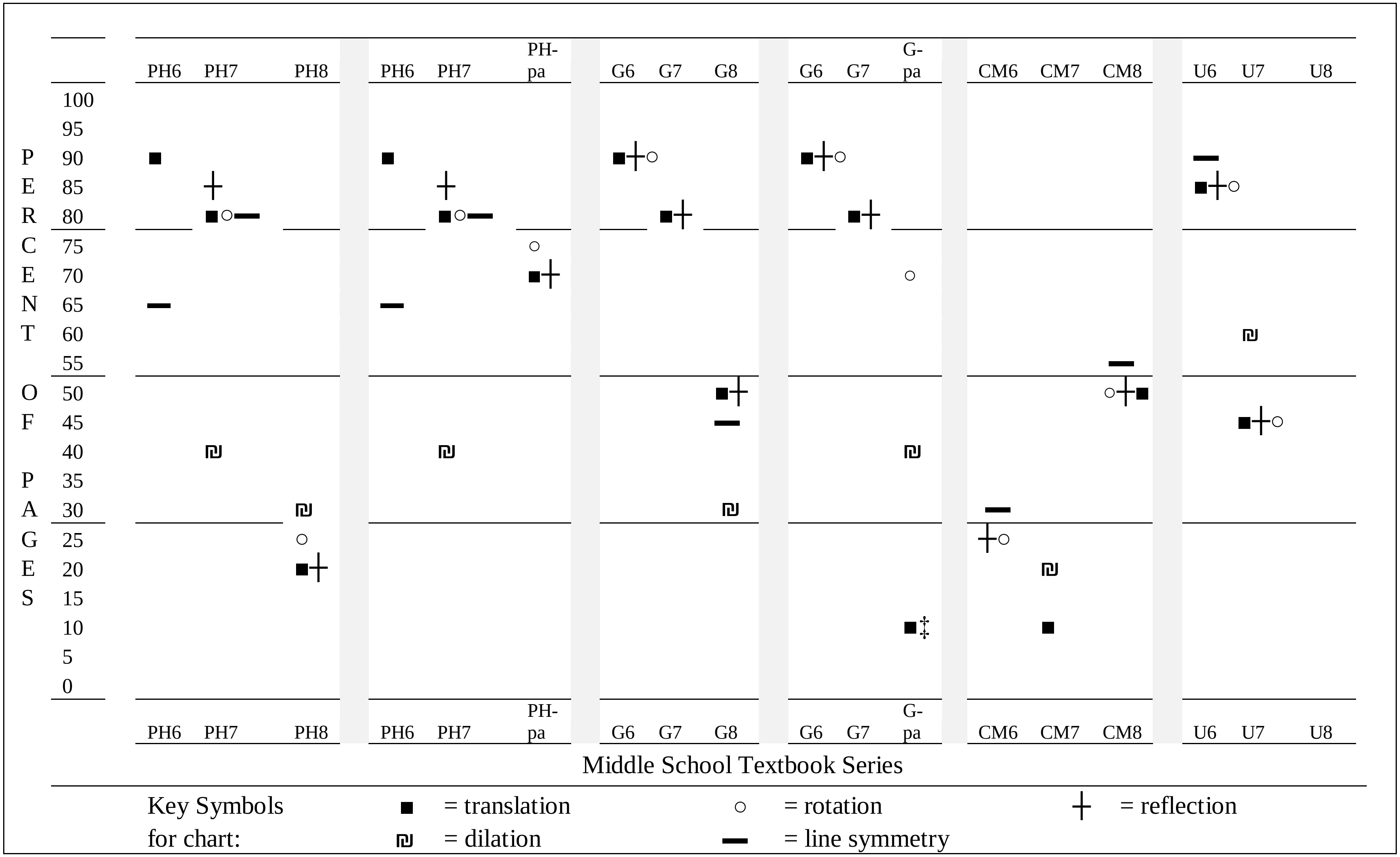


*Figure 7.* Placement of Transformation Topics in Textbooks by Percent of Pages Covered Prior to Lesson

rearranged by the teacher or district curriculum specialist). Four other textbooks first presented transformations in the second quartile range of textbook pages (Glencoe, G8; Connected Mathematics 2, CM6 and CM8; UCSMP, U7-Transition Mathematics), and two textbooks placed the first transformation lesson in the third quartile range of pages (Prentice Hall, PH6 and Pre-Algebra). Four textbooks placed the first transformation lesson in the fourth quartile of pages (Prentice Hall, PH7, Glencoe, G6 and G7; and UCSMP U6- Pre-Transition Mathematics). Because research has pointed out that lessons placed within the fourth quartile of textbook pages are not likely to be studied during a school year (Tarr et al., 2006; Weiss et al., 2001, 2003) it is unlikely that students would have the opportunity to study these lessons.

**Lesson pages related to each type of transformation.** The types and quantity of pages of each type of transformation are listed in Table 8. The types of transformation lessons, both narrative and exercises, contained in each textbook were listed by the total number of pages dealing with the construct. The pages were coded using linear measure, and each lesson page assessed was subdivided to the closest fourth of the page when more than one topic was included. The total page number listed in this table for each type of transformation was complied by adding the pages that primarily dealt with a specific transformation concept. Some approximation was necessary when more than one type of transformation was presented in the lesson narrative and exercises. The data in the table were rounded to the tenths place.

The proportion of pages devoted to transformation topics varied by textbook. The Prentice Hall textbooks predominately focused on the rigid transformations (reflections,

Table 8

*Number of Pages of Narrative and Exercises by Transformation Type*

| | *Translations* | *Reflections/ Reflectional Symmetry* | *Rotations/ Rotational Symmetry* | *Dilations* | *Composite Transformations* |
|---|---|---|---|---|---|
| PH6 | 1.3 | 3.0 | 3.0 | 0.0 | 0.0 |
| PH7 | 3.8 | 3.8 | 3.8 | 0.0 | 0.0 |
| PH8 | 3.8 | 3.8 | 3.5 | 3.5 | 0.0 |
| PHpa | 4.5 | 4.0 | 4.0 | 2.0 | 0.0 |
| G6 | 5.5 | 5.0 | 5.0 | 0.0 | 0.0 |
| G7 | 4.5 | 5.0 | 0.0 | 0.0 | 0.0 |
| G8 | 4.5 | 4.8 | 4.5 | 5.5 | 0.0 |
| Gpa | 2.8 | 2.8 | 5.8 | 5.5 | 0.0 |
| CM6 | 0.0 | 2.8 | 2.8 | 0.0 | 0.0 |
| CM7 | 0.0 | 0.0 | 0.0 | 25.8 | 0.0 |
| CM8 | 14.8 | 16.8 | 15.0 | 0.0 | 7.3 |
| U6 | 5.0 | 4.5 | 10.5 | 0.0 | 0.0 |
| U7 | 6.8 | 8.0 | 7.5 | 7.0 | 0.0 |
| U8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

translations, rotations); the Glencoe series was similar but included an equal proportion of pages devoted to dilations in the G8 and G-pa textbooks. The Connected Mathematics 2 series focused exclusively on dilations in the CM7 textbook and on the three rigid transformations in the CM8 textbook. The UCSMP series also included the topic of dilations in the U7 textbook. Yet, the overall proportions of lesson pages on each transformation did not appear to adhere to any systematic order or arrangement within the textbooks.

To summarize, within the textbooks of each series and across the textbooks of the four publishers the order of presentation of transformation topics and the appearance of all types of transformation topics appeared to be generally inconsistent. Translations were offered first in seven of the textbooks, but only five of these offered reflection as the second topic. The topic of rotations appeared in 11 of the textbooks, but the topic of dilation appeared in only six of the fourteen textbooks examined.

The order of the transformation topics varied among grade levels and across the four publishers. The findings indicated that each of the thirteen middle school mathematics textbooks presented topics of transformations, but the inclusion of all concepts, the order of presentation, and the location within the textbooks were inconsistent among grade levels and across published series. Inconsistency in the particular transformation topics included the order of presentation of the topics, but a rationale for the order of lesson topics within the student textbook editions was not included and could not be determined by the focus of this study's findings.

**Characteristics and Structure of Transformation Lessons**

This section presents data addressing the research question: What is the nature of the lessons on geometric transformation concepts in student mathematics textbooks from sixth grade through eighth grade, within a textbook series and across different publishers? The following section discusses the findings related to the components of the transformation lessons, the structure of the lesson and the narratives, how the components were typically organized in each series, and the characteristics of the presentations of the transformation constructs.

**Components of transformation lessons.** Of the four series analyzed in this study, the formats of the lessons in three of the textbook series were similar. The Prentice Hall, Glencoe, and UCSMP series basically contained the same types of components although they have been given slightly different titles. Differences within lessons were observed in the titles of the sections within a lesson, for example, Prentice Hall labeled student exercise questions as *Homework Exercises*, Glencoe labeled the questions as *Practice and Problem Solving*, and UCSMP used the label *Questions*.

The Prentice Hall, Glencoe, and UCSMP series started lessons with the objectives or a listing of the *Big Ideas* for the lesson. The topic was then discussed with vocabulary defined within the body of the lesson; terms were sometimes highlighted or bolded in the script. Most often the terms were defined within the narrative portion and the wording remained exactly the same or similar in presentations from the sixth through the eighth grades within a series. When a topic was repeated in the next grade level the depth of content did not increase. It was observed that the definitions of terms appeared to be presented in a mathematically formal form with accompanying explanations in the UCSMP series textbooks.

The narrative of the lessons contained discussion of the transformation topic with illustrations or graphs, a range of two to four examples worked out for student study within the narrative section, and exercises for student practice. Typically, examples presented steps for students to follow when completing the given questions; then a similar sample problem was provided for the student to answer orally or complete in written form. The Prentice Hall series offered some student activities at the beginning of the lesson, whereas the Glencoe lessons sometimes began with a Mini Lab. All three series kept the same structure for the middle grades textbook sequence with few exceptions. UCSMP textbooks presented framed blocks or highlighted sections for properties, rules, and important key concepts. The U6, U7, G-pa textbooks were found to contain increased amounts of discussion and explanations about transformation concepts in the narrative of the lessons, as well as more detail in the diagrams that accompanied the student examples than was found in the other textbook examined.

The narratives of the lessons were followed by student exercises to be completed in or out of class. Both the Prentice Hall and the Glencoe series typically included 3 to 7 questions to check for student understanding within the set of student exercises. The number of student exercises within the lessons of the three series varied from 14 to 35, with each series individually averaging approximately 22 exercise problems over the total number of lessons on transformations.

In contrast, the Connected Mathematics 2 series textbooks and lessons appeared in a different format. The unit modules in the CM2 series were similar to chapters in the other three series. The modules were stand alone bound paperback modules to be presented in an order determined by the teacher or school curriculum specialist. Each module began with pages numbered starting with one and included a glossary and index for the unit topics. The module (chapter) was divided into sections called investigations and each contained up to five sub-investigations. The objectives were presented at the beginning of the module and were not delineated for individual investigations. In the units (chapters) analyzed in this study, not all of the investigations (lessons) contained within a unit were in direct correlation to the transformation concepts under investigation, and hence were not included.

Each CM2 investigation was subdivided into problem activities which began with a short discussion and a list of student questions to be worked to expose students to the topics and concepts. Each investigation was subdivided into student activities designed to enhance the topic of the investigation. There was little narrative discussion or worked-out examples for student study; rather, it appeared that students were expected to work on assigned questions designed to have students discover the material important for the concepts. It was noted that few terms were defined in the lessons examined in the Connected Mathematics 2 series, likely because the format of the textbooks were based on student discovery through investigation. The student exercises were placed at the end of the investigations without designation as to which questions accompanied which subdivision of concepts. The activities in each subtopic numbered from two to eight questions, each with multiple parts. Approximately 30 to 60 student exercises followed at the end of all of the investigation questions, with an average of 43 questions.

**Characteristics of transformation constructs in each textbook series.** In the following sections transformations found in each textbook series is discussed.

***Prentice Hall textbook series.*** The Prentice Hall textbooks contained lessons on symmetry, line of symmetry, reflections, translations, rotations, and dilations. Each type of transformation is discussed below.

*Symmetry, line of symmetry, and reflection.* Prentice Hall presented the topic of line symmetry in each of the four sample textbooks. In the PH6 textbook, lesson 8.7 focused on line symmetry, with both the term *line symmetry* and *line of symmetry* defined. Examples were given showing line figures and drawings. No specific instructions were indicated with the examples. Students were asked to determine if a line of symmetry was present and how many lines of symmetry a figure had. Reflection in PH6 is presented in lesson 8.8 on transformations where this topic was mixed with translations and rotations.

PH7 lesson 10.6 included line of symmetry with reflections. This section started by identifying lines of symmetry to introduce the topic of reflection. Similar examples, drawings, and graphs were used in PH8 lesson 3.7, for example explain the line of reflection. The lessons in both PH7 and PH8 presented the same sequence by first reflecting a point, then a triangle over the y-axis from left to right. The questions for students following the examples asked for a response on the same type tasks. PH-pa, lesson 9.9 addressed line symmetry with the topic of reflections. An illustration of a pattern for clothing illustrated the line of reflection; other diagrams and graphs were similar to what was presented in the previous textbooks examined in this series. The PH-pa did add an example of reflection over a horizontal line of symmetry that was not previously observed. The instructions for reflections were written in the body of the examples and the properties of reflection were not highlighted or delineated in the lesson.

*Translations.* The second section examined in PH6, lesson 8.8, presented the topic of translations mixed with reflections and rotations. The examples offered for student study show drawings of figures translated from left to right. This example provided two line drawings and questions to determine if the figures appeared to be transformed by translation. The student oral example asked a similar question. The lesson in PH7, lesson 10.5, used the vocabulary of image and prime notation. Examples were provided to illustrate the concept; one was translation of a point, the other of figures translated to the right and down direction. Instructions were provided within the body of the examples to provide work for the student to follow. The student oral questions were similar to the provided examples with figures translated up and to the left. The examples provided in PH8, lesson 3.6, were similar in presentation and use of figures and illustrations. The same terminology (transformation, translation, and image) and definitions were used in both textbooks; the term *prime notation* was defined in the PH7 textbook. The lesson in

PH-pa, lesson 9.8, mirrored the previously presented examples for translating points and figures with the exception that one example illustrated the translation of a point to the left and up.

*Rotations***.** The textbook PH6 included one example on rotation mixed in with transformations, in lesson 8.8. This example showed a flower with petal rotation of 120°. No explanation was offered for determining the number of degrees and no instructions were offered in the example (Figure 8).

Example Application: Nature

3. Through how many degrees can you rotate the flower at the left so that the image and the original flower match?

(similar picture)

The image matches the original flower after rotations of 120°, 240°, and 360°.

Prentice Hall, Course 1, ©2010, p 403

*Figure 8.* Rotation Example

The student oral example asked the student to determine if a given figure had rotational symmetry, but this topic is not covered further. In lesson 10.7 of PH7, rotational symmetry and finding the angle of rotation were discussed. The rotation examples and exercises were all presented in the counterclockwise direction. In PH7, the narrative of lesson 10.7 states:

> "The direction of every rotation in this book is counterclockwise unless noted as clockwise. If a figure can be rotated 180° or less and match the original figure, it has **rotational symmetry**." (bold in original, p. 519)

No explanation or reasoning was offered for these parameters placed on the rotation examples or exercises and most of the problems followed the counterclockwise direction for movement. The example illustrated rotation displays on two graphs, one with 180° left hand rotation about the origin, the other with 90° left hand rotation about the center of the figure. The angles used in the textbooks focus on angles of rotation based on 90°, 180°, 270°, and 360°, and remained the same through the PH8 textbook. The example in the PH8 textbook on graphing rotations showed steps for graphing an image. This depth of discussion did not appear in the previous grade levels. Few exercises within this series asked for angle of rotation, or rotation about a point other than the origin or vertices of the figure. The method to determine the angle is not described. Lesson 9.10 in the PH-pa textbook defined the terms using the same wording, diagrams, and graphs offered for student study, and were similar to what was presented in both the PH7 and PH8 textbooks. In this series of textbooks, the topic of dilations was addressed only in the PH8 textbook and delineation of rotation properties was not evident. Both enlargements and reductions were presented as well as questions on scale factor.

*Dilations.* In this series of textbooks, dilations were presented only once in PH8 lesson 4.5, entitled *Similarity Transformations*. Three examples were provided, one on a reduction dilation of a triangle with instructions for finding the side lengths of the image. The second example illustrated an enlargement and gave steps to find the coordinate points of the vertices. The last example showed finding the scale factor in a reduction problem. The three oral student questions were similar to the example problems.

***Glencoe textbook series.*** The Glencoe textbooks contained lessons on symmetry, reflections, translations, rotations, and dilations. Each type of transformation is discussed below.

*Symmetry.* The Glencoe series presented lesson 6.5 on symmetry in the G8 textbook. The lesson began with a Mini Lab where the students were asked to trace the outline of The Pentagon. Students were instructed to draw a line through the center and one vertex of The Pentagon, fold the paper at the line, and examine the results. Within the same Mini Lab, students were instructed to trace the Pentagon on tracing paper and then to hold the center point and rotate the figure from its original position to find rotational matches. Instructions were provided to expose the student to the concepts of line symmetry, lines of symmetry, rotational symmetry, and angle of rotation. Three additional examples were provided, each with similar reinforcement questions following each example.

*Reflection and translations.* The Glencoe G7 lesson began with an example on line symmetry. Both G6 lesson 11.9 and G7 lesson 10.10 provided examples on reflecting figures over the x-axis in both the upward and downward direction and asked the student to reproduce similar reflections. In the G6 textbook, students reflected a figure to the left, but the textbook did not provide instructions; in contrast, the G7 textbook provided instruction for completing the movement of the figure to the left. Similar coordinate graphs were provided in the examples in both of the textbooks. In the G6 lesson, a highlighted block was provided for student study on terminology and illustrations of figures reflected over the x- and y- axes. Lesson 6.6 on reflection in the G8 textbook provided an example of reflection with movement to the left, and one with movement upward. The third example on reflection added line symmetry to the concept by having one point of the figure placed on the y-axis. The narrative drew students' attention to the fact that the line of reflection was also the line of symmetry in this example.

The structure of G-pa lesson 2.7 was different from the lessons previously reviewed in textbooks in this series. In this lesson, both reflections and translations were presented in the discussion portions prior to the presented examples. The terms *flip* and *slide* were provided in a key concept highlighted block. The coordinate plane diagrams were detailed showing the coordinates of the figures. Following each discussion was a detailed example with instructions to complete the transformation. One example was provided to reflect a figure downward and then to the right. The second student example included a new element with the student reflecting the figure over the y-axis and then onto the figure itself. This type of direction had not been discussed previously within this lesson or the previous textbooks in this series. Translations were also discussed in G-pa lesson 2.7 by providing illustrations of the movements of the figures, and an example with movement of a figure to the right and downward.

Textbook lessons G6-11.8, G7-10.9, and G8-6.7 presented examples that appeared to be similar across all three textbooks. The specific topics covered were translating figures to the left, right and down, and left and down. Finding the coordinates of the figure after it was translated was also presented. G6 included a key concept block with terminology and a model drawing of a translation.

*Rotations.* Lesson 11.10 on rotation in the G6 textbook begun with a Mini Lab that directed an activity in which students attached a piece of tracing paper to a coordinate plane with a fastener. A figure was traced onto the tracing paper and then the rotation was illustrated by the movement of the figure on the tracing paper around the fastener as the origin. Both clockwise and counterclockwise rotations were used with angle measure of 90°, 180°, 270°, and an explanation was provided that any measure may be used from 0° to 360°. The topic of rotational symmetry was covered in one example using a drawing of a snowflake. Lesson 11.3 on rotation in the G-pa textbook followed a slightly different format. More discussion and graphs were provided in the explanation. The center of rotation was discussed and illustrated, and there was an example of a rotation about a point other than the origin. However, the angle measures of rotation remain a multiple of 90°. Here, also, rotational symmetry was presented in one example using a drawing of a snowflake.

*Dilations.* Lesson 4.8 on dilations was introduced in the G8 textbook with a Mini Lab that gave instructions to dilate a figure by increasing the size of the grid on the paper. Both lesson 4.8 and lesson 6.8 in the G-pa textbook provided examples with instructions to shrink a figure and another to enlarge a figure. All of the examples used the origin as the center of dilation. Both lessons provided examples on finding the scale factor of the size change. The G8 textbook provided a real-world example of the size in change of a person's pupils when having an eye exam.

***Connected Mathematics 2 textbook series.*** The units under investigation in the textbooks began with a list of objectives for the unit, but the list was not delineated to align each objective to a particular lesson or activity. The divisions in the units were called investigations. The typical unit contained up to five investigations, although all were not in direct correlation to the concepts under investigation in this study. An investigation was subdivided into problem activities which began with a discussion and a list of student questions to be worked for the student to explore the concept ideas. The problem activities numbered from two to eight questions with multiple parts each. The student exercises followed all of the problem activities contained in the investigation and numbered from approximately 30 to 60 questions.

*Symmetry and line of symmetry*. A module entitled *Shapes and Designs: Two-Dimensional Geometry* was included in the CM6 curriculum. The second part of Investigation 1 discussed reflection symmetry (also called mirror symmetry) and rotation symmetry. The student was asked to identify reflection symmetry and rotation symmetry in drawings, in triangles, quadrilaterals, polygons, and other shapes found in the classroom. Three types of symmetry were discussed again in the CM8 module entitled *Kaleidoscopes, Hubcaps, and Mirrors: Symmetry and Transformations*. Reflectional symmetry and rotational symmetry were discussed and the topic was expanded to include center of rotation and angle of rotation. The subject of kaleidoscope designs and tessellations were included to describe the basic design elements. This module continued and discussed translational symmetry.

*Reflections, translations, and rotations.* Reflections, translations, and rotations were discussed in Investigation 2 of the CM8 *Kaleidoscopes, Hubcaps, and Mirrors* module. This Investigation presented symmetry transformations and began with reflections over the y-axis. In an example for students to answer, there was a problem where the figure was reflected onto itself. The topic of rotation and then translation was presented in the student questions. The topic of these transformations and symmetry was related to describing tessellations. Investigation 5 in this module discussed transforming coordinates and the rules used for reflections. Next the rules for translation of figures were presented followed by the rules for rotations. The fourth part of this Investigation presented rules for combinations of transformations. This was the only direct reference to composite transformations observed in all CM2 sampled textbooks. The narrative sections in these units presented limited terminology and information about the mathematical concept. The student was directed to work on problems to achieve the specifics that were presented in the examples in the other three series examined in this study.

*Dilations.* A unit in the CM7 textbook was dedicated to the topic of dilations; the title of this unit is *Stretching and Shrinking*. Investigation 1 immersed the student in solving a mystery. This activity centered on identification of a person by enlarging diagrams using a two-band stretcher. Next the topics of scaling up and down were explored. Investigation 2 presented work with similar figures and the student was to explore scaling by construction of a table of points showing scaling and distorted scaling (one coordinate is changed but the other was not). Different scaling examples were provided using a cartoon character, and scaled figures as cartoon family members.

***UCSMP textbook series.*** The UCSMP textbooks contained lessons on symmetry, reflections, translations, rotations, and dilations. Each type of transformation is discussed below.

*Symmetry and reflections.* The topics of symmetry and line symmetry were presented in lesson 2.3 of the U6 textbook. The list of vocabulary included *symmetric*, *reflection-symmetric*, *symmetry line*, *rotation-symmetric, rotation symmetry,* and *center of symmetry*. This lesson addressed the topics of reflection and rotation symmetry in a general discussion about symmetry, the advantages of recognizing symmetry in a figure was included. The narrative points out that if a figure was reflected over a line through its center, it is not possible to distinguish the image from the preimage. Rotational symmetry was defined as the center of symmetry. Tracing paper was suggested for use in the practice for rotational symmetry.

Lesson 6.2 in U7 continued this topic with reflections and reflection symmetry. Examples were given for reflecting a figure over a line (not present in the example), and reflecting a figure over-onto itself. In an example of reflecting a point over a line, the property of the line being the perpendicular bisector of the distance between the points was illustrated and discussed. Additional examples included reflecting a figure over an oblique line, reflection symmetry of a figure over/onto itself, and symmetry in regular geometric figures. Although there were no specific lessons on transformations in U8, the terms *reflection-symmetric* and *axis of symmetry* were discussed within the topics of quadratic equations and graphing.

*Translations***.** In U6 lesson 11.6 a translation was defined using the term *slide*. The term *vector* was defined and used to indicate the movement of the translation and the parts of the arrow were delineated with their meaning. Examples showed translation drawings, translations of a polygon on dot paper, and on a coordinate plane. Explanation was provided by using the *addition model* (adding values to each coordinate) to transform the coordinates of the preimage figure.

In U7, the topic of translations began in lesson 6.1, with an example of translations of repetitive patterns on cloth. Examples were provided on a detailed coordinate plane and the rule for finding the image coordinates were provided. Horizontal and vertical translations were discussed as well as translations in a diagonal direction. The last example in this lesson illustrated the use of a graphing calculator and the steps to perform the translation with this technology.

*Rotations.* The topics of angles and rotations were presented in U6, lesson 11.4 which begun with instructions for construction of a triangle with one given side length and two given angle measures. Instructions for duplicating an angle using a ruler and protractor, and using a compass and a straightedge were discussed and illustrated step by step. The topic of rotation of a figure was accompanied with a detailed drawing and the direction of the rotation about a fixed point was indicated. An example in this lesson included suggestions for tracing a figure and in another example using a computer program to show the movement of the figure in a counterclockwise and clockwise direction about a point.

The U7 textbook included the topic of understanding rotation in the second half of lesson 5.2. This narrative discussed rotation in a plane about a point called its center. The magnitude of rotation was indicated to show both positive and negative partial revolutions as well as the addition and subtraction of the number of degrees of the angle measures. A highlighted block drew attention to the fundamental property of rotations (angle measures may be added). Next in lesson 6.3, the topics of rotations and rotation symmetry were continued. Examples included rotation of a point and of figures. A highlighted block illustrated the rotation property. Rotational symmetry was discussed and examples were given with instructions for finding the measure of the angle of symmetry.

*Dilations.* Dilations were presented in U7 lesson 7.7 in a section called *The Size-Change Model for Multiplication* (p. 470). The terms in this section included expansion, size-change factor, contraction, and size change of magnitude k, but the term dilation itself was not used. Students were provided with two activities in the narrative portion of this lesson. The students were instructed to graph a figure and its enlargement in one activity and to graph the figure and its reduction in the other. As students answer questions within the activity's sequence of steps, they were guided to discovery of the concepts. The example using scale factor was presented in word problem form and related the meaning to an example using increased earnings. This lesson continued with an activity for size change performed on a graphing calculator. The activity provided delineated instructions on calculator use and screen shots for each step. This lesson ended with a discussion of a size change of one. The term identity was not used.

***Summary of textbook series.***

In summary, across the four textbook series, translations, reflections, rotations, and dilations lessons were present in at least one textbook in a three year sequence. Little was observed in any lesson that would assist in correcting or eliminating the issues that students experience with topics of transformations as identified in the literature. The Glencoe and UCSMP textbook series appeared to contain more direct instruction that would assist students with various kinds of specific types of transformations by including more explanations and detailed illustrations. Yet, the fact that a lesson was contained in a textbook is not a guarantee that it will be used in the classroom and some of the lesson locations within the textbooks appeared in a location that would limit student exposure to study the constructs.

**Number of Transformation Tasks**

This section presents the answer to the question: To what extent do the geometric transformation lessons' student exercises incorporate the learning expectations in textbooks from sixth grade through eighth grade within a published textbook series, and across textbooks from different publishers?

The student exercise data is reported in this section. A total of 1149 student exercises following the lessons were analyzed over the four textbook series. The student exercises were located at the end of each lesson, with the exception of the Connected Mathematics 2 series in which questions occurred at the end of the complete unit (chapter). The questions within the CM 2 textbooks were typically multi-part questions and each part was counted as one question in the coding process.

When evaluating questions that contained multi-parts, each part of the question, either numbered or lettered, was counted as one question. A total of 336 in the four Prentice Hall textbooks, 352 in the four Glencoe textbooks, 251 in the Connected Mathematics 2 series; and 210 student exercises in the UCSMP series were analyzed. Figure 9 displays the total number of transformation tasks in each textbook and each series, including the textbooks designated for the alternate pre-algebra course for grade 8.

**Number of tasks in each series.** Both the Prentice Hall and Glencoe textbook series were analyzed with the two textbook sequences that show the variations available for district textbook curriculum choice for their middle grades. The grade eight textbook would be chosen from either the Course 3 or the Pre-Algebra textbook and was presented to illustrate the content of each curriculum depending on the choice of textbooks and to provide a visual comparison. The Prentice Hall series (PH678) contained an average of 71 transformation questions in each textbook, and the PH67-pa sequence contained an average of 81 questions in each. Notice that the Glencoe series G678 offered students the greatest number of transformation tasks for practice of concepts over the three year curriculum which contained a total of 265 transformation questions, or an average of 88

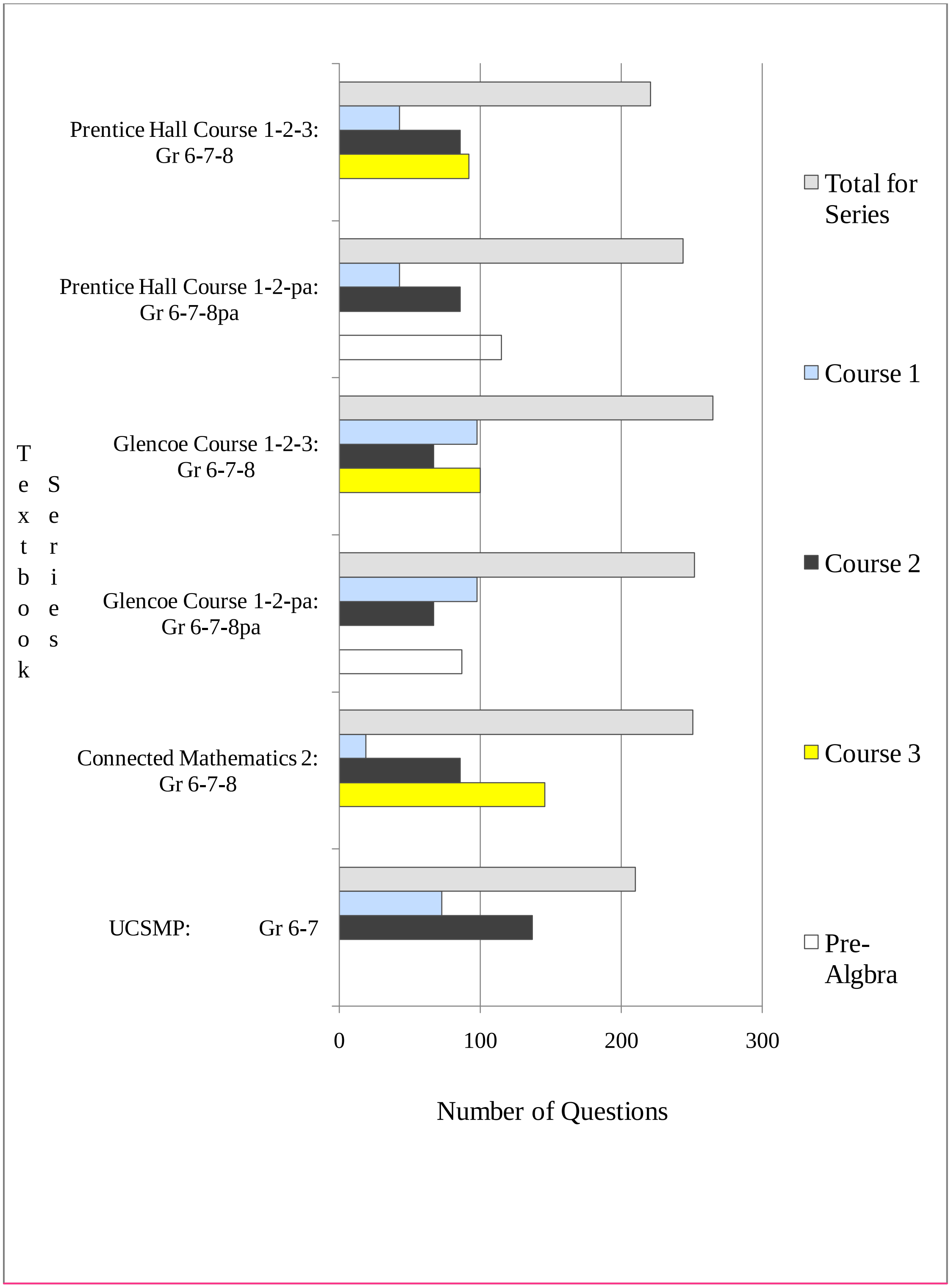


*Figure 9*. Number of Transformation Tasks in Each Series by Grade Level

questions per textbook. The Glencoe G6 textbook contained more than twice the number of exercises offered in the PH6 textbook. Both the Glencoe series, G67-pa, and the Connected Mathematics 2 series had approximately 250 transformation questions each, or an average of 83 questions per textbook. The Connected Mathematics 2, CM8 textbook offered 59% more exercises than offered by PH8 and 50% more than the number offered by the G8 textbook. The UCSMP series contained a total of 210 student exercises on transformations, an average of 105 questions per textbook (the U8 textbook did not contain transformation questions and was not used in these calculations).

**Number of each type of transformation task presented in student exercises.** The data collected on tasks included the specific type of transformation that the student was asked to perform in the exercises. In exercises that contained multiple parts, each part was counted as one task. Figure 10 presents the number of student tasks that addressed each transformation construct in each of the textbook series. The data presents the actual number of exercises for each type of transformation to facilitate comparing the types of transformations within each textbook.

The type of task least represented in all of the textbook series was composite transformations. The types of transformations represented most frequently were translations and reflections, followed by rotations. Dilation tasks were presented in fewer exercises than the rigid transformations except in the CM2 series. The Prentice Hall series placed a larger concentration of questions on reflections, translations, and rotations. The Glencoe series concentrated on translations and reflections; the Connected

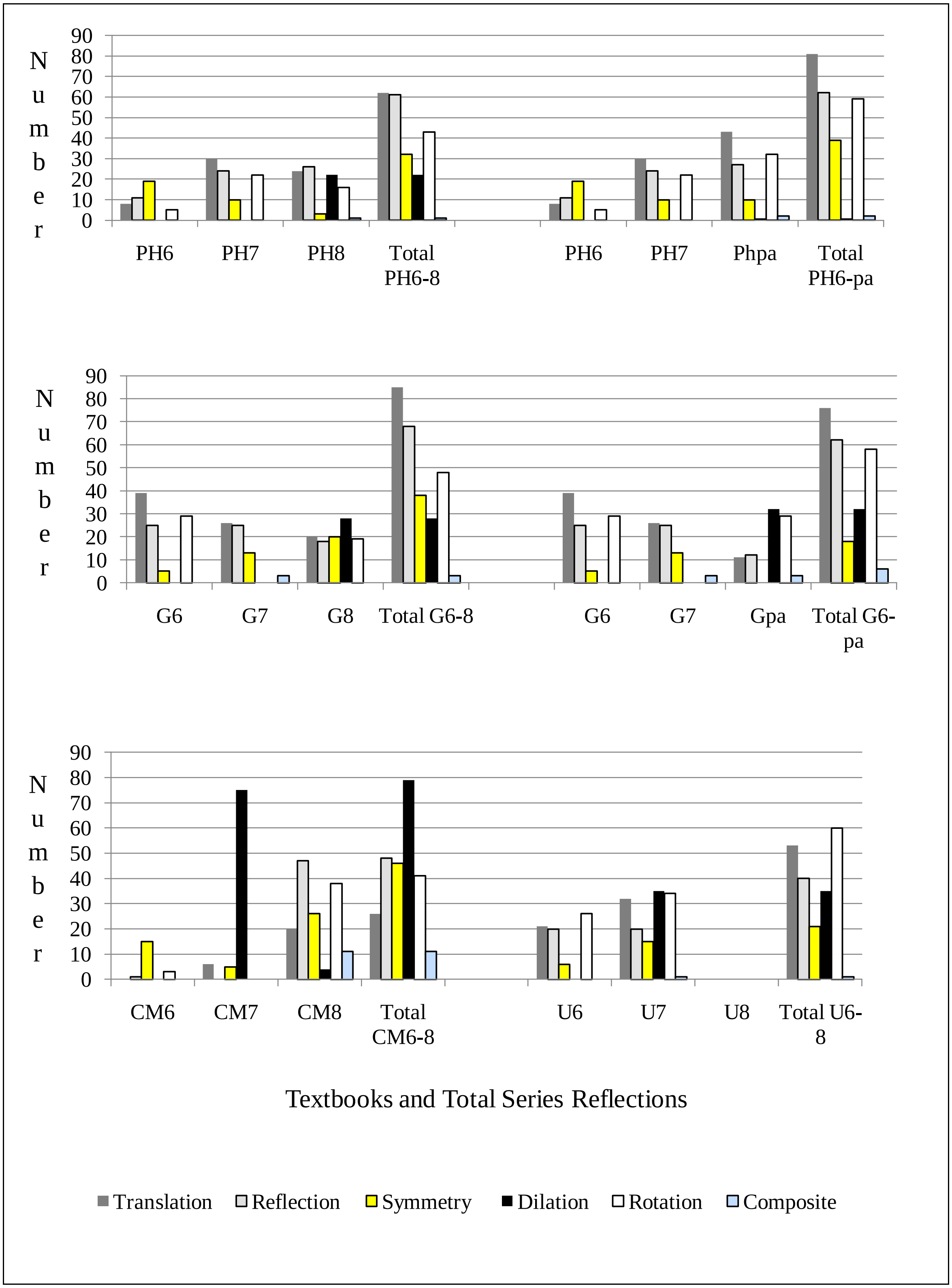


*Figure 10.* Number of Each Transformation Type in Each Textbook by Series

Mathematics 2 series concentrated on dilations in the CM7 textbook and appeared to have an even number of the other types of transformations in the grade 8 textbook. The UCSMP series covered transformations in the 6th and 7th grade textbooks, and did not present transformation topics in the 8th grade textbook. Figure 11 illustrates the relative importance that each textbook series placed on each of the transformation concepts by the specific number of questions presented in each series. This presentation provides a relative comparison over the series, whereas Figure 10 provided a comparison across textbooks within a series. Table 9 presents the total number and percent of the types of transformation tasks in each textbook and in each textbook series. The type and amount of tasks contained in each textbook series varied since it was dependent on the transformation concepts included in each of the textbooks. The most frequently presented transformation in any series, with over 30% of the tasks in each, was translations.

The Prentice Hall textbook series focused close to 30% of student exercises on translation, and 27% on reflection tasks. This approximate percentage applied to both the PH678 sequence and the PH67-pa sequence. A larger percent of tasks were devoted to symmetry in the PH6 textbook, but symmetry tasks remained approximately constant with either sequence of textbooks by Prentice Hall. Rotation tasks numbered less than 20% in the Prentice Hall, PH678 textbook sequence, but increased to almost 25% with the pre-algebra textbook sequence. Dilations accounted for about 10% in the PH678 textbook sequence, but less than 1% with the choice of the Prentice Hall textbooks, 67-pa curriculum. With the choice of the pre-algebra textbook for the PH series the topic of dilations was $< 0.1\%$ of the total transformation tasks.

The Glencoe textbook series presented approximately 30% of the transformation

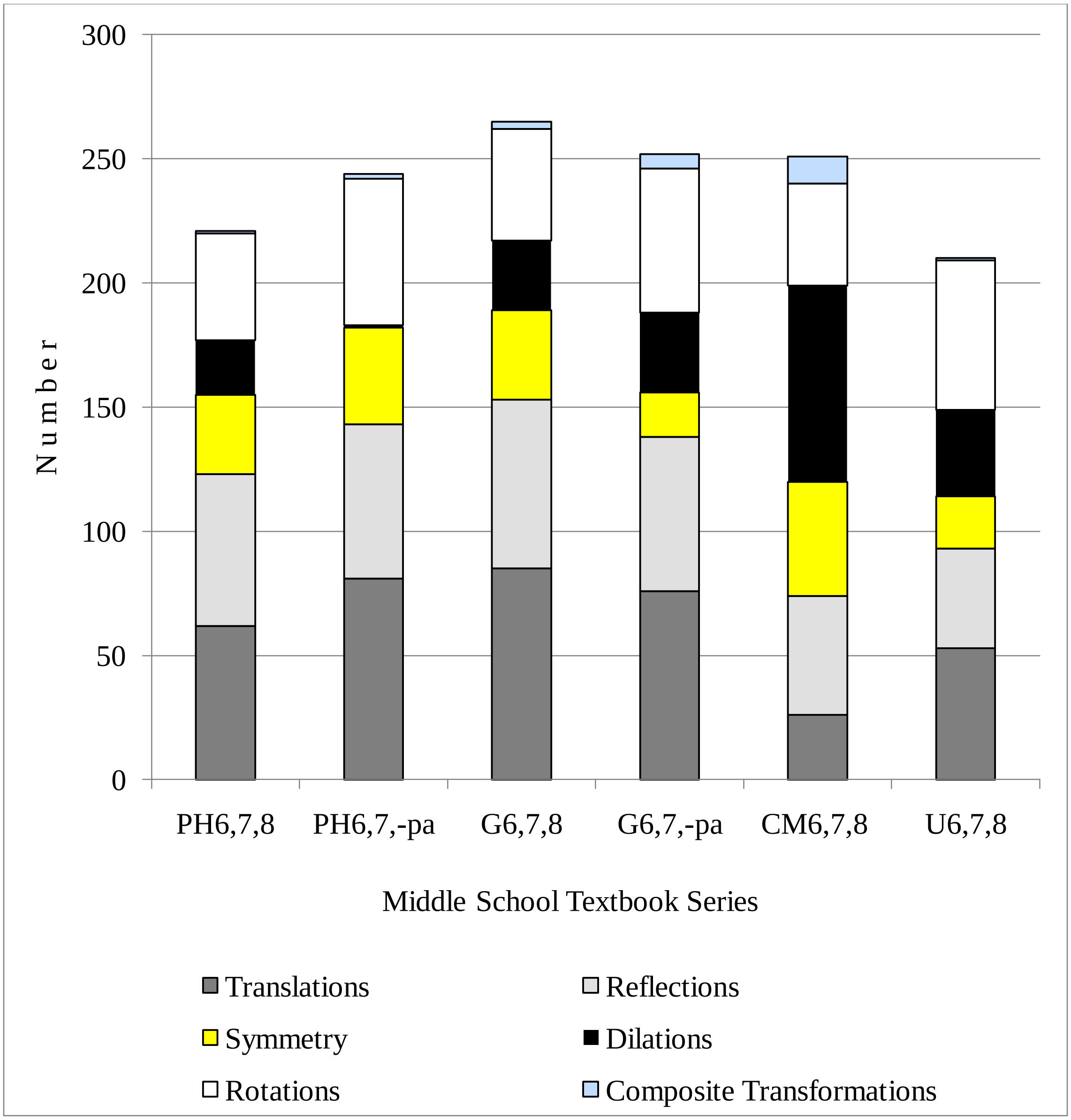


*Figure 11.* Total Number of Transformation Exercises in Each Textbook Series

tasks on translations, with either textbook series choice. Additionally the Glencoe series presented approximately 25% on reflections. Rotation tasks were addressed in 16% of the tasks in the G8 textbook, and 33% in the G-pa textbook. Again, composite transformation tasks were seldom represented with 1% in the G678 series of textbooks and 2.4% for the G67-pa alternative textbook sequence.

Table 9

*Number and Percent of Each Transformation Type to the Total Number of Transformation Tasks in Each Textbook*

| *Text book* | *Total Tasks* | *Translation Tasks* | | *Reflection Tasks* | | *Rotation Tasks* | | *Symmetry Tasks* | | *Dilation Tasks* | | *Composite Tasks* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | # | % | # | % | # | % | # | % | # | % | # | % |
| PH6 | 43 | 8 | 18.6 | 11 | 25.6 | 5 | 11.6 | 19 | 44.2 | 0 | 0.0 | 0 | 0.0 |
| PH7 | 86 | 30 | 34.9 | 24 | 27.9 | 22 | 25.6 | 10 | 11.6 | 0 | 0.0 | 0 | 0.0 |
| PH8 | 92 | 24 | 26.1 | 26 | 28.3 | 16 | 17.4 | 3 | 3.3 | 22 | 23.9 | 1 | 1.1 |
| PH-pa | 115 | 43 | 37.4 | 27 | 23.5 | 32 | 27.8 | 10 | 8.7 | 1 | 0.9 | 2 | 1.7 |
| | Prentice Hall Textbook Series Total for Grades 6, 7, 8 | | | | | | | | | | | | |
| PH-678 | 221 | 62 | 28.1 | 61 | 27.6 | 43 | 19.5 | 32 | 14.5 | 22 | 10.0 | 1 | <0.1 |
| PH-67pa | 244 | 81 | 33.2 | 62 | 25.4 | 59 | 24.2 | 39 | 16.0 | 1 | <0.1 | 2 | 0.8 |
| G6 | 98 | 39 | 39.8 | 25 | 25.5 | 29 | 29.6 | 5 | 5.1 | 0 | 0.0 | 0 | 0.0 |
| G7 | 67 | 26 | 38.8 | 25 | 37.3 | 0 | 0.0 | 13 | 19.4 | 0 | 0.0 | 3 | 4.5 |
| G8 | 100 | 20 | 20.0 | 18 | 18.0 | 16 | 16.0 | 18 | 18.0 | 28 | 28.0 | 0 | 0.0 |
| G-pa | 87 | 11 | 12.6 | 12 | 13.8 | 29 | 33.3 | 0 | 0.0 | 32 | 36.8 | 3 | 3.4 |
| | Glencoe Textbook Series Total for Grades 6, 7, 8 | | | | | | | | | | | | |
| G-678 | 265 | 85 | 32.1 | 68 | 25.7 | 45 | 17.0 | 36 | 13.6 | 28 | 10.6 | 3 | 1.1 |
| G-67-pa | 252 | 76 | 30.2 | 62 | 24.6 | 58 | 23.0 | 18 | 7.1 | 32 | 12.7 | 6 | 2.4 |
| CM6 | 19 | 0 | 0.0 | 1 | 5.3 | 3 | 15.8 | 15 | 78.9 | 0 | 0.0 | 0 | 0.0 |
| CM7 | 86 | 6 | 7.0 | 0 | 0.0 | 0 | 0.0 | 5 | 5.8 | 75 | 87.2 | 0 | 0.0 |
| CM8 | 146 | 20 | 13.7 | 47 | 32.2 | 38 | 26.0 | 26 | 17.8 | 4 | 2.7 | 11 | 7.5 |
| | Connected Mathematics 2 Textbook Series Total for Grades 6, 7, 8 | | | | | | | | | | | | |
| CM series | 251 | 26 | 10.4 | 48 | 19.1 | 42 | 16.3 | 46 | 18.3 | 79 | 31.5 | 11 | 4.4 |
| U6 | 73 | 21 | 28.8 | 20 | 27.4 | 26 | 35.6 | 6 | 8.2 | 0 | 0.0 | 0 | 0.0 |
| U7 | 137 | 32 | 23.4 | 20 | 14.6 | 34 | 24.8 | 15 | 10.9 | 35 | 25.5 | 1 | 0.7 |
| U8 | 0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 | 0 | 0.0 |
| | UCSMP Textbook Series Total for Grades 6, 7, 8 | | | | | | | | | | | | |
| U series | 210 | 53 | 25.2 | 40 | 19.0 | 60 | 28.6 | 21 | 10.0 | 35 | 16.7 | 1 | 0.5 |

In the Connected Mathematics 2 series, symmetry exercises were the focus in almost 80% of the CM6 transformation tasks, and more than 87% in the CM7 textbook. Additionally, in Connected Mathematics 2 series, dilations tasks represented more than 31% of the transformation exercises. Composite transformation tasks were present in 4.4% of the transformation exercises and represented the highest concentration of all the series examined.

The UCSMP textbook series contained transformation lessons in the grade 6 and 7 textbooks, transformations were not covered in the UCSMP textbook for 8th grade. The transformation exercises focused on reflections in 19% of the exercises and translations in 25% of the transformation tasks. The UCSMP series placed the largest emphasis on rotation (28.6%). Dilation tasks were presented in approximately 16% of the exercises. Composite transformation tasks appeared in a negligible percentage of exercises in all four of the textbook series. Notice that composite transformation tasks were negligible in number in most of the textbook series examined. The findings show a small amount of content on composite transformations presented in some textbooks with the highest value of 4.4% found in the CM2 series.

**Characteristics of the transformation tasks in the student exercises.** This section expands on the student exercise data to address the specific characteristic of the transformation tasks within each exercise. In addition to differences comparing the types of transformations covered in each text, detailed study of each transformation type was conducted to understand the nature of how each transformation was structured. Specific characteristics and sample examples are illustrated in Appendix K.

Following each type of transformation topic a summary graph is presented showing the number of exercises in each textbook by series on the specific transformation types. The categories of tasks were grouped specifically into three to four categories in relation to the student issues identified from the literature review. When an exercise required a response that was not specific or could not be grouped into the specifically defined categories it was labeled as a general transformation type. A general transformation type would include filling in vocabulary or identifying the direction of movement of the transformation from a diagram or picture. Typical general translation sample problems were provided within the transformation type sections to further explain how the exercises were classified. Appendix I provides examples to illustrate each of the categories of the specific transformation tasks.

***Translations***. Table 10 displays the tasks related to translations with the direction of movement of the figure determined by instructions in the student exercises in each of the textbooks. Notice the Prentice Hall PH6 textbook focused entirely on nonspecific translation tasks and the propensity to single directional movements in the PH7 textbook. General translation tasks were those that gave instructions for a translation but not direction or axis over which to move the figure. Figure 12 illustrates an example of this type of exercise. Other types of general translation exercises asked the student to write the rule for the translation, or describe the translations used in an illustrated pattern. The PH-pa textbook presented general translation questions and figures translated in a downward/right direction.

Table 10

*Percent of Each Type of Translation Task to the Total Number of Translation Tasks in Each Textbook*

| | | Task and direction of movement (±x, ±y) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *Text book* | *Total Number* | *General-Tr* | *Tr +y* | *Tr +x* | *Tr -y* | *Tr -x* | *Tr (+,-)* | *Tr (-,-)* | *Tr (+,+)* | *Tr (-,+)* |
| PH6 | 8 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| PH7 | 30 | 13 | 3 | 30 | 20 | 17 | 0 | 7 | 7 | 3 |
| PH8 | 24 | 29 | 8 | 8 | 4 | 13 | 8 | 4 | 13 | 13 |
| PH-pa | 43 | 26 | 7 | 7 | 5 | 5 | 33 | 9 | 7 | 2 |
| G6 | 39 | 10 | 3 | 3 | 3 | 8 | 18 | 21 | 15 | 2 |
| G7 | 26 | 34 | 0 | 7 | 3 | 3 | 17 | 14 | 14 | 7 |
| G8 | 20 | 20 | 0 | 0 | 0 | 5 | 20 | 15 | 20 | 20 |
| G-pa | 11 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CM6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CM7 | 6 | 50 | 0 | 0 | 0 | 0 | 50 | 0 | 0 | 0 |
| CM8 | 20 | 100 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| U6 | 21 | 48 | 0 | 10 | 14 | 5 | 5 | 5 | 0 | 14 |
| U7 | 32 | 41 | 6 | 9 | 13 | 6 | 9 | 13 | 3 | 0 |
| U8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Note:* *The direction of movement of the translation is designated by the signs of the coordinate directions (±x, ±y). Hence, (+, -) indicates to the right and down.

**The number of exercises reported herein does not reflect the total number of questions presented in the textbook exercises, but only those relating to the specific transformation characteristics. The numbers reported in the tables are rounded to a whole percentage and hence do not necessarily total 100 percent because a task could be coded as having more than one type of characteristic (e. g., translate from left to right, reflect a figure upward over a horizontal).

26. Writing in Math
Why is it helpful to describe a translation by stating the horizontal change first?
Prentice Hall, Course 2, © 2010, p. 513

*Figure 12.* Example of General Translation Exercise

The Glencoe series offered many questions on general translations with G7 and G-pa listing the highest percentages in each. The G6, G7, and G8 textbooks contained nearly twice as many questions on translations moving to right/down, left/down, and right/up than any of the other directions of movements of exercises in this series. The G8 textbook contained a nearly equal distribution of questions asking for translation movement upward and downward in combination with right and left movements. Notice that the G-pa text focused only on general translation exercises.

In the Connected Mathematics 2 series, the CM6 textbook did not offer translations in a lesson, while the CM7 textbook focused 50% of questions on tasks for translations to the right/down in 50% of the exercises and the remaining 50% were general translation questions. The CM8 presented 100% general translation questions.

The UCSMP U6 textbook offered approximately 50% of its transformation exercises on general translations, and a combination of right, left, and mixed directions. Exercises with translating a figure upward or to the right/up were not present. The U7 textbook focused over 40% on general translation questions, and a combination of directions except upward and to the left. As stated earlier, the UCSMP grade 8 textbook did not contain transformational lessons.

Figure 13 summarizes the translation exercises in each textbook series. This figure groups the types of translations into four groups. General translation problems and single direction movement of translation exercises are easier for students to perform than translations with dual direction of movement, and those with translations upward and/or to the left.

***Reflections.*** Table 11 presents information on the nature of the tasks related to reflection with the direction of movement of the figure in each of the textbook series.

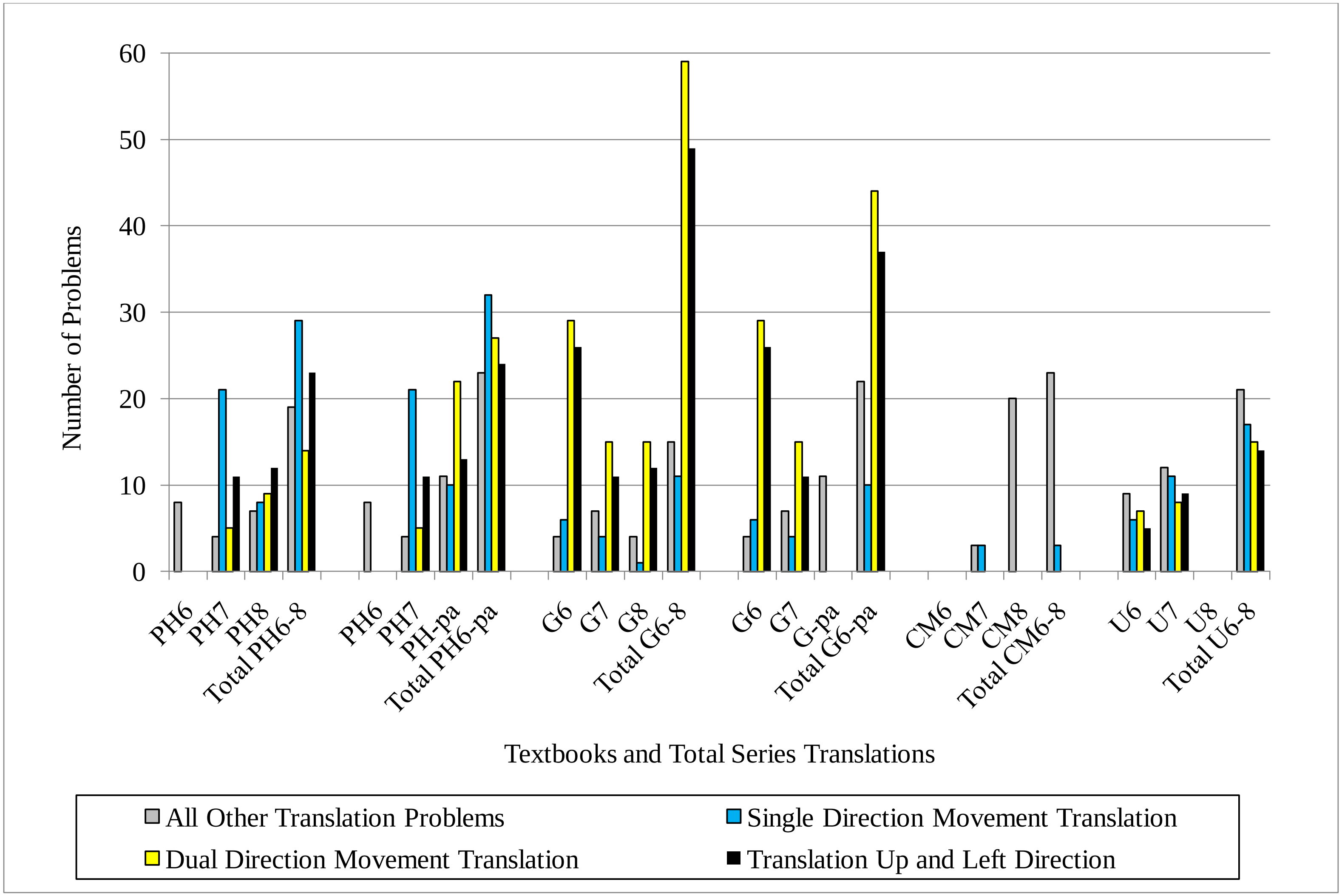


*Figure 13.* Summary of Translation Exercises in the Middle School Textbook Series

Table 11

*Percent of Each Type of Reflection Task to Total Number of Reflection Tasks in Each Textbook*

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Task and direction of movement over axis* | | | | | | | | | | | | |
| *Text book* | *Total Num -ber* | *Rf* | *Rf up* | *Rf down* | *Rf right* | *Rf left* | *Rf over line* | *Rf on to* | *Rf Right down* | *Rf Right up* | *Rf Over x* | *Rf Over y* | *Rf sym* |
| PH6 | 11 | 45 | 0 | 9 | 9 | 0 | 9 | 0 | 0 | 0 | 9 | 18 | 0 |
| PH7 | 24 | 29 | 4 | 17 | 25 | 17 | 0 | 4 | 0 | 0 | 0 | 0 | 4 |
| PH8 | 26 | 0 | 12 | 23 | 12 | 23 | 12 | 0 | 0 | 0 | 0 | 0 | 19 |
| PH-pa | 27 | 26 | 4 | 30 | 4 | 19 | 15 | 4 | 0 | 0 | 0 | 0 | 0 |
| G6 | 25 | 0 | 16 | 16 | 20 | 12 | 0 | 0 | 0 | 12 | 8 | 16 | 0 |
| G7 | 25 | 0 | 8 | 24 | 8 | 12 | 0 | 20 | 0 | 0 | 0 | 28 | 0 |
| G8 | 18 | 11 | 6 | 17 | 17 | 0 | 11 | 12 | 0 | 0 | 17 | 11 | 0 |
| G-pa | 12 | 0 | 8 | 25 | 17 | 0 | 0 | 16 | 0 | 0 | 0 | 33 | 0 |
| CM6 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 100 |
| CM7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CM8 | 47 | 17 | 0 | 2 | 2 | 11 | 6 | 15 | 0 | 0 | 0 | 6 | 40 |
| U6 | 20 | 20 | 0 | 5 | 0 | 0 | 20 | 0 | 0 | 0 | 20 | 10 | 25 |
| U7 | 20 | 25 | 0 | 0 | 15 | 0 | 0 | 5 | 5 | 0 | 30 | 15 | 5 |
| U8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Note:* Direction of movement of the reflected figure is indicated by *up*, *down*, *right*, *left*, etc., *over the axis* or a *line*, or of a figure translated to overlap (*onto*) some part of the pre-image.

Both the Prentice Hall and Glencoe series focused most student exercises on the reflection of figures in one direction and offered few problems with reflections over a line other than the x- or y-axis. Some of the exercises examined did not specify the direction of the reflection to the right/left or up/down, hence the coding symbols on the tables as *Rf over x* or *Rf over y* were needed; this type of exercise typically instructed the student to draw a figure and perform a reflection. Figure 14 provides a sample of this classification.

Rf right/down or Rf right/up indicated a diagonal movement of the reflection on the graph. The symbol *Rfo* indicated examples where the student was to perform a reflection of the figure over/onto the pre-image itself. The Glencoe series textbooks G7, G8 and G-pa, as well as the Connected Mathematics 2 textbook CM8 contained numerous problems coded as *Rfo*. Figure 15 presents a typical problem that was coded as reflection over/onto itself. For this type of exercise the pre-image was reflected over a line and is super-imposed on top of itself in whole or in part.

> 10 b. When a point (x,y) is reflected over the x-axis, what are the coordinates of its image?
>
> UCSMP, Pre-Transition Mathematics (U6), ©2009, p.647

*Figure 14.* Example of Reflection Exercise - *Rf over x*

> Graph each figure and its reflection over the x-axis. Then find the coordinates of the reflected image.
> 6. quadrilateral DEFG with vertices D(-4, 6), E(-2, -3), F(2, 2), and G(4, 9)
>
> Glencoe, Course 2 ©2009, p. 560

*Figure 15.* Example of Reflection Exercise - *Rfo (over/onto preimage)*

Figure 16 summarizes the reflection exercises in each middle school textbook series. This figure groups the types of reflections into four groups: general reflection problems, reflections upward and/or left movement exercises, reflections over an oblique line, and reflection over/onto the pre-image. Directions of reflection pre-image movement to the right and downward are easier for students to perform than reflections over an oblique line or reflections of the image overlapping onto the pre-image figure.

The Prentice Hall, PH6 textbook included approximately 45% of the total

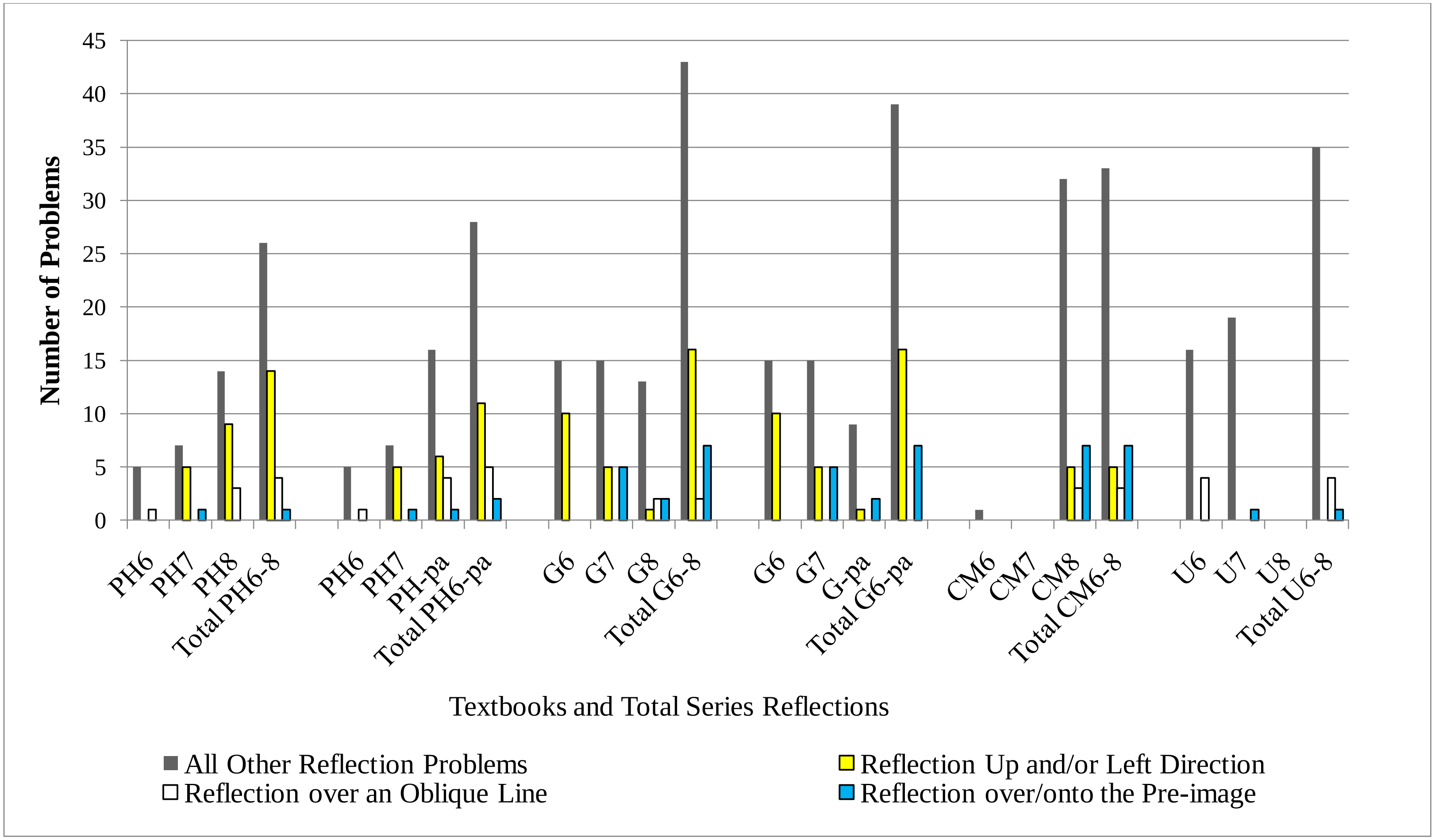


*Figure 16.* Summary of Reflection Exercises in the Middle School Textbook Series

reflection questions on general reflections and PH7, PH-pa, contained approximately 25%. The percentage of the remaining exercises decreased in frequency of reflections from downward/right, to the left/up direction. Exercises containing reflection of a figure over a line other than an axis, or of a figure reflected over/onto the figure itself were seldom present. The Glencoe textbook G8 contained 11% general reflection questions. All four of the Glencoe textbooks contained problems for single or double directional movements of reflections, as to the right and downward, and for a figure reflected over/onto the pre-image of the figure.

The results showed that reflection exercises were seldom included in the CM6 and CM7 textbooks and were presented essentially in only the CM8 textbook, additionally the CM8 textbook presented reflection problems with movement of the figure to the left/up, or downward, as well as reflections of figures over/onto the pre-image.

The UCSMP series textbooks presented approximately one quarter of the transformation tasks on general reflections, and the same amount on reflecting figures either right and left, or up and down. The U6 textbook presented another fourth of the exercises on reflective symmetry.

***Rotations.*** Student exercises on rotations were found in eleven of the fourteen textbooks as shown in Table 12. In the Prentice Hall series all instructions indicated that rotations were in a counterclockwise direction. The G6 and G-pa textbooks presented rotation tasks in both the clockwise and counterclockwise directions, as well as exercises on rotation symmetry. The Glencoe textbook G7 did not contain exercises on rotation tasks. The G8 textbook's exercises center 94% of all transformation tasks on rotation

Table 12

*Percent of Each Type of Rotation Task to Total Number of Rotation Tasks in Each Textbook*

| | | | | Task and direction of movement | | | |
|---|---|---|---|---|---|---|---|
| *Textbook* | *Total Number* | *Ro* | *Ro-right* | *Ro-left* | *Ro symmetry* | *Ro exterior point* | *Ro angle* |
| PH6 | 5 | 0 | 0 | 60 | 40 | 0 | 0 |
| PH7 | 22 | 23 | 0 | 32 | 45 | 0 | 0 |
| PH8 | 16 | 63 | 0 | 0 | 31 | 0 | 6 |
| PH-pa | 32 | 13 | 0 | 41 | 25 | 6 | 16 |
| G6 | 29 | 7 | 34 | 34 | 24 | 0 | 0 |
| G7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G8 | 16 | 6 | 0 | 0 | 94 | 0 | 0 |
| G-pa | 29 | 21 | 52 | 3 | 24 | 0 | 0 |
| CM6 | 3 | 0 | 0 | 0 | 100 | 0 | 0 |
| CM7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CM8 | 38 | 18 | 5 | 13 | 58 | 0 | 5 |
| U6 | 26 | 35 | 15 | 12 | 38 | 0 | 0 |
| U7 | 34 | 24 | 32 | 18 | 26 | 0 | 0 |
| U8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

*Note:* Direction of movement of the rotated figure is indicated by *right*, *left*, rotation in respect to an *exterior point*, or finding the *angle* of rotation.

symmetry. Few exercises within this series asked for finding the angle of rotation, or rotation about a point other than the origin or vertices of the figure.

The Connected Mathematics 2 textbook CM6 presented 100% of the rotation exercise tasks on rotational symmetry. The CM7 textbook did not address the topic of rotation, while the CM8 textbook contained exercises on both clockwise and counterclockwise directions, 5% on angle of rotation, and 58% on rotational symmetry.

The UCSMP textbooks, U6 and U7, also presented rotation problems with both clockwise and counterclockwise directions as well as on the topic of rotational symmetry. Few exercises within this series asked for angle of rotation, or rotation about a point other than the origin or a vertex of the figure.

Figure 17 summarizes rotation exercises in each middle school textbook series. This figure groups the types of rotations into three categories: general rotation exercises; finding angle of rotation; and rotation about a point other than the origin or a vertex, which is the most difficult for students as indicated by the research. Over the four textbook series, no exercises were observed that included rotation of a figure about a point exterior to the given figure.

***Dilations.*** Dilation exercises were found in five of the fourteen textbooks, at least one in each series (Table 13). In the Prentice Hall series only the PH7 textbook offered exercises on dilations and scale factor. Similarly, dilations were found in the Glencoe series in both the G8 and the G-pa textbooks, which contained questions on enlargements and reductions. The Connected Mathematics 2 series included dilation as a topic in the CM7 textbook and presented questions on enlargements, reductions of figures, and scale factor. Dilation tasks were represented in almost 32% of the transformation exercises. The UCSMP textbook U7 included the topic of dilation with the property of identity when the scale factor was equal to one. The other three series of textbooks did not include this concept. The U7 textbook was the only one observed to contain a scale factor of one used with a reference to *identity*.

Research indicated dilations to be the most difficult of the four transformations. Performing dilations in relationship to a point other than the coordinate plane origin or a

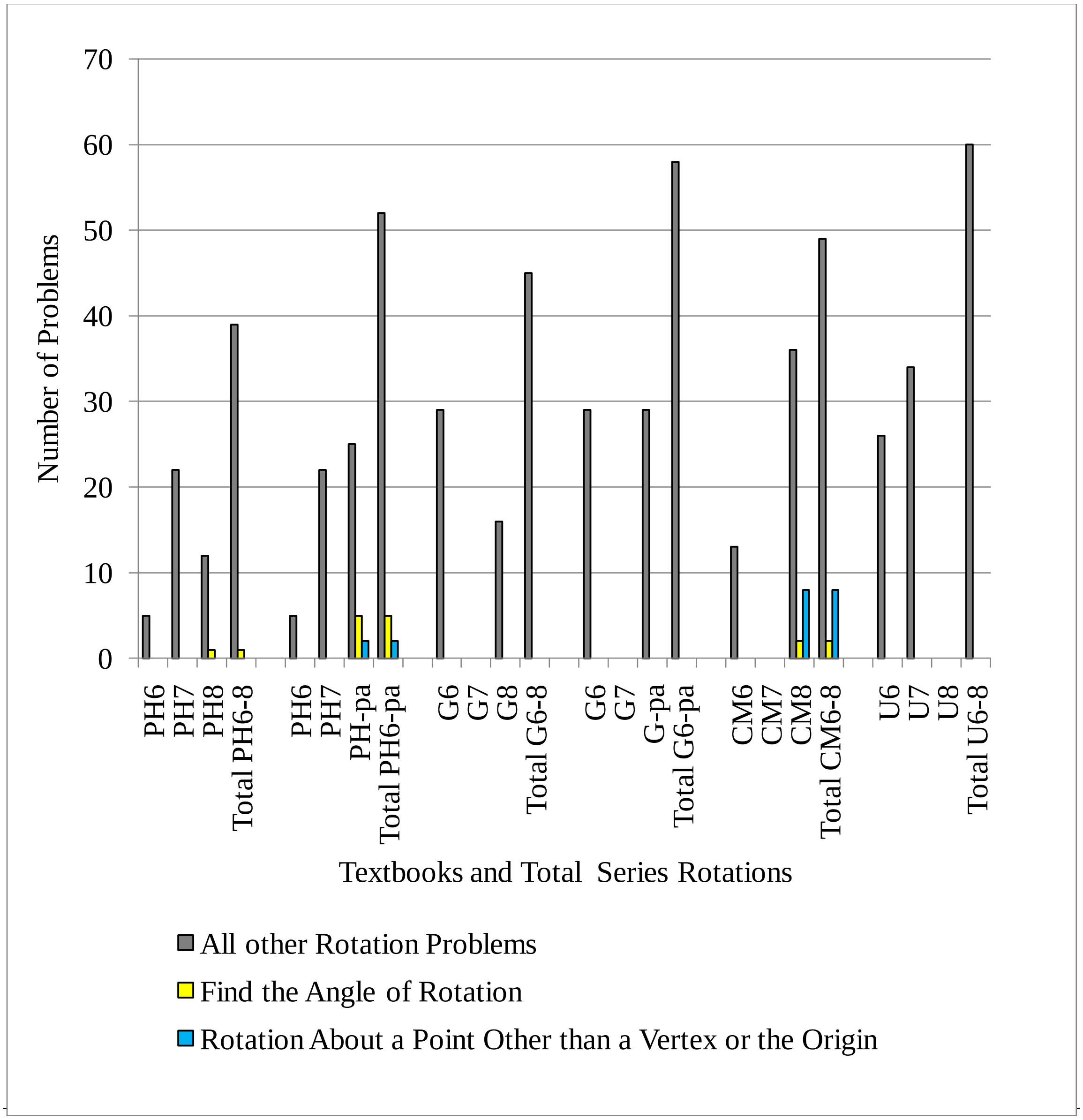


*Figure 17.* Summary of Rotation Exercises in the Middle School Textbook Series

vertex of the figure were typically difficult for student to perform. These types of dilations were not observed in any of the textbooks. Figure 18 summarizes the dilation exercises in each middle school textbook series, and groups types of dilations into four categories, enlarge or shrink, scale factor, and identity.

Table 13

*Percent of Each Type of Dilation Task to Total Number of Dilation Tasks in Each Textbook*

| *Textbook* | *Total* | *Di* | *En* | *DiEno* | *Sf* | *Identity* |
|---|---|---|---|---|---|---|
| PH6 | 0 | 0 | 0 | 0 | 0 | 0 |
| PH7 | 0 | 0 | 0 | 0 | 0 | 0 |
| PH8 | 22 | 36 | 41 | 0 | 23 | 0 |
| PH-pa | 1 | 0 | 0 | 0 | 100 | 0 |
| G6 | 0 | 0 | 0 | 0 | 0 | 0 |
| G7 | 0 | 0 | 0 | 0 | 0 | 0 |
| G8 | 28 | 32 | 32 | 0 | 36 | 0 |
| G-pa | 32 | 34 | 34 | 0 | 31 | 0 |
| CM6 | 0 | 0 | 0 | 0 | 0 | 0 |
| CM7 | 75 | 24 | 52 | 0 | 24 | 0 |
| CM8 | 4 | 0 | 0 | 0 | 100 | 0 |
| U6 | 0 | 0 | 0 | 0 | 0 | 0 |
| U7 | 35 | 31 | 51 | 0 | 11 | 6 |
| U8 | 0 | 0 | 0 | 0 | 0 | 0 |

*Key:* Di - shrink dilation, En - enlarge dilation, Sf - scale factor,
DiEno = dilation center other than the origin or vertices,
Identity = resulting image is congruent to the pre-image.

***Composite Transformations.*** Table 14 displays the number of composite transformation exercises in each textbook. Of a total number of student exercises evaluated over the four textbook series, only 21 exercises were found that included this type of task. Two student exercises on composite transformations are illustrated in Figure 19.

The inclusion of composite transformation exercises in all textbooks was negligible. The CM8 textbook presented at least three times the number of tasks on composite transformations than what was identified in any other textbooks series, with a total of 11 questions.

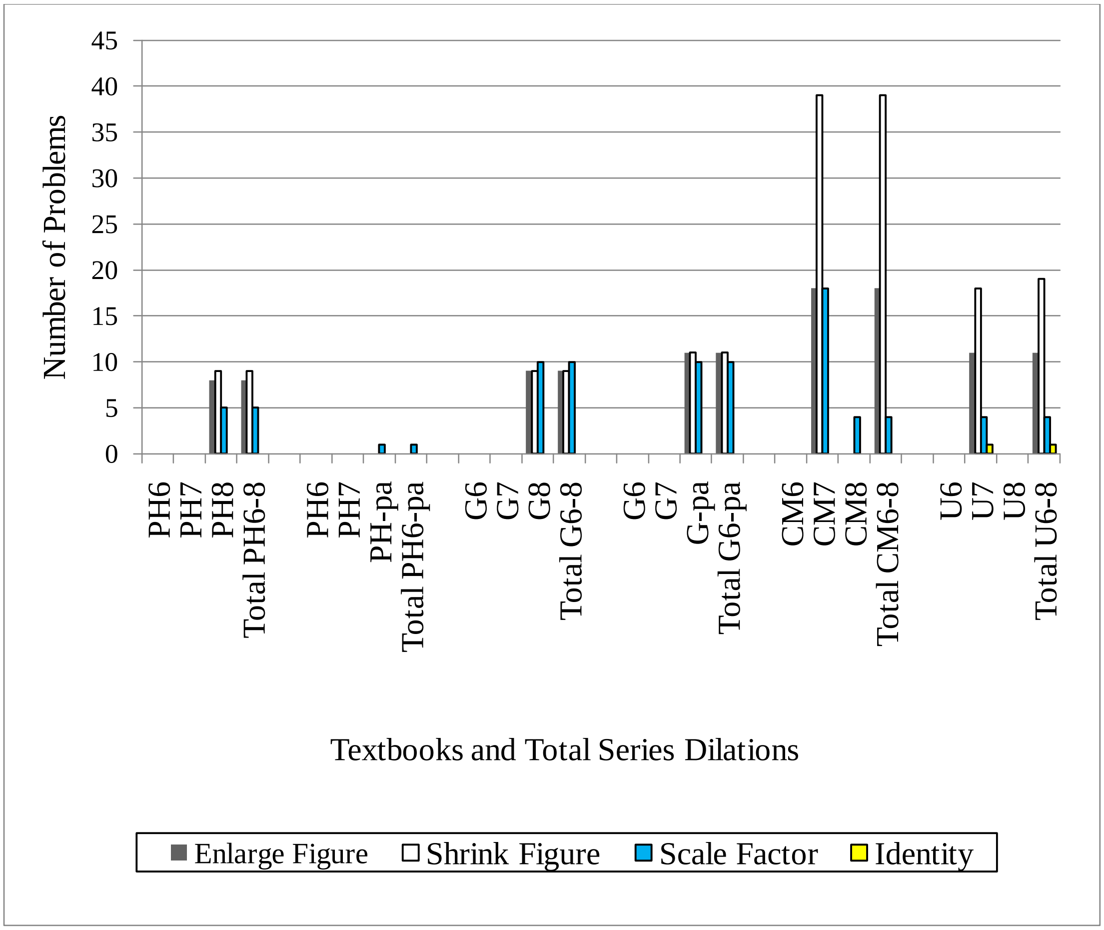


*Figure 18.* Summary of Dilation Exercises in the Middle School Textbook Series

Table 14

*Number of Composite Transformation Exercises in Each Textbook Series*

| | *Grade 6* | *Grade 7* | *Grade 8* | *Pre-Algebra* |
|---|---|---|---|---|
| Prentice Hall | 0 | 0 | 1 | 2 |
| Glencoe | 0 | 3 | 0 | 3 |
| Connected Mathematics 2 | 0 | 0 | 11 | n/a |
| UCSMP | 0 | 1 | 0 | n/a |

17. What single transformation is equivalent to a reflection in the y-axis followed by a reflection in the x-axis followed by another reflection in the y-axis?

18. Draw a figure on a coordinate grid. Perform one transformation on your original figure and a second transformation on its image. Is there a single transformation that will produce the same final result?

Connected Mathematics 3, Grade 8 © 2009, Module: *Kaleidoscopes, Hubcaps, and Mirrors*, p 90)

*Figure 19.* Sample Composite Transformation Student Exercises

**Student exercises analyzed by the characteristics of performance expectations.** This *s*ection presents data addressing the research question: To what extent do the geometric transformation lessons' student exercises incorporate the performance expectations in textbooks from sixth grade through eighth grade within a published textbook series, and across textbooks from different publishers? The student exercises were analyzed by the type of performance expected to answer the exercises. Figure 20 presents the data collected on the type of responses including: applying vocabulary, applying steps previously given, graphing the answer, making a drawing, finding angle measures or coordinates, matching content or assessing true/false statements, providing a written answer, working a problem backwards, and correcting an error in a given problem. Where a question asked for more than one type of response, each type was recorded in the analysis. The type of question that required a student to suggest a counterexample was not found in any of the transformation exercises. Appendix E illustrates examples of each type of student response question.

The types of performance expectations found in exercises predominately focused

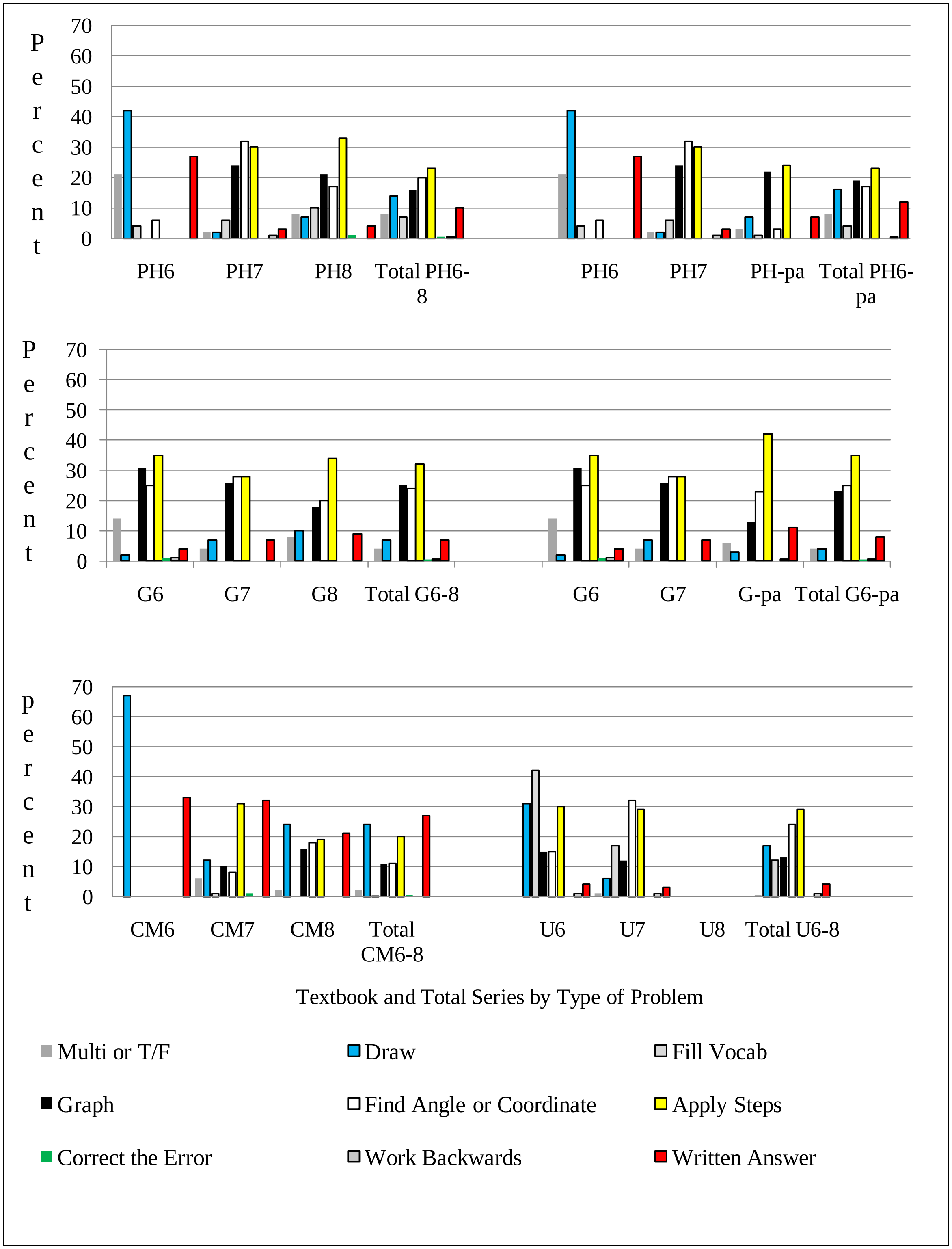


*Figure 20.* Analysis by Number and Type of Performance Expectations in the Transformation Exercises in the Textbook Series

on students applying steps previously given in the narrative of the lesson, graphing the image of a figure, finding coordinates and the measure of the angle of rotation. The types of tasks that seem to embody the ideas in the process standards, such as requesting a written response were occasionally included and those such as working a problem backwards and correcting an error were found on few or no occurrences across all textbooks examined.

**Suggestions for instructional aids and real-world connections.** Table 15 presents the findings in each textbook indicating the suggestions for the use of mathematics manipulatives (M), a computer software program (computer), the internet, or a calculator. Also presented is the number of references found to real-world connections. The number of instances where real world topics were found was divided into two categories. Occurrences of content, pictures or drawings that were referenced in the problem but seemed to be extraneous to the transformation concept were listed as being without connections; an example of this type of exercise is given in Figure 21. The problem illustrated was considered to be without connections because the idea of the candle was not necessary to complete the problem.

Candles: A decorative candle on a table has vertices R (-5, -4), S (-1, -2), and T (1, -5). Find the vertices of the candle after each translation. Then graph the figure and its translated image.

9. 3 units right
10. 2 units right, 4 units up

Glencoe, Course 1, © 2009, p. 607

*Figure 21.* Example of Exercise with Real World Relevance without Connections

Real world suggestions that seemed to be an integral part of the transformation concept was listed as being with connections and an example is shown in Figure 22. There were no data found where instructions or exercises related to other academic subjects.

> 14. Eyes: During an eye exam, an optometrist dilates her patient's pupils to 7 millimeters. If the diameter of the pupils before dilation was 4 millimeters, what is the scale factor of the dilation?
>
> Glencoe, Pre-Algebra, © 2010, p. 310

*Figure 22.* Example of Dilation Exercise with Real World Connections

Across the Prentice Hall, Glencoe, and UCSMP series, many of the transformation tasks were set in mathematical context without real-world connections. Some references were used to illustrate transformations, including illustrations of snowflakes, fabric patterns, puzzle pieces, or mirror images, but few were offered with connections to the use of transformations in actual settings. One memorable example offered an explanation of dilation in the context of the change of the size of the pupil of a patient's eye in a doctor's exam (Figure 23). Next to this example, in the margin, an explanation of the eye dilation procedure is provided with photographs of an eye before and after the dilation.

> Real-World Example
> 4. Eyes: An optometrist dilates a patient's pupils by a factor of 5/3. If the pupil before dilation has a diameter of 5 millimeters, find the new diameter after the pupil is dilated. Glencoe, Course 3 ©2009, p.

*Figure 23.* Example of Dilation with Real-World Connections

Table 15

*Number of Suggestions for the Use of Manipulatives, Technology, and Real World Connections to Mathematics Concepts*

| *Textbook* | *Manipulative* | *Technology* | *Real World w/o Connections* | *Real World with Connections* |
|---|---|---|---|---|
| PH6 | 1-M | 1-internet | Art | Fabric, windmill |
| PH7 | - | - | - | Chess, nature |
| PH8 | - | - | Skater, art, pictures | Chess |
| PH-pa | - | - | Pictures | Flower, snowflake, butterfly |
| G6 | - | - | Candles, rugs, flower, button, patch | Sailboat, video game, bedroom, art, nature |
| G7 | - | 1-computer | Flags, violin, insect | Map, game board, art research, letters, gate |
| G8 | - | - | Pictures, hubcaps, cars, window | Overhead sheet, pentagon, flags, symbols, folk art, instruments, orchid |
| G-pa | - | - | Art, turtle | Chess, stamps, microscope, eye exam |
| CM6 | - | - | Rug, flag | Bee, clock |
| CM7 | - | 1-computer | - | Video cartoon characters |
| CM8 | 1-M | 3-computer | - | - |
| U6 | - | 2-computer | - | - |
| U7 | 3-M | 1-calculator | Arch | Belt, fabric, hubcap |
| U8 | - | - | - | - |

The number and types of tasks in each series varied in number, but possibly a closely related and informative issue is the *level of cognitive demand* required for students to complete the exercises. The level of cognitive demand required to complete the transformation exercises is discussed in the next section.

**Student exercises summarized by textbook series.** The types of transformation exercises presented in each of the textbook series will be discussed in the following sections.

***Prentice Hall.*** The PH6 textbook presented a majority of general translation questions, i.e., as multiple choice or true/false, and drawing a figure. Also in the PH6 textbook 27% of the transformation tasks required a written answer. The PH7 textbook focused 30% of transformation exercises on applying steps that were given in the narrative examples, 32% on labeling a coordinate point for finding an angle measure, and 24% on graphing a response. In both the PH8 and PH-pa textbooks, students were to apply steps 33% and 24% of the time, respectively. The PH8 exercises focused 21% on graphing a response, whereas the PH-pa exercises on graphing occurred 22% of the time. Overall, either curriculum choice of 678, or 67-pa, placed greater emphasis on tasks of drawing figures, finding an angle or coordinates, and applying steps previously given in the narrative of the lesson and less emphasis on correcting an error, or working a problem backwards.

***Glencoe.*** In the Glencoe series textbooks, the student was expected to respond by applying steps previously presented in 35% of the transformation exercises in G6, 28% in G7, 34% in G8, and 42% in G-pa. Graphing a response was represented in 13% to 31% of the exercises on transformations. Also, finding a coordinate or the measure of an angle was presented 25% of the time in G6, 28% in G7, 18% in G8, and 13% in G-pa in the transformation exercises. Overall, either curriculum choice of 678, or 67-pa, placed greater emphasis on the less demanding tasks of finding an angle or coordinates, graphing a figure, and applying steps previously given in the narrative of the lesson and less emphasis on correcting the error, working a problem backwards, or providing a written response.

***Connected Mathematics 2.*** In the Connected Mathematics 2 series, the CM6 textbook exercises requested a written answer 33% of the time and the balance of exercises involved the student with drawing an answer. All three textbooks in this series focused on having students respond with a written answer for an overall series average of 27%. Across the three textbooks in this series, applying steps previously given was represented in 20% of the exercises, and drawing figures in 24%. The performance expectations of correcting the error and working a problem backwards were not presented.

***UCSMP.*** In the UCSMP series, textbook U6 students were expected to apply steps in 30% of the exercises, graph in 15%, find a coordinate or angle measure in 15%, and produce a written answer in 4% of the exercises. The U7 textbook provided exercises to apply steps in 29%, fill-in vocabulary terms in 17%, find a coordinate or angle measure in 32%, and graph an answer in 12% of the exercises. Over the two books in this series that presented transformation concepts, finding angle measures or coordinate points and applying steps previously presented appeared most frequently; working a problem backwards and responding with a written response was sporadically observed.

## Level of Cognitive Demand Expected by Students in the Transformation Exercises

This section presents data addressing the research question: What *level of* *cognitive demand* is expected by student exercises and activities related to geometric transformation topics in middle grades textbooks? The *level of cognitive demand* was identified using the parameters and framework established by Stein, Smith, Henningsen, and Silver (2000), and hence the *levels of cognitive demand* were divided into four sub-levels. The *Lower-Level* (LL) exercise demands are represented in memorization type tasks; the *Lower-Middle Level* (LM) tasks are characterized by examples using procedures without connections; the *Higher-Middle Level* (HM) tasks are characterized by examples using procedures with connections; and the *Higher- Level* (HH) tasks are examples involving tasks of doing mathematics.

Table 16 shows the percent of each level of cognitive demand required by the student to complete the transformation exercises in each of the textbooks. A total of 1149 student exercise tasks were evaluated. Overall, 522 tasks or approximately 45% were evaluated to be *Lower-Level* tasks, those in which students applied vocabulary, answered yes or no, or gave a short answer. The tasks classified as *Lower-Middle Level* totaled 562 tasks (49%); these tasks generally required students to apply steps illustrated in the body of the lesson. Questions that were evaluated to require *Higher-Middle Level* and *Higher-Level* demand represented a total of approximately 5% of all student exercises across the four series of textbooks.

Of all of the fourteen textbooks analyzed, G-pa presented the highest share of tasks in the *Lower-Middle Level* (83%), while the rest offered approximately similar percentages of tasks in both the *Lower-Level* and *Lower-Middle Level*. Differences were noted for the PH6, CM6 and CM8 textbooks with a larger percent (more than 50%)

Table 16

*Percent of Each Level of Cognitive Demand Required by Student Exercises on Transformations in Each Textbook and Textbook Series.*

| *Textbook* | *Total Tasks* | *Level of Cognitive Demand by Percentage* | | | |
|---|---|---|---|---|---|
| | | *Lower Level* | *Lower Middle* | *Higher Middle* | *Higher Level* |
| PH6 | 43 | 93 | 7 | 0 | 0 |
| PH7 | 86 | 50 | 50 | 0 | 0 |
| PH8 | 92 | 35 | 59 | 7 | 0 |
| PH-pa | 115 | 38 | 59 | 3 | 0 |
| | Prentice Hall Textbook Series Total for Grades 6, 7, 8 | | | | |
| PH678 | 221 | 52 | 45 | 3 | 0 |
| PH67-pa | 244 | 52 | 47 | 1 | 0 |
| G6 | 98 | 24 | 69 | 5 | 1 |
| G7 | 67 | 42 | 49 | 9 | 0 |
| G8 | 100 | 35 | 58 | 4 | 3 |
| G-pa | 88 | 9 | 83 | 8 | 0 |
| | Glencoe Textbook Series Total for Grades 6, 7, 8 | | | | |
| G678 | 265 | 33 | 60 | 6 | 2 |
| G67-pa | 253 | 24 | 69 | 7 | 0 |
| CM6 | 19 | 63 | 26 | 11 | 0 |
| CM7 | 86 | 56 | 40 | 4 | 0 |
| CM8 | 146 | 74 | 21 | 5 | 0 |
| | Connected Mathematics 2 Textbook Series Total for Grades 6, 7, 8 | | | | |
| CM678 | 251 | 67 | 17 | 5 | 0 |
| U6 | 73 | 41 | 45 | 12 | 1 |
| U7 | 137 | 51 | 43 | 6 | 0 |
| U8 | 0 | 0 | 0 | 0 | 0 |
| | UCSMP Textbook Series Total for Grades 6, 7, 8 | | | | |
| U678 | 210 | 48 | 44 | 8 | 0 |

*Note:* The numbers reported in the tables are rounded to a whole percentage and hence do not necessarily total 100 percent.

of tasks in the *Lower-Level* category; PH-pa and G6 had more than 50% in the *Lower-Middle Level* category. All textbooks showed a low percentage of transformation tasks in the *Higher-Middle Level* and *Higher-Level* categories. U6 and CM6 contained 12% and 11% of *Higher-Middle Level* tasks. U8 contained no student exercises to be analyzed related to transformation tasks.

Figure 24 displays an overall analysis for each sub-level of cognitive demand as required by the presented exercises. This display allows for a visual comparative analysis from one textbook to another.

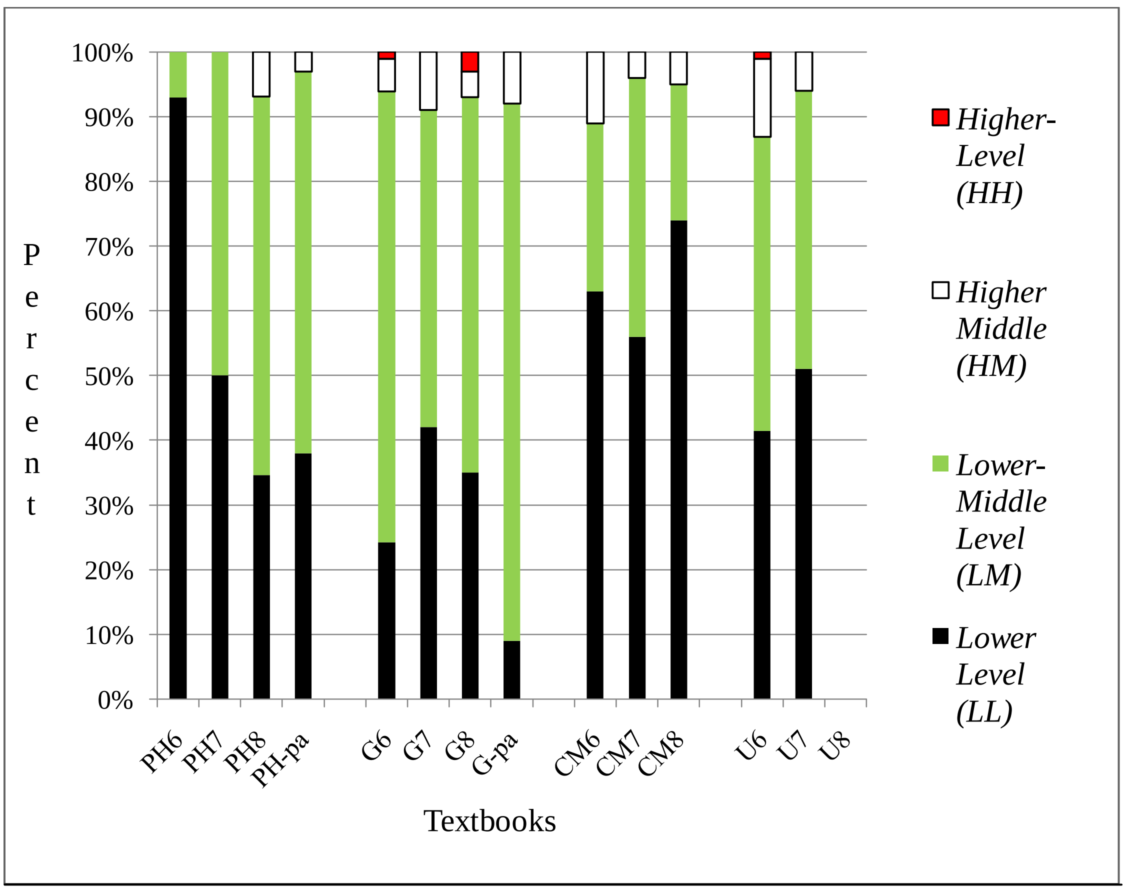


*Figure 24.* Level of Cognitive Demand Required by Students on Transformation Exercises in Each Textbook

In summary, the majority of transformation tasks presented in the textbooks examined were classified as *Lower-Level* or *Lower-Middle Level* of cognitive demand required by students to complete the exercises. The Connected Mathematics 2 series was found to have the highest percentage of *Lower-Level* tasks, and the smallest percent of *Lower-Middle Level* Tasks, and a few exercises were found in the range of *Higher-Middle Level*. The Prentice Hall and USCMP series contained approximately 50% of each *Lower-Level* and *Lower-Middle Level* tasks. Glencoe offered approximately 30% *Lower-Level* tasks and 65% *Lower-Middle Level*. Overall, the small number of exercises that required *Higher-Middle Level* demand and the lack of *Higher-Level* demand exercises in all four of the textbook series may indicate that the work set out for student practice is not as challenging as it should be to produce high achievement and increase interest in the content of this area of mathematics.

**Summary of Findings**

The summary of findings related to the analysis of the treatment of geometric transformations in the four series of middle grades textbooks were presented in this chapter. All four series contained lessons on the concepts of translations, reflections, rotations, and dilations. This chapter has also presented data comparing and contrasting transformation content, lesson narratives and student exercises on transformations; this included the physical characteristics of the textbooks, such as location and page counts on transformation topics as well as number and kinds of tasks asked of students. The structure and components of the transformation lessons were also compared and contrasted. Student exercises were analyzed for the expected student level of cognitive demand required to complete these tasks.

The summary of findings was presented in four sections. The first section presented data on the findings related to “Where” the content of transformations lessons were located in the textbooks within a publisher and across different publishers. This section related to research question number one and the first segment of the conceptual framework for content analysis.

The second section presented summary of findings related to “What” was included in the narrative of the lesson. This discussion related to the second research question and the second segment of the conceptual framework. The third section presented summary of findings related to “How” student exercises were presented with the lessons and included specific characteristics of exercises and the processes employed to encourage student learning. The fourth section presented a summary of findings on the level of cognitive demand required by the student exercises and relates to research question number four.

**“Where” the content of transformations lessons are located in the textbooks.** Overall the physical characteristics of the 14 textbooks were similar in total number of pages, instructional pages, chapters, lessons, and transformation lessons (in 13 textbooks) with few exceptions. As previously mentioned, the U8 textbook did not contain lessons on transformation concepts and the number of chapters and lessons in the Connected Mathematics 2 textbook series were fewer in number, but CM2 contained a similar number of pages.

In each textbook, the number of pages devoted to transformation concepts varied from approximately 6 to 54 with an overall average of 18 pages with standard deviation of 13, hence a large variance in the concentration on transformation topics in each textbook was found. Table 17 presents the averages and standard deviations of the number of textbook pages in each of the series examined. Note the page average for the CM and the UCSMP series, indicates that each of these series devoted more page area to the transformation concepts than either of the other publishers' series.

Table 17

*Transformation Page Number Average and Standard Deviation in Each Textbook Series*

| *Textbook Series* | *Percent of Transformation Pages Across the Series* | *Transformation Page Average* | *Standard Deviation (with in textbook series)* |
|---|---|---|---|
| PH 6-7-8 | 1.82 | 11.0 | 3.61 |
| PH 6-7-pa | 1.66 | 10.4 | 2.72 |
| G 6-7-8 | 2.18 | 14.8 | 4.94 |
| G 6-7-pa | 1.95 | 13.9 | 3.89 |
| CM 6-7-8* | 4.45 | 27.9 | 24.33 |
| U 6-7** | 3.17 | 24.7 | 6.58 |
| U 6-7-8 | 2.06 | 16.4 | 15.00 |
| All Textbook*** | 2.50 | 17.1 | 13.03 |

*Curriculum constructed on student discovery structure.
**Calculations exclude the U8 textbook.
***Calculations include the U8 textbook.

The total number of transformation pages, in each textbook, was approximately evenly divided between the narrative of the lesson and the student exercises, with the exception of the CM7 and CM8 textbooks in which 71% and 77% of transformation pages were for student exercises. The CM2 series provided almost three times more page area devoted to student exercises than to instructional pages. The large proportion of presented student questions, both within the lessons and the exercises, appeared to be due to the curriculum format based on the philosophy of student discovery.

**“What” is included in the transformation lessons of each textbook series.** In 10 of the examined textbooks, the transformation lessons were presented following or in close proximity to one another; in 3 textbooks lessons were found in different parts of the curriculum sequence (G-pa, CM8, and U7). Translations were offered first in seven of the textbooks, but only five of these textbooks offered reflection as the second topic. The topic of rotations appeared in 11 of the textbooks, and the topic of dilation appeared in only six of the fourteen textbooks examined.

The characteristics of the lessons in three of the series of textbooks (PH, G, and U) examined were similar, that being of a traditional presentation with objectives, topic discussion, defined vocabulary, examples illustrating worked out problems, followed by student exercises. Most often the vocabulary and definitions presented were the same over the span of the series. Over the four series of textbooks very few transformation properties were included. The differences occurred most often in the UCSMP textbook series. The UCSMP textbook lessons appeared to include more sophistication in the mathematical language used in the narrative of the lessons, and an increase in detail in graphs, explanations of terminology, and properties.

In contrast to the traditional presentation of the three series above, Connected Mathematics 2 is a curriculum built on the philosophy of student discovery. The units (chapters) were stand alone paperback modules that contained student investigations subdivided into sections that focused on specific topics and activities. The investigations typically began with a short introduction followed by questions the students were to discuss and answer to develop the concepts of the lesson. In the examined lessons of this textbook series, few vocabulary or worked out examples were provided for student study. The student exercises were placed at the end of the investigation and were not delineated as to which sub-investigation section they were to accompany.

**"How" transformation exercises are presented in the lessons.** In summary, all four of the textbook series contained general type questions on translations and translating a figure downward and to the right. The Glencoe series included translations upward and to the left, and the UCSMP included translating figures to the left.

General reflection exercises were noted in the UCSMP series. Reflections to the right and downward were predominately identified in the Glencoe and Connected Mathematics 2 series. Reflections of figures over an oblique line were noted in the Connected Mathematics 2 series, and reflections over/onto the pre-image were observed in both the CM2 and the Glencoe series. Research identified the difficulties that students experience with reflections (Burger & Shaugnessy, 1986; Kuchemann, 1980, 1981; Perham, Perham, & Perham, 1976; Rollick, 2009; Schultz, 1978), particularly reflections over an oblique line, and over/onto the pre-image (Edwards & Zazkis, 1993; Soon, 1989; Yanik & Flores, 2009); hence, one would expect to see more attention to these issues in the curriculum of each series.

Rotation of figures in a counterclockwise direction was noted in all four textbook series, but rotation of figures in a clockwise direction was seldom used in the Prentice

Hall series. The topic of angle of rotation was mostly limited to angle measures that were multiples of 90°. Exercises of rotation about a point other than a vertex or origin of a figure were not observed in any series. Research indicated that students experience difficulties with the measure of angle of rotation (Clements & Battista, 1989, 1990, 1992; Clements, Battista & Sarama, 1998; Clements & Burns, 2000; Kidder, 1979; Krainer, 1991; Olson, Zenigami, & Okazaki 2008; Soon, 1989; Wesslen & Fernandez, 2005), rotation about a point other than the center of the figure (Edwards & Zazkis, 1993; Yanik & Flories, 2009; Soon & Flake, 1989; Wesslen & Fernandez, 2005), finding the location of the center of rotation (Clements, Battista & Sarama, 1998; Edwards & Zazkis, 1993; Soon, 1989; Soon & Flake, 1989; Wesslen & Fernandez, 2005; Yanik & Flories, 2009), and the direction of turn (Clements et al., 1996; Soon, 1989; Wesslen & Fernandez, 2005).

Dilations were presented in all four series, yet the UCSMP series was the only one to include the concept of identity and the scale factor 1. Research indicated that students do not understand that a positive scale factor indicates an enlargement, and a fraction (not a negative number) scale factor indicates a reduction of the figure (Soon, 1989). Clarification of these issues was not observed in any series.

Composite transformations were negligibly studied in all four textbook series. Research indicated that students have difficulty identifying and understanding composite transformations (Burke, Cowen, Fernandez & Wesslen, 2006; Schattschneider, 2009; Wesslen & Fernandez, 2005). Because of this, it would be expected to see more work with composite transformations presented in the curriculum than what was observed in all

14 textbooks of the sample.

In all of the textbooks across the four series very few suggestions were included for the use of manipulatives or technology in the narrative or the student exercises portion of the lessons. The occurrence of real-world connections was sub-divided into two categories, one real-world with connections, and another real-world without connections. The occurrences of pictures, drawings, or content in problems that seemed to be extraneous to the transformation concept were listed as being without connections. The PH, CM, and U series were found to present few real-world related topics in either category. The G series presented some recommendations for real-world related topics, more in the category of with connections than without.

The number of each type of transformation included in the student exercises in each series is presented in Figure 25. Across all of the textbook series examined in this study, students would have an opportunity to experience tasks in all four of the transformations (translations, reflections, rotations, dilations), except in the PH67-pa sequence of textbooks that provided a very limited number of dilation exercises. Otherwise, all of the series contained transformation exercises for student experience, with an average of 204 questions per series and a standard deviation of 18.

In all four textbook series the specific characteristics of the exercises and the processes employed to encourage student learning were found to be dominated by exercises that required students to answer exercises by graphing, applying steps previously given, and finding a coordinate point or an angle measure. Additionally, the

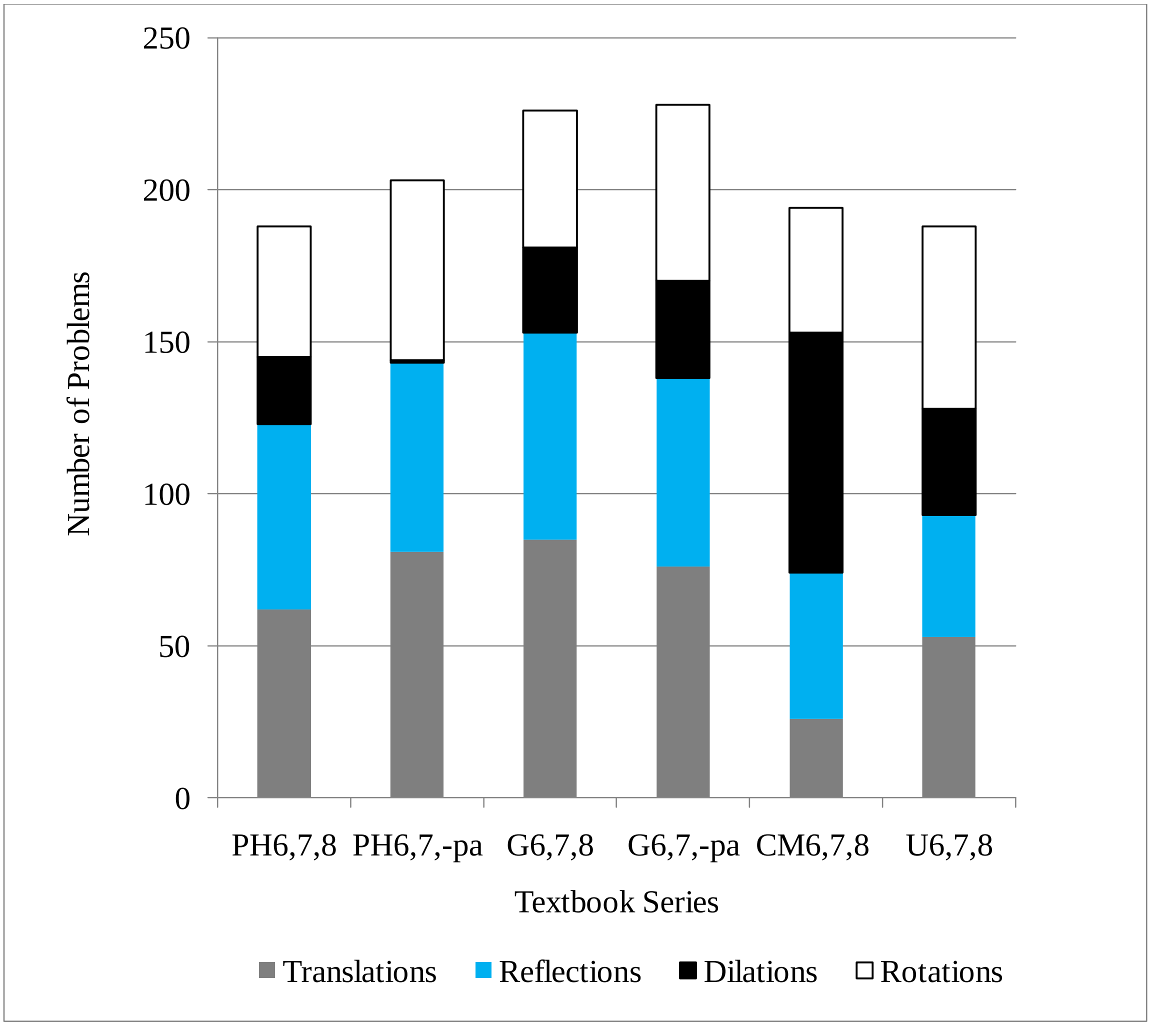


*Figure 25.* Total Number of the Four Transformation Type Exercises in Each Textbook Series

Connected Mathematics 2 Series included many exercises where the student was required to provide a written answer. The processes of correcting the error, and working a problem backwards were almost non-existent.

**Level of cognitive demand required by student exercises.** In this section the summary of findings for the fourth research question are discussed. The level of cognitive demand, as defined by Boston and Smith (2009), Smith and Stein (1998), Stein and Smith (1998), and Stein, Smith, Henningsen, and Silver (2000), is the level of demand that was required by the student to complete a mathematical task. The four levels have been previously defined: *Lower-Level* (LL); *Lower-Middle Level* (LM); *Higher-Middle Level* (HM); and *Higher-Level* (HH).

Figure 26 presents an overview of the percent of levels of cognitive demand in the student transformation exercises in each textbook series. Of all of the transformation exercises analyzed over the four textbook series, 45% were categorized as *Lower-Level* cognitive demand and 49% were categorized as *Lower-Middle* level. Overall, approximately 5% of the transformation exercises were categorized as *Higher-Middle*, and 0.04% tasks were classified as requiring *Higher-Level* cognitive demand for task completion. The textbook series with the most transformation exercises requiring the *Lower-Level* was the CM series with approximately 67%. The textbook series with the most transformation exercises requiring the *Lower-Middle* level of cognitive demand was the Glencoe series G67-pa containing 69% and next G678 with 60%. Additionally, the Glencoe basal series presented a few transformation exercises requiring *Higher-Middle Level* of cognitive demand. The four Prentice Hall series and the Connected Mathematics 2 series offered no transformation exercises that were classified as requiring *Higher-Level* cognitive demand.

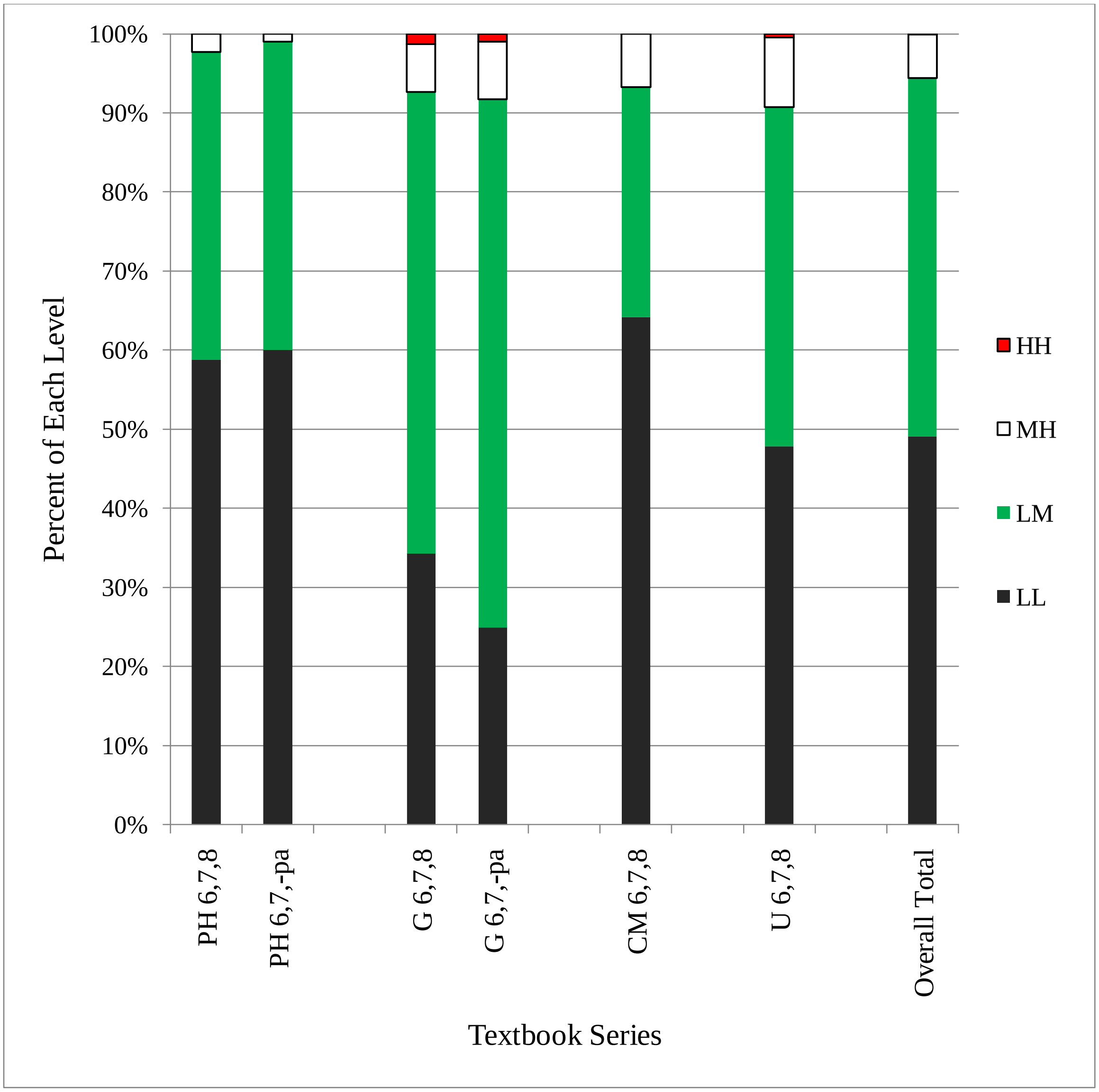


*Figure 26.* Percent of Levels of Cognitive Demand in Student Exercises in Each Textbook Series

Overall, the representation of *Lower-Level* and *Lower-Middle Level* tasks seemed disproportionally high in comparison to the number of tasks in the *Higher-Middle Level* and *Higher-Level* categories. Cognitively demanding tasks promote thought and reasoning and provide students with a potential *opportunity to learn* (Henningsen &

Stein, 1997; Stein & Smith, 1998) while improving student performance (Boston & Smith, (2009). Hence one might expect to see a larger proportion of cognitively demanding tasks provided in the exercises of the lessons.

The next and final chapter discusses the results, limitation, and significance of this study. Implications for future research are also delineated.

## Chapter 5: Summary and Conclusions

Spatial reasoning is needed for everyday life and one way of achieving the life skills necessary and expected for the demands in today's society is to study geometric transformations. One way of addressing students' accessibility to the study of geometric transformations is through curriculum content analysis for the inclusion of transformation topics. Textbooks are a common and often used element in U. S. classrooms and the textbook is heavily relied upon by teachers for making instructional decisions, including the scope and sequence of a mathematics course (Grouws & Smith, 2000; Hunnell, 1988; NEAP, 2000). Therefore, the content of the textbooks used in the classroom is a determining factor that influences students' *opportunity to learn* geometric transformation concepts. What needs to be determined is how, if at all, the content and presentation of the topics of transformations are handled in textbooks and if the topics are addressed in a manner to clarify persistent student difficulties identified in the research literature.

**Overview of the Study.** The purpose of this study was threefold: to describe the content of geometric transformation lessons (narrative and exercises) to identify the components of these lessons within a series of textbooks that span from grades 6 through grade 8, and across different publishers; to determine if student exercises with the transformation lessons facilitate student achievement by the inclusion of processes that encourage conceptual understanding with performance expectations; and to conduct an analysis of the nature of the presentations of geometric transformations by considering the nature of the narratives and exercises and the relative emphasis on transformation topics to determine students' potential *opportunity to learn* concepts of transformations.

The data were collected from 14 middle school textbooks from four publishers using a coding instrument developed from existing research techniques in the field of textbook content analysis. Specific details on the coding instrument and procedures were presented in Chapter 3: Research Design and Methodology. The findings were reported using both descriptive statistics and qualitative methods in order to address the research questions.

This chapter discusses the findings presented in the previous chapter. A synopsis of the study has been provided including a description of the textbooks sampled. Next, the research questions are revisited followed by the results and discussion based upon the research findings in Chapter 4. Limitations, significance, and implications for future research conclude this chapter.

**Research Questions**

This study sought to answer the following questions:

1. What are the physical characteristics of the sample textbooks? Where within the textbooks are the geometric transformation lessons located, and to what extent are the transformation topics presented in mathematics student textbooks from sixth grade through eighth grade, within a published textbook series, and across different publishers?
2. What is the nature of the lessons on geometric transformation concepts in

student mathematics textbooks from sixth grade through eighth grade, within a published textbook series?

3. To what extent do the geometric transformation lessons' student exercises incorporate the learning expectations in textbooks from sixth grade through eighth grade within a published textbook series, and across textbooks from different publishers?

4. What *level of cognitive demand* is expected by student exercises and activities related to geometric transformation topics in middle grades textbooks? The *level of cognitive demand* is identified using the parameters and framework established by Stein, Smith, Henningsen, and Silver (2000).

Together, these four questions give insight into potential *opportunity to learn* that students have to study geometric transformations in middle grades textbooks.

## Purpose of the Study

The purpose of this study was to examine the nature and treatment of geometric transformation topics and tasks in middle grades students' textbooks to determine students' potential *opportunity to learn* transformation concepts. Specifically, the research questions were posed to examine the contents, location, sequence, and scope of the topics in transformation lessons from textbooks that were designed for grades 6 through 8 from four published series available for use in the United States. The coding instrument for analysis was based on national recommendations for the inclusion of geometric transformation topics in the middle grades as well as from research findings on issues that students experience when working with transformations to determine whether textbooks cover the concepts in ways to address these difficulties.

**Summary of Results**

Data from this study revealed that each middle grades mathematics textbook examined contained lessons on the concepts of geometric transformations with the exception of one textbook (grade 8) from the UCSMP series. The presentation of the transformation topics varied by textbook and all topics did not appear in each of the textbooks. No consistency was found in terms of order, frequency, or location of the topics within the textbooks by publisher or grade level.

But potential *opportunity to learn* (OTL) is related to many factors: placement of lessons within the sequence of the textbook, sequence and scope of the transformation lessons, nature of the way content is introduced, types and expectations of student exercises, and level of cognitive demand or challenges expected of students. When these issues were considered, students' OTL across the series varied.

Research indicates that approximately 75% of the textbook is typically covered in the middle grades mathematics classroom during a school year (Jones & Tarr, 2004; Valverde et al., 2002; Weiss et al., 2001); hence it is possible that students may not have an opportunity to experience transformation topics when using a textbook series where the lessons are positioned in the fourth quartile of pages. Therefore, when this placement of lessons occurs, the potential *opportunity to learn* mathematical concepts becomes close to non-existent. In the next four sections, potential *opportunity to learn* transformation concepts is reviewed and discussed in each of the textbook series.

**Opportunity to learn transformation concepts in the Prentice Hall textbook series.** The Prentice Hall textbook series contained one textbook for each of grades 6 to 8 (PH678) and a pre-algebra textbook alternative (PH67-pa) to accommodate choice on curriculum content for the study of pre-algebra concepts in grade 8. Each of the textbooks included two to four lessons on geometric transformations that were contained in 1.2% to 2.4% of the total instructional pages in the textbooks. The structure of the lessons typically started with lesson objectives, terminology defined, discussion of concepts, and illustrated examples followed by student exercises. Over all four textbooks the narrative of the transformation lessons and the student exercises shared approximately equal amounts of page area. Content on translations, reflections and rotations topics were present in all four of the textbooks in this series, although PH6 contained one third the amount of page coverage on translations as the other three textbooks. Dilations were studied in the two textbooks designated for use in grade 8. Composite transformations were not included in the Prentice Hall middle grades series textbooks. The content, diagrams, and examples within the narrative of the transformation lessons appeared to be repetitive over the grade levels of the textbooks examined.

The relative location of the transformation lessons within the pages of this textbook series raised concern about potential opportunities to study transformations. In the PH678 textbook sequence, the topics of translations, reflections, rotations, and dilations were placed in the 22% to 32% range in the PH8 textbook, and dilations in the 42% range in the PH7 textbook. Hence, students using the PH basal series would likely have the potential opportunity to study dilations in grade 7 and again in grade 8 along with the three rigid transformations. Because other transformation lessons in this series were placed predominately in the fourth quartile, it is unlikely that students would have additional opportunities for experience. In the PH67-pa textbook sequence, all transformation lessons but one were placed in the fourth quartile, hence students are not likely to have the opportunity to study transformations during their middle grades experience except for dilations in grade 7.

Both Prentice Hall textbook sequences contained approximately the same number of student transformation exercises when all transformation lessons were considered. More exercises were offered on translations and reflections than on rotations. Dilation exercises appeared to be somewhat limited in the PH678 sequence, and almost non-existent in the PH67-pa sequence. Translation and reflection exercises predominately deal with one directional movement and the majority of rotation exercises used counterclockwise direction without the inclusion of rotations about a point other than the center of the figure. The types of transformations that were shown to be the most difficult for students to perform, as indicated by the literature, were not included.

Student performance expectations included many transformation exercises where students were to apply steps that were previously illustrated in the narrative of the transformation lessons, graph a response, and find an angle measure or coordinates of points. The performance expectation that required a written response was observed in approximately 10% of the exercises. The types of problem that utilized correcting an error in a given solution or working a problem backwards were not observed. Few occurrences were found in the transformation lessons that suggested the use of manipulatives or technology. The level of cognitive demand required to complete the transformation exercises in both sequences of textbooks was 52% in the *Lower-Level* category and approximately 46% in the *Lower-Middle Level*. Occurrences of *Higher-Middle Level* and *Higher-Level* of cognitive demand categories were negligible.

***Opportunity to learn*** **transformation concepts in the Glencoe textbook series.** The Glencoe textbook series contained one textbook for each of grades 6 to 8 (G678) and a pre-algebra textbook alternative (G67-pa) to accommodate choice on curriculum content for the study of pre-algebra concepts in grade 8. Each of the textbooks included two to four lessons on geometric transformations that were contained in 1.4% to 2.8% of the total instructional pages in the textbooks. The structure of the lessons typically started with mathematics objectives, terminology defined, discussion, and illustrated examples followed by student exercises. Over all four textbooks in this series the narrative of the transformation lessons and the student exercises were approximately equal in amount of page area. Content on the topics of translations and reflections were present in all four of the textbooks in this series. Rotations were not evident in the G7 textbook and dilations were presented in the two textbooks designated for use in grade 8. Composite transformations were not included in the Glencoe middle grades series textbooks. The content, diagrams, and examples within the narrative of the transformation lessons appeared similar over the grade levels of the textbooks examined with the exception of the G-pa textbook that contained increased amounts of transformation discussion, explanations, and increased detail in the illustrations and diagrams.

The relative location of the transformations lessons within the pages of these textbooks was a concern in terms of opportunity to learn because approximately 75% of textbook content is studied during a school year at the middle grades level (Jones & Tarr, 2004; Valverde et al., 2002; Weiss et al., 2001). In the Glencoe series, both the G6 and G7 textbooks placed transformation topics following 90% and 81% of the textbook pages, respectively; in contrast the Glencoe textbooks, G8 and G-pa, placed some topics in the 45% and 12% range except for the topic of rotations which was placed at the 75% mark in the G-pa textbook. Therefore, students who use either choice of the Glencoe textbook sequence were likely to have an opportunity to study transformations in grade 8 because of their location within the textbook pages, but may miss the study of rotations if the PH67-pa sequence was used. Hence, student potential *opportunity to learn* transformation topics in the Glencoe series appeared likely with the choice of either the G678 basal or the G67-pa textbook sequence. Another concern was the limited page area on the topics translations and reflections in the G-pa textbook in comparison to the presentations in the other three textbooks in this series; this might indicate a lack of concept coverage in the G-pa textbook.

Both Glencoe textbook sequences contained approximately the same number of student transformation exercises when all transformation lessons were considered. More exercises were offered on translations and reflections than on rotations and dilations. Translation exercises included one and two directional movements, utilizing both movements to the right/left and up/down. Reflection exercises included the type indicated to be the most difficult for students (i.e., when the reflection overlaps the preimage figure). Rotation exercises included both clockwise and counterclockwise directions, but not rotations about a point other than the center of the figure. Dilation exercises included scale factor questions.

Student performance expectations included many exercises where students were to apply steps that were previously illustrated in the narrative of the transformation lessons, find an angle measure or coordinates of points, and graph a response. The performance expectation that required a written response was observed in approximately 8% of the transformation exercises. The problem types that utilized correcting an error in a given solution or working a problem backwards were not observed. Few occurrences were found in the transformation lessons that suggested the use of manipulatives or technology. The level of cognitive demand required to complete the transformation exercises was found to be 33% *Lower-Level* and 60% *Lower-Middle Level* in the G678 sequence; and 24% *Lower-Level* and 69% *Lower-Middle Level* in the G67-pa sequence of textbooks. The occurrence of *Higher-Middle Level* tasks was approximately 6% in either of the textbook sequence and *Higher-Level* tasks were observed in 2% of the exercises in the G678 sequence.

**Opportunity to learn transformation concepts in the Connected Mathematics 2 textbook series.** The CM2 textbook series is a National Science Foundation funded Standards-based series utilizing modular consumable units (workbooks) that are quite different from more familiar curricula formats. The CM2 series have pre-algebra and algebra topics embedded within the curriculum. The philosophy of this curriculum is that student learning utilizes an investigative approach with problem centered investigations of mathematical ideas in a discovery setting employing small group collaborative explorations. The analysis in this study was based on the order of unit presentations and transformation topics in the publisher's single bound edition, but the units were stand-alone soft covered workbooks, and the order of use can be rearranged by the classroom teacher or district curriculum specialist.

The modular units were structured as investigations that were divided into sub-investigations of mathematical concepts. Each modular unit (workbook) contained a list of objectives which were not delineated for each investigation. The transformation investigations contained a small amount of narrative discussion on the concepts, posed situations, and questions for the students to consider and address. Terminology was not evident and may be left for the teacher to introduce. Student exercise questions were offered at the end of a set of investigations; the exercises were not delineated for each sub-investigation. Over the three textbooks, there was an approximate ratio of 25/75 pages of lesson investigations to student exercises. Content on translations were contained in the CM7 and CM8 textbooks; reflection and rotation topics appeared in the CM6 in a limited amount and also in the CM8 textbook.  One CM7 unit module was mostly dedicated to the study of dilations. Composite transformations were included in the CM8 transformation content.

In this study the student editions of the textbook modules were the only materials examined. It is possible that related transformation terminology, concept specifics, and topic examples were offered in the teacher's edition of the publisher series to assist the classroom teacher with the inclusion of related terminology, specific transformation concepts and related transformation properties. That is, because of the student investigative nature of the textbook the teacher needs to focus a summary discussion of students' findings to ensure that students have learned the essential mathematics of the investigation. Examination of teacher's editions was outside of the scope of this study, therefore no conclusions were offered regarding what may or may not have been included in the additional resources.

In regard to the relative location of transformation lessons within the pages of the Connected Mathematics 2 series, students' potential *opportunity to learn* transformation concepts was viable with the publisher suggested order of topics as found in the single bound edition. The CM6 placed transformation topics at the beginning of the second quartile, CM7 in the first quartile (14%), and CM8 in the second and third quartiles (50% and 60%) range. So, students were likely to have an opportunity to study transformation topics in all three middle grades years.

The CM2 series contained approximately equal numbers of student exercises on reflections, rotations, and dilations, with about one half of this number on translation exercises. Translation and reflection exercises predominately dealt with one directional movement and the rotation exercises used both clockwise and counterclockwise directions, but rotations about a point other than the center of the figure were not observed. Exercises did not include the type indicated to be the most difficult for students when the reflection overlaps the preimage figure. Rotation exercises included the topic of the measure of the angle of rotation, and dilation exercises included scale factor questions. Composite transformations were included in the CM8 textbook.

Student performance expectations included many exercises where students were to apply steps or procedures that were previously indicated in the investigations, draw or graph a response, and find an angle measure or coordinates of points. The performance expectation that required a written response was observed in approximately 27% of the exercises. The problem types that utilize correcting an error in a given solution or work a problem backwards were not observed. Few occurrences were found in the transformation lessons that suggested the use of manipulatives or technology. The level of cognitive demand required to complete the transformation exercises was found to be 64% *Lower-Level* and 29% *Lower-Middle Level* in the CM2 textbook series. The occurrence of *Higher-Middle Level* tasks was approximately 6% over the textbook series.

**Opportunity to learn transformation concepts in the University of Chicago School Mathematics Project textbook series.** The UCSMP textbook series contained one textbook for each of grades 6 to 8. The UCSMP series has pre-algebra and algebra topics embedded within the curriculum, hence with the completion of the three textbook sequence students have completed the equivalent of middle grades algebra by the end of 8$^{th}$ grade. Since the U8 textbook did not contain any lessons on transformations, the content of transformations was analyzed only in the U6 and U7 textbooks. The two textbooks included four and five lessons respectively on geometric transformations that were contained in 2.6% and 3.7% of the total instructional pages in the textbooks. The structure of the transformation lessons was somewhat traditional and typically started with mathematics objectives, terminology defined, discussion, and illustrated examples which included learning strategies and student exercises. Over the two textbooks the narrative of the lessons and the student exercises shared approximately equal amounts of page area. Lessons on translations, reflections, and rotations were presented in both textbooks, and dilations were included in the U7 textbook. Content with composite transformations was not evident in either textbook.

In the UCSMP series, the topics of transformations were placed following 84% of the U6 textbook pages, and 45% of the pages in the U7 textbook. If the students were to miss the topics in grade 6, it would be likely that they would be exposed to the transformation topics in grade 7. The U7 textbook contained almost twice the number of student transformation exercises as was found in the U6 textbook. The number of problems in the U7 textbook on each of the types of transformation exercises was proportionally larger, and the addition of the dilation exercises in the U7 textbook accounts for the larger number.

Translation exercises included one and two directional movements, utilizing both movements to the right/left and up/down. Reflection exercises included over an oblique line, and those indicated to be the most difficult for students when the reflection overlaps the preimage figure. Rotation exercises included both clockwise and counterclockwise directions, but not rotations about a point other than the center of the figure. The lesson on rotations in the U7 textbook included detailed instructions with extensive diagrams explaining the angle of rotation. Dilation exercises included scale factor questions, and the only reference in any of the sampled textbooks to a scale factor identity concept.

Student performance expectations included many exercises where students were to find an angle measure or coordinates of points, apply steps that were previously illustrated in the narrative of the lesson, and draw or graph a response. The performance expectation that required a written response was observed in approximately 4% of the exercises. The problem types correct an error in a given solution or work a problem backwards, were not observed. Few occurrences were found in the transformation lessons that suggested the use of manipulatives or technology. The level of cognitive demand required to complete the transformation exercises was found to be 49% *Lower-Level* and 44% *Lower-Middle Level* in the U67 textbook sequence. The occurrence of *Higher-Middle* level tasks was approximately 8% over the two textbooks.

**Discussion**

This study examined geometric transformations in four middle grades textbook series available for classroom use in the United States. The purpose was to analyze the nature and characteristics of geometric transformation lessons in middle grades textbooks to determine the extent to which these textbooks provided students the potential *opportunity to learn* geometric two-dimensional transformation concepts.

Many variables must be considered when decisions are made to adopt a mathematics textbook series to support delivery of the standards of a district or state. Some of the variables that must be considered are the population of students that will be served, including past achievement levels and previous exposure to mathematics curricula. Academically, choices must be made as to what kinds of work would be most beneficial to obtain highest student achievement and future student success.

Also to be considered with textbook series choice is the relative importance of various mathematics concepts and the amount of attention each topic receives in the curricula of choice because students do not learn mathematics to which they are not exposed (Begle, 1973; Stein, Remillard, & Smith, 2007; Tornroos, 2005). Because the literature indicates exposure to spatial sense through geometric transformations prior to the study of formal geometry provides students with an advantage for higher achievement and success (Clements, 1998), it should be important to ensure students an opportunity to study geometric transformations prior to the study of formal geometry.

All of the middle school textbook series examined presented topics of geometric transformations (translations, reflections, rotations, and dilations). The sequence and scope of the transformation lessons varied by textbook and by series. Some topics repeated exactly from one grade to the next, as with the Prentice Hall and Glencoe series. Some topics received no treatment in some individual grade level course textbooks. Because students' experience with transformation content is based on the middle school textbook series adopted, the potential *opportunity to learn* transformations was further considered across the 3 year middle school experience.

The location of the transformation lessons in the sequence of textbook pages was of some concern because research indicates that content placed at the end of the textbook can easily be omitted, and students most likely will not learn it (Stein, Remillard, & Smith, 2007). Research indicates that approximately 75% of the textbook is covered in the middle grades classroom during a school year (Jones & Tarr, 2004; Valverde et al., 2002; Weiss et al., 2001), so it is possible that students may not have an opportunity to experience transformation topics when using a textbook where lessons are placed in the fourth quartile of pages. Therefore, when the positioning of transformation lessons occurs in the fourth quartile of textbook pages, student potential *opportunity to learn* transformation concepts becomes close to non-existent.

The placement of the majority of transformation topics in the textbooks examined (Prentice Hall grade 6, 7, and Prealgebra; Glencoe grade 6 and 7; and UCSMP grade 6) was in the fourth quartile of pages. Additionally UCSMP grade 8 did not contain any transformations lessons. Hence, the *opportunity to learn* when using Prentice Hall 6, 7, and Prealgebra; Glencoe 6 and 7, or UCSMP 6 textbooks is extremely low. The PH8, G8, G-pa, and U7 textbooks placed transformation content prior to the third quartile of pages. Hence, students using series that included these four textbooks would likely have an opportunity to study transformations in the 7$^{th}$ or 8$^{th}$ grade curriculum. Content coverage in the Connected Mathematics 2 series was located within the first 55% of the textbook pages, therefore an opportunity to study transformations was provided if the textbook modules were studied in the order as suggested in the publisher single bound edition.

All transformation topics were not presented in all of the textbooks, and some transformation topics received less attention in some of the textbooks. This, coupled with the placement of the concepts within the fourth quartile of textbook pages, may lead to some topics of transformations being abbreviated or missed entirely during the middle school years. For example, dilation received limited coverage overall and was only studied at any depth in the CM7 textbook of the Connected Mathematics series. These results further highlight the limited opportunities for students to investigate specific concepts of transformations and provides confirmation that developers could increase and/or expand content coverage.

With few exceptions, lessons were found to repeat content from one year to the next with little or no evidence of an increase in content development or depth of knowledge. Generally, vocabulary definitions were found to repeat from one year to the next and often relevant mathematical properties and connections were not included in the lessons. Many of the narratives were observed to lack sophistication, and did not include applications that would have drawn students into the structure of ‘doing mathematics’. Exceptions were noted in UCSMP grade 6 and 7, and Glencoe Pre-Algebra textbooks which contained increased amounts of discussion and explanations about transformation concepts as well as more detail in the diagrams that accompanied the narrative.

During the last decade, from NCTM’s *Principles and Standards for School Mathematics* through 2006 with the publication of the NCTM *Focal Points* to the present movement with the Common Core State Standards Initiative adopted by the majority of states in the Union, the placement of transformation concepts and content has been realigned. What might have been delineated for seventh grade focus in one set of recommendations might now be designated for eighth grade focus. The adoption and implementation of the *Common Core State Standards,* in 2010, will likely assist in the (re)organization of the specific topics and depth of coverage as recommended during specific grades of middle school. Hopefully, the suggested standards for transformation concepts will follow with alignment in new editions of published textbooks.

All four textbook series presented a similar number of exercises that were generally found to encompass routine tasks with many repetitions. The types of tasks where students have been observed to have issues, misconceptions, and difficulties were represented in smaller numbers. Additionally, composite transformation tasks were seldom offered in any of the four series. Hence, this analysis provides a hypothesis as to why students' achievement levels are low, since the prevalent types of transformations were overly presented in the textbooks.

Most student exercise performance expectations included applying steps previously illustrated in the lesson, finding angle measures or coordinate points, drawing or graphing, and filling in vocabulary terms. Few exercises expected students to correct the error, work a problem backwards, or provide a written answer. Across the four series, few suggestions were included for the use of manipulatives in the study of transformation concepts. Some connections were made to real-world connections, but approximately half of those examined appeared to include extraneous references to something in the real-world that was not necessary to complete the exercise.

The levels of cognitive demand required for students to engage with the transformation exercises were found to be predominately *Lower-Level*, and *Lower-Middle Llevel*. Very few transformation exercises were found to require *Higher-Middle Level* of cognitive demand, and a negligible number was found to demand the *Higher-Level* (Stein, Smith, Henningsen, & Silver 2000). Just as Li (2000) and Mesa (2004) found low levels of cognitive demand, this study also found lower levels of cognitive demand than might be expected in regard to the present recommendations and standards for the learning of mathematics.

The levels of cognitive demand required by exercises should stimulate students to make mathematical connections and offer opportunities for student thinking while making a difference in how students come to view mathematics. The Smith and Stein

(1998) framework was used to analyze the nature of the student tasks by the level at which they provided student engagement in high levels of cognitive thinking and reasoning. It is possible that if a different framework had been used for analysis, as for example Webb's (1997) *Depth of Knowledge* framework, the results of the findings on student exercises would have been different.

Of the four textbook series analyzed many variables could be satisfied with choice of one of the four series analyzed. Therefore, no conclusion is offered as to which textbook series is best because choice is a value judgment; but, there are clearly different *opportunities to learn* geometric transformation in each series.

**Limitations of the Study**

This study has several limitations. The first is the relative sample size of textbooks that were analyzed. It was the intent of this researcher to include textbooks widely used by middle school children in the United States. However, because market share data are not available, the choice of publishers was based on recommendations from university mathematics teacher educators, knowledge of the relative size of the publishing firms (Reys & Reys, 2006), and the reputation of the textbook authors. Nevertheless, different types of textbooks were chosen to illustrate variance among middle grades textbooks. The student discovery philosophy and general format of the Standards-based textbook series created a struggle collecting data on the lesson portion. Clarification of lesson strategies might have been possible with the inclusion of analysis of the teacher's edition of the textbooks; unfortunately these resources were not included in the focus of this study.

The total sample included three grade level textbooks from four publishers, with the addition of an alternative title from two of the publishers who offer options for individual purchaser preferences. Although the sample used in this study was manageable to enact an in-depth analysis, the findings may not be generalizeable to all middle grades textbooks presently published in the United States.

Another limitation of this study was its focus on student textbooks. The premise for the study was to examine the material to which the student is directly exposed. Therefore it was not possible to account for other resources and materials that influence the content of classroom instruction, including the content of the teacher's editions. Textbooks have a definite influence on the content of the mathematics that is presented in the classroom; however, the incorporation of learning goals in a textbook does not insure that the potential *opportunity to learn* will be provided by the inclusion of the material in the enacted curriculum. What is presented in the classroom is also dependent on other numerous demands, including but not limited to, district mandated curriculum, teachers' beliefs, teachers' pedagogical and content knowledge, time constraints, and teachers' choice for the inclusion or exclusion of textbook chapters, particular lessons, mathematical concepts, or student exercises.

A third limitation of this study is the strength of the framework document to delineate all content on the topic of transformations and capture the concepts of the narrative and student examples and exercises. The framework was developed using the work of other researchers (Clements, Battista & Sarama, 1998; Fischer, 1997; Flanagan, 2001; Flanders, 1987; Jones, 2004; Jones & Tarr, 2007; Schultz & Austin, 1983; Smith &

Stein, 1998; Soon, 1989; Stein & Smith, 1998; Wesslen & Fernandez, 2005) and was intended to correlate all concepts and topics that were the focus in this study. Although related terminology cross-referencing sections was included to account for the inclusion of all pertinent concepts, an additional limitation is that the contents on prerequisite skills, mixed reviews, activities not within the lessons, and isolated student exercises in cumulative reviews and assessments were not included.

A fourth limitation relates to the use of the Stein and Smith (1998) framework for determining the level of cognitive demand required by student exercises. It is possible that results would have been different if an alternate framework for investigating cognitive complexity had been used.

**Significance of the Study**

Research indicates that U. S. students' achievement lags in growth in many areas of mathematics, including in spatial reasoning (Battista, 2007; Silver, 1998; Sowder, Wearne, Martin, & Strutchens, 2004) which is needed for understanding our three-dimensional world. Spatial reasoning, taught through transformations, has been neglected as an area for study by students in the middle grades and is in need of development within mathematics learning. In response to the improvement needed in the mathematics curriculum in the United States, professional organizations have put forth recommendations (NCTM, 1989, 2000) in the form of mathematics standards that set criteria for teaching and learning of worthwhile mathematical tasks related to further mathematics achievement and future success.

Many studies suggest that textbooks are common elements in mathematics classrooms (Begle, 1973; Driscoll, 1980; Haggarty, & Pepin, 2002: Porter, 1989; Reys, Reys, Lapan, Holliday, & Wasman 2003; Robitalle & Travers, 1992; Schmidt, McKnight, & Raizen, 1997; Schmidt et al., 2001; Schmidt, 2002; Tornroos, 2005) and that textbook content influences instructional decisions (Grouws & Smith, 2000; Lenoir, 1991, 1992; Pellerin & Lenoir, 1995; Reys, Reys, & Lapan, 2003) and directly affects students' *opportunity to learn*. Because the textbook is such an influential factor on student learning, it becomes important to document the opportunities presented in textbooks for students to gain competency on important mathematical concepts at a level beyond procedural skills. It is important to identify "*Where"* the mathematical concepts are placed in the textbook, and in what sequence; "*What"* content is presented in the narrative of the lessons, and "*How"* the processes are utilized to assist students to attain highest achievement. The areas of concern are aligned with the conceptual framework on Content Analysis for the written curriculum. If the content is not present in the textbook, because it is lacking or placed at the end of the textbook where it is easily omitted, then students most likely will not learn it (Stein, Remillard, & Smith, 2007).

This study's focus was on middle grades textbook content analysis on the nature and treatment of spatial reasoning through geometric transformations. The findings of this investigation add to the body of knowledge about curriculum analysis for the mathematics education research community as well as for curriculum developers. Curriculum developers and textbook authors might find it helpful to familiarize themselves with the research findings on students' misconceptions and difficulties in understanding transformation concepts and to develop specific content in the curriculum to address these issues.

The results obtained herein can provide information to school district personnel, curriculum specialists, and teachers on the content within their student textbooks regarding the location, sequence, narrative presentation, development of geometric transformation concepts, as well as on the characteristics of the student exercises and the level of cognitive demand of the tasks provided for student practice. The methodology used in this study may be used to apply content analysis techniques to other content areas within mathematics curriculum with adjustments to specific terminology and specifics for performance expectations. In addition, the outline provided in the development of the methodology may be useful in the planning and execution for future research on curriculum content analyses.

The importance of textbook content analysis extends beyond the specific content that was analyzed in this study. Textbook authors, curriculum developers, curriculum specialists, and teachers might use the conceptual framework and collection documents presented herein, adjust them to specific mathematical content in question and use these instruments to perform content analysis of other topics with an eye to what is contained in classroom textbooks, for textbook series adoption processes, or for classroom curriculum to align with district or state directives. Adjustments to the framework presented here would include the compilation of a complete terminology list for the specific topics and adjustments for the types of learning processes included in the specific mathematical strand being examined. Additionally, examinees would need to make the determination as to which textbook resources would be examined in addition to the student textbooks.

**Implications for Future Research**

The process of curriculum analysis may be undertaken for numerous reasons, among which is informed choice when decisions for textbook adoption are planned, or for the purpose of curriculum alignment to mathematical standards. Hence, it is insufficient to only analyze superficial characteristics when making decisions about textbooks and the curriculum adopted for use within a state or district. Information from in-depth curricula analyses is an important aspect to consider in the textbook selection process. The results from this study indicate that textbooks, although similar in page numbers and lesson topics, may be very different in terms of depth of concept presentation, inclusion of specific relevant properties, and required levels of cognitive demand in student exercises. As a result of these differences students have different *opportunities to learn*.

The curricula examined herein illustrate the differences that can be found when an examination is executed. The results of this study, together with the knowledge supplied from existing research, lead to implications for mathematics education in areas including those of curriculum development, future content analyses, and recommendations for future studies.

Future research could provide curriculum developers information on the order of introduction of the topics of transformations so that the content knowledge from one form builds into the next transformation objective. As observed in the analysis of this study, in some textbooks the topics of transformations were placed in isolation from one another in sequence. The presentation of transformation lessons together might lead to a rich environment for the development of concepts. Hence, not only is research needed on the effects of the order of introduction of transformation topics but also on the influence that the proximity of transformation topics in the curriculum sequence has on student learning and achievement.

This study found that transformation concepts were seldom connected to other strands of mathematics. Curriculum development could include relationships to other areas of mathematics to increase student conceptual understanding and student achievement, for example; relating dilations with similarity and proportions. Additionally, transformation topics may be used together to develop interesting and in-depth activities that provide closure to the system of transformations with the inclusion of composite transformations as suggested by Wesslen and Fernandez (2005). The inclusion of composite transformations may also motivate students to see and understand connections between transformations and real world applications while encouraging students to become more involved in the problem solving aspect of the activities and a level of higher order thinking in preparation for high school geometry.

Curriculum developers might also find it helpful to familiarize themselves with the issues that students experience with transformation topics as identified in the research, and to add activities that address these student issues. Examples of student difficulties include translating figures from the right to the left, reflecting a figure over an oblique line, and rotating a figure about a point outside of the figure. Because research indicates that the direction of movement of the translation has a definite impact on the difficulty of the task (Rollick, 2009; Schults & Austin, 1983; Shultz, 1978), one might expect to see more attention given to the direction of movement of the figure in translation lessons and exercises. Additionally, the nature and treatment of the topics of transformations could be developed with accompanying properties to build a foundation for student understanding and for later success in the study of formal geometry in high school.

The study of transformations can be enhanced by the inclusion of interesting and explicit activities designed to illustrate the link to real world connections, the connections between mathematical strands, as well as to make a rich and interesting mathematical experience by the inclusion of composite transformations. Developers might also relate transformations of two dimensional objects to the study of three dimensional objects to assist students in the spatial visualization of drawing such figures in two dimensions. These topics might also be related to figures on a net, cross sectional drawings, and the constructions of three dimensional figures from two dimensional drawings.

This study highlighted the levels of cognitive demands that were most prevalent in student exercises. In light of the recommendations in *Principles and Standards for School Mathematics* and the *Process Standards* (NCTM, 2000) it might be expected that student exercises include tasks that are more demanding, not only to facilitate increase in student conceptual understanding but also to assist in keeping students interested in mathematics.

This study was designed to analyze the written curriculum in the form of the textbooks to which students have direct exposure. Future research might expand this focus and include the support provided in the teacher's editions and additional publisher resources that accompany the textbooks to analyze content that is provided by the publisher for lesson strategies and planning. Additionally, analyses might include analysis on how middle grades transformation concepts are expanded from the elementary school curriculum, and how the high school curriculum on transformations builds on the middle grades content. Future research might consider the analysis of the content of transformations contained in textbooks used in the United States with textbooks from other countries.

Future research might consider content analysis of a larger sample size of middle school textbook series. The use of a larger sample size would provide wider coverage on the treatment of transformations in middle school textbook series and provide a complete picture of the scope and sequence of topics that students experience in K-12, as well as provide content analysis for textbook series not included in this study; a larger sample size might also allow for greater generalization of results. Additionally, the developed conceptual framework as well as the coding instrument developed for this study may provide a foundation for future content analyses. Researchers may also consider analysis of classroom use of manipulative materials with the transformation lesson concepts to determine the influence on students' level of engagement and the effects on student conceptual understanding and achievement.

As indicated in the findings of this study, the appearance and sequence of the concepts of transformations were different across the four publishers. The review of research on transformations did not address the sequence or proximity of lessons as significant for student achievement; hence, the inconsistency of order of types of transformation lessons offered indicates the need for further research on the introduction of specific transformation concepts to assist in student learning and to produce highest student achievement. In doing so, future research might examine if there exists a specific order of introduction and presentation of the topics to help students develop understanding of these concepts to maximize student achievement.

Additionally, the *level of cognitive demand* as assigned by the professional on evaluation of tasks in an analysis situation may or may not be the level at which the student engages with the cognitively demanding tasks in an actual classroom setting. Further study might examine the distribution of the required *levels of cognitive demand* in assignments and how the student perceives, interacts, and enacts the tasks. Research might also analyze the *levels of cognitive demand* for examples and exercises and how they compare with the distribution of the levels of cognitive demand across all of the topics presented in the series.

Other aspects of research might include the comparison of transformation topics presented in the curriculum with the assessments that accompany the textbook series as well as those on state and national standardized assessments. Research might also examine the nature and treatment in the curricula from other countries to determine the international perspective on these concepts.

Research is needed to investigate the curriculum that is enacted in the classroom since there are likely fewer opportunities for students to learn about transformations in the implemented curriculum than in the intended curriculum (Tarr, Chavez, Reys, &

Reys, 2006), and having the topics of transformations present in the written curriculum is not a guarantee that they are presented in the classroom. Research is also needed on teachers' familiarity with the concepts of transformations presented in the written curriculum as well as how teachers interpret students' misconceptions and issues. Research might also investigate why teachers chose to include, or omit, particular parts of the transformation curriculum from instruction, as well as the levels of inclusion of manipulatives, activities utilized, and student exercises assigned.

And finally, the concentration of mathematics curriculum is largely defined by the textbooks that students and teachers use. A content analysis of mathematical topics is required to gauge the treatment and level of sophistication of concepts available for student study as well as the processes included to support student learning. Specific portions of the framework utilized in this study were useful in capturing differences found in the middle grades mathematics textbooks examined. In particular, the qualitative portion of the analysis included delineation of terminology, objectives, properties, and examples offered for student study; and the analysis of the performance expectations within the student exercises with the levels of cognitive demand provided a finer level of detail than would have been achieved through analysis of the transformation constructs alone. This study contributed to these areas of analysis and provided an important perspective into the treatment of transformations in middle grades textbooks and the specifics areas where development or improvements are needed. This study has provided an illustration of the potential of curriculum content analysis and hopefully will encourage others to continue content analysis in other areas of mathematics.

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# Appendices

# Appendix A

## Pilot Study

This pilot study was designed to determine the extent and variations of treatment of geometric transformations in middle school textbooks, and to determine if there were enough differences that a more extensive analysis would be worthwhile and informative. The following research questions were devised for the pilot study.

### Research Questions

- What are the opportunities for students to study geometric transformations in eighth grade mathematics textbooks?
- How does the presentation of geometric transformations differ across textbooks from different publishers?
- What level of cognitive demand related to geometric transformation topics (Stein, Smith, Henningsen, & Silver, 2000) is required by the student exercises and activities in eighth grade textbooks?

### Sample

Two textbooks from different publishers with similar educational philosophies were chosen to establish the possible existence of variations in their presentations. The two books reviewed for the pilot study were *Prentice Hall Course 3 Mathematics* © 2004 (PH), and *Glencoe Mathematics Applications & Concepts Course 3 © 2004* (Glencoe).

### Procedures

The first step in conducting the pilot study was to review similar content analysis research and compile a list of themes to examine to determine what data to collect to investigate the treatment of geometric transformations in middle school textbooks. The variations of data included the physical locations of the lessons within the textbooks and the order of presentation of the transformation concepts, nature of the narrative of the lessons with properties and terminology presented, number and specific types of student exercises, and the level of cognitive demand required by the student exercises. From this collection of themes a query list was designed to collect data from the two textbooks.

The designed list of information collected was organized into a format to collect data on the location of lessons, the narrative of the lesson, and on the student exercises presented. This collection document was adjusted during the textbook examination process to provide sufficient space to record the lesson's definition of terms and observations specific to the lesson; two additional types of student exercise headings were added to the initial list: matching and true/false. Additionally, in order to provide a reliability check for the inclusion of all transformation lessons within the textbook, a list of related transformation terms was developed from a list of terms located in these textbooks and the list was added to the data collection document to be used as a cross reference. This list of terms was used to search the textbook glossary to provide a page comparison for inclusion of all presented information on transformations. As the pilot study progressed through the data collection process, changes to this collection document were followed by recoding the lessons to ensure accuracy of findings. The collection document fabricated and refined during the pilot study generated the coding instrument for this study.

**Analysis of Transformations in Two Middle School Textbooks**

The two books were similar in regard to the number of instructional pages; the Glencoe textbook had 81.96% and the Prentice Hall had 82.3% instructional pages of the total page count of the textbooks. Both textbooks contained similar resources for prerequisite skills, selected answers, and glossary (both in English and Spanish); differences in the number of pages appeared to be attributable to type set, page layout, and amount of white space provided on each page in the selected answers and extra problems pages.

Table A1

*Total Textbook Page Count Analysis for Glencoe and Prentice Hall Grade 8 Textbooks*

| | *Glencoe* | | *Prentice Hall* | |
|---|---|---|---|---|
| | *N* | *%* | *N* | *%* |
| Total Page Count | 715 | | 844 | |
| Instructional Pages in Textbook | 586 | 81.9 | 695 | 82.3 |
| Number of Transformation Sections | 17 | 2.9 | 26 | 3.7 |
| Instructional Page Total | 10.5 | 1.8 | 8.8 | 1.2 |
| Percent of Textbook Prior to First Transformation Section | | 32.9 | | 22. 5 |
| Total Number of Student Transformation Exercises | 127 | | 149 | |

The total number of instructional pages dealing with the topic of transformations was 17 in the Glencoe textbook and 26 in the Prentice Hall textbook. Because the overall total student exercise count per section in these textbooks was not under review in this pilot study, it was not possible to present data on the equivalence of the number of exercises presented. Proportionally, Glencoe presented approximately 12 problems per each instructional page whereas Prentice Hall presented 17 per each instructional page determined by total transformation exercises divided by total transformation pages.

**Summary of Student Exercise Problems in Transformations**

Figure A1 presents a summary of all of the transformation exercises offered in the two sampled middle school textbooks. Each of the four categories of transformation exercises (translation, reflection, rotation, and dilation) are shown with the percent of exercises given in each textbook for each type of transformation concept. The Glencoe textbook presented 61 problems on reflection concepts this represented almost 50% of the total exercises in this textbook offered on transformations. Following with the next largest quantity, on the concept of dilation, was 34 exercises or 21.26% of the total (or approximately ½ of the amount found for reflection). The Glencoe textbook presented the largest number on student exercises in the transformation topic of reflection, and a small amount on the topic of translations (see Figure A1).

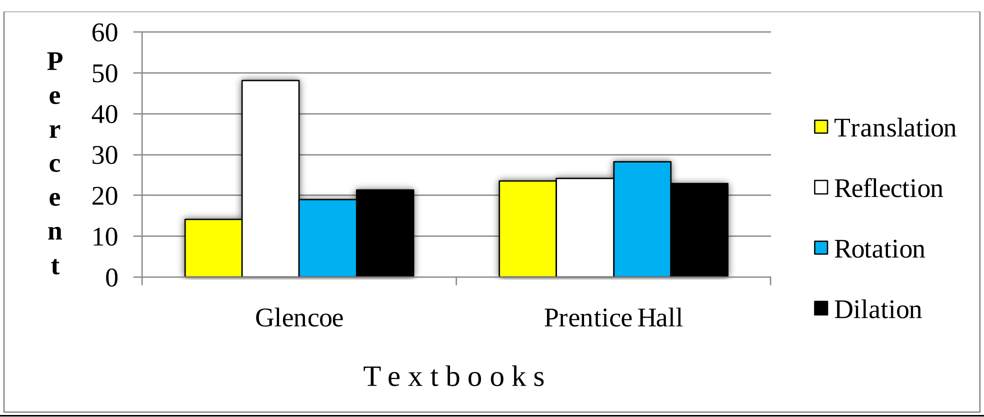



*Figure A1*. Percentage of Student Exercises by Type of Transformation in Glencoe and Prentice Hall Grade Eight Textbooks

Prentice Hall appeared to present a different perspective by the nearly equivalent proportions of each of the types of transformation exercises.

**Order of presentation of transformation topics.** The presentation order of transformation topics in each of the two textbooks is shown in Table A2. Prentice Hall introduced translation followed by reflection with symmetry and rotation. These topics were studied in the first 25% of the textbook pages and in a chapter related to graphing in the coordinate plane. This chapter was 68 pages in length and was 10 pages longer than the average chapter length in the Prentice Hall textbook. Additionally, the transformational concepts were taught in the context of graphing and not taught or reviewed in the context of geometry. The topic of dilation was presented in a chapter on proportions and application; this section had 32% of the textbook pages prior to it.

The Glencoe textbook first introduced dilations for study following the first 27% of the instructional pages. Dilations were covered in a chapter with the topics of ratios, rates, proportions, similar figures, scale drawings and models. The concepts of reflection with symmetry, translation, and rotation were grouped in a later chapter dealing with topics in geometry; this material was offered following 40% of the instructional pages in the sequence.

**Analysis of textbook lesson narratives about translations.** In the Prentice Hall textbook, Chapter 3 was entitled “Graphing in the Coordinate Plane”, and section 8 presented “Translations”. This section started on page 157, and extends for 5½ pages including the student exercises. The narrative portion of the lesson covers 2½ pages. Approximately 23% of the instructional pages in this text come before this lesson.

Table A2

*Placement of Presentation of Transformations Topics within the Two Textbooks by Percent of Pages Covered Prior*

| *Percent of Textbook Pages Covered* | *Glencoe* | *Prentice Hall* |
|---|---|---|
| 17 | | |
| 18 | | Translation |
| 19 | | Reflection |
| 20 | | Rotation |
| 21 | | |
| 22 | | |
| 23 | | |
| 24 | | |
| 25 | | |
| 26 | | |
| 27 | Dilation | |
| 28 | | |
| 29 | | |
| 30 | | |
| 31 | | |
| 32 | | Dilation |
| 33 | | |
| 34 | | |
| 35 | | |
| 36 | | |
| 37 | | |
| 38 | | |
| 39 | | |
| 40 | Reflection | |
| 41 | Translation | |
| 42 | Rotation | |
| 43 | | |
| 44 | | |

Two learning objectives for the lesson were listed: (1) to identify and perform translations on a coordinate plane; and (2) to perform translations of a given figure. The section began by defining transformations and showing pictures of puzzle pieces to illustrate translations, reflections, and rotations. The terms translation, image, and prime

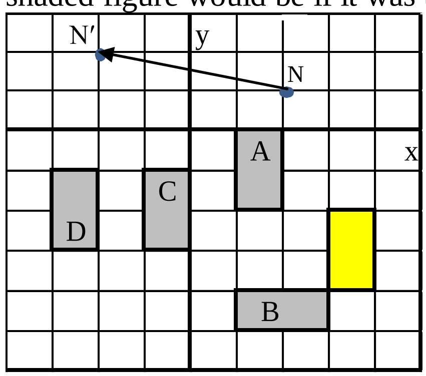
(A′) were defined. The definition of translation specified that each point of the figure was moved the same distance and in the same direction; an accompanying illustration shows one point on a coordinate plane being moved to a new location on the graph. In this lesson, after Example 2, the student was told that an arrow may be used to show a translation, as A → A′. However, there are no defined steps given for performing the transformation. The third example covered the second objective, with the caption "Describing Translations". In this example, students add or subtract translation values to the coordinate points to write a rule to indicate the movement of the coordinate graphed figure.

In the Glencoe text, Chapter 6 was titled "Geometry" and section 8 presented "Translations." The section started on page 296, and extended for 3½ pages including the student exercises. The narrative portion of the lesson covered two pages. Approximately 50% of the instructional pages in this text came before this lesson. Section 8 was the fourth section on transformations and presented one of the topics that was the focus of this investigation. The section began with an example to answer the question of how this material would be used in a real world setting. A drawing was of a chess board and a chess piece movement. One objective was addressed, that is, to graph a translation on a coordinate plane. The term *translation* was defined as the term for movement and the term *slide* was included. Next was a shaded block to draw student attention to the properties of transformations. The key concepts in the block were that the original figure and the image were "congruent", and have the same orientation; also, every point on the image moved the same distance from the original figure.

The narrative example offered specific steps for performing a translation, and the written work showed that both the x-coordinate and the y-coordinate are changed when each point was moved to the image location. In the last narrative example, students used a coordinate plane to locate an image from an original rectangular block to match a movement of a figure to its translated image. This question might be interpreted as working the problem backwards (Figure A2).

Example: Use a Translation

3. Point N is moved to a new location, N′. Which white shape shows where the shaded figure would be if it was translated in the same way?




(Glencoe, 2004, p297)

*Figure A2.* Sample Exercise to Work a Problem Backwards

**Analysis of student exercise sets.** This section presents results on the content of problems analyzed from the student exercise sets in each textbook.

***Translations.*** The types of problems on translations were comparable across both the Prentice Hall and the Glencoe textbooks. Both textbooks focused on the student performing translations on a coordinate plane. The Glencoe textbook presented 18 student exercise problems on translations, or 14% of the total problems on transformations; and the Prentice Hall textbook presented 33 translation problems, which represented 24% of the total transformation exercises (see Table A3). Prentice Hall presented approximately 80% more problems than the Glencoe textbook and included two problems on composites of translations whereas the Glencoe textbook includes none.

Table A3

*Percent of Student Exercises on Translations by Textbook*

| | *Glencoe* | | *Prentice Hall* | |
|---|---|---|---|---|
| *Transformation Type* | *N* | *% - (based on 127)* | *N* | *% - (based on 149)* |
| Translations | 18 | 14 | 33 | 22 |
| Composite Translations | 0 | 0 | 2 | 1 |
| Total Translation Exercises | 18 | 14 | 35 | 24 |

***Reflections.*** Reflections (see Table A4) were analyzed differentiating the type of reflection with reference to the reflection line. Research shows that variations according to the reflection line create different levels of difficulty for students (Clements, Battista, & Sarama, 2001; Clements & Sarama, 1992; Denys, 1985; Grenier, 1988; Hollebrands, 2003, 2004; Soon, & Flake, 1989); hence, the analysis separated exercises by characteristics of the reflection line for analysis. According to research, reflection over the y-axis is an easier concept for middle school students than is reflection over the x-axis. Reflection over a line other than the x- or y-axis is a more difficult concept than either of the two aforementioned. Reflectional symmetry was presented in the Prentice Hall textbook as an integral part of reflection; hence the reflection and symmetry topics were presented together.

The highest concentration of questions in the Prentice Hall textbook, in this set of exercises, was on the topic of reflectional symmetry. These questions may not completely capture the specific properties of the concept of reflection. Composite reflections were represented in both textbooks with Glencoe presenting four problems or 3%, and Prentice Hall offering two problems or 1% of the total.

Table A4

*Percent of Student Exercises on Reflections by Type of Reflection and Textbook*

| | *Glencoe* | | *Prentice Hall* | |
|---|---|---|---|---|
| *Transformation Type* | *N* | *% - (based on 127)* | *N* | *% - (based on 149)* |
| Reflection Over the X- axis | 17 | 13 | 8 | 5 |
| Reflection Over the Y-axis | 8 | 6 | 6 | 4 |
| Reflection Over a Line other than the X or Y-axis | 24 | 19 | 4 | 3 |
| Reflectional Symmetry | 8 | 6 | 14 | 9 |
| Composite Reflection | 4 | 3 | 2 | 1 |
| Total Reflection Exercises | 61 | 40 | 34 | 23 |

***Rotations.*** According to research, the concept of rotation creates problems for many students. Students seem to focus on the movement being a turning motion, but they have difficulty understanding the center of the rotation and the number of degrees in the angle of rotation (Freudenthal, 1971; Hollebrands, 2003, 2004; Soon & Flake, 1989). So, this analysis categorized the rotation problems by the center of rotation (see Table A5).

The Prentice Hall textbook had 15 exercises on rotation of a figure about the origin, which represented 10% of the total exercises on transformations. One problem containing a composite of rotations was offered. Nineteen student exercises were given representing 15% of the total problems on rotations in the Glencoe textbook. Note that the Glencoe textbook did not include any student exercises on rotation about a point other than the origin or a vertex of a figure.

Table A5

*Percent of Student Exercises on Rotations by Type of Rotation and Textbook*

| | *Glencoe* | | *Prentice Hall* | |
|---|---|---|---|---|
| *Transformation Type* | *N* | *% - (based on 127)* | *N* | *% - (based on 149)* |
| Rotation with Center of Origin or a Vertex of the Figure | 19 | 15 | 15 | 10 |
| Rotational Symmetry | 4 | 3 | 11 | 7 |
| Rotation about a Point Other than the Origin or a Vertex of the figure | 0 | 0 | 11 | 7 |
| Composite Rotation | 0 | 0 | 1 | 1 |
| Total Rotation Exercises | 23 | 18 | 38 | 28 |

***Dilations***. The student exercises were coded categorizing enlarging or shrinking (reduction) problems separately. The Glencoe textbook presented equal numbers of both enlargements and reductions, and included the concept of scale factor. Prentice Hall presented approximately the same number of problems on both types of dilations but did not discuss scale factor at the same time. Prentice Hall presented two problems on composite dilations, and Glencoe included none. The total number of dilation exercises in both textbooks was nearly equivalent (see Table A6).

Table A6

*Percent of Student Exercises on Dilations by Type and by Textbook*

| *Transformation Type* | *Glencoe* *N* | *% - (based on 127)* | *Prentice Hall* *N* | *% - (based on 149)* |
|---|---|---|---|---|
| Dilation – Enlargement | 10 | 8 | 15 | 10 |
| Dilation – Shrink | 10 | 8 | 17 | 11 |
| Scale Factor | 7 | 6 | 0 | 0 |
| Composite Dilation | 0 | 0 | 2 | 1 |
| Total Dilation Exercises | 27 | 21 | 34 | 23 |

***Composite transformations.*** Composite transformations were a specific content focus area in some of the literature (Glass, 2004; Hollebrands, 2003; Wesslen & Fernandez, 2005). The NCTM recommends students focuses on composites of transformations, hence, composite transformations are an integral part to the closure of the topic of transformations. Glencoe presented a total of four composite exercises representing 3% of a total of 127 transformation problems. Prentice Hall presented a total of seven composite exercises representing 5% of 149 transformation problems.

**Student Exercise Content Analysis by Expected Student Performance**

The student exercises were also analyzed according to the type of performance expected from the student. This data was discussed in three categories. The first category included data on the specific work the student was to perform to answer the problem, such as applying vocabulary, applying steps previously given, finding coordinates or angle measures, graphing the answer, making a drawing, matching content, correcting an error in a given problem, assessing true/false statements, presenting a written answer, working a problem backwards, giving a counterexample, and the real-world relevance/subject related matter of the exercises (Table A7). The next section included textbook suggestions for the inclusion of manipulatives or technology. This section also focused on recent standards and recommendations for inclusion of these resources. The last section addressed the level of cognitive demand (Table A8) required by the student to complete the exercises. The level of cognitive demand was based on the framework developed by Stein and Smith (1998).

***Student exercises.*** The total number of student exercises for each of the textbooks, Glencoe and Prentice Hall, were analyzed for expected student performance to complete each question (Table A7). The exercises were categorized for the following performance type: filling in vocabulary, applying steps previously given, finding coordinates or angle of rotation measures, graphing the answer on a coordinate plane, making a drawing, matching content, correcting an error in a given problem, and assessing true/false statements. A sample of an exercise that requires the student to apply steps previously given and to graph the answer is shown in Figure A3. A list of textbook references to real-world relevance and other subject related matters, within the material, was included.

The Glencoe textbook focused 29% of the transformation exercises on using vocabulary and Prentice Hall asked the same in a total of 4% of their questions. Both examples in the textbooks required the students to apply steps that were illustrated in the narrative portion of the lesson. Glencoe asked students to apply illustrated steps in 75% of exercises, and Prentice Hall used this strategy in 92% of their exercises. Questions that asked students to find the coordinates of an image, or the angle of rotation, were represented in 51% of the exercises in Prentice Hall, and 17% in the Glencoe textbook.

Table A7

*Percent of the Total Exercises in the Transformations Sections by Type and by Textbook*

| | *Glencoe* | | *Prentice Hall* | |
|---|---|---|---|---|
| *Exercise Type* | *N* | *% - (based on 127)* | *N* | *% - (based on 149)* |
| Apply Vocabulary | 37 | 29 | 6 | 4 |
| Apply Steps Previously Given | 95 | 75 | 137 | 92 |
| Find Coordinates or Angle | 21 | 17 | 76 | 51 |
| Graph Answer | 41 | 32 | 65 | 44 |
| Draw Answer | 18 | 14 | 11 | 7 |
| Matching | 1 | 1 | 1 | 1 |
| Correct the Error | 1 | 1 | 2 | 1 |
| True/False | 2 | 2 | 0 | 0 |
| Written Answer | 11 | 9 | 20 | 13 |
| Work Backwards | 1 | 1 | 4 | 3 |
| Give a Counterexample | 0 | 0 | 0 | 0 |
| Real-World Relevance | 3 | 2 | 15 | 10 |
| Total Number of Student Exercises in Transformations | 127 | | 149 | |

*Note:* Exercises may require students to perform more than one of the performance types in the same question. For example, graph an answer and give the coordinates.

Example:

Graph the figure with the given vertices. Then graph the image of the figure after the indicated translation, and write the coordinates of its vertices.

1. Triangle XYZ with vertices X( -4 , -4), Y( -3 , -1), and Z(2 , -2) translated 3 units right and 4 units up. (Glencoe, 2004, p. 298)

*Figure A3.* Example of Type of Problem to Apply Steps Previously Given

The Glencoe text either asked the student to graph the answer in 32% of the exercises, or draw an answer in 14% of the total. Prentice Hall asked the student to graph the answer in 44% of the total exercises or draw an answer in 7% of their questions. The three exercise designations of matching, correcting the error, or deciding if information presented was true or false were negligibly presented in both textbooks.

The inclusion of transformation exercises that required a written answer numbered 11 out of 127 in the Glencoe textbook as compared to 20 out of 149 exercises in the Prentice Hall textbook. Working an exercise backwards (Figure A4) was only used in the Glencoe textbook once, and four times in the Prentice Hall textbook within the transformation exercises. A sample exercise is illustrated for working a problem backwards.

Example:

23. *Writing in Math*

Suppose you translated a point to the left 1 unit and up 3 units. Describe what you would do to the coordinates of the image point to find the coordinates of the preimage.

*Figure A4.* Working an Exercise Backwards

Neither textbook used the strategy of having the students find a counterexample. Glencoe used the inclusion of real-world relevance topics in a total of three questions, and overall included topics of designing shirts, business logos, patterns in rugs, and symmetry of letters. Additionally, Glencoe's presentations related to academic subjects of art, language arts, science, and music. Prentice Hall related the concepts and questions to the real-world relevance topics in fifteen questions that included topics of games, pictures, animals, boats, and puzzles, with related subject inclusion of algebra, language arts, and art.

**Manipulatives and technology.** Only one exercise in the Glencoe textbook and seven in the Prentice Hall offered suggestions for the inclusion of manipulatives or technology. Neither the Glencoe nor Prentice Hall textbook suggested the use of specific mathematics manipulatives, such as attribute blocks, geoboards, mirrors or miras®, in any of the student exercises. The Glencoe textbook included one question in which the student was expected to use the internet as a resource for finding the answer. The Prentice Hall textbook included an activity at the end of the section presenting the topic of similarity in transformations. This section suggested that the students use a computer software program (not listed) to investigate dilation of a figure.

**Level of cognitive demand.** Table A8 reports results on the level of cognitive demand performance required by the student to complete the exercises. The framework for coding the questions was patterned after the work of Stein and Smith (1998) and Jones (2004). Stein and Smith presented a Mathematical Tasks Framework with four levels: (1) *Lower-Level* designates tasks that include memorization or exact reproduction of learned facts; (2) *Lower-Middle Level or* "Procedures without Connections" applies algorithms from prior tasks and no connections to mathematical concepts; (3) *Higher-Middle Level or* "Procedures with Connections" requires some degree of cognitive effort, has connections to mathematical concepts and ideas; (4) *Higher-Level* or "Doing Mathematics" requires cognitive effort, exploration of mathematical relationships, analysis of the task, and an understanding of concepts to properly answer.

The analyzed student exercises in the transformation lessons of the Glencoe and Prentice Hall textbooks are presented in Table A8. Glencoe's exercises are predominantly in the category of "Procedures without Connections", with more than 61% of the overall transformation questions in this category. Prentice Hall's student exercises were predominately in the Lower-Level category, with 84% of the exercises at this level. Both textbooks offered only one (1) exercise that was coded in the category of "Doing Mathematics".

Table A8

*Percent of Transformation Exercises by Level of Cognitive Demand by Textbooks*

| | *Glencoe* | | *Prentice Hall* | |
|---|---|---|---|---|
| *Level of Cognitive Demand* | *N* | *% - (based on 127)* | *N* | *% - (based on 149)* |
| Lower – Level | 31 | 24 | 124 | 83 |
| Procedures without Connections | 78 | 61 | 18 | 12 |
| Procedures with Connections | 17 | 13 | 6 | 4 |
| Doing Mathematics | 1 | 1 | 1 | 1 |
| Total Exercises | 127 | | 149 | |

**Pilot Study Findings Discussion**

In this pilot study, two eighth grade textbooks were analyzed to determine the extent to which students had an *opportunity to learn* geometric transformations. The coding instrument was constructed using similar content analysis research. The coding instrument was refined during the process of data collection. Data collected included information for both quantitative and qualitative characteristics of the textbooks. The coding instrument was designed to collect data in three major sections. The focus of the first section was on the physical characteristics of the textbook, including page counts and location of transformation lessons within the structure of the textbook. The first section also recorded the location of transformation lessons that were analyzed; a vocabulary list was developed to support an index check to insure that all pertinent concept locations were identified. The second section collection concentrated on lesson analysis and recorded the nature and characteristics of the narrative, objectives, vocabulary, and illustrated examples in the lessons. The third section of the document focused on student exercises provided with the lesson including characteristics of expected student performance and level of cognitive demand needed for students to complete the questions.

Findings indicate that the two books are similar in regard to the number of instructional pages, which was approximately 80% of the total number of pages within each textbook. Both textbooks also provided similar resource pages, including prerequisite skills, selected answers, and a glossary in both English and Spanish. Both textbooks also provided transformation lessons on reflections, translations, rotations, and dilations.

The locations of the transformation lessons within the textbooks differed. The Prentice Hall © 2004 textbook placed the translation, reflection, and rotation lessons within the first 20% of the textbook pages, whereas Glencoe © 2004 placed emphasis on these same three lessons after the first 42% of the textbook pages. Additionally the Prentice Hall textbook placed approximately equal emphasis on all four of the transformation topics, while the Glencoe textbook placed almost 50% of the entire transformation emphasis on the concept of reflection, and the least amount, 14%, on translations.

In the narrative portion of the lessons, both textbooks provided written objectives and definitions of terms. In the Prentice Hall textbook, a discrepancy was found between the definition of translation and the coordinate graph offered as an example. This textbook did not offer a defined list of steps to follow, and students had to interpret what was required. The symbol for translation (→) was mentioned. An explanation of the arrow symbol to indicate movement on the coordinate plane and the term vector were not used, nor was the notation for movement of a point as (± x, ± y). The Prentice Hall lessons on transformations did not delineate the properties being used in the concepts.

In the narrative portion of the lessons in the Glencoe textbook the students were offered a list of defined steps for performing the transformation; properties were also discussed. The term "slide" was included in the definitions, but the symbol for translation (→) was not included, nor was the term vector.

The analyses of expected student performance on exercises in the Glencoe textbook indicated that emphasis was placed on applying steps that were given during the instructional portion of the lesson (75%), on graphing an answer (32%), and on finding coordinates or angle measures (17%). Close to 9% of the transformation questions asked the student to write out their answer. In approximately 2.5% of the transformation questions, topics focused on real-world relevance. In the Prentice Hall textbook, 91% of the transformation questions asked the student to apply previously given steps, 44% were on graphing an answer, and 51% on finding coordinates or angle measures. In 10% of the problems the transformation questions integrated real-world relevant topics.

The level of cognitive demand required in student questions was similar for both textbooks for the highest level; each incorporated less than 1% in the category of “doing mathematics.” Glencoe placed the most emphasis “Procedures without Connections” (61%) and 24% in questions rated as “Lower-Level.” The Prentice Hall textbook contained 83% of the questions in the lowest rating “Lower-Level”, and 12% rated as “Procedures without Connections.”

**Summary**

The purpose of this pilot study was to explore and analyze the presentation and development of the concept of geometric transformations in two middle grades textbooks. I set out to determine the extent to which these textbooks provided opportunities for students to learn transformations. The particular focus was to collect data from these textbooks to compare the presentations of transformation topics within the examples and exercises. The preceding work presented an analysis of the *Prentice Hall Course 3 Mathematics* ©2004 and *Glencoe Mathematics Applications & Concepts Course 3* © 2004. The work was intended to be similar to the work that would present the findings in chapter four of a dissertation.

The results from this pilot investigation were important to confirm existing differences in the presentations and the treatment of the topics between selected textbooks. Thus, the results suggest that it would be worthwhile to expand the study to a more detailed analysis of a larger variety of textbooks.

## Appendix B

### Composite Transformation Sample Conversions and Properties List

| *Transformation 1* | *Transformation 2* | *Resulting Single Transformation* |
|---|---|---|
| Translation | Translation | Translation or Identity |
| Reflection | Translation | Glide |
| Translation | Glide Transformation | Glide or Identity |
| Rotation 90º Clockwise | Rotation 90º Clockwise | Reflection |
| Rotation 90º Clockwise | Rotation 90º Counterclockwise | Identity |
| Rotation 180º Clockwise | Rotation 180º Clockwise | Identity |
| Glide Transformation | Glide Transformation | Translation or Identity |
| Reflection | Reflection over Parallel Lines | Translation |
| Reflection | Reflection over Perpendicular Lines | Rotation |
| Rotation Aº Clockwise | Rotation Bº Clockwise | Rotation (A + B)º |

Properties of Composite Transformations.

A composite transformation made up of two or more transformations performed one after the other will exhibit the following properties (Burke, Cowen, Fernandez, & Wesslen, 2006).

1. When two transformations result in an image being identical to the preimage there was no change and the composite was called an identity transformation.

2. When the same result of the composite transformations can be achieved with one single transformation forming the same image, this property is called closure.
3. Every combination of composite transformations has an inverse.
4. Three combinations of transformations can be combined in any order, called the associative property.

## Appendix C

### Properties of Geometric Transformations Expected to be Present in Lessons

| Transformation | Properties |
|---|---|
| Translation | • All points of the preimage move the same distance and direction.<br>• Orientation of object remains the same, just location changes.<br>• Preimage and image are congruent. |
| Reflection | • Preimage and image are same shape and size. Orientation of figure is reversed (Rollick, 2009).<br>• Corresponding vertices of preimage and image are equidistant from and perpendicular to the line of reflection (Rollick, 2009). |
| Rotation | • Corresponding points on preimage and image are the same distance from center of rotation.<br>• Resulting image is congruent to the preimage.<br>• Orientation of preimage and image are changed with respect to angle of rotation.<br>• The farther a figure is from the center of rotation, the farther the figure moves in rotating the same angle measure (Keiser, 2000).<br>• Direction of rotational movement may be in clockwise (right) or counterclockwise (left) direction. Two different rotations may result in the same image: Example, rotation 270 degrees in one direction or 90 the other direction (Olson, 2008). |
| Composite | • When two transformations, performed one after the other, result in an image identical to the preimage, no change occurs and the composite is called an identity.<br>• When two transformations result in an image, the composite may be replaced with one transformation to form the same image.<br>• Every combination of composite transformations has an inverse. |
| Dilation | • Preimage and image are similar figures.<br>• The orientation is the same for both figures. |

## Appendix D

### Aspects of Transformations and Student Issues

| *Transformation* | *Issues* | *Research Study* |
|---|---|---|
| Transformation Constructs | • Misconceptions and Difficulties | Bouler & Kirby, 1994; Kidder 1976; Moyer, 1978; Shah, 1969; Soon, 1989; Soon & Flake, 1989; Usiskin et al., 2003; Yanik & Flores, 2009 |
| | • Problems with Vocabulary Use | Meleay, 1998; Soon, 1989 |
| | • Properties of Figures | Boulter & Kirby, 1994; Hollebrands, 2004; Kidder, 1976; Laborde, 1993 |
| Translation | • Issues with direction of movement, right, left, or over a diagonal line | Rollick, 2009; Schults & Austin,1983; Shultz, 1978 |
| | • Movement of figure in same direction and length of vector shaft | Flanagan, 2001; Hollebrands, 2003;Wesslen & Fernandez, 2005 |
| Reflection | • Issues with direction of movement, right, left, and over a diagonal line | Burger & Shaugnessy, 1986; Kuchemann, 1980, 1981; Perham, Perham, & Perham, 1976; Rollick, 2009; Schultz, 1978 |
| | • Interpretation of the line of reflection as cutting the figure in half, or alternatively the line of reflection falling along the long edge of the figure being reflected | Edwards & Zazkis, 1993; Yanik & Flores, 2009 |
| | • Reflection onto/over the Preimage | Edwards & Zazkis, 1993; Soon, 1989; Yanik & Flores, 2009 |

| *Transformation* | *Issues* | *Research Study* |
|---|---|---|
| Rotation | • Center of Rotation | Clements, Battista & Sarama, 1998; Edwards & Zazkis, 1993; Soon, 1989; Soon & Flake, 1989; Wesslen & Fernandez, 2005; Yanik & Flories, 2009 |
| | • Rotation point other than origin or vertices | Edwards & Zazkis, 1993; Yanik & Flories, 2009; Wesslen & Fernandez, 2005 |
| | • Center of rotation - external to figure | Soon & Flake, 1989 |
| | • Angle of Rotation, understanding, measuring, determining | Clements & Battista, 1989, 1990, 1992; Clements, Battista & Sarama, 1998; Clements & Burns, 2000; Kidder, 1979; Krainer, 1991; Olson, Zenigami, & Okazaki 2008; Soon, 1989; Wesslen & Fernandez, 2005 |
| | • Direction of Rotation- clockwise and counterclockwise directionality | Clements et al., 1996; Soon, 1989; Wesslen & Fernandez, 2005 |
| Dilation | • Scale Factor and Identity | Soon, 1989 |
| Composite Transformation | • Misconceptions and Difficulties | Burke, Cowen, Fernandez & Wesslen, 2006; Schattschneider, 2009; Wesslen & Fernandez, 2005 |

## Appendix E

### Examples of Student Performance Expectations in Exercise Questions

Student performance expectations are defined as the type of response expected from students to answer each type of exercise question. The following chart provides an example for each of the exercise types. The types of performance expectations include: apply steps previously given in the lesson, apply/fill in vocabulary, make a drawing, graph the answer, find coordinates of point(s), find measure of the angle, match the given content or multiple choice, determine if the statement is true or false, provide a written answer, work a problem backwards, correct an error in a given problem, and give a counterexample. The type of question that required a student to construct a counterexample was not found in any of the transformation exercises and hence an example is not provided in the following table.

In the table, examples are offered to illustrate specific types of performance expectations. Frequently in one example more than one type of performance was expected, and hence the expected performance for each type of work was coded for the exercise; for example, an exercise might have required the student to graph the answer and also find/list the coordinates. In this case, both graph the answer and find the coordinates of the point(s) would have been recorded

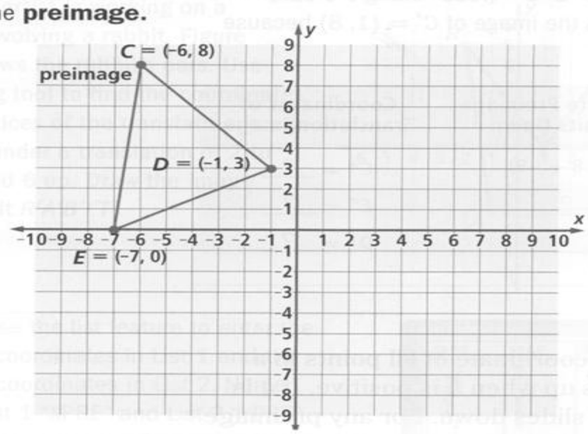
<table>
<tr><th>Type of Performance Expectation</th><th>Illustration of Exercise<br>(actual number of exercise given from textbook)</th></tr>
<tr><td>Apply/fill in vocabulary</td><td>1. Vocabulary - A translation is a type of _____? (Prentice Hall, Course 2, p. 512)<br>2. Name three types of transformations. (Prentice Hall, Course 2, p521)<br>1. Vocabulary - A (transformation, image) is a change in the position, shape, or size of a figure. (Prentice Hall, Course 3, p. 137)</td></tr>
<tr><td>Apply steps previously given in the lesson</td><td>

**Horizontal or Vertical Translations**

**Activity 1**

**Let $C = (-6, 8)$, $E = (-7, 0)$, and $D = (-1, 3)$. Translate $\triangle CED$ 7 units to the right.**

Step 1 Graph $\triangle CED$ on a coordinate grid as shown below. $\triangle CED$ is called the **preimage.**

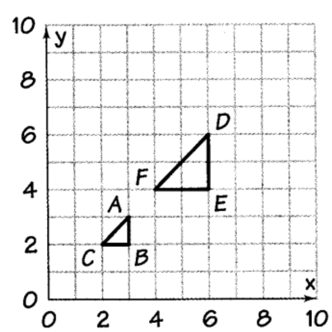


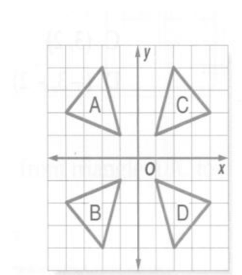
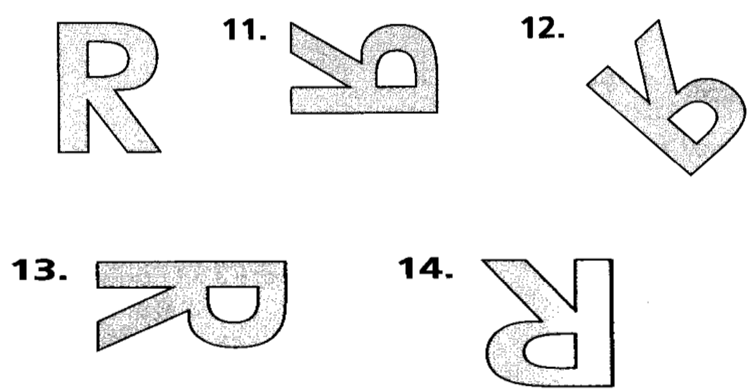

Step 2 To translate the preimage 7 units to the right, add 7 to each *x*-coordinate. The result is a triangle 7 units to the right of $\triangle CED$. We call this $\triangle C'E'D'$ (read "triangle *C*-prime, *E*-prime, *D*-prime"). $\triangle C'E'D'$ is the **translation image** of $\triangle CED$.

<table>
<tr><th>Coordinates of Preimage</th><th>Translate Preimage 7 Units to the Right</th><th>Coordinates of Translation Image</th></tr>
<tr><td>$C = (-6, 8)$</td><td>$(-6 + 7, 8)$</td><td>$C' = (1, 8)$</td></tr>
<tr><td>$E = (-7, 0)$</td><td>?</td><td>?</td></tr>
<tr><td>$D = (-1, 3)$</td><td>?</td><td>?</td></tr>
</table>

Step 3 Graph each image point on the same grid. Each image point is 7 units to the right of the preimage point. For instance, $C' = (1, 8)$ is 7 units to the right of $C = (-6, 8)$.

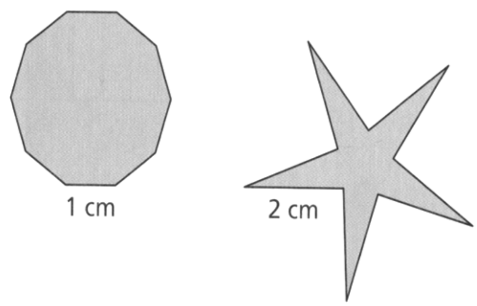
(UCSMP, Transition Mathematics, p. 359)
</td></tr>
</table>

| *Type of Performance Expectation* | *Illustration of Exercise*<br><br>*(actual number of exercise given from textbook)* |
| --- | --- |
| | **15. a.** Draw quadrilateral $PQRS$ with $P = (0, 0)$, $Q = (6, 0)$, $R = (6, 2)$, and $S = (0, 4)$.<br>**b.** On the same axes, draw the image of $PQRS$ when 2 is subtracted from each *first* coordinate and 4 is subtracted from each *second* coordinate.<br>**c.** The preimage and image are __?__.<br><br>(UCSMP, Transition Mathematics, p. 364) |
| Make a drawing | 26. In parts (a) and (b), use a capital letter as the basic design element.<br>a. Sketch a strip pattern with reflection symmetry only.<br>b. Sketch a strip pattern with reflection symmetry and rotation symmetry. (CM8, Unit 5, p18) |
| Graph the answer | **26.** In *Stretching and Shrinking,* you used rubber bands to enlarge shapes. In the figure below, a rubber band was used to enlarge triangle $ABC$. The anchor point for the rubber band was at the origin. The knot traced around triangle $ABC$ as the pencil drew triangle $DEF$.<br><br><br>**a.** What is the scale factor from triangle $ABC$ to triangle $DEF$?<br>**b.** On grid paper, sketch an enlargement of triangle $ABC$ by a scale factor of 1.5 with the anchor point at the origin. Call this enlargement triangle $XYZ$.<br>**c.** How is triangle $XYZ$ like or unlike a translation image of triangle $ABC$?<br><br>(Connected Mathematics 2, Grade 8, Unit 5, p43) |

| *Type of Performance Expectation* | *Illustration of Exercise*<br>*(actual number of exercise given from textbook)* |
| --- | --- |
| Find the coordinates of point(s) | Example 2 Triangle JKL has vertices J(-4, 2), K(-2, -4), and L(3, 6). Find the vertices of $\Delta J'K'L'$ after a dilation with the given scale factor. Then graph $\Delta JKL$ and $\Delta J'K'L'$.<br>3. scale factor: 3 4. Scale factor: ¼<br>(Glencoe, Course 3, p. 248) |
| Find the measure of an angle | 31. In Parts a-d, what is the magnitude of the rotation of the hand of a clock in the given amount of time?<br>a. 3 hours<br>b. 1 hour<br>c. 10 minutes<br>d. 1 day<br>(UCSMP, Transition Mathematics, p. 293) |
| Match the given content, multi choice | For Exercises 17 - 19, use the graph shown at the right.<br>17. Which pair(s) of figures is reflected over the x-axis?<br>18. Which pair(s) of figures is a reflection over the y-axis?<br>19. If figure B was reflected over the x-axis, which figure(s) would the resulting image look like?<br><br>(Glencoe, Course 1, p. 613) |
| Determine if the statement is true or false | Tell whether each shape is a rotation of the shape at the left.<br><br>(Prentice Hall, Course 1, p. 404) |

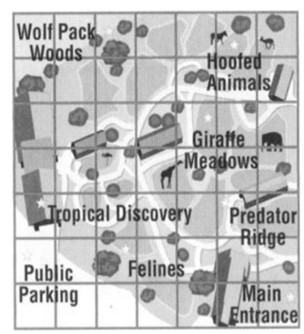
<table>
<tr><th>Type of Performance Expectation</th><th>Illustration of Exercise<br>(actual number of exercise given from textbook)</th></tr>
<tr><td>Correct an error in a given problem</td><td>

**13.** Suppose a student draws the figures below. The student says the two shapes are similar because there is a common scale factor for all of the sides. The sides of the larger figure are twice as long as those of the smaller figure. What do you say to the student to explain why they are *not* similar?



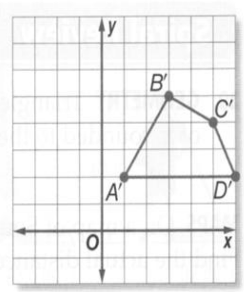

(Connected Mathematics 2, Grade 7, Unit 2, p. 33)
</td></tr>
<tr><td>Provide a written answer</td><td>

**4. MAPS** Nakos explores part of the Denver Zoo in Colorado as shown. He starts at the felines and then visits the hoofed animals. Describe this translation in words and as an ordered pair.




(Glencoe, Course 2, p. 555)
</td></tr>
<tr><td>Work a problem backwards</td><td>

**20. CHALLENGE** Suppose a figure *ABCD* is dilated by a scale factor of $\frac{1}{2}$ and then reflected over the $x$-axis and the $y$-axis. The final image is shown on the graph at the right. Graph the original image and list the coordinates of points *A*, *B*, *C*, and *D*.




(Glencoe, Pre-Algebra, p. 311)
</td></tr>
</table>

## Appendix F

## Coding Instrument

Segment 1a: "Where" (Content)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Title: | | | | Publisher: | | | |
| Grade Level: | | Copyright Date: | | | ISBN: | | | |
| # pages in text | | # pages Instructional | | | | | | |
| | Number of pages | | | | | | | |
| Ch # | # Sections | Instruct-ional | Ch Rev | Practice Test | Stnd Test Practice | Excluded pg(s) | Projects | Total |
| 1 | | | | | | | | |
| 2 | | | | | | | | |
| 3 | | | | | | | | |
| 4 | | | | | | | | |
| 5 | | | | | | | | |
| 6 | | | | | | | | |
| 7 | | | | | | | | |
| 8 | | | | | | | | |
| 9 | | | | | | | | |
| 10 | | | | | | | | |
| 11 | | | | | | | | |
| 12 | | | | | | | | |
| 13 | | | | | | | | |
| 14 | | | | | | | | |
| 15 | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

Back of Book: Total number of pages of each of the following

| | | |
|---|---|---|
| Selected Answers | Word Problems | Skills |
| Tables | Extra Practice | Projects |
| Glossary | Index | |
| 2nd Lang | Last printed page# | |

Segment 1b: Location of Topics

| Textbook: | Grade Level: | Lesson: |
|---|---|---|

| Focus Content for Analysis |
|---|

| Ch/Section # | start pg # | # pages | Section Title |
|---|---|---|---|
| | | | |

| Ch/Section # | start pg # | # pages | Section Title |
|---|---|---|---|
| | | | |

| Ch/Section # | start pg # | # pages | Section Title |
|---|---|---|---|
| | | | |

| Ch/Section # | start pg # | # pages | Section Title |
|---|---|---|---|
| | | | |

| Ch/Section # | start pg # | # pages | Section Title |
|---|---|---|---|
| | | | |

| Ch/Section # | start pg # | # pages | Section Title |
|---|---|---|---|
| | | | |

| Ch/Section # | start pg # | # pages | Section Title |
|---|---|---|---|
| | | | |

Segment 1c

| Textbook | Grade | Lesson |
| --- | --- | --- |
| Terminology: | Page numbers: | |
| bilateral symmetry | | |
| congruence | | |
| coordinate plane | | |
| dilation, dilate | | |
| enlarge, expand | | |
| flip | | |
| geometry | | |
| glide | | |
| line symmetry | | |
| pivot | | |
| reduction | | |
| reflection | | |
| Rotation, rotation motion, rotary motion | | |
| rotational symmetry, angle of | | |
| Scaling/ scale model, scale/drawings | | |
| Scale factor | | |
| Slide | | |
| Stretch | | |
| tessellation | | |
| Turn | | |
| transformations | | |
| translation | | |
| turn symmetry | | |
| two dimensional figures | | |
| vector | | |
| vertex/vertices | | |

Segment 2: “What: (Lesson Analysis)

| Textbook: | Grade Level: | Lesson: |
|---|---|---|
| Objectives: | | |
| | | |
| Vocabulary: | | |
| | | |
| Properties: | | |
| | | |

Lesson Narrative:

| Amount of Page(s) |
|---|

| Contents | | | Number related review problem/type |
|---|---|---|---|

| Example Number | Intro | 1 | 2 | 3 | 4 | | | Notes: |
|---|---|---|---|---|---|---|---|---|
| amount of page | | | | | | | | |
| topic of example | | | | | | | | |
| follow # steps | | | | | | | | |
| Graphic | | | | | | | | |
| student performance? | | | | | | | | |
| Counterexample | | | | | | | | |
| Correct the error | | | | | | | | |
| real world related | | | | | | | | |
| manipulatives | | | | | | | | |
| technology | | | | | | | | |
| | | | | | | | | |

Other features:

Segment 3: “How” (Processes)

| Student Exercises | | | Textbook | | Grade: | | | Lesson: | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Example Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| type of transformation | | | | | | | | | | |
| apply steps given | | | | | | | | | | |
| fill/apply vocab | | | | | | | | | | |
| graph answer | | | | | | | | | | |
| find coordinates (angle) | | | | | | | | | | |
| written answer | | | | | | | | | | |
| work backwards | | | | | | | | | | |
| give a counterexample | | | | | | | | | | |
| matching | | | | | | | | | | |
| correct the error | | | | | | | | | | |
| true/false | | | | | | | | | | |
| real-world relevance | | | | | | | | | | |
| subject related | | | | | | | | | | |
| type manipulatives | | | | | | | | | | |
| type technology | | | | | | | | | | |
| level of cognitive demand | | | | | | | | | | |
| | | | | | | | | | | |

## Appendix G

### Instrument Codes for Recording Characteristics of Student Exercises

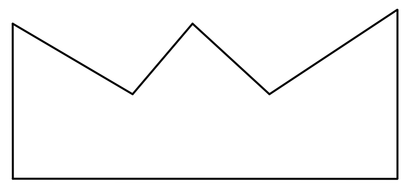
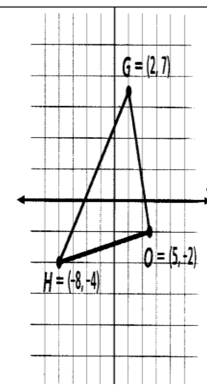
| *Feature* | *Categories* | *Symbol* |
|---|---|---|
| Amount of Page | Linear Measure in Inches | 0, ¼, ½, ¾, 1 |
| Graphic: Identifies the presence and/or nature of a graphic | Diagram or Drawing | D |
| | Coordinate Graph or Photograph or | G |
| | Picture | P |
| | None | No (or blank) |
| Nature of Transformation in Example or Exercise | Transformation | T |
| | Translation | Tr w/ arrow(s)* |
| | Reflection | Rf |
| | over X axis | Rfx w/ |
| | over Y axis | arrow(s)* |
| | over oblique line | Rfy w/ |
| | into/over preimage | arrow(s)* |
| | | Rfl |
| | *arrows for direction of transformation | Rfo |
| | Reflectional symmetry | Rfsy |
| | Line symmetry | lsy |
| | Dilation (reduce/enlarge) | Di/En |
| | other than origin or vertices | Di/Eno |
| | Rotation | Ro right=r, left=l |
| | around point other than origin/vertices | Roo |
| | angle of | R< |
| | rotational symmetry | rosy |
| | Composite Transformation | Comp |
| | Tessellation | tess |

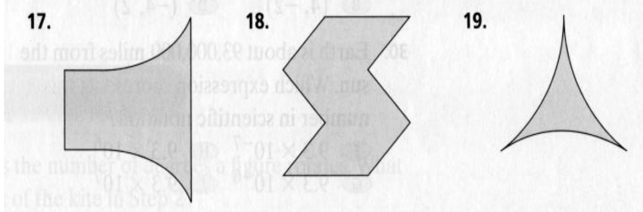
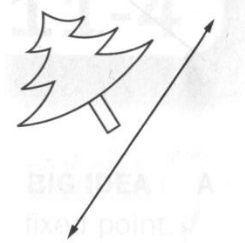
| *Feature* | *Categories* | *Symbol* |
|---|---|---|
| Use of Manipulatives | Classroom supplies (graph paper, ruler, pencil, straightedge, trace paper, etc.) | ·=[dot] (no mark) |
| | Craft type manipulative (straws, string, etc.) | Cr |
| | Math Manipulative (patty paper, mira®, attribute blocks, mirror, etc.) | M |
| Use of Technology | Calculator/Computer Program | Calc/Cp |
| Expected Student Response | Correct the Error in the Given Problem | Yes =Y, or no entry |
| | Apply Vocabulary | |
| | Graph Answer | |
| | Apply Steps Previously Given | |
| | Find Coordinates or Angle | |
| | Work a Problem Backwards | |
| | Provide a Counterexample | |
| Other Characteristics | Real-world relevance | Name topic |
| | Subject related | List Subject |
| Level of Cognitive Demand* | Memorization, lower-level | LL |
| | Procedures without connections, lo-mid | LM |
| | Procedures with connections, hi-middle | HM |
| | Doing mathematics, higher-level | HH |
| Properties of Geometric Transformations | | List Properties |

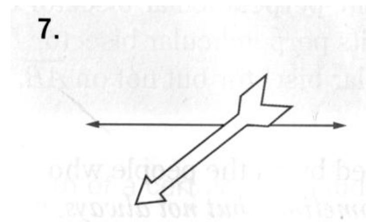
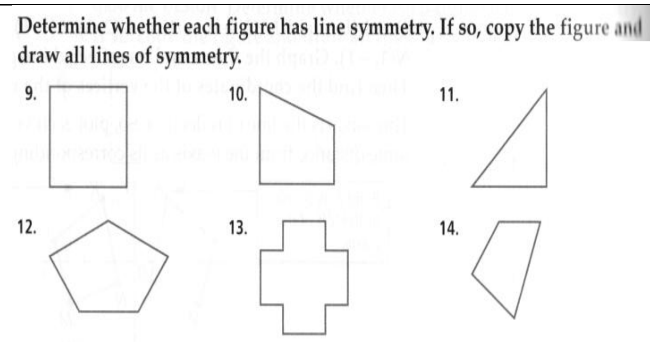
*Note:* * A complete list of Levels of Cognitive Demand descriptions are incorporated in the Chapter 2 review.

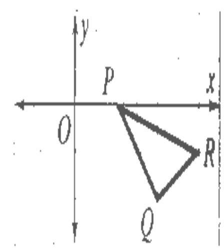
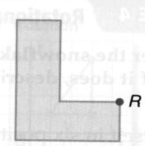
## Appendix H

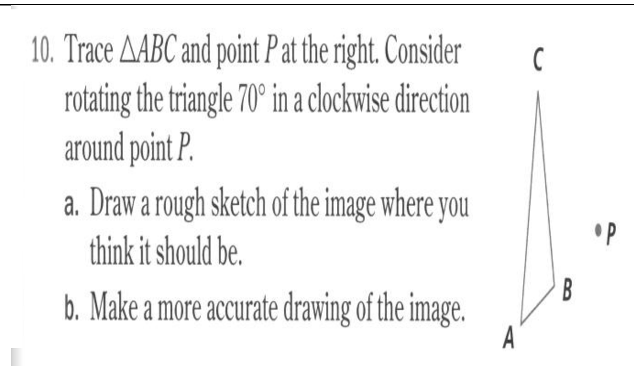
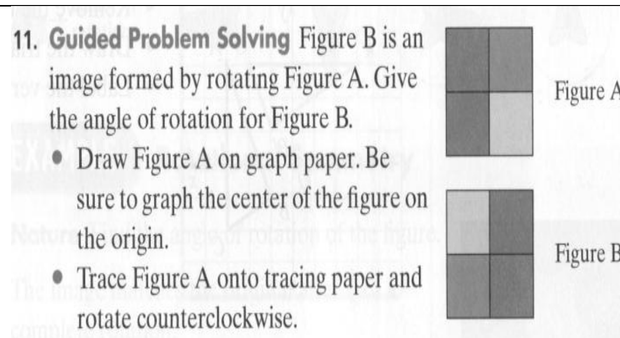
### Transformation Type Sub-grouped Categories and Examples

| *Transformation* | *Symbol* | *Direction Movement* | *Example* |
|---|---|---|---|
| Translation | Tr | | 17. Draw a translation of the figure.<br><br>(Prentice Hall, Course 1, p. 405) |
| | Tr w/ arrow(s) | Direction of Movement | **In 7 and 8, copy the grid. Then graph $\triangle HOG$ and its image under the translation that is described.**<br>**7.** The image of $(x, y)$ is $(x + 3, y)$.<br>**8.** The image of $(x, y)$ is $(x + -3, y + -2)$.<br><br>(#7- Tr→) (#8 - Tr ↓← )<br>(UCSMP, Transition Mathematics, p. 362) |
| Reflection | Rf | | 26. Writing in Math<br>When you find the coordinates of an image after a reflection over the x-axis or the y-axis, what do you notice about the coordinates of the new image in relation to the coordinates of the original image?<br>(Glencoe, Course 1, p 614) |

| *Transformation* | *Symbol* | *Direction Movement* | *Example* |
| --- | --- | --- | --- |
| Reflectional Symmetry | Rfsy | | **Copy each figure that has reflectional symmetry. Draw the lines of symmetry. Write *no reflectional symmetry* where applicable.**   (Prentice Hall, Course 3, p. 143) |
| | Rfx w/ arrow(s) | over X axis | Graph each point and its reflection over the indicated axis. Write the coordinates of the image. 10. D (4 , 2) (Rfx↓) |
| | | over Y axis | 11. F ( -1 , 5 ) (Rfy→) (Prentice Hall, Course 2, p516) |
| | Rfl | over oblique line | In 6 and 7, trace the figure and use paper folding to find the reflection image. Rfl 6.  (UCSMP, Pre-Transition Mathematics, p. 647) |

| *Transformation* | *Symbol* | *Direction Movement* | *Example* |
|---|---|---|---|
| | Rfo | into/over preimage | Rfo<br><br>(UCSMP, Pre-Transition Mathematics, p. 647) |
| Line Symmetry | Lsy | | Determine whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.<br>9. 10. 11.<br>12. 13. 14.<br>(Glencoe, Course 2, p. 560) |
| Dilation | Di/En | reduce enlarge | 19. Publishing<br>To place a picture in his class newsletter, Joquin must reduce the picture by a scale factor of 3/10. Find the dimensions of the reduced picture if the original is 15 centimeters wide and 10 centimeters high.<br>(Glencoe, Course 3, p 229) |
| | Di/Eno | other than origin or vertices | None found |

| *Transformation* | *Symbol* | *Direction Movement* | *Example* |
|---|---|---|---|
| | sf | scale factor | **21.** A movie projector that is 6 feet away from a large screen shows a rectangular picture that is 3 feet wide and 2 feet high.<br>**a.** Suppose the projector is moved to a point 12 feet from the screen. What size will the picture be (width, height, and area)?<br>**b.** Suppose the projector is moved to a point 9 feet from the screen. What size will the picture be (width, height, and area)?<br><br>(Connected Mathematics, Grade 7, Unit 2, p17) |
| Rotation | Ro | | **Copy $\triangle PQR$. Draw the image of $\triangle PQR$ after a rotation of the given number of degrees about the origin.**<br>**8.** 90° **9.** 180° **10.** 270°<br><br><br><br>(Prentice Hall, Course 3, p148) |
| | | right=r,<br><br>left=l | **Draw each figure after the rotation described.**<br>**5.** 90° clockwise rotation about point *R*<br><br><br><br>(Glencoe, Pre-Algebra, p. 608) |

| *Transformation* | *Symbol* | *Direction Movement* | *Example* |
| --- | --- | --- | --- |
| | Roo | around point other than origin/ vertices | <br>10. Trace $\triangle ABC$ and point $P$ at the right. Consider rotating the triangle 70° in a clockwise direction around point $P$.<br>a. Draw a rough sketch of the image where you think it should be.<br>b. Make a more accurate drawing of the image.<br><br>(UCSMP, Pre-Transition Mathematics, p 653) |
| | R< | angle of rotation | <br>**11. Guided Problem Solving** Figure B is an image formed by rotating Figure A. Give the angle of rotation for Figure B.<br>• Draw Figure A on graph paper. Be sure to graph the center of the figure on the origin.<br>• Trace Figure A onto tracing paper and rotate counterclockwise.<br><br>(Prentice Hall, Course 3, p. 148) |
| Rotational symmetry | rosy | | 24. Challenge<br>State the least number of degrees you must rotate an equilateral triangle for the image to fit exactly over the original triangle.<br>(Prentice Hall, Course 1, p 405) |
| Composite Transformation | Comp | | 19. Writing in Math<br>Suppose you reflect a figure over the x-axis and then you reflect the figure over the y-axis. Is there a single transformation using reflections or translations that maps the original figure to its image? If so, name it. Explain your reasoning.<br>(Glencoe, Pre-Algebra, p. 105) |

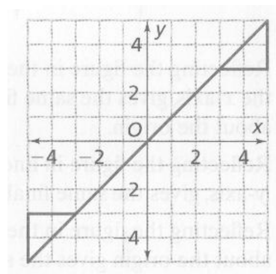
| *Transformation* | *Symbol* | *Direction Movement* | *Example* |
|---|---|---|---|
| Tessellation | tess | | 24. Tessellations<br>A tessellation is a pattern formed by repeating figures that fit together without gaps or overlays. Use the information at the left to describe how tessellations and translations were used to create the pattern on the egg.<br><br>**Real-World Link .** People in the Ukraine developed a technique called Pysanka for decorating eggs. The largest Pysanka egg was created using 524 star patterns and 2,206 triangular pieces.<br>(Glencoe, Course 1, p 608) |

# Appendix I

## Examples of Tasks Characterized by Levels of Cognitive Demand in Exercise Questions

The level of cognitive demand required by the student to complete performance expectations in the exercises is based on the framework developed by Stein and Smith (1998) and Smith and Stein (1998). Their framework document identified the level of cognitive demand in mathematical tasks by providing an evaluation of student thinking and reasoning required by the types of questions. Four categories of the level of cognitive demand identified are illustrated.

| Level of Cognitive Demand | Characteristics | Example Exercise |
|---|---|---|
| *Lower-Level (LL) demands* (memorization): | Memorization, exact reproduction of learned facts, vocabulary, formulas, materials, etc., lack of defined procedures, no connections to mathematical facts, rules. | 1. Vocabulary<br>How is a line of reflection a line of symmetry?<br>2. How many lines of symmetry does an equilateral triangle have?<br><br>(Prentice Hall, Course 2, p. 516) |

| | | |
|---|---|---|
| *Lower-Middle Level (LM) demands* (procedures without connections): | Procedures lacking mathematical connections requires use of algorithm, no connection to mathematical concepts, no explanations needed. | **22.** The diagram at the right has rotation symmetry.<br>**a.** What center and angle of rotation will rotate each flag onto the other?<br>**b.** Compare the coordinates of key points on one flag with the coordinates of their images on the other flag. Describe the pattern you see.<br><br>(Connected Mathematics 2, Grade 8, Unit 5, p 90) |
| *Higher-Middle Level (HM)* demands (procedures with connections): | Procedures with connections, procedures for development of mathematical understanding of concepts, some connections to mathematical concepts and ideas, multiple representations with interconnecting meaning, effort and engagement in task required. | 24. Writing in Math<br>Triangle ABC is translated 4 units right and 2 units down. Then the translated image is translated again 7 units left and 5 units up. Describe the final translated image in words.<br>(Glencoe Course 2, p. 556) |

| | | |
|---|---|---|
| *Higher-Level (HH)* demands (doing mathematics): | Doing mathematics, requires non-algorithmic procedures, requires exploration of mathematical relationships, requires use of relevant knowledge and analysis of the task requires cognitive effort to achieve solution required. | 24. A home copy machine had 5 settings: 122%, 100% 86%, 78%, and 70%. By using these settings as many times as you wish, show how you can make copies of 10 different sized between 100% and 200% of the original. (UCSMP, Transition Mathematics, p. 476) |

## Appendix J

**Background for Content Analysis and Related Research Studies**

| *Instrument Sections* | *Research Studies and Implications for Framework Development* |
|---|---|
| Segment 1a: "Where" (Physical Characteristics) | |
| Grade levels | Flanders, 1987, 1994a; Herbel-Eisenmann, 2007; Jones, 2004; Jones & Tarr, 2007; Li, 2000; Mesa, 2004; Remillard, 1991; Stylianides, 2005, 2007; Sutherland, Winter, & Harris, 2001; Wanatabe, 2003 |
| Publisher | Lundin, 1987 |
| Number of textbook pages | Flanders, 1987; Jones, 2004; Jones & Tarr, 2007; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002 |
| Number of instructional pages | Lundin, 1987; Jones, 2004; Jones & Tarr, 2007; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002 |
| Second Language | McNeely, 1997 |
| Segment 1b: | |
| Focus Content for Analysis | Lundin, 1987 |
| Section: Number of Pages | Flanders, 1994a, 1994b; Jones, 2004; Jones & Tarr, 2007; Lundin, 1987 |
| Section Start Page Number | Jones, 2004; Jones & Tarr, 2007 |
| Number of Pages in Section | Shields, 2005 |
| Amount of Narrative Pages | Shield, 2005 |
| Segment 1c: | |
| Terminology | Meleay, 1998; Soon, 1989 |

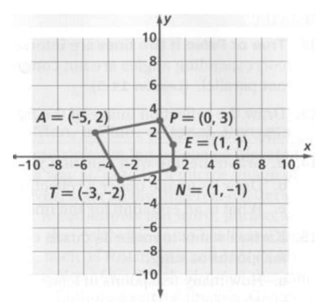
| *Instrument Sections* | *Research Studies and Implications for Framework Development* |
|---|---|
| Segment 2: "What" Narrative | |
| Objectives | Kulm, 1999; Kulm, Roseman, and Treistman, 1999; Tarr, Reys, Barker, and Billstein, 2006 |
| Vocabulary | Meleay, 1998; Soon, 1989 |
| Lesson Narratives | Haggarty & Pepin, 2002; Herbel-Eisenman, 2007; Johnson, Thompson, & Senk, 2010; Jones, 2004; Jones & Tarr, 2007; Mesa, 2004; Rivers, 1990; Shield, 2005; Soon, 1989; Sutherland  Winter & Harries, 2001; Tornroos, 2005; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002 |
| Segment 3: "How" Processes | |
| Transformation Characteristic | Boulter & Kirby, 1994; Kidder, 1976; Moyer, 1978; Shah, 1969; Soon, 1989; Soon & Flake, 1989; Usiskin, et al., 2003; Yanik & Flores, 2009 |
| Direction of Translation | Rollick, 2009; Shultz, 1978; Schults and Austin, 1983 |
| Movement Related to Vector | Flanagan, 2001 |
| Reflection over Diagonal | Burger & Shaugnessy, 1986; Kuchemann, 1980, 1981; Perham, Perham, & Perham, 1976; Schultz, 1978 |
| Reflection onto Preimage | Edwards & Zazkis, 1993; Soon, 1989; Yanik & Flores, 2009 |
| Center of Rotation | Clements, Battista & Sarama, 1998; Edwards & Zazkis, 1993; Soon, 1989; Soon & Flake, 1989; Wesslen & Fernandez, 2005; Yanik & Flories, 2009 |
| Angle of Rotation | Wesslen & Fernandez, 2005 |
| Direction of Rotation | Wesslen & Fernandez, 2005 |

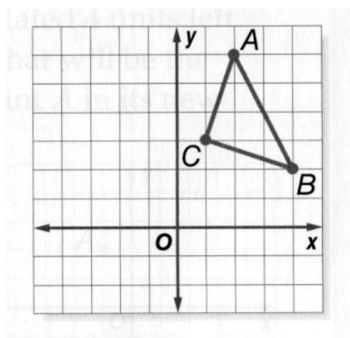
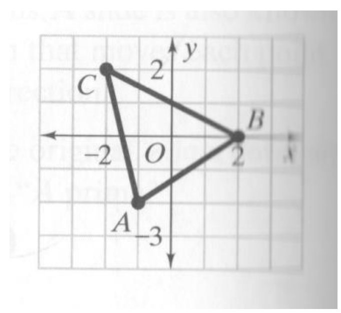
| *Instrument Sections* | *Research Studies and Implications for Framework Development* |
|---|---|
| Dilations | Soon, 1989 |
| Composite Transformations | Burke, Cowen, Fernandez & Wesslen, 2006; Schattschneider, 2009; Wesslen & Fernandez, 2005 |
| Graph/graphic | Sutherland, Winter & Harries, 2001 |
| Real World Relevance | NCTM, 1989, 2000 |
| Technology Inclusion | NCTM, 1989, 2000; Rivers, 1990 |
| Manipulatives | Jones, 2004; Jones and Tarr, 2007; Kieran, Hillel, & Erlwanger, 1986; Magina & Hoyles, 1997; Martinie & Stramel, 2004; Mitchelmore, 1998; NCTM, 1989, 2000; Stein & Bovalino, 2001; Weiss, 2006; Williford, 1972 |
| Exercise Performance Demands | Jones & Tarr, 2007; Li, 2000;Tornroos, 2005; Valverde, Bianchi, Wolfe, Schmidt, and Houang, 2002 |
| Written Answer | NCTM, 1989, 2000 |
| Work Backwards | NCTM, 1989, 2000 |
| Give a Counterexample | NCTM, 1989, 2000 |
| Level of Cognitive Demand | Doyle, 1988; Jones, 2004; Jones & Tarr, 2007; Li, 2000; NCTM, 1991; Porter, 2006; Resnick, 1987; 1996; Smith & Stein, 1998; Stein, Grover, & Henningsen, 1996; Stein, Lane, and Silver 1996; Stein & Smith, 1998 |

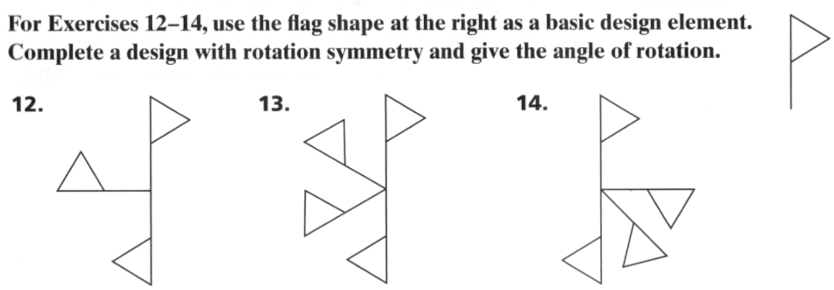
## Appendix K

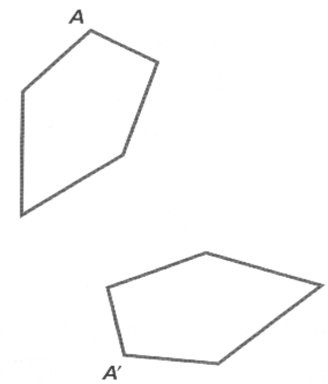
## Examples of Transformation Tasks in Exercise Questions

Specific characteristics of transformation tasks were identified in the research as causing student errors and misconceptions in student learning; this necessitated the need to subdivide the tasks: translation, reflection, rotation, and dilation, to identify the characteristics of tasks that address these student problems in the textbook content. Examples of each of the types of exercises are illustrated.

| *Type of Transformation* | *Illustration of Exercise* |
|---|---|
| Translation | |
| • all other translation problems | 26. Writing in Math<br>Why is it helpful to describe a translation by starting the horizontal change first?<br>(Prentice Hall, Course 2, p513) |
| • single direction movement translation | 7. a. Translate *PENTA* 3 units down. Label the image $P''E''N''T''A''$.<br>b. What are the coordinates of $P''$, $E''$, $N''$, $T''$, and $A''$?<br>c. If the point $(x, y)$ is translated $b$ units vertically, what are the coordinates of its image?<br> |

| *Type of Transformation* | *Illustration of Exercise* |
| --- | --- |
| | (UCSMP, Pre-Transitions, p 663) |
| • double direction movement translation | 1. Translate $\triangle ABC$ 3 units left and 3 units down. Graph $\triangle A'B'C'$.<br><br>(Glencoe, Course 2, p. 555) |
| • translation up and/or left direction | **Graph each translation of $\triangle ABC$. Use arrow notation to show the translation.**<br>**14. left 3 units, up 3 units**<br><br>(Prentice Hall, Course 2, p512) |
| Reflection | |
| • all other reflection problems | 1. Give a real-world example of a reflection.<br>(UCSMP, Pre-Transitions, p. 646) |
| • reflection up and/or left direction | 21. Writing in Math<br>Suppose you translate a point to the left 1 unit and up 3 units.<br>Describe what you would do to the coordinates of the original point |

| *Type of Transformation* | *Illustration of Exercise* |
|---|---|
| | to find the coordinates of the image.<br>(Prentice Hall, Course 3, p. 139) |
| • reflection over an oblique line | **Figure *EFGH* has vertices *E*(2, 5), *F*(4, 5), *G*(6, 1), and *H*(3, 1). Graph figure *EFGH* and its image after a reflection over each line. Name the coordinates of the vertices of the reflected figure.**<br>**25.** line through (0, 2) and (−3, 2)<br>(Prentice Hall, Course 3, p. 144) |
| • reflection over/onto the pre-image | 5. The vertices of FGH are F (-3, 4), G (0 , 5), and H (3 , 2). Graph the triangle and it image after a reflection over the y-axis. (Glencoe, Pre-Algebra, p. 104) |
| Rotation | |
| • all other rotation problems | Mental Math<br>A triangle lies entirely in Quadrant I. In Which quadrant will the triangle lie after each rotation about (0 , 0)?<br>18. 90° 19. 180° 20. 270° 21. 360°<br>(Prentice Hall, Course 2, p. 522) |
| • find the angle of rotation | **For Exercises 12–14, use the flag shape at the right as a basic design element. Complete a design with rotation symmetry and give the angle of rotation.**<br><br><br>(Connected Mathematics 2, Grade 8, Unit 5, p. 16) |

| *Type of Transformation* | *Illustration of Exercise* |
|---|---|
| • rotation about a point other than a vertex or the origin | **12.** Give the magnitude and draw the center for the rotation that maps figure $A$ onto figure $A'$. *Hint:* Use your circle finder.   (UCSMP, Transitions Mathematics, p. 378) |
| Dilation | |
| • enlarge figure | 29. Select a drawing of a comic strip character from a newspaper or magazine. Draw a grid over the figure or tape a transparent grid on top of the figure. Identify key points on the figure and then enlarge the figure by using each of these rules. Which figures are similar? Explain.<br>a. (2x, 2y)   b. (x, 2y)   c. (2x, y)<br>(Connected Mathematics 2, Grade 7, Unit2, p. 24) |
| • shrink figure | Graph the coordinates of quadrilateral EFGH. Find the coordinates of its image after a dilation with the given scale factor. Graph the image.<br>13. E (-3, 0), F(1, -4), G(5, 0), H(1,4); scale factor of ½<br>(Prentice Hall, Course 3, p. 190) |
| • scale factor | 14. Eyes<br>During an eye exam, an optometrist dilates her patient’s pupils to 7 millimeters. If the diameter of the pupil before dilation was 4 millimeters, what is the scale factor of the dilation?<br>(Glencoe, Pre-Algebra, p. 310) |