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Modeling alternate strategies for airline revenue management

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Modeling Alternate Strategies for Airline Revenue Management

by

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MODELING ALTERNATE STRATEGIES FOR AIRLINE REVENUE MANAGEMENT

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ABSTRACT

Ever since the deregulation of the airline industry in 1978, fierce competition has made every airline to try and gain a competitive edge in the market. In order to accomplish this, airlines are turning to advanced optimization techniques such as revenue management. Revenue management is a way for airlines to maximize capacity and profitability by managing supply and demand through price management.

Over the last few years research in the field of revenue management has steadily progressed from seat inventory control techniques such as single leg seat inventory and network inventory control to ticket pricing techniques. Ticket pricing techniques involve setting ticket prices according to the time remaining to depart and inventory level conditions at that point in time. These models can be solved either by dynamic or mathematical programming. However, these models in addition to having increased complexity are based on several assumptions which may not be valid in real life situations thereby limiting there applicability

In this research, we have developed computer simulation models using Arena software as a tool to solve airline revenue management problem. Different models based on factors such as customer behavior, which would involve the probability of a customer accepting a ticket and relevant pricing methods such as seats remaining and time

remaining have been developed with the objective to reach an optimal revenue management policy.

Initially, the strategies have been developed and tested for a single flight leg for different types of destinations such as tourist, business and mixed tourist and business. It was found that models where pricing was based on seats remaining generated the most revenue for the tourist destinations, time remaining for the business destinations and pricing based on time and seats remaining for the mixed type. Two different strategies, one where the ticket price for the indirect (stop-over) flight increases as more seats for direct flight are sold and the second where the ticket price for the indirect flight decreases have been developed for a network of three cities with direct and stop-over flights. It was found that the first strategy works well for the business destination. There was no significant difference between the two strategies for the other two destinations. Also the model was run where a set percentage of seats on the direct flight are sold prior to the opening of indirect flight bookings (blocking). It was found that blocking of seats did not increase the total revenue generated.

CHAPTER 1

INTRODUCTION

The airline industry has become extremely competitive in recent years. The number of airlines operating within the United States has increased tremendously. Since the deregulation of the airline industry in 1978, airlines have been allowed to choose their own market segments, decide their own routes and set their own fares as long as they comply with the regulations laid down by the Federal Aviation Authority (FAA) [Yu, 1998]. This fierce competition has made most airlines turn to advanced optimization techniques to develop decision support systems for management and control of airline operations.

An important aspect considered in any airline industry is the maximization of revenue from the sale of seats in the aircraft. This is called revenue management. Originally known as yield management, revenue management has been successfully adapted to numerous industries in recent years including utilities, cruise lines, trucking, amusement parks, hotels, rental cars and others.

Revenue management is a business practice that enables companies to increase revenue by accurately matching product availability and pricing to the market demand. Basic principle of revenue management is to maximize the revenue by controlling inventory levels and pricing of perishable products.

Airline revenue management other than the maximization of revenue allows an airline a chance to operate a large variety of fares so as to enhance the attractiveness of that airline to the consumers.

1.1 Revenue Management in the Airline Industry

Over the years airline revenue management systems have progressed from simple leg control through segment control and finally to the origin-destination or network control. The problem of revenue management is divided into Seat or Discount Allocation and Ticket Pricing.

1.1.1 Seat or Discount Allocation

Also known as seat inventory control, it is the determination of optimal booking limits for the seats in each fare class such that total revenue is maximized. Two approaches namely single leg control and network control have been explored till now. In single leg control the flights legs are optimized separately or one at a time where as in network inventory control all the flight legs including connecting and direct flights between a pair of cities are optimized simultaneously. Hence, network revenue management is to manage the sales of ticket to local passengers as well as connecting passengers in order to maximize revenue for the entire airline network. Typically in case of all major airlines 25 - 50% of passengers will have at least one connection. Thus when connecting traffic is a significant portion of total traffic, leg based revenue management can result in allocation that are clearly sub-optimal.

Seats on an aircraft are categorized as Executive class with high fares and Economy class with low fares. However if you consider the economy section of the aircraft, although all seats are physically identical they are never priced identically. This gives rise to different fare classes.

So the question is how to and how many tickets to sell within the coach class to different customers. In the seat inventory control approach it is assumed that prices for different fare classes are given according to some predetermined criteria and only seat allocation needs to be determined so as to maximize the total revenue. A system called nested reservation system [Belobaba, 1989] for determining booking limits for the fare classes is the most common system used by airlines today.

A nested reservation system is one in which fare class inventories are structured such that a high fare request will not be refused as long as any seats remain available in lower fare classes. A nested reservation system is thus binding in its limits on lower fare classes but its limits are transparent from above (for higher fare classes). Booking limit for a fare class is maximum number of seats that can be sold for that fare class. For example, if a three-fare class nested reservation system is considered then the booking limit for the highest fare class will be the total capacity of the cabin and the next fare class will have the booking limit equal to the total cabin capacity less the seats protected for the higher fare class from the lower classes.

By having a nested reservation system the airline ensures that higher fare class demands are always accepted as long as there are seats available in the cabin. In a nested reservation system the difference between the binding limit of a higher fare class and binding limit of the immediate lower class is called the protection level for the higher fare class. These are the seats that are reserved or protected from sale in the lower classes.

It is desirable for the airline to sell as many tickets in the highest fare class as possible. But just increasing number of seats that are allocated for the highest fare class would not be beneficial because some of the seats in the highest fare class may remain vacant when the flight takes off thus generating no revenue. On the other hand had these seats been allocated to a lower fare class for which there may possibly be more demand than a higher fare class, more revenue would have been generated. Hence objective of the airline is to allocate seats for each fare class such that the mix of seats sold on the aircraft generates max revenue.

1.1.2 Overbooking

If an airline accepts reservations only for the number of seats available then there is always a risk of flight departing with vacant seats because of cancellations or ‘no shows.’ However if the airline sells seats more than its capacity, then there is a possibility that the airline may have to bump some ticket holding passengers. Such passengers are usually rebooked on a later flight and given some compensation. However there is a loss of good will and a bumping cost is incurred. Usually a fixed percentage is used as an overbooking factor.

1.1.3 Ticket Pricing

Differential pricing is the determination of prices for each class of tickets such that the total revenue is maximized. The profit maximization price of a ticket depends on market reactions and marginal cost, i.e., both the market and the company's internal structures are determinants of a ticket price. There are two key elements to a price: the market side or the demand and supplier side or supply [Yeoman and Ingold, 1997].

Market side is the relative perceived value of a product and the consumer's willingness and the ability to buy the product. Sales volume represents the amount consumed at various price levels and when combined with the value (price) indicates the turnover generated. This relationship reflects the principles of the demand curve D1 shown in Figure 1 [Yeoman and Ingold, 1997]. Here P1 and P2 on the Y-axis represent the two price levels, P2 being a greater price than P1 and Q1, Q2 and Q3 on the X-axis represent the sales volume wherein Q3 is the most number of seats sold, Q2 least and Q1 in between them.

The total turnover is calculated by multiplying Q1 and P1 or Q2 and P2. The revenue can be increased in two ways, either lower prices and raise volumes or raise prices and accept lower volumes. These are called movements along the demand curve. As demand is an independent variable, these movements can only result in an increase or decrease in price. This figure basically represents the price elasticity and explains the relationship between a change in price and change in quantity demanded.

The main thing to be considered here is that the price-volume relationship can vary considerably between and even within markets, making the pricing decision difficult, yet critical. In addition to such movements along the demand curve, the curve can also shift to the right or left. When the demand curve shifts to the right (D2), it represents an increase in demand, whereas a shift to the left (D1) represents reduced demand. The cases of such shifts arise due to changing business environments such as good marketing, offering promotional fares, lower rates offered by competing airlines etc.

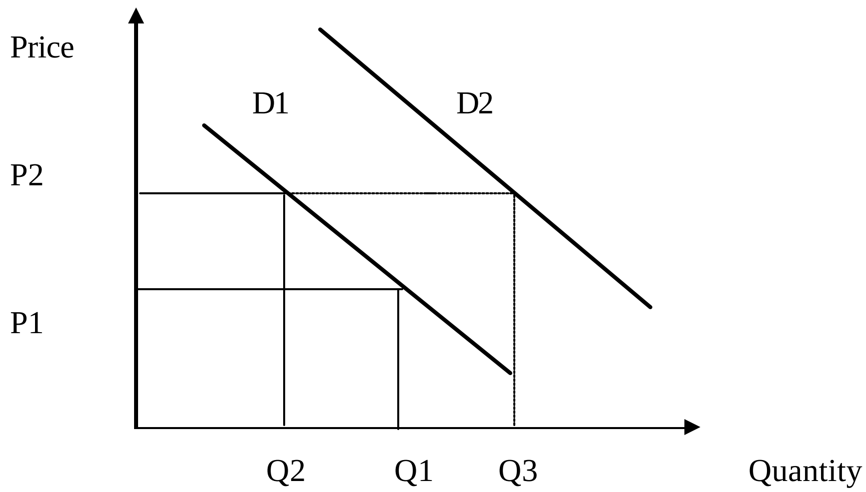


Figure 1 The Demand Curve

Hence, a shift in the demand curve to the right can result in a greater revenue generation without a reduction in price (D2) or a potential to raise price and maintain volume, perhaps raising profitability.

1.2 Characteristics of Revenue Management

The characteristics of revenue management are

1. *Relatively fixed capacity.* Only a fixed amount of capacity is available and cannot be easily added or reduced, e.g. an aircraft has fixed number of seats due to cabin restrictions and a hotel has fixed number of rooms when it is built.
2. *Perishable inventory*. This means there is a deadline up to which the inventory can be sold. After that the inventory is worthless just like food items and cannot be reused. The seats on an aircraft after it takes off cannot be sold and will not generate any revenue.
3. *Fluctuating demand*. In most service industries demand is seasonal. Revenue management can be used to generate more demand than usual during off-peak periods and can help to increase revenue during peak demand period.
4. *Product differentiation*. This important characteristic is the main reason for a price differential. In the coach class of an aircraft even though the seats are physically the same, they cost different as the two individuals occupying the seats have purchased them at a different point in time.

1.3 Revenue Management in Other Industries

Since American Airlines pioneered revenue management, many industries have tried to adopt it. Not far behind the airline industry are the hotel industry and the rental car industry. Cruise lines and tour operators are looking at revenue management too. The movie industry and on the same lines the sporting industry would hugely benefit from revenue management.

1.4 Thesis Organization

The organization of the rest of the thesis is as follows. Chapter 2 reviews the prior work done in the area of airline revenue management. Chapter 3 states the problem of airline revenue management and also discusses the major assumptions that have been made with their justifications. Chapter 4 discusses the modeling approach and the two main factors namely pricing structure and customer behavior that affect the model. Chapter 5 presents the results for a single flight leg model and an analysis of variance is conducted to verify significant factors. Chapter 6 presents the strategies used in the modeling of a network of three cities with direct and stop-over flights between them and also present their results. Finally, Chapter 7 gives the summary and conclusions and also states the further research that can be done in this area.

CHAPTER 2
LITERATURE REVIEW

This chapter presents an overview of the research done by various authors in the area of revenue management. Most of the material presented in this chapter is adapted from two excellent reviews of airline revenue management by McGill and VanRyzin [1999] and Pak and Piersma [2002]. As stated before the problem has been more or less divided into the seat inventory control problem and the ticket-pricing problem.

2.1 Seat Inventory Control

The seat inventory control problem involves allocation of finite seat inventory to the demand that occurs over time before the flight is scheduled to depart. Here the objective is to find the right mix of passengers to maximize the revenue. The problem is approached either as single leg seat inventory control or as network inventory control.

2.1.1 Single Leg Seat Inventory Control

Here the flight legs are optimized separately. Consider a passenger traveling from A to C through B and offering to pay $800 for his entire journey. That is, traveling from A to C using flight legs from A to B and from B to C. It is assumed that the airline is charging this passenger $500 for the first flight leg from A to B and $300 for the second flight leg from B to C. Now consider a second passenger traveling from A to B and offering to pay $600 for his journey. If the single leg approach is used, the first passenger can be rejected on the flight leg from A to B because the second passenger is willing to pay a higher fare on this flight leg and the airline stands to increase its revenue by $100. But by rejecting the first passengers offer, the airline looses an opportunity to create revenue for the combination of the two flight legs. But if the second flight leg from B to

C did not get filled up, then it could have been more profitable to accept the first passenger to create revenue for both flight legs. This is the main drawback of the single leg inventory control. Bandla [1998] proposes a solution for such an approach using reinforcement learning. There are two categories of single leg solution methods: static and dynamic solution methods.

2.1.1.1 Static Solution Methods

In a static model a booking period is regarded as a single interval and a booking limit for every booking class is set at the beginning of every booking period. A drawback of the static solution method is that it considers all the bookings done up to and at a particular point in time and as we know the booking process is a continuous one. Hence this is not exactly an optimal approach although it is a popular one as it can handle large problems and also multiple leg problems.

Littlewood [1972] was the first to propose a solution method for the airline revenue management problem for a single flight leg with two fare classes. His idea was to equate the marginal revenue in each of the two fare classes. He suggests closing down the low fare class when the revenue from selling another low fare seat exceeds the expected revenue of selling the same seat at a higher fare. Belobaba [1987] extends Littlewood's rule to multiple fare classes and introduces the term expected marginal seat revenue (EMSR). His method is called EMSRa and incorporates nested protection level, i.e., the number of seats to be sold to each fare class. However his method does not yield optimal booking limits when more than two fare classes are considered.

2.1.1.2 Dynamic Solution Methods

Dynamic solution methods for the seat inventory control problem do not determine a booking control policy at the start of the booking period as the static solution methods do. A dynamic model sets the booking limit for each booking class according to the actual bookings throughout the entire booking process. However a limitation of this approach is that the model developed is computationally intensive.

Lee and Hersh [1993] consider a discrete time dynamic programming model. A non-homogenous Poisson process models demand for each fare class. Use of Poisson process gives rise to Markov Decision Process model where in booking requests at time *t* are independent of the decisions made before time *t*, except available capacity. The entire booking period is divided in to a number of decision periods and each request constitutes a period. The decision rule says that a booking request is accepted only if its fare exceeds the expected cost of seats at time *t*. Multiple seat bookings, which are a practical issue in airline seat inventory control are also considered. Subramanian et al [1999] also formulate and analyze a Markov Decision Process model for airline seat allocation on a single leg flight with multiple fare classes. They have incorporated cancellations, no shows and overbooking.

Lautenbacher and Stidham [1999] link the dynamic and static approaches of the single leg seat inventory control model. They demonstrate that a common Markov Decision process underlies both the approaches and formulate an omnibus model that yields the static and dynamic models as special cases.

2.1.2 Network Inventory Control

Network seat inventory control is aimed at optimizing the complete network of flight legs offered by the airline simultaneously. As explained in the example in Section 2.1.1 Single Leg Seat Inventory Control, consider that the second flight leg from B to C do not get filled up. The first passenger flying from A to C, was obviously paying more than the second passenger traveling from A to B for the entire journey, but was still rejected. Hence the airline would be flying with an empty seat on flight leg B to C and thereby losing potential revenue on flight leg B to C. In this process the airline increased its revenue by $100. However if the first passenger was accepted, then there would be no vacant seats on any of the flight legs and the airline would have increased its revenue by $200 instead. Thus accepting the first passenger would maximize total revenue of both the flight legs. This is network revenue management. Network inventory control takes in to account the overall revenue the passenger creates from its origin to its destination.

Singh [2002] proposes a stochastic approximation approach to such an airline network revenue management problem and solves it using reinforcement learning algorithm.

2.2 Ticket Pricing Models

It is now common for airline practitioners to view pricing as part of the revenue management process. The reason for this is pretty clear - the existence of differential pricing for airline seats is the starting point of airline revenue management and price is generally the most important determinant of passenger demand behavior. There is also a natural duality between price and seat allocation decisions. If price is viewed as a variable that can be controlled on a continuous basis, raising the price sufficiently high can shut down a booking class. Also when there are many booking classes available, shutting down a booking class can be viewed as changing the price structure faced by the customer.

2.2.1 Dynamic Pricing Models

Treatments of revenue management as a dynamic pricing model can be found in the work done by Carvalho and Puterman [2003]. They considered a problem of setting prices dynamically to maximize expected revenues in a finite horizon model in which the demand distribution parameters are unknown. The authors suggests a promising pricing policy called the "one step look ahead rule" where in a Taylor series expansion of the future reward function illustrates the tradeoff between short term revenue management and future information gains.

Chatwin [1999] proposes an optimal dynamic pricing model of perishable products with stochastic demand. A finite set of allowable prices is assumed. A continuous time dynamic programming model is employed in which at any given time the state of the model is the number of items in the inventory and the retailer's decision is to choose the price to sell at. Demand is assumed to be Poisson with decreasing rate. This model verifies the intuition that optimal price is non-increasing in the remaining inventory and non-decreasing in the time to go. Gallego and Van Ryzin [1994] suggest a dynamic pricing policy of inventories with stochastic demand. Their formulation uses intensity control and obtains structural monotonic results for the optimal price as a function of the stock level and the amount of time left. However they allow only a finite number of prices.

Feng and Gallego [1995] investigate the problem of deciding the optimal timing of a single price change from a given initial price to either a given lower or higher second price. They show that the optimal policy is to decrease the initial price as soon as the time to go falls below a time threshold which depends on the number of yet unsold items. While the model is realistic for retailers of seasonal goods and for certain nonstop flights, it does not extend to multiflight, multileg situation where customers from different itineries compete for the capacity of the flight legs. Feng and Xiao [2000a] generalize the results from the above policy by incorporating risk analysis and multiple price changes. Also Feng and Gallego [2000] also extend their original work to address the problem of deciding the optimal timing of price change within a given menu of allowable, possibly time dependent price paths each of which is associated with a general Poisson process with Markovian, time dependent predictable intensities.

Feng and Xiao [2000b] present a continuous time yield management model with multiple prices and reversible changes in price. Demand at each price is Poisson with constant intensities. The problem is formulated as an intensity control model and optimal solution in closed form is derived. The model further improves the one proposed in Feng and Gallego [1995], as an exact solution rather than a deterministic one is obtained. Gallego and VanRyzin [1997] also propose a multi product dynamic pricing problem with its application to network yield management. They start with a demand for each product, which is a stochastic point process with an intensity that is a vector of the prices of the products and the time at which they are offered. An upper bound for the optimal expected revenue is established by analyzing the deterministic version of the problem.

From the review of the literature done in this chapter it can be summarized that most of the research that has been done is in the area of seat inventory control which again could be categorized in to single leg and network seat inventory control. Whatever little has been done in the field of ticket pricing has been using mathematical or dynamic programming models that are computationally intensive and time consuming. Also most of these models are fairly complicated and they make simplifying assumptions such as pre-determined prices, no batch/multiple seat bookings, stochastic demand, fixed number of seats assigned to each fare class, lower fare class requests arrive before higher fare class requests etc**.** Hence the validity of these models is under question and their exact solutions may not be worked out. In the next chapter we state our research objectives along with the parameters and the assumptions made.

CHAPTER 3
RESEARCH STATEMENT

In this chapter the problem of network revenue management is stated. The main objectives of this research are also discussed. The major assumptions that have been made are explained with their justifications.

3.1 Problem Statement

The problem considered in this research constitutes a network of three cities with multiple origin-destination combinations. There are direct and stop-over flights in between them. The revenue generated from the sale of tickets on all flights in their coach class in the network is to be maximized. Passengers request reservations in the coach class of the flight depending upon their preferred itineraries. Every time a passenger requests a reservation, the airline checks for the availability of seats for that itinerary. If seats are available, the airline provides the passenger with a fare and the passenger decides whether to accept or reject the fare.

Customer arrivals are assumed to follow a non-stationary Poisson process (an arrival process, which has a rate that varies over time). The objective here is to maximize the revenue generated from sale of seats over the entire network. Also policies to be followed for the sale of tickets on the direct and the stop-over flight for the same final destination have to be developed.

3.2 Research Assumptions

The following parameters have been considered in this research

- *Network Details*

A network of three cities has been considered in this research. Only one-way travel between the cities is considered for modeling purposes and no round trip fare option is offered. However the return part of the journey could be modeled as a percentage of the cost of the first part of the journey and it would be also dependent on the date of the return trip. A request for a booking is always associated with a particular origin-destination. For example consider a network of three cities namely Tampa, Atlanta and New York. A flight from Tampa to New York via Atlanta (stop-over flight) would be a different origin destination combination from a flight flying directly between Tampa and New York (direct flight).

- *Fair Structure*

The range of fares for different origin destination combinations is different and is pre-determined. Considering the above example the range of fares for a flight from Tampa to New York via Atlanta would be different from the fares for a direct flight between the two cities. Hence the fare structure would vary depending on whether it is a direct flight or with a layover and also factors such as the distance between the cities etc.

- *Arrival Process*

Many systems are subject to experience arrival loads that can vary dramatically over the time frame of the simulation. There is a specific probabilistic model for this called the non-stationary Poisson process, which provides an accurate way to reflect time-varying arrival patterns. Hence the passengers in our model are assumed to arrive with a nonstationary Poisson process and each origin destination combination has its own arrival process.

3.3 Factors and Strategies Considered

Three main factors and three individual strategies will be considered in this model. The factors are pricing strategy, acceptance probability and customer arrival rate.

3.3.1 Pricing Strategy

The pricing strategy basically means the different ways the customer is charged a price. This could be categorized on the basis of the time left to depart (*time remaining strategy)*, or up on the number of seats left to be sold, i.e. *seats remaining strategy* or could be a combination of both called the *hybrid strategy*.

3.3.2 Acceptance Probability

Acceptance Probability reflects up on the probability that a customer will purchase a ticket. It could be classified according to the price being offered to the customer or *probability w.r.t. price strategy* or according to time left to depart called *probability w.r.t. time strategy* or could be a combination of both called *composite probability.*

3.3.3 Customer Arrival Rate

Three different customer arrival rates of low, medium and high each suggesting a market with a low demand, medium and high demand for tickets has been experimented with.

3.4 Research Objectives

Our objective is to develop an optimal ticket pricing policy for the airline industry. Different pricing strategies such as seats remaining, time remaining, and hybrid strategy as well as acceptance strategies such as probability of a customer buying a seat with respect to time, price or a combination of both are developed and tested using simulation models. Initially the pricing policy is developed for a single flight leg and then for a network of cities to explore the alternatives for direct and indirect flights that airlines can offer to maximize their revenue. The comparison of results from these strategies can help in determining the optimal ticket pricing policy for the airline industry.

The factors considered and the strategies used is presented in the next chapter and explained in detail with the aid of an example.

CHAPTER 4
MODELING AND SOLUTION METHODOLOGY

This chapter discusses in detail the main factors that are used in the development of a single leg model. The three main factors are the pricing strategy, acceptance probability and customer arrival rate.

4.1 Pricing Strategy

The pricing strategy that we have used in this research is based on the prices being offered online by some popular airlines. Generally it was observed that the price of a fare for a 30-day period varied from 2 times to a maximum of 3 times the cheapest fare. However what is more interesting is as to how these prices are offered to the customers and at what point of time in to the booking period. The pricing strategy can be explained using three different approaches.

4.1.1 Time Remaining Approach

At the start of the booking period, when the entire capacity of the aircraft is available and in order to attract more customers to sell as many seats as possible, airlines offer the cheapest fares. The fares go on increasing as the time to depart nears and the number of seats available becomes less. In such a scenario last minute customers will end up paying the most expensive fares the airline has to offer. The relationship between time remaining and price offered is a linear one and is shown in Figure 2. The equation that describes this relationship is as follows.

$$\textit{Price Offered} = P_{max} - (\textit{Time Remaining}) * j \quad (1)$$

where,

Price Offered is the price at which the ticket is sold to the customer.

P_{max} is the maximum ticket price set by the airline.

P_{min} is the minimum ticket price set by the airline.

Time Remaining is the time left for the flight to depart.

j is a normalizing constant such that *Price Offered* will be P_{min} , when *Time Remaining* is 30 days.

This equation satisfies our initial condition that price offered is least at the start of the booking period and is the highest when the flight is about to depart.

4.1.2 Seats Remaining Approach

When the entire seat inventory is available at hand and in order to kick-start the booking process, the cheapest fare is offered. The price goes on increasing as the number of seats reduces and the last seat is offered at the highest price. The relationship which is linear is shown in Figure 3. The equation used is as follows

$$\textit{Price Offered} = P_{max} - (\textit{Seats Remaining}) * k \qquad (2)$$

where in,

Seats remaining are the number of seats still available for sale at that time.

k is a normalizing constant such that Price Offered will be P_{max} , when *Seats Remaining* will be zero.

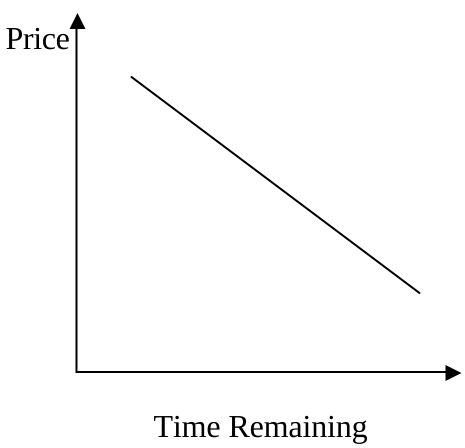


Figure 2. Graph of Price Offered and Remaining Time

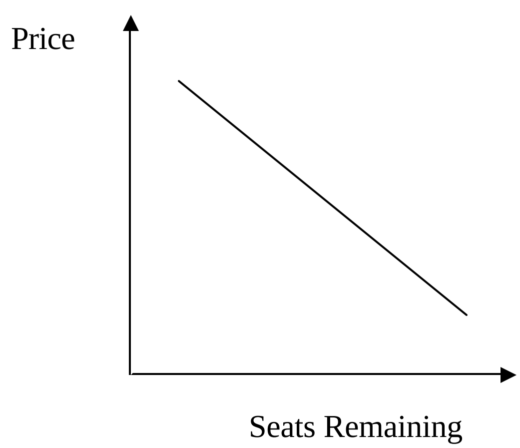


Figure 3. Graph of Price Offered and Remaining Seats

This equation also satisfies the conditions that price offered is lowest when most of the seats are remaining and vice versa.

4.1.3 Hybrid Approach

Both the approaches are logical in their own ways. However consider the case when after a few days in to the booking period, very few or almost no seats are sold. The first approach would entail the airline to charge a higher price as the time to depart nears where as the second approach would require the airline to charge a lower price since very few seats were sold. Hence the need for a hybrid model was felt. The hybrid approach would charge fares depending on the number of seats sold as well as the number of days into the booking period. Here we use the equation

$$\textit{Price Offered} = P_{max} - (\textit{Time Remaining}) * j - (\textit{Seats Remaining}) * k \qquad (3)$$

where,

j and *k* are normalizing constants such that *Price Offered* is $P_{max,,}$ when *Time Remaining* is equal to the booking period and *Seats Remaining* is close to zero.

4.2 Acceptance Probability

The probability of a customer accepting or rejecting a seat is called acceptance probability and this probability can be classified into three different types depending up on the price and the time at which the customer accepts it and their combination.

4.2.1 Probability with Respect to Price Offered

If a lower price is offered by the airline, the tickets are likely to be sold easily. Hence a very high acceptance probability is assumed when the cheapest fare is offered. However, as the fare increases, the probability of acceptance decreases leading to the lowest probability when the fare offered is the highest. The equation used is

$$\textit{Probability of Acceptance} = 100 - (\textit{Price Offered} - \textit{Cheapest Price}) * l \qquad (4)$$

where,

l is the normalizing constant such that *Probability of Acceptance* is 100% when *Price Offered* is the *Cheapest Price.*

This equation will return a 100% probability of acceptance when price offered is the lowest and vice versa. Figures 4 and 5 show the graphs of the probability of acceptance with respect to the price offered and time remaining respectively.

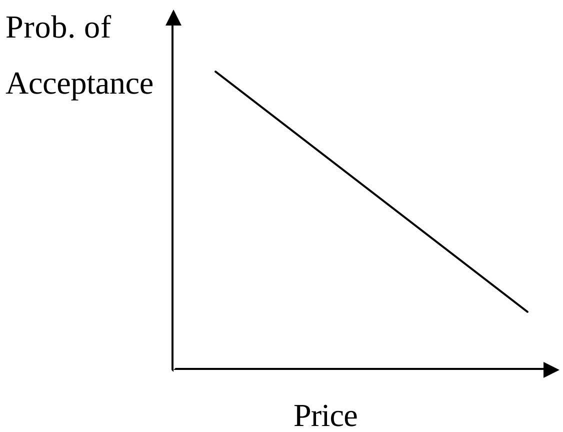


Figure 4 Graph of Probability of Acceptance and Price Offered

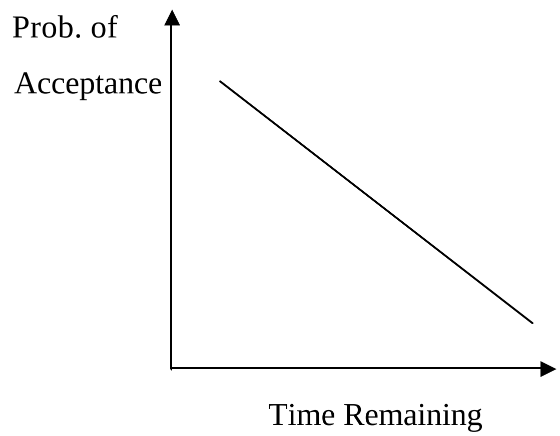


Figure 5 Graph of Probability of Acceptance and Time Remaining

4.2.2 Probability with Respect to Time Remaining

When the bookings open say 30 days before departure, there is very little rush to buy and hence the probability of acceptance is also very low. However as the date of departure approaches, more customers especially business travelers tend to buy tickets at whatever price. Hence the probability of acceptance is greater towards the end. The equation we have used here is,

$$Probability\ of\ Acceptance = 100 - (Time\ Remaining)*m \quad (5)$$

This equation follows the initial as well as the final conditions of 100% and 50% probability of acceptance. The graph of time remaining and probability is linear and is shown in Fig. 5.

4.2.3 Composite Probability

Both the probability approaches mentioned above are correct in their individual capacity. However consider the case when the bookings are just opened and at the same time the lowest fares are offered by the airline. The probability with respect to price would suggest a higher probability of acceptance as the ticket price is the lowest; where as the probability with respect to time would suggest a lower probability of acceptance as it is too early in the booking period and the customer is in no particular hurry to book. Hence the need for a composite probability approach that models the customer behavior on the basis of price offered and time remaining. Here we have used the equation

$$\textit{Probability of Acceptance} = 100 - (\textit{Price Offered} - \textit{Cheapest Price})*l - (\textit{Time Remaining})*m \qquad (6)$$

This equation satisfies the initial conditions of higher probability when price offered is the cheapest and Time Remaining is 30 days and vice versa.

l and *m* are the respective normalizing constants.

4.3 Customer Arrival Rate

The booking process is assumed to start 30 days in advance and this 30 day period is divided in to 6 time slots of 5 days each. Three different customer arrival rates of low, medium and high have been used and the customers arrive according to non-stationary Poisson process.

4.4 Simulation as a Tool

There have been several models based on mathematical programming techniques to tackle the airline revenue management problem. However, these models are based on many simplifying assumptions such as pre-determined prices, no batch/multiple seat bookings, fixed number of seats assigned to each fare class, lower fare class requests arrive before higher fare class requests, etc. which are not realistic. In spite of these assumptions the models are quite complex to build, understand and solve. The strategies and policies developed earlier in this chapter are tested using computer simulation models in this research. The reason for using simulation is that it allows the models to represent the real world system faithfully. However, the results are based on statistical foundations. Therefore, while using simulation models one needs to verify and validate them. Also, the statistical issues should be resolved properly in order for the results to be valid and meaningful.

4.5 Model Development

Using equations (1) through (6), we consider the development of a single leg airline revenue management model. Based on the outcome of this model, we develop a network revenue management model of three cities. Booking period in all the models starts 30 days in advance. Customers arrive according to Poisson process, which has a rate that varies with respect to time (non-stationary Poisson process). The number of seats on the aircraft is fixed.

A range of fares with an upper limit equal to the maximum price offered and lower limit equal to the minimum price offered is set by the airline. Each arriving customer is offered a ticket price by the airline booking system according to the pricing structure i.e., according to the time remaining, seats remaining and hybrid approach. It is up to the customer to decide whether to accept or reject the offered ticket price. This is called acceptance probability. Also this decision is a two way by chance probability. If the customer rejects the offer, he exits the booking system.

If the customer accepts the offer price, a seat is reserved for him/her and the total number of seats available for sale is reduced by one. The ticket price offered and the total revenue generated at this stage is recorded. Our objective in this model is to maximize revenue within the above-mentioned boundaries and conditions.

Here we are considering nine different models depending on the pricing structure (seats remaining, time remaining and hybrid approach) and customer behavior (probability with regards to price and time as well as hybrid probability) and their combinations. Also depending on the probability of accepting a ticket, an approximation of the type of destination served by the flight such as a tourist destination, business destination or a mix of both can be obtained. Hence the model combinations could be as shown in Table1.

In this chapter we have presented the strategies for ticket pricing and probability of accepting the price offered for different types of customers. The corresponding equations were formulated as well. The numerical analyses of these strategies and their results are presented in the next chapter.

Table 1 Model Combinations

Model Type	Pricing Strategy based on	Type of Destination
Seats Rem. & Price Prob.	Seats Remaining	Mainly Tourist Destination (e.g. Las Vegas)
Time Rem & Price Prob.	Time Remaining	
Hybrid Price & Price Prob.	Time and Seats Remaining	
Seats Rem & Time Prob	Seats Remaining	Mainly Business Destination (e.g. Detroit)
Time Rem & Time Prob	Time Remaining	
Hybrid Price & Time Prob.	Time and Seats Remaining	
Seats Rem. &Hybrid Prob.	Seats Remaining	Could be a mix of both (e.g. New York)
Time Rem & Hybrid Prob.	Time Remaining	
Hybrid Price & Hybrid Prob	Time and Seats Remaining	

CHAPTER 5
EXPERIMENT DESIGN AND ANALYSIS OF RESULTS

In this chapter the policies and strategies as discussed in the previous section are tested using simulation models and their results are presented. Also an analysis of variance is performed.

5.1 Single Leg Models

In this section we have considered nine different models with their assumptions and necessary details and have compared their results. For this flight leg, a flight capacity of 200 passengers is assumed. The minimum price offered by the airline is $125 per ticket whereas the maximum is $400. The booking of this flight leg starts 30 days in advance. A fare offered is for the first part of the round trip and only the forward part of the journey is modeled. The return part can be modeled in a similar fashion. The time duration of 30 days is divided in to 6 time slots of 5 days each. The customer arrivals are assumed to follow Poisson distribution with arrival rates that vary with respect to time. Three different arrival rates have been used. The terminating condition for this model could be either when all the seats are sold out or when the end of the booking period is reached.

5.1.1 Normalizing Constants

In this section we describe how we have calculated the values of the normalizing constants *j, k, l and m*.

5.1.1.1 Time Remaining Approach

$Price\ Offered = P_{max} - (Time\ Remaining) * j$

Initially when we open the bookings, the time remaining is 30 days and price offered is the cheapest price. Hence,

$125 = 400 – (30)*j$

which gives us a value of $j = 9.167$

5.1.1.2 Seats Remaining Approach

Price Offered = P_{max} *– (Seats Remaining) * k*

Initially when the booking is opened the entire seat inventory is available and hence, the cheapest fare is offered. Therefore,

$125 = 400 – (200)*k$

which gives us $k = 1.375$

5.1.1.3 Hybrid Approach

Price Offered = P_{max} *- (Time Remaining) * j – (Seats Remaining) * k*

The total price differential between the minimum and the maximum price offered is (400-125) = 275, the time remaining is 30 days and seats are 200.

Case 1. Balanced Weights. If we decide to assign equal weight to both the time remaining and seats remaining then we have

$275*0.5 = 30j$ which gives a value of $j = 4.58$. Also

$275*0.5 = 200k$ which gives a value of $k = 0.6875$.

Case 2. Weighted towards Seats Remaining. Suppose we decide to assign 20% weight to the time remaining and the remaining 80% weight to the seats remaining, we have

$275*0.2 = 30j$ which gives $j = 1.8333$ and

$275*0.8 = 200k$ which gives $k = 1.1$

Case 3. Weighted towards Time Remaining. Instead, if 80 % weight is attached to the time remaining and 20 % to seats remaining, then

$275*0.8 = 30j$ or, $j = 7.3333$

$275*0.2 = 200k$ or, $k = 0.275$

5.1.1.4 Probability of Acceptance with Respect to Price Offered

*Probability of Acceptance = 100 – (Price Offered –Cheapest Price)*l*

The probability of acceptance is set to range between a high level of 100% and a low level of 50%. When price offered is the highest probability of acceptance is the lowest. This gives us,

$50 = 100 - (400\text{-}125)*l$ or, $l = 0.1818$.

5.1.1.5 Probability of Acceptance with Respect to Time Remaining

*Probability of Acceptance = 100 – (Time Remaining)*m*

If the time remaining is 30 days then the probability of accepting a ticket is on the lower side considering all the time the customer has to choose a flight. Therefore we have,

$50 = 100 - (30)*m$ or, $m = 1.6667$

5.1.1.6 Composite Probability

*Probability of Acceptance = 100 – (Price Offered – Cheapest Price)*l – (Time Remaining)*m*

The probability of acceptance is set at two levels 100 % and 50 % and their differential is 50. The price offered differential is 275 and time remaining is 30 days. Thus,

Case1. Balanced Weights. The probability of acceptance depending up on the price offered and time remaining is given equal weight. This is an example of a mixed type of market where the demand by tourists as well as business travelers is equal (e.g. New York).

$50*0.5 = 275l$ or, $l = 0.0909$

$50*0.5 = 30m$ or, $m = 0.83$

Case2. Weighted Towards Price Offered. Here the price offered is given 80 % weight and time remaining is given 20 % weight. This is an example of a tourist driven market where majority of the passengers are price conscious tourists (e.g. Las Vegas).

$50*0.8 = 275l$ which gives $l = 0.1454$

$50*0.2 = 30m$ or, $m = 0.3333$

Table 2 Table of Normalizing Constants and Their Values

Equation	Criterion	Normalizing Constant
Price Offered = P_{max} – *(Time Remaining) * j*	Time Remaining	*j* = 9.167
Price Offered = P_{max} – *(Seats Remaining) * k*	Seats Remaining	*k*= 1.375
Price Offered = P_{max} - *(Time Remaining) * j – (Seats Remaining) * k*	Time and Seats Remaining	*j* = 4.58 & *k* = 0.6875 (Equal Weights) *j* = 1.8333 & *k* = 1.1 (Weighted Towards Seats) *j* = 7.3333 & *k* = 0.275 (Weighted Towards Time)
*Probability of Acceptance = 100 – (Price Offered –Cheapest Price)*l*	Price Offered	*l* = 0.1818
*Probability of Acceptance = 100 – (Time Remaining)*m*	Time Remaining	*m* = 1.6667
*Probability of Acceptance = 100 – (Price Offered - Cheapest Price)*l – (Time Remaining)*m*	Price Offered and Time Remaining	*l* = 0.0909 & *m* = 0.83 (Equal Weights) *l* = 0.1454 & *m* = 0.3333 (Weighted Towards Price) *l* = 0.0363 & *m* = 1.3333 (Weighted Towards Time)

Case3. Weighted Towards Time Remaining. The time to depart is given more weight (80%) and price offered is given 20 % weight. This example could represent a market where majority of customers are business travelers (e.g. Detroit).

$50*0.2 = 275l$ or, $l = 0.0363$

$50*0.8 = 30m$ or, $m = 1.3333$

The different policies we discussed so far and their corresponding normalizing constants have been stated in Table 2.

5.1.2 Arrival Rate

The customer arrivals follow a non-stationary Poisson arrival process with three different arrival rates of low, medium and high. The booking period of 30 days is divided in to 6 time slots of 5 days each and each time slot having a different arrival rate. A low arrival rate can be 0.16, 0.25, 0.33, 0.41, 0.25, 0.33 per hour which corresponds to 4, 6, 8, 10, 6, 8 customers per day. A medium arrival rate can be 0.20, 0.30, 0.35, 0.45, 0.38, 0.33 which corresponds to 5, 7, 8, 11, 9, 8 customers per day. A high arrival rate is 0.35, 0.40, 0.45, 0.48, 0.30, and 0.35 which is 8, 9, 10, 11, 7 and 8 arrivals per day. The arrival rates can be summarized from the following table.

Table 3 Arrival Rates

Type	Arrival Rate(Number of customers per day for every 5 days)						
	0-5 days	5-10 days	10-15 days	15-20 days	20-25 days	25-30 days	Total Customers
Low	4	6	8	10	6	8	210
Medium	5	7	8	11	9	8	240
High	8	9	10	11	7	8	265

The arrival rates are also calculated in terms of number of customers per hour as Arena software is unable to accept the arrival rate in terms of customers per day. If the number of customers per day is 6, then 6/24 = 0.25 would be the arrival rate per hour.

Table 4 Comparison of Single Leg Models for Low Arrival Rate

Model Type	Customer Arrivals	Tickets Purchased	Customers balked (high price)	Customers lost due to no seats	Seats Vacant	Average Revenue ($)	Half Width** ($)	Avg. Ticket Price($)	Time Seats get Full*	Type of Destination
Seats Rem & Price Prob	207	162	44	0	37	38,274	570	235	n/a	Mainly tourist destination e.g. Las Vegas
Time Rem & Price Prob	207	151	56	0	49	39,431	585	261	n/a	
Hybrid Price & Price Prob	207	157	49	0	43	38,915	575	247	n/a	
Seats Rem & Time Prob	207	160	46	0	40	37,685	754	234	n/a	Mainly business destination e.g. Detroit
Time Rem & Time Prob	207	160	46	0	40	46,086	668	287	n/a	
Hybrid Price & Time Prob	207	160	46	0	40	39,343	732	245	n/a	
Seats Rem & Hybrid Prob	207	162	45	0	38	38,203	723	235	n/a	Could be a mix of both e.g. New York
Time Rem & Hybrid Prob	207	153	54	0	47	40,802	598	267	n/a	
Hybrid Price & Hybrid	207	159	47	0	41	43,511	636	273	n/a	

*days into the booking period **based on 95% Confidence Interval

Table 5 Comparison of Single Leg Models for Medium Arrival Rate

Model Type	Customer Arrivals	Tickets Purchased	Customers balked (high price)	Customers lost due to no seats	Seats Vacant	Average Revenue ($)	Half Width** ($)	Avg. Ticket Price($)	Time Seats get Full*	Type of Destination
Seats Rem & Price Prob	241	182	59	0	18	45,396	633	249	n/a	Mainly tourist destination e.g. Las Vegas
Time Rem & Price Prob	241	177	64	0	23	46,116	624	261	n/a	
Hybrid Price & Price Prob	241	180	61	0	20	45,778	616	254	n/a	
Seats Rem & Time Prob	243	188	54	0	12	47,731	808	253	n/a	Mainly business destination e.g. Detroit
Time Rem & Time Prob	241	186	54	0	14	53,328	677	286	n/a	
Hybrid Price & Time Prob	241	186	54	0	14	48,301	786	259	n/a	
Seats Rem & Hybrid Prob	243	189	54	0	11	47,954	752	254	n/a	Could be a mix of both e.g. New York
Time Rem & Hybrid Prob	241	182	59	0	18	49,852	649	274	n/a	
Hybrid Price & Hybrid	241	185	55	0	15	51,104	669	275	n/a	

*days into the booking period **based on 95% Confidence Interval

Table 6 Comparison of Single Leg Models for High Arrival Rate

Model Type	Customer Arrivals	Tickets Purchased	Customers balked (high price)	Customers lost due to no seats	Seats Vacant	Average Revenue ($)	Half Width** ($)	Avg. Ticket Price($)	Time Seats get Full*	Type of Destination
Seats Rem & Price Prob	279	197	73	9	0	51,285	370	259	17	Mainly tourist destination e.g. Las Vegas
Time Rem & Price Prob	280	199	56	24	0	47,255	310	237	23	
Hybrid Price & Price Prob	280	198	66	16	0	49,603	275	250	20	
Seats Rem & Time Prob	279	198	69	12	0	51,569	354	260	22	Mainly business destination e.g. Detroit
Time Rem & Time Prob	279	198	69	12	0	52,986	303	267	21	
Hybrid Price & Time Prob	279	198	69	12	0	51,853	317	261	21	
Seats Rem & Hybrid Prob	280	198	70	0	2	51,532	341	260	n/a	Could be a mix of both e.g. New York
Time Rem & Hybrid Prob	280	198	64	0	2	49,998	313	252	n/a	
Hybrid Price & Hybrid	280	198	68	0	2	51,808	289	261	n/a	

*days into the booking period **based on 95% Confidence Interval

5.2 Results and Analysis

Using the above values of j, k, l and m, the nine models are run for a replication length of 30 days and 100 replications each for low, medium and high arrival rates. The number of replications have been calculated to obtain a half width of less than 2% of the revenue generated and has been explained in Appendix B. The results are shown in Tables 4, 5 and 6 respectively and are average for 100 replications. The first column indicates the model type as explained in detail in Table 1 and the last column gives the exact time in to the booking period when the seats get full.

Tourists are mostly price conscious people and hence tend to book their flights well in advance. Hence their acceptance probability of a ticket would be mostly based on price. From Table 4 we observe that for the tourist destination any of the three models could be used as the average revenue generated is pretty much the same. The half widths of all the three revenue generated values overlap and hence, there is no significant difference between the three values. Since average ticket price is the least for Seats Rem & Price Prob model, this could be the optimal strategy. Also out of the three models the Seats Rem & Price Prob model sells the most seats.

Business travelers generally tend to book late in the booking period and price is not really the deciding factor for them. Hence their acceptance probability of a ticket would be mostly based on time. From Table 4 we observe that for the business destination, the revenue generated by all the three models is significantly different as their half widths do not overlap. Time Rem and Time Prob generates the most revenue as price is charged according to time remaining and business customers tend to book late when the price is higher. Also average ticket price is the most for this model. Hybrid Price & Time Prob model generates the second most revenue and Seats Rem & Time Prob the least revenue. Numbers of seats remaining vacant are the same in all the models.

For a mixed type of destination which would have both tourist and business travelers, the probability of acceptance would be based on both time and price offered. The revenue generated by Seats Rem & Hybrid Prob and Time Rem & Hybrid Prob is not significantly different as their half widths overlap. But the revenue generated by Hybrid Price and Hybrid Prob is significantly different from the other two models. This model generates the most revenue as price offered is according to both time and seats remaining and both types of customers, tourists and business book on this flight. Also the average ticket price is not very high and hence, this could be the optimal policy. Time Rem & Hybrid Prob generates the second most revenue. Average ticket price is the slightly less for Time Rem & Hybrid Prob, but since less customers book this flight average revenue generated is less. The most number of customers purchasing tickets is in the Seats Rem & Hybrid Prob model, but as the average ticket price is least the revenue generated is also the least.

Similar conclusions can be drawn from Table 5 and 6 with the only exception that for the high arrival rate, the Seats Rem. and Price Prob. generates the most revenue for the tourist destination. This could be attributed to the high rate of arrival which fills up the seats faster when the ticket price is low. All these results have been summarized according to the arrival rate and destination type in Table 7.

Table 7 Best Pricing Strategy

Arrival Rate	Destination Type		
	Tourist	Business	Mixed
Low	Seats Rem & Price Prob	Time Rem & Time Prob	Hybrid Price & Hybrid Prob
Medium	Seats Rem & Price Prob	Time Rem & Time Prob	Hybrid Price & Hybrid Prob
High	Seats Rem. & Price Prob	Time Rem & Time Prob	Hybrid Price & Hybrid Prob

5.2.1 Sensitivity Analysis

To test the sensitivity of the results obtained from our model we have taken a second example and verified if the results and conclusions drawn are consistent with the original example. In this second example we have assumed a flight capacity of 300 passengers with the lower price limit being set at $200 and the higher limit at $425. The booking period was kept at 30 days and two arrival rates (low and medium) were adjusted corresponding to the increase in flight capacity. The adjusted low arrival rate gives 0.35, 0.45, 0.53, 0.50, 0.38 and 0.43 customers per hour or 8, 11, 13, 12, 9 and 10 customers per day. The adjusted medium arrival rate gives 0.38, 0.45, 0.50, 0.63, 0.55 and 0.50 customers per hour or 9, 11, 12, 15, 13 and 12 customers per day. All the nine models were run for 100 replications for the adjusted low and medium rate of arrivals. The revenues generated by the hybrid probability models were used for verifying the sensitivity of the results. The revenue generated by the original example and this second example are shown in Table 8 and 9.

Table 8 Revenues for Low and Adjusted Low Rate of Arrival

Model Type	Revenue Generated by Original Example ($)	Revenue Generated by Second Example ($)
Seats Remaining & Hybrid Probability	38,203 (13.8%)*	60,866(8.8%)*
Time Remaining & Hybrid Probability	40,802 (6.6)*	62,326(6.3%)*
Hybrid Price & Hybrid Probability	43,511	66,267

*proportion by which this revenue is less than the maximum revenue

Table 9 Revenues for Medium and Adjusted Medium Rate of Arrival

Model Type	Revenue Generated by Original Example ($)	Revenue Generated by Second Example ($)
Seats Remaining & Hybrid Probability	47,954(6.5%)*	72,228(7.2%)*
Time Remaining & Hybrid Probability	49,852(2.5%)*	74,940(3.3%)*
Hybrid Price & Hybrid Probability	51,104	77,465

*proportion by which this revenue is less than the maximum revenue

From Table 8 it can be observed that the hybrid price policy generates the most revenue followed by the time remaining model. The seats remaining model generates the least revenue for both the examples. The percentage loss in the revenue compared with the best policy is given in parentheses for both the examples. It can be seen that the relative performance of all the policies in the second example is consistent with that of original problem. Similar observations can be drawn from Table 9. The outcome of the second example reinforces our belief that the strategies that have been modeled are robust with regard to assumptions that have been made regarding ticket pricing, plane capacity and arrival population.

5.2.1 Analysis of Variance

In order to compare the results produced by different simulations runs and to find out the impact the parameters that are varied (controls) have on the results (response) we perform analysis of variance also called ANOVA. The hybrid price and hybrid probability model is considered as a sample example. We believe that there are two factors that if varied will give significant changes in the revenue generated. These two factors are the price offered and probability of acceptance. The price offered and probability of acceptance are determined by the following equations,

$$\textit{Price Offered} = P_{max} - (\textit{Time Remaining}) * j - (\textit{Seats Remaining}) * k$$

$$\textit{Probability of Acceptance} = 100 - (\textit{Price Offered-Cheapest Price})*l - (\textit{Time Remaining})*m$$

where j, k, l and m are normalizing constants as shown in Table 2. Another factor we believe most certainly has an impact on the revenue generated is the arrival rate and hence we intend to conduct an analysis of the significant factors at all three levels of the arrival rate. However for the sake of our proposal we have used the low rate of arrival.

Table 10 Controls and Their Levels

Control	Level 1	Level 2	Level 3
Pricing Strategy	Weighted towards Seats Remaining	Balanced Weights	Weighted towards Time Remaining
Acceptance Probability	Weighted Towards Price Offered	Balanced Weights	Weighted Towards Time Remaining
Arrival Rate	Low	Medium	High

The ANOVA for this 2 factorial, 3 level design is performed using Minitab software for 10 replicates at each level for the low arrival rate. The analysis of variance is as follows.

Multilevel Factorial Design

```
Factors:       2     Replicates:     10
Base runs:     9     Total runs:     90
Base blocks:   1     Total blocks:    1
```

Number of levels: 3, 3

Analysis of Variance for Revenue Generated, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Pricing Strategy	2	133526695	133526695	66763348	6.69	0.002
Acceptance Prob.	2	1071964396	1071964396	535982198	53.72	0.000
Pricing Strategy*AcceptProb	4	25370324	25370324	6342581	0.64	0.638
Error	81	808115814	808115814	9976738		
Total	89	2038977229				

From the above analysis we see that the F values for Pricing Strategy (6.69) and the F-value for Acceptance Probability (53.72) are greater than $F_{0.05, 2, 81}$ (3.15). Hence we can conclude that both Pricing Strategy and Acceptance Probability are significant factors and their interaction Pricing Strategy*Acceptance Probability is not significant as its F value (0.64) is less than 3.15. The ANOVA for the medium and high arrival rates also indicate similar results. They have been attached in the Appendix portion of this document. From Table 3 we can observe that the time remaining and probability based on time strategy generates the most revenue ($46,086). However, in all models other than the hybrid model either the pricing strategy or acceptance probability can be adjusted but not both. In the sample hybrid model which we have used for the purpose of calculations, high revenue of $52,755 can be obtained by setting the pricing strategy and acceptance probability at level 3 as shown in Table 10. This is done using the Process Analyzer part of the Arena simulation software. Hence the hybrid price and hybrid probability model out performs all the other models in terms of revenue generated.

Other strategies where the probability of acceptance is based on the price offered are more or less meant for the price conscious leisure travelers (tourists) who tend to book their tickets much in advance when the price offered is the lowest where as the strategies where the probability of acceptance is based on time are applicable to the business travelers who don't mind paying a high fare as long as they get to their destination at the right time. Such customers usually tend to book their tickets late in the booking period. The hybrid price and hybrid probability strategy covers the scenarios mentioned above into one model.

In the next chapter we will develop some new strategies for a network of three cities with direct and stop-over flights and suggest the optimal strategy to be used.

CHAPTER 6
NETWORK MODELS

This chapter discusses the main factors used in the development of the network model of three cities. The main factors to be considered here are the pricing of all the flight legs, their acceptance probability and also different arrival patterns of customers.

6.1 Flight Network

A network of three cities is considered as shown in Figure 6. There are four origin-destinations, Tampa-Atlanta (1-2), Atlanta-New York (2-3) and Tampa-New York (1-3) and Tampa-Atlanta-New York (1-2-3). Their assumed flight capacities and price ranges are shown in Table 11.

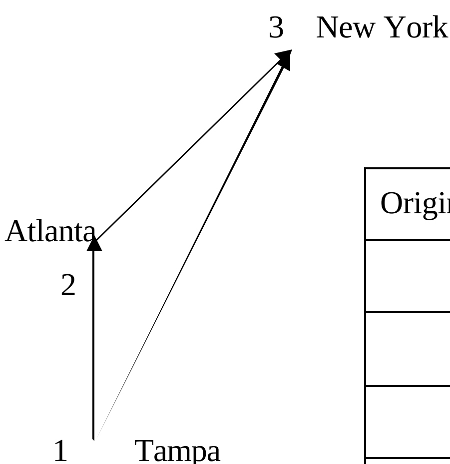


Figure 6 Flight Network

Table 11 Flight Capacities and Price Ranges

Origin-Destination	Flight Capacity	Min Price	Max Price
1-2	200	100	275
2-3	125	100	225
1-3	150	150	350
1-2-3	125	100	300

6.2 Pricing Strategy

The pricing strategy for the individual flight legs 1-2 and 2-3 will remain the same as in the single leg approach as these customers are flying only on these single legs and do not have a connecting flight. Hence the pricing for these flight legs will be dependent upon the time remaining, seats remaining and their combination. As observed in Chapter

5 there could be nine different combination equations for each flight leg and here we have used only the hybrid combinations as it was found that the combination of time and seats remaining model gives the maximum revenue. These have been summarized in Table 10.

However for a customer flying from Tampa to New York can either book on the direct flight (1-3) or the connecting flight with a stop over in Atlanta (1-2-3). Hence the problem comes down to pricing the origin-destinations 1-3 and 1-2-3.

6.2.1 Pricing Strategy for Tampa New York (1-3) Direct Flight

The pricing for Tampa New York direct flight (1-3) is assumed to be independent of the seats remaining on the indirect route (1-2-3). Here we have assumed flight leg 1-3 to be independent and hence the price offered for 1-3 would be similar to the single flight legs 1-2 and 2-3. It can be either dependent on time remaining, seats remaining or their combination. Here also we have used the hybrid equations and these have been summarized in Table 12.

6.2.2 Pricing Strategy for Tampa New York (1-2-3) Indirect Flight

The indirect flight 1-2-3 is offered to generate extra revenue from the vacant seats on flight legs 1-2 and 2-3. But at the same time it has to be made sure that this flight does not diminish the revenue generated by the direct flight. Thus, the price offered for the indirect flight has to be dependent on the seats available for that flight leg, seats remaining on the direct flight and the price offered for the direct flight. Two strategies have been used.

Strategy1. In the first strategy the price offered when the booking begins is the cheapest and there after increases as the seats remaining on the direct flight decrease. The equation developed for this approach is,

$$Price\ Offered123 = Price\ Offered_{13} – (Seats\ Remaining_{123})*j - (150 - Seats\ Remaining_{13})*k \quad (7)$$

Initially, when the booking starts we have

$$100 = 150 – (125)*j - (150 – 150)*k$$

Which gives us $j = 0.4$

Towards the end of the booking process,

$$Price\ Offered123 = 350 – (1)*j - (150 – 1)*k$$

$$300 = 350 - 0.4 - 149*k \text{ which gives us } k = 0.3328$$

Strategy2. It was observed from the websites of some popular airlines that when the bookings were opened the indirect flight was priced much higher than the direct flight. The explanation for this strategy could be that the airline wants to sell the seats on the direct flight first and then the remaining demand is absorbed of by the indirect flight. Hence, the price offered for the indirect flight when the booking is opened is the highest and the ticket price decreases as the booking period advances. The equation developed for this approach is

$$Price\ Offered123 = Price\ Offered_{13} – (Seats\ Remaining_{\ 123})*j - (50 - Seats\ Remaining_{13})*k \quad (8)$$

Initially, the booking starts and all the seats are available, the cheapest price will be offered for 1-3 and price offered for 1-2-3 will be max. Thus,

$300 = 150 – 125*j – (50 - 150)*k$ which is

$100k – 125j = 150$

Also towards the end the following condition could prevail,

$100 = 350 – 1*j – (50 – 1)*k$ which is

$49k + j = 250$

Equating the above two equations we can solve for j and k.

$j = 2.8353$ and $k = 5.0441$

6.3 Acceptance Probability Strategy

Acceptance probability equations for the individual flight legs 1-2 and 2-3 will remain the same as in the single leg approach as these customers are flying only on these single legs and do not have a connecting flight. These are dependent on the time remaining to depart, price offered or their combination. Here we have used only their hybrid combination and it has been stated in Table 13. A customer flying from Tampa to New York can either book on the direct flight (1-3) or the connecting flight with a stop over in Atlanta (1-2-3). Hence the question is whether to accept itinerary 1-3 or 1-2-3 or not to accept the fare at all. The acceptance probability cannot be based on time as both

Table 12 Normalizing Constants for Pricing Strategy

Flight Leg	Equation	Criterion	Normalizing Constant
1-2	*Price Offered12* = P_{max12} - *(Time Remaining) * j – (Seats Remaining12) * k*	Time and Seats Remaining	*j* = 2.9166 & *k* = 0.4375 (Equal Weights)
2-3	*Price Offered23* = P_{max23} - *(Time Remaining) * j – (Seats Remaining23) * k*	Time and Seats Remaining	*j* = 2.0833 & *k* = 0.5 (Equal Weights)
1-3	*Price Offered13* = P_{max13} *– (Time Remaining)*j- (Seats Remaining 1 3)*k*	Time and Seats Remaining	*j* = 3.3333 & *k* = 0.6666 (Equal Weights)

Table 13 Normalizing Constants for Acceptance Strategy

Flight Leg	Equation	Criterion	Normalizing Constant
1-2	*Probability of Acceptance12 = 100 – (Price Offered12 - Cheapest Price12)*l – (Time Remaining)*m*	Price Offered and Time Remaining	*l* = 0.14 & *m* = 0.83 (Equal Weights)
2-3	*Probability of Acceptance23 = 100 – (Price Offered23 - Cheapest Price)*l – (Time Remaining)*m*	Price Offered and Time Remaining	*l* = 0.2 & *m* = 0.83 (Equal Weights)

flights are assumed to depart at the same point in time. Hence the only deciding factors are the price differential between the direct and stop-over flight and the inability to buy even the cheapest fare offered for the Tampa New York.

6.3.1 Probability of Not Buying

As mentioned before the customer will decide not to fly Tampa New York (1-3 or 1-2-3) if he is not able to even purchase the lowest offered fare which could be either 1-3 or 1-2-3. Hence two equations are used to determine whether the customer will accept the fare.

Case 1. Acceptance Probability if the lower price offered is 1-3

$$\text{Probability of Acceptance}_{T\text{-}NY} = 100 - (\text{Price Offered13} - \text{Cheapest Price13}) * l \quad (9)$$

If price offered is maximum, acceptance probability is lower. Hence,

$$50 = 100 - (350 - 150) * l$$

Which gives $l = 0.25$

Case 2. Acceptance Probability if the lower price offered is 1-2-3

$$\text{Probability of Acceptance}_{T\text{-}NY} = 100 - (\text{Price Offered123} - \text{Cheapest Price123}) * m \quad (10)$$

Similar to *Case1*, we have

$50 = 100-(300 - 100)*m$ and $m = 0.25$

6.3.2 Price Differential

If the price difference between the direct and the indirect route is \$50 or less than \$50, it is assumed that the customer would rather fly direct route than the stop-over route. However there would still be some passengers who would want to save that \$50 and we have assumed them to be 10% of this population.

If the price differential is \$150 or greater than \$150, it is assumed the customers would rather fly the stop-over route and save some money. However there would still be some passengers who would want to fly directly and we have assumed them to be 10% of this population.

If the price differential is between \$50 and \$150 then the acceptance probability equation is

$$\textit{Probability of Acceptance} = 10 + (\textit{Price Differential} - 50)*0.8 \qquad (11)$$

This equation suggests that lower the differential, lower is the probability of flying the indirect route and higher the differential, higher is the probability of flying the indirect route.

6.4 Arrival Distribution

The customer arrivals are assumed to follow Poisson distribution with arrival rates that vary with respect to time and different arrival rates have been used for each origin-destination. As in the single leg models the booking period of 30 days is divided in to 6 time slots of 5 days each and each time slot having a different arrival rate.

Three different arrival patterns have been experimented with. The first pattern shows a mixed type of market with both tourist and business concentration. The second pattern signifies a business market with customers not really price conscious and booking towards the end of the booking period. This is shown by an increase in the customer arrival rate towards the end of the booking process. The third pattern shows a tourist market where the customer arrivals are concentrated towards the beginning of the booking process as price conscious tourists generally tend to book at the start. The three different arrival patterns for on flight leg (1-2) are shown in Figures 7 and the arrival rates are shown in Table 14.

6.5 Model Development

The booking of all flight legs is assumed to start 30 days in advance and all flights depart at the same point in time. The fares offered are for the first part of the round trip and only the forward part of the journey is modeled and return part can be modeled in similar way. The pricing and acceptance of fares on flight legs 1-2 and 2-3 follows the same assumptions and parameters used in the single leg approach. The hybrid price and hybrid probability strategies have been used. Now when a customer for the Tampa-New York itinerary enters the booking process, two separate fares namely 1-3 and 1-2-3 are offered to him. If the customer cannot afford even the minimum of the two offered fares he leaves the system. If the customer can afford to purchase either of the fares, then the question is whether he will fly the direct or the indirect route. If the price differential between the direct and the indirect fares is less than or equal to $50, he is assumed to buy the direct flight fare (1-3). However there will be some passengers who would still fly the indirect route (1-2-3) and we have assumed them to be 10% of this population. If the price differential is $150 or more than $150, the customer is assumed to fly the indirect route. And there will be some passengers (10%) who would want to avoid the indirect route and still fly directly. If the price differential is between $50 and $150, the lower the differential, lower is the probability of flying the indirect route and higher the differential, higher is the probability of flying the indirect route.

6.5.1 Blocking of Seats in 1-2-3

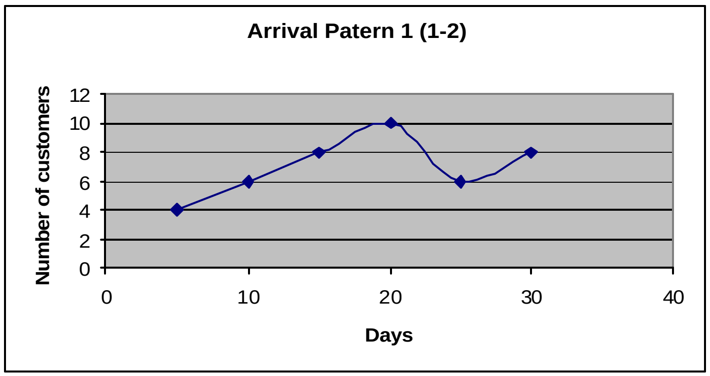
In this model we have also attempted to exclusively sell the seats on the direct route (1-3) when the booking process starts, by blocking the seats on the indirect route (1-2-3) till a certain number of seats on the direct route are sold and then opening up the seats to be sold on the indirect route. This exclusive reservation of seats on the direct route is done for 50% (75 seats), 33% (50 seats), 16% (25 seats) and zero seats out of the total flight capacity of 150 seats. The models developed have been run for a replication length of 30 days and 200 replications each for the three different arrival patterns and using both the pricing strategies for *Price Offered 123*. By setting the number of replications at 168, the half width for the total revenue generated is less than 1% of its value. The method for calculating the number of replications is shown in Appendix B. The results are shown in Table 15, 16 and 17.

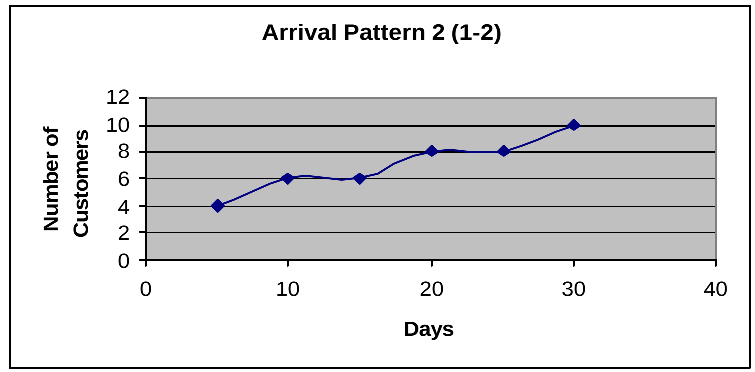
6.6 Results and Analysis

In Table 12, the first column indicates the percentage of seats initially reserved for the direct flight 1-3. This implies that the indirect flight between the destinations (1-2 and 2-3 in our example) will not be opened until certain percentage of direct flight seats is sold. The model was run with 50% (75 seats), 33% (50 seats) and 16% (25 seats) of the

Table 14 Arrival Rates and Pattern

Itinerary	Arrival Rate (Number of customers per day for every 5 days)			Total No of Customers	Total Flight Capacity
	Arrival Pattern 1	Arrival Pattern 2	Arrival Pattern 3		
Tampa-Atlanta	4, 6, 8, 10, 6, 8	4, 6, 6, 8, 8, 10	10, 8, 8, 6, 6, 4	213	200
Atlanta-New York	2, 5, 7, 8, 6, 4	2, 4, 5, 6, 7, 8	8, 7, 6, 5, 4, 2	155	125
Tampa-New York	5, 7, 8, 11, 9, 8	5, 7, 8, 8, 9, 11	11, 9, 8, 8, 7, 5	232	150





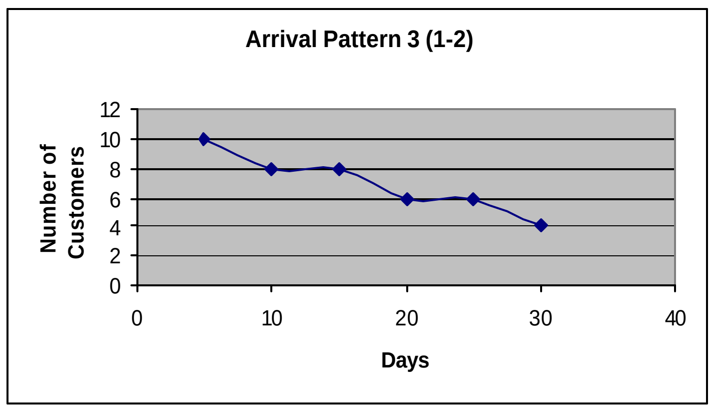


Figure 7 Arrival Pattern

Table 15 Results for Arrival Pattern 1

% Seats Reserved for 1-3	Strategy 1					Strategy 2				
	Flight Leg	Revenue Generated	Average Ticket Pr	Seats Vacant	Customer Balked	Flight Leg	Revenue Generated	Average Ticket Pr	Seats Vacant	Customers Balked
50 (75 seats)	1-2	$23,560	$179	48	76	1-2	$23,738	$180	35	77
	2-3	14,013	158	16	67	2-3	13,675	157	5	67
	1-3	30,611	243	24	95	1-3	26,989	236	36	94
	1-2-3	4,533	230	16		1-2-3	6,753	208	5	
	Total	72,719	(829)*	-		Total	71,157	(775)*	-	-
33 (50 seats)	1-2	23,643	180	45	77	1-2	23,886	182	29	77
	2-3	13,967	159	13	68	2-3	13,289	158	1	67
	1-3	31,835	245	20	88	1-3	27,692	238	13	86
	1-2-3	5,161	216	13		1-2-3	8,805	226	1	
	Total	74,608	(826)*	-	-	Total	73,674	(764)*	-	-
16 (25 seats)	1-2	23,576	182	41	76	1-2	23,506	182	29	76
	2-3	14,093	162	8	69	2-3	13,077	158	0	64
	1-3	33,437	247	15	78	1-3	29,614	240	27	76
	1-2-3	5,784	197	8		1-2-3	9,391	222	0	
	Total	76,892	(782)*	-	-	Total	75,589	(701)*	-	-
0	1-2	24,102	185	35	77	1-2	23,957	185	23	78
	2-3	14,076	164	4	69	2-3	12,218	158	0	63
	1-3	34,689	247	9	64	1-3	31,336	240	20	63
	1-2-3	6,327	181	4		1-2-3	10,406	217	0	
	Total	79,196	(739)*	-	-	Total	77,920	(684)*	-	-

* half width

Table 16 Results for Arrival Pattern 2

% Seats Reserved for 1-3	Strategy 1					Strategy 2				
	Flight Leg	Revenue Generated	Average Ticket Pr	Seats Vacant	Customer Balked	Flight Leg	Revenue Generated	Average Ticket Pr	Seats Vacant	Customers Balked
50 (75 seats)	1-2	$24,071	$182	47	77	1-2	$23,977	$182	38	77
	2-3	14,757	163	13	67	2-3	14,457	162	6	66
	1-3	30,433	245	25	95	1-3	27,784	239	34	95
	1-2-3	4,792	229	13		1-2-3	5,813	195	6	
	Total	74,054 (858)*		-	-	Total	71,995 (785)*		-	-
33 (50 seats)	1-2	24,356	184	42	77	1-2	24,435	185	28	77
	2-3	14,470	165	11	68	2-3	13,673	163	1	65
	1-3	31,393	247	23	88	1-3	27,447	239	35	86
	1-2-3	5,436	208	11		1-2-3	8,850	224	1	
	Total	75,656 (821)*		-	-	Total	74,406 (702)*		-	-
16 (25 seats)	1-2	24,284	186	38	77	1-2	24,379	187	25	77
	2-3	14,751	168	6	68	2-3	13,166	164	0	63
	1-3	33,006	249	17	78	1-3	29,160	241	29	77
	1-2-3	5,940	190	6		1-2-3	9,717	220	0	
	Total	77,982 (802)*		-	--	Total	76,424 (684)*		-	-
0	1-2	24,491	189	33	78	1-2	24,511	189	20	78
	2-3	14,424	170	2	67	2-3	12,307	164	0	63
	1-3	34,701	249	11	64	1-3	30,867	241	22	58
	1-2-3	6,627	177	2		1-2-3	10,790	216	0	
	Total	80,244 (738)*		-	-	Total	78,477 (666)*		-	-

* half width

Table 17 Results for Arrival Pattern 3

% Seats Reserved for 1-3	Strategy 1					Strategy 2				
	Flight Leg	Revenue Generated	Average Ticket Pr	Seats Vacant	Customer Balked	Flight Leg	Revenue Generated	Average Ticket Pr	Seats Vacant	Customers Balked
50 (75 seats)	1-2	$21,381	$167	52	80	1-2	$21,501	$168	38	80
	2-3	13,041	149	17	70	2-3	12,678	149	5	69
	1-3	29,329	233	24	96	1-3	25,598	225	36	95
	1-2-3	4,534	224	17		1-2-3	6,979	205	5	
	Total	68,286	(811)*	-	-	Total	66,758	(735)*	-	-
33 (50 seats)	1-2	21,650	169	47	80	1-2	21,743	170	31	80
	2-3	13,037	151	13	71	2-3	12,318	149	1	68
	1-3	30,578	235	20	86	1-3	26,423	227	34	84
	1-2-3	5,201	209	13		1-2-3	8,819	217	1	
	Total	70,468	(771)*	-	-	Total	69,305	(675)*	-	-
16 (25 seats)	1-2	21,827	171	41	80	1-2	21,977	172	26	80
	2-3	13,001	152	9	71	2-3	11,726	148	0	66
	1-3	32,145	236	14	73	1-3	28,167	229	27	72
	1-2-3	5,960	195	9		1-2-3	9,682	212	0	
	Total	72,935	(753)*	-	-	Total	71,554	(649)*	-	-
0	1-2	21,841	174	36	81	1-2	22,050	175	22	81
	2-3	12,998	155	3	70	2-3	10,810	148	0	62
	1-3	33,772	236	7	57	1-3	30,576	231	17	57
	1-2-3	6,795	178	3		1-2-3	10,689	205	0	
	Total	75,407	(651)*	-	-	Total	74,127	(637)*	-	-

* half width

seats being reserved. The entire table is divided in to two parts, the results for strategy 1 and 2 which is nothing but the two equations we have developed for *Price Offered* $_{123}$.

From Table 15, we see that the revenue generated progressively increases as the number of seats reserved for the direct flight goes on decreasing. Reserving seats for the direct flight does not increase the revenue and the model with no seats reserved gives the maximum revenue. This is true for both the strategies. When the two strategies are compared, strategy 1 outperforms strategy 2 in terms of the total revenue generated. However this difference in the revenue generated is not significant for arrival patterns 1 and 3 (Tables 15 and 17 respectively) as the half widths for the average revenue generated overlap. But this difference is significant for the arrival pattern 2 as seen from Table 16.

For strategy 1, the average ticket prices and revenue generated for leg 1-2 and 2-3 remain constant irrespective of the blocking. The revenue generated for direct and indirect flights increases as the number of seats blocked goes on reducing. This could be due to the fact that when the most number of seats are blocked (75 seats), the direct flight seats get sold out faster as the ticket price is lower initially. Also, when the indirect flight bookings are opened, certain number of seats on the direct flight has been sold and the direct flight is more costly than the indirect flight. Therefore, there is more demand to purchase the indirect flight thereby reducing the revenue generated for the direct flight. But as the blocking of seats goes on reducing and as the bookings for direct and indirect flight are opened at the same time this direct flight revenue increases.

Average ticket price for the indirect flight when more number of seats are blocked (75 seats) is higher resulting in less customers buying and hence, lower revenues. But when no seats are blocked the average ticket price is much lower resulting in more demand and hence more revenue generated. This combined increase in the revenue of direct and indirect flight results in a higher total revenue generated when no seats are blocked. Similar conclusions can be dram from strategy 2. The best pricing strategy to be used for different type of markets can be summarized according to Table 18.

In this chapter we have seen a flight network of three cities. Pricing strategies and customer acceptance strategies for the four origin destinations have been discussed.

Further, the sale of the indirect flight seats was blocked until a percentage of the direct flight seats were sold. These strategies were tested for the three different patterns of arrival. The results suggested the optimal strategy to be followed. In the final conclusion chapter of this thesis, we will summarize the entire thesis and discuss the future extensions that can be carried out.

Table 18 Best Pricing Strategy for Network Model

Strategy to be followed	Arrival Pattern		
	Mixed	Business	Tourist
	Strategy 1 or 2	Strategy 1	Strategy 1 or 2

CHAPTER 7
CONCLUSIONS

In this chapter we will briefly summarize the research undertaken in this thesis and also state the scope for future research.

7.1 Summary and Conclusions

In this research, a very important problem faced by the airline industry namely ticket pricing was considered. Different strategies such as pricing strategy, customer acceptance probability strategy and factors such as customer arrival rates and arrival distribution were considered. Initially the pricing policy for a single flight leg was developed. Three different pricing strategies namely time remaining, seats remaining and their combination were developed. Also, customer behavior such as probability of acceptance based on price offered and the time remaining to depart was studied. The pricing strategies were tested using simulation models for three different customer arrival rates. Following conclusions were drawn.

- For a tourist destination where the probability of acceptance was based on price, the pricing according to seats remaining was the optimal policy. This policy gave a lower average ticket price and higher revenues thus benefiting both the customer and the airline.
- For a business destination where the acceptance probability was based on time, pricing according to time remaining generated the most revenue.
- For a mixed type of destination where the acceptance probability was based on both time to depart and the price offered, the pricing according to both seats remaining and time remaining outperformed all the other strategies.

We also investigated the impact of offering indirect (stop-over) flights on the revenue generated by considering a network of three cities where travel can be made both direct and with a stop-over. Two different strategies were developed. According to the first strategy the pricing for both the direct and indirect flights was cheapest at the start of the booking period and ended with the last ticket being sold at the maximum price. The second strategy suggested a reverse path with the indirect flight being sold at the maximum price at the start of the booking period and the price reducing there after. This was done to discourage the selection of indirect flights early in the booking process. The first strategy always outperformed the second strategy in terms of revenue generated with their difference being significant for an arrival pattern resembling a business destination and insignificant for arrival patterns for the tourist and mixed destinations.

Also the effect of blocking of indirect route until a certain proportion of seats on the direct route were sold was investigated. It was observed that this approach did not increase the revenue. The model with no seats blocked generated the most revenue.

The single leg models for the low and medium rate of arrival were tested using another example with a different price range, flight capacity and corresponding arrival rates. The results were found to be consistent indicating the robustness of the models we have developed. Thus, an attempt was made in this research to develop a set of ticket pricing policies that could benefit the airline industry.

7.2 Scope for Further Research

Some of the extensions that can be undertaken to make this research more widely useful are:

1. It is known that every airline overbooks its flights to compensate for cancellations, no-shows etc. This extra revenue obtained from overbooking could contribute to the overall revenue generated. Hence, the factors such as cancellations, no-shows and overbooking could be integrated with the policies that have been developed in this research. The impact of these factors on the relative performance of different strategies can be investigated.

2. These days airlines offer fare prices to customers by taking into account the fares offered by their competitors. This competition aspect in the airline industry with regards to ticket pricing could be considered. Game theory based models could probably be used to investigate this aspect of the problem. The other related aspect that could be studied is the impact of alliance or code sharing. Code sharing provides a way for both major carriers and established regional carriers to expand their customer base by feeding in to each other's flight networks.
3. The ticket pricing strategies that we have developed are with the expectation of one customer buying one ticket. Discount could be given to large groups buying together and hence, this aspect of group bookings may impact the revenue generated specially in low demand markets.

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APPENDICES

Appendix A Analysis of Variance

A1 The Analysis of Variance for the Medium Rate of Arrival.

Multilevel Factorial Design

```
Factors:      2     Replicates:    10
Base runs:    9     Total runs:    90
Base blocks:  1     Total blocks:   1

Number of levels: 3, 3
```

General Linear Model: Revenue Generate versus Price Offered, Probability

Factor	Type	Levels	Values
Pricing Strategy	fixed	3	1, 2, 3
Acceptance Probability	fixed	3	1, 2, 3

Analysis of Variance for Revenue Generated, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Pricing Strategy	2	116590110	116590110	58295055	4.57	0.013
Acceptance Prob	2	1685751160	1685751160	842875580	66.01	0.000
Pricing Strat*Accept Probability	4	13230612	13230612	3307653	0.26	0.903
Error	81	1034331612	1034331612	12769526		
Total	89	2849903494				

```
S = 3573.45   R-Sq = 63.71%   R-Sq (adj) = 60.12%
```

From the above analysis we see that the F values for pricing strategy (4.57) and acceptance probability (66.01) are greater than $F_{0.05, 2, 81}$ (3.15). Hence we can conclude that both Pricing Strategy and Acceptance Probability are significant factors and their interaction Pricing Strategy*Acceptance Probability is not significant as its F value (0.26) is less than 3.15.

A2 The analysis of Variance for the High Rate of Arrival.

Multilevel Factorial Design

```
Factors:      2     Replicates:    10
Base runs:    9     Total runs:    90
Base blocks:  1     Total blocks:   1

Number of levels: 3, 3
```

Appendix A (continued)

General Linear Model: Revenue Generate versus Price Offered, Probability

Factor	Type	Levels	Values
Price Offered	fixed	3	1, 2, 3
Probability	fixed	3	1, 2, 3

Analysis of Variance for Revenue Generated, using Adjusted SS for Tests

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Pricing Strategy	2	135062050	135062050	33765512	4.30	0.002
Acceptance Prob	2	1502155964	1502155964	751077982	64.49	0.000
Pricing Strat*Accept Probability	4	1220663	1220663	305166	0.03	0.999
Error	81	943289950	943289950	11645555		
Total	89	2454497317				

S = 3412.56   R-Sq = 61.57%   R-Sq (adj) = 57.77%

From the above analysis we see that the F values for pricing strategy (4.30) and acceptance probability (64.49) are greater than $F_{0.05, 2, 81}$ (3.15). Hence we can conclude that both Pricing Strategy and Acceptance Probability are significant factors and their interaction Pricing Strategy*Acceptance Probability is not significant as its F value (0.03) is less than 3.15.

Appendix B Method for Calculating the Number of Replications

The equation used for calculating the number of replications to obtain a specific value of half width is

$n = n_0 * h_0^2 / h^2$ [Kelton, Sadowski and Sadowski, 2002]

where,

n = number of replications

n_0 = number of initial replication

h_0 = half width from the initial replications

h = half width required

If the total revenue generated from 10 replication is 39,941 and the half width is 2,329, to obtain a half width of 2% of 39,941 which is 798, we have

$n = 10 * (2{,}329 / 798)^2$

n = 85.17 which we can round off to 100 replications.

Similarly, if the total revenue generated from 10 replication is 79,885 and the half width is 3,279, to obtain a half width of 1% of 79,885 which is 798 we have

$n = 10 * (3{,}279 / 798)^2$

n = 168