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Using Visual Analogies To Teach Introductory Statistical Concepts

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Using Visual Analogies To Teach Introductory Statistical Concepts

Abstract

Introductory statistical concepts are some of the most challenging to convey in quantitative literacy courses. Analogies supplemented by visual illustrations can be highly effective teaching tools. This literature review shows that to exploit the power of analogies, teachers must select analogies familiar to the audience, explicitly link the analog with the target concept, and avert misconceptions by explaining where the analogy fails. We provide guidance for instructors and a series of visual analogies for use in teaching medical and health statistics.

Keywords

quantitative literacy, health numeracy, analogies, education, cognition, statistics

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Cover Page Footnote

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INTRODUCTION

Statistical concepts are integral to quantitative literacy, but some of the most basic are the most challenging for students to master. Many statistics instructors have had the discouraging experience of realizing, at the end of a full semester course, that students who can follow instructions to correctly compute a P -value or a confidence interval still cannot explain what they are.

Like many other teachers, we have observed the power of analogies for introducing new statistical concepts. An analogy is a systematic comparison of the similarities between two concepts, one of them familiar (the “analog”) and one of them unfamiliar (the “target”) (Duit 1991, Martin 2003, Glynn 2008, Behar, Grima, and Marco-Almagro 2013). The concept of a criminal trial, for example, is routinely used in textbooks to set the stage for hypothesis testing or to illustrate specific concepts such as false positives and false negatives (Bangdiwala 1989, Brewer 1989, Sowe 2001). Our experience suggests how useful these analogies can be, and we have found it particularly effective to develop teaching materials that exploit visual learning by illustrating analogies with photographs and diagrams.

Although we have noticed that visual analogies seem to be welcomed by students, we found ourselves uncertain about why they appear to work so well, how to select and introduce them for optimal learning, and how to avoid student misunderstandings. Our objectives in this paper are to synthesize research in education and cognitive psychology about the analogy; summarize some evidence to explain how analogies work in learning; and list elements of the teaching analogy that make it effective. We conclude with three worked-out examples of visual analogies that we have used in teaching statistics. Because our experience is in teaching medical students and graduate students in health informatics and public health, we illustrate our paper with examples from health and medicine. However, we believe the concepts are easily applicable for instructors teaching in the social sciences, business, or other domains.

METHODS

We conducted a literature search using the terms “analogy/ies,” “metaphor,” and “learning/education.” We conducted the initial search within websites of journals that regularly publish articles on teaching statistics (*Teaching Statistics*, *The American Statistician*, *Journal of Statistics Education*). Articles were included if they reported novel experiments or synthesis of previous experiments in statistics or science, rather simply teaching examples. Additional articles and books were then identified from citation lists; these sources were in the cognitive psychology

and learning literature, and they pertained to teaching scientific concepts in general rather than statistics specifically.

RESULTS

Experimental Evidence

Evidence that analogies can be useful in learning comes from a diversity of experimental classroom studies. In a series of experiments, Donnelly and McDaniel (1993) demonstrated that undergraduates performed better when asked to draw inferences about new scientific knowledge when they had been taught with analogies rather than literal descriptions. The experiments compared test performance between groups of students who had read literal descriptions and other groups who had read analogical versions of the same information. Although students could answer basic-level questions accurately with the literal descriptions, they answered inferential questions more accurately after reading the analogies. The effect was limited to novices, i.e., students who knew little about the field in question (Donnelly and McDaniel 1993).

In another series of college classroom studies, Braasch and Goldman (2010) demonstrated that the effect of teaching with analogies was largest when the analog was familiar to the students. College students were divided into an experimental group that read literal text descriptions (in this case, about weather science) or text descriptions that included analogies. In the analogy group, students with stronger prior knowledge about the analog performed better on post-intervention knowledge tests (Braasch and Goldman 2010).

Analogies can improve immediate recall as well as long-term retention (Glynn and Takahashi 1998). With middle school students, students who studied a detailed graphic drawing parallels between an analog (a factory) and a target concept (an animal cell) had better performance on immediate knowledge tests as well as those administered two weeks later (Glynn and Takahashi 1998).

Analogies have also been shown to help motivate and engage students and show the relevance of the material to their lives and work (Sowey 2001, Martin 2003). For example, in communities with traditional sex roles, linking math skills to the complex operations women routinely apply while knitting, sewing, or cooking can demonstrate relevance (Harris 2000). Summaries of other experimental evidence are presented by Glynn and Takahashi (1998), Duit 1991) and Glynn (2008), who synthesized the results into the framework described below under “Effectiveness of analogies.”

Mechanisms of action

Analogies provide a structure for students to understand novel concepts by linking them to more familiar ones. In other words, analogies help students engage in

elaboration, the cognitive process of clarifying the relationships between what the student already knows and what is being learned (Glynn 2008). A good analogy can be extremely useful in helping students construct effective “mental models” or cognitive representations of complex concepts (Martin 2003, Glynn 2008).

For this reason, analogies are particularly effective with novices (Martin 2003). Once equipped with an effective basic mental model, students can apply it to solve progressively more complex problems, while updating the model to make it increasingly sophisticated and able to handle new types of problems (Mayer, Mathias, and Wetzell 2002).

Analogies and their close relations, metaphors (in which one thing is said to *be* another thing), appear to be central to the way the human brain conceptualizes abstractions (Lakoff and Johnson 1980). One theory is that we construct mental representations of abstract concepts by basing them upon concrete and tangible physical experience. So, for the concept of “in,” the concrete metaphor of a physical container is applied to a wide variety of abstractions, such as being “in” a statistics course, “in” a bad mood, or “in” the category of patients with diabetes (Lakoff and Johnson 1980). The Venn diagram employs the container metaphor as a visual representation of an abstract set (Lakoff and Núñez 2000). The number line is a visual metaphor that shows how integers are ordered but also implies that the real numbers are connected by a sort of a continuous substrate, which facilitates ways to imagine even more abstract concepts such as irrational numbers falling between them (Lakoff and Johnson 1980).

Effectiveness of analogies

A synthesis of the evidence suggests that instructors should follow certain guidelines when selecting and applying analogies in teaching.

- **Choose an analog familiar to the audience** (Sowey 2001, Glynn 2008, Braasch and Goldman 2010). A baseball analogy is likely to be unfamiliar to many non-U.S. students. Cards might be unfamiliar in cultures that prohibit gambling (Martin 2003).
- **Choose an analog that shares important similarities with the target** (Glynn and Takahashi 1998, Glynn 2008). The analog should have relatively deep, structural similarities with the target, rather than simply superficial similarities (Martin 2003). The features of the target and analog should resemble each other, and relationships between the features should also be similar.
- **Explain the mapping** (Sowey 2001, Glynn 2008). Simply claiming that hypothesis testing is like a criminal trial does not provide students with enough grounding. In addition to mentioning the similarity between the two concepts, the instructor should spend some time explaining in detail

how characteristics of the trial are similar to characteristics of the hypothesis test. In other words, the instructor's explanations must promote *elaboration* (active linkage of novel information with what the student already knows).

- **Explain where the analogy fails** (Glynn and Takahashi 1998, Glynn 2008). Analogs by definition never map perfectly onto the target concept. Teachers must show students where the analogy breaks down, by, for example, pointing out features of the analog that do not map to the target or situations in which the analogy is inappropriate.
- **Exploit visual imagery** (Glynn and Takahashi 1998, Glynn 2008). As has been demonstrated in other teaching situations, comprehension and retention can be improved when complex information is presented in two modes, with visuals supplementing text or oral information (Mayer 2001, Carney and Levin 2002). Illustrations are especially useful for supporting the elaboration process (Carney and Levin 2002).

TEACHING EXAMPLES

This synthesis of the literature on teaching and learning in statistics has helped us choose and refine the analogies we routinely use in teaching. We conclude with three teaching examples from our own experience, elaborated according to the principles described above.

Example 1: Cooking with spinach

The spinach anecdote (Fig. 1) is one that we have used in teaching characteristics of diagnostic and screening tests. Here is an example of how it can be used in class, explained according to Glynn's "teaching with analogies" framework (Glynn 2008).

1. **Introduce the target.** Explain that the topic is characteristics of tests such as those used to screen for or diagnose disease. Examples include the antibody test used to test for HIV, or mammograms used to screen for breast cancer. In these diseases, there is a "gold standard" test that provides the true answer about whether the patient has disease (for example, polymerase chain reaction to detect the virus, or a biopsy to determine whether a lump is cancer). But the "gold standard" test is either time-consuming and costly (as in the case of HIV) or painful and somewhat dangerous (as in the case of a biopsy). As a result, we have the need for a cheaper or less burdensome test, which is called the diagnostic/screening test. Screening and diagnostic tests are topics that medical and public health students understand to be important.

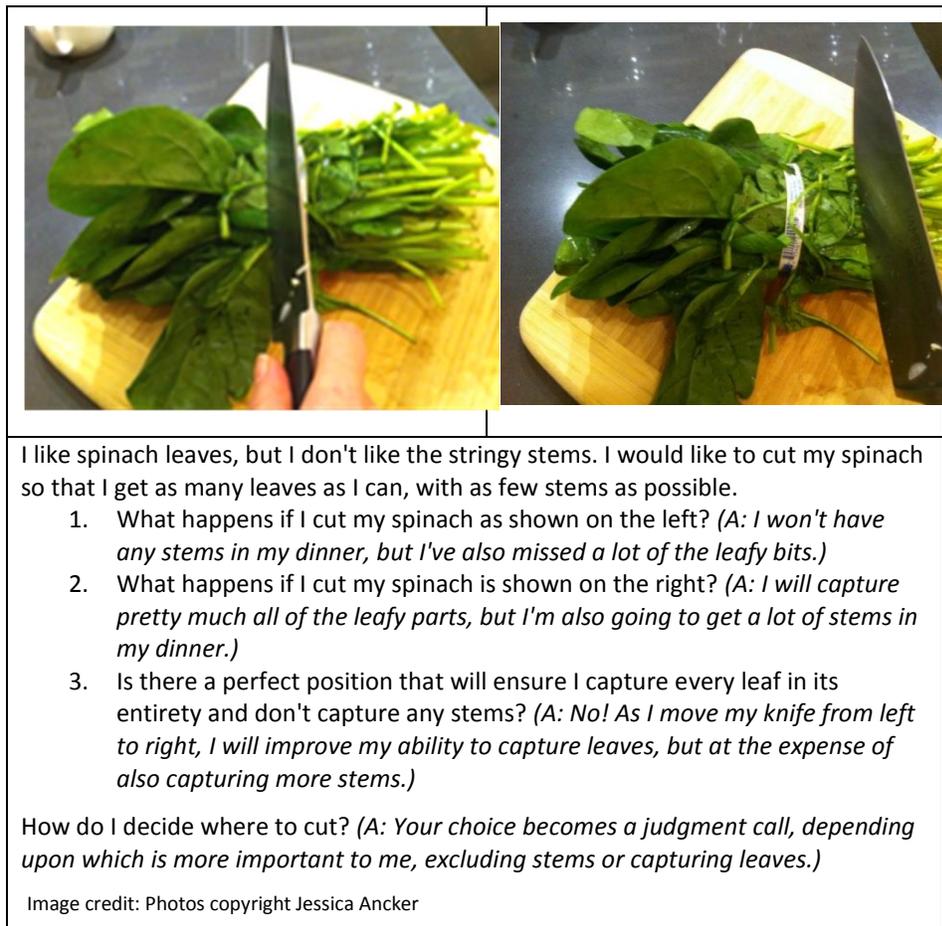


Figure 1. Images and discussion points showing how cutting spinach can be used as an analogy for the trade-off between sensitivity and specificity in the development of a diagnostic or screening test.

2. **Discuss what students already know about the analog.** Figure 1 provides questions and answers that elicit what students already know about the analog.
3. **Identify relevant features of the analog and the target, and explain their mapping.** Explicitly describe how the stems are patients with disease, the leaves are those without disease, the knife is the diagnostic test that separates patients with disease from patients without disease, and the knife position is the threshold selected for the diagnostic test.
4. **Where does the analogy break down?** The cook has the option of cutting the stem off each leaf individually to cut more accurately. Unfortunately, students may erroneously infer that administering

diagnostic tests to patients one at a time will give more accurate results. In fact, the reason that cutting spinach leaves one at a time works effectively is that it enables the cook to apply visual inspection to determine where to cut. It is crucial for the instructor to explain that it is visual inspection that is the analog of applying the “gold standard” test, not the process of testing patients one at a time.

5. **Summary and building a more sophisticated mental model.** The process of chopping a bundle of spinach can effectively illustrate concepts such as the trade-off between sensitivity (proportion of diseased patients correctly diagnosed by the test) and specificity (proportion of healthy patients correctly identified as disease-free by the test). Students can be invited to imagine how they would feel about eating tender stems from very young spinach or tough and stringy stems from older spinach. In the latter case, they might really dislike the stems, so they might place the knife in such a way as to get all the stems; the photograph shows that they will probably end up cutting off a lot of edible spinach leaf as well, demonstrating that maximizing sensitivity reduces specificity. We can talk to students about how, in the real world, we make similar condition-specific value judgments about whether it is more important to capture cases of the disease (requiring a test with high sensitivity) or more important to rule out disease (requiring a test with high specificity). We can then discuss situations in which we would want to develop a test optimized for screening (a high-sensitivity test that captures most cases of the disease, analogous to a knife positioned so that it captures most of the stems, even though this position inevitably will include a lot of leaves) or for diagnosis (a high-specificity test would be analogous to a knife positioned to exclude almost all of the leaves).

Example 2: Hurricanes and Uncertainty

We have used the visualization of Figure 2 productively as an analogy for the concept of a confidence interval. We teach on the East Coast of the United States, where it is not uncommon for weather forecasters to broadcast images such as Figure 2 two or three times per year. The cornucopia-shaped projected hurricane path represents a prediction interval, which can be used as an analogy for the general idea of a confidence interval (Broad et al. 2007).

1. **Introduce the target.** Explain that the topic is uncertainty in medical statistics.
2. **Discuss what students already know about the analog.** We first ensure that students recognize that the pale swath represents

uncertainty about where the hurricane will strike in the future. It is often helpful to pick an image from a very recent named hurricane so students can be prompted to recall their own uncertainty about where it would strike.

3. **Identify relevant features of the analog and the target, and explain their mapping.** One useful feature of this analog is that the map simultaneously depicts multiple versions of the confidence interval, growing wider as they move farther into the future. The image depicts increase in uncertainty with increasing unreliability of predictions in the future, and the instructor can explicitly map the image to how increases in the standard error widen a 95% confidence interval. The path image also shows the eastern-most and western-most estimates of the hurricane's position on the basis of current data, just as the lower and upper bounds of the confidence interval can be interpreted as the largest and smallest plausible estimates for a population statistic.

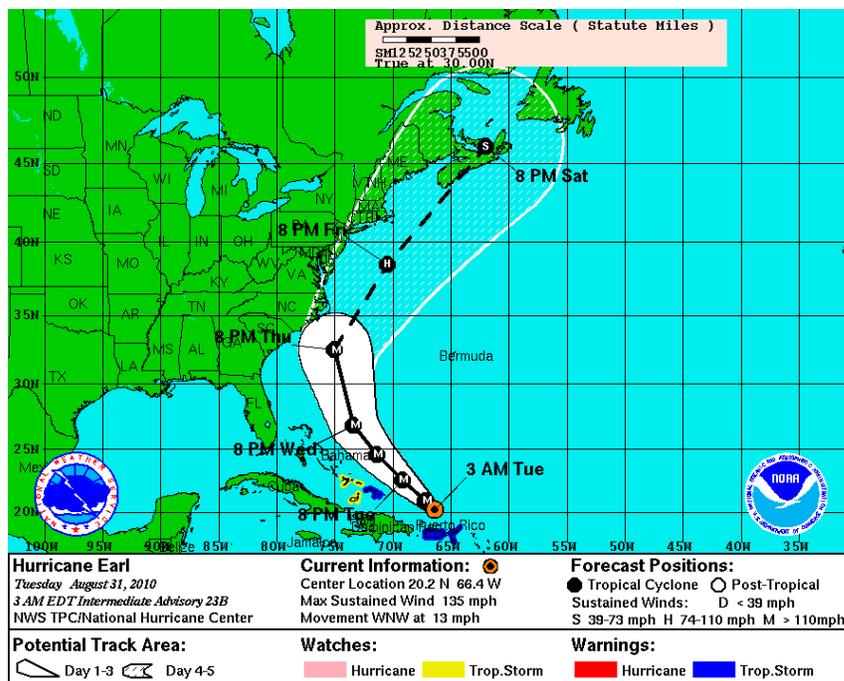


Figure 2. Projected path of Hurricane Earl, 2010

Image credit: Public domain, published by the National Oceanic and Atmospheric Administration, http://www.nhc.noaa.gov/archive/2010/graphics/al07/loop_5W.shtml

4. **Where does the analogy break down?** In the hurricane projection, our uncertainty arises because we are predicting the future. By contrast, in a 95% confidence interval, our uncertainty arises because

we are estimating a population statistic on the basis of sample data, and sampling error is inevitable. In teaching, we need to make sure that students do not think that a 95% confidence interval around the sample mean is a prediction of the future of that sample.

5. **Summary and building a more sophisticated mental model.** After fully discussing the analog, we show students examples of 95% confidence intervals in health and medical statistics. By placing the hurricane map next to a graphical or text report of a 95% confidence interval, we encourage students to apply similar concepts in interpreting them. For example, Figure 2 shows that our estimate of the hurricane location is so uncertain that it might hit New York City (which can be thought of as a “clinically significant” result in a medical context, or a “pragmatically important” one in other contexts) or might end up in the middle of the ocean (where it would not be especially important to people living on the East Coast). Similarly, if a drug company reports that mean 12-month weight loss with its new drug was 5 kg, with a 95% confidence interval of 1 kg to 9 kg, students can be encouraged to interpret the upper and lower confidence limits in light of their clinical significance.

Example 3. Science Fiction Null Hypothesis

One of the most challenging concepts in introductory statistics is the P -value. Many instructors have been disappointed to find that at the conclusion of the course, after students have successfully calculated dozens of hypothesis tests, they still cannot explain what a P -value is. We have constructed an analogy around the idea of visiting a science fiction world. Figure 3 shows one visualization, but in class we might select screenshots from a popular science fiction movie such as *Avatar*.



Figure 3. A parallel or fantasy universe as an analogy for the null hypothesis in hypothesis testing.

Image credit: <http://www.publicdomainpictures.net/view-image.php?image=169033&picture=fantasy-landscape>

1. **Introduce the target.** When we teach medical statistics, we find that medical students have already heard the phrase “statistical significance,” and so we introduce hypothesis testing as a method of understanding what is statistically significant. We introduce the concept of formulating a null hypothesis by saying it is like traveling to a fantasy world that is similar to our own except in a few important ways.
2. **Discuss what students already know about the analog.** If we have chosen screenshots from a science fiction movie, we first double check to make sure that it is familiar to the students. We then discuss how the science fiction world is like our own except in a few important respects. For example, the world of Pandora in *Avatar* has terrestrial plants that are like ours but are bioluminescent, and mountains that look like terrestrial mountains but float in the air.¹ In testing a hypothesis about whether a new vaccine prevents Ebola, students can be encouraged to (mentally) travel to a science fiction world in which Ebola exists but this particular vaccine has already been proven to be completely useless.
3. **Identify relevant features of the analog and the target, and explain their mapping.** In the science fiction world, we know that the Ebola vaccine has no effect. But we also know that random chance affects any experiment, so we would not be terribly surprised if the test statistic by chance rose slightly above or below zero. In other words, we know that the rate of Ebola infection is the same in vaccinated and unvaccinated populations in the fantasy world, because the vaccine doesn't work there. But if we took samples from the fantasy world, we might see that the proportion of people who got Ebola was slightly lower in the vaccinated sample than the unvaccinated sample, or slightly higher, simply by chance. We can then return to the target concept (the null hypothesis) and explicitly show the distribution of the test statistic under the null.
4. **Where does the analogy break down?** We have found that this analogy often provokes students to ask why the standard error in the science fiction world should be assumed to be the same as the standard error in our own world. We think this question demonstrates that the students have fully embraced the idea that the null hypothesis world is not the same as our own world, but it also shows that we need to guide

¹ See <http://www.avatarmovie.com/images.html#11>

them about which assumptions are routinely employed in order to conduct hypothesis testing.

5. **Summary and building a more sophisticated mental model.** After the conceptual discussion of the null hypothesis world, we return to the target (hypothesis testing) and work through a number of examples of formulating a null hypothesis, computing a test statistic, and making a decision. We keep reinforcing the idea that the P -value is the probability that the observed test statistic would occur *in the fantasy world*, not in our own world. We continue reinforcing this association by repeating the fantasy world image on every slide mentioning the null hypothesis or the P -value.

SUMMARY AND CONCLUSIONS

Introductory statistics courses expose students to unfamiliar, complex, and challenging concepts. Effectively chosen analogies, especially when illustrated, are powerful tools for teaching.

In addition to the examples here, a rich literature provides other suggestions, many easily translatable to visual representations. Some of these include:

- For the power of a statistical test: the power of an optical lens (Sowey 2001, Behar, Grima, and Marco-Almagro 2013) or the power of a lightbulb casting a sharp or fuzzy shadow (Hoquette 2004).
- For standard error and confidence interval: assessing today's grocery price in relationship to ordinary weekly fluctuations in prices (Sowey 2001).
- For a histogram: Putting apples through a size sorting machine (Martin 2003).
- For expected value of a probability distribution: The balance point of a seesaw (Martin 2003, Behar, Grima, and Marco-Almagro 2013).

To be effective in teaching, the analogy must be familiar to the audience and must resemble the target concept in important ways. We recommend that before trying a new analogy in class, the teacher spend some time working out the mapping between analog and target to ensure that the similarities are rich and not merely superficial.

In addition, the teacher must introduce the analogy thoughtfully, spending some time to elicit what students already know about the analog, explicitly link this knowledge to features of the target concept, and explain when the analogy fails. These instructional steps ensure that students understand the analogy,

engage in the process of creating a mental model of the new material, and avoid misconceptions that might arise from following the analog too closely.

Analogies appear to be especially effective as a way of giving novices a basic understanding of an unfamiliar concept. More advanced students may have mental models of the material that are so well-developed and nuanced that they no longer need to rely upon the analogy. Finally, illustrating the analogy visually is highly effective, directing students' attention to relevant features of the analogy and helping make it memorable. We also routinely repeat these visual images later in the lecture or in later lectures to provide a gentle visual reminder of important concepts from earlier in the course.

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