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Sheryl Stump
Ball State University, sstump@bsu.edu

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Quantitative Reasoning for Teachers: Explorations in Foundational Ideas and Pedagogy

Abstract
This note describes a course designed to prepare community college instructors and K-12 teachers for teaching foundational aspects of quantitative reasoning. A body of literature on quantitative reasoning and quantitative literacy informed the course design. The note describes the course content, which includes engaging in case studies, reading and discussion, writing assignments, group problem solving, and news-of-the-day presentations. Details of these assignments are provided. The capstone assignment for the course is for participants to design a set of case studies for their own students. Details of this assignment are also provided as well as specific examples of participants' learning.

Keywords
quantitative reasoning, online education, teacher education, Ball State University

Cover Page Footnote
Sheryl Stump is a professor in the Department of Mathematical Sciences at Ball State University. She teaches undergraduate and graduate-level mathematics and mathematics education courses for prospective and practicing teachers. She focuses on blending problem-based and proficiency-based instruction and assessment.

Introduction

According to the Mathematical Association of America's (MAA) report, *Quantitative Reasoning for College Graduates: A Complement to the Standards*, "The foremost objective of both liberal and professional types of higher education should be to produce well-educated, enlightened citizens, who can reason cogently, communicate clearly, solve problems, and lead satisfying, productive lives" (MAA 1994). While the MAA report argues that quantitative literacy is not achieved in a single course, many colleges and universities have developed courses to provide a foundation for quantitative literacy. This article describes an online graduate course that was originally designed to prepare community college instructors for *teaching* those foundational courses. The course has also become an elective for K-12 classroom teachers working on a master's degree in mathematics education.

The course, Quantitative Reasoning for Teachers, was designed to help participants broaden their understanding of quantitative reasoning, develop their skills in quantitative reasoning, and develop knowledge and skills for teaching quantitative reasoning. It includes features of quantitative reasoning that are notably in contrast to the features of traditional mathematics: Quantitative reasoning is carried out in real-life, authentic situations; the problems are ill defined, estimation is crucial, and an interdisciplinary approach is often needed (Madison 2006). The course also includes features of traditional mathematics—problem solving, the development of conceptual understanding and procedural fluency, and the development of mathematical models within particular content areas. Because written communication is such an important aspect of quantitative reasoning (Lutsky 2008), participants engage in the reading and writing of quantitative arguments throughout the course. The course also guides participants through explorations of pedagogical issues related to the development of quantitative literacy. Ultimately, they design instructional materials to develop their own students' quantitative literacy.

Quantitative Reasoning for Teachers is a required course in the Master of Arts in the Post-Secondary Foundational Mathematics Teaching program at Ball State University.[1] The three-credit course is one of two required mathematics content courses in this new masters program, which also includes nine credit

[1] Founded in 1918, Ball State is a state-assisted residential university in Muncie, Indiana, a midsize Midwestern city (2014 population, about 70,000), one hour northeast of Indianapolis. About 21,000 undergraduate and graduate students enroll each year in diverse academic programs on and off campus. Ball State is ranked a research university, high research activity by the Carnegie Foundation for the Advancement of Teaching and is accredited by the Higher Learning Commission. Individual programs are accredited by various regional and national organizations.

hours of research and pedagogical knowledge for teaching mathematics, six credit hours of pedagogical knowledge for teaching adults, and nine additional elective credit hours of mathematics content. The course has been offered in a 16-week online format once every two years and has typically had between 14 and 20 students enrolled.

Course Content

Engaging in Case Studies

The heart of the course is participants' engagement in case studies of media articles accompanied by warm-up exercises and study questions. The book *Case Studies for Quantitative Reasoning* (Madison et al. 2012) provides explorations involving numbers and quantities; percent and percent change; measurement and indices; linear and exponential growth; graphical interpretation and production; and counting, probability, odds, and risk. Participants explore these topics in authentic situations in political, economic, entertainment, health, historical, and scientific contexts. They interpret and represent quantitative information in various mathematical forms — equations, graphs, diagrams, tables, and words — and they recognize the strengths and limitations of these models. They use quantitative information to evaluate arguments, determining whether stated conclusions should be inferred. They also use quantitative information to construct organized and logically cohesive written arguments of their own.

Two of these case studies are assigned each week. Participants type their responses into a Word document, sometimes attaching an Excel document, and submit them online. Using the rubric that appears in the *Case Studies* textbook as a guide, the assignments are scored for completion, representation, calculation, and communication.

Additional Assignments

Each week, the participants also complete an additional assignment. These assignments — readings and discussion, writing assignments, group problem solving, and news-of-the-day presentations rotate in four-week cycles throughout the semester.

Readings and Discussion/Writing Assignments. The purpose of the reading and discussion assignments is for participants to broaden their understanding of quantitative reasoning and quantitative literacy as constructs and to explore related pedagogical issues. An online discussion board provides a forum for participants to summarize the ideas in each assigned reading, pose specific questions or comments to their classmates, pose another specific question to the instructor, and respond to the questions and comments of at least three of their

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classmates. In the week following each reading assignment, participants complete a related writing assignment.

The first reading assignment is the first chapter, "Examples and Principles," of the book *Innumeracy: Mathematical Illiteracy and Its Consequences* (Paulos 2001). In this chapter, Paulos describes fascinating examples of innumeracy in various historical and contemporary settings. He highlights applications of the multiplication principle for determining large quantities and calculating probabilities. In the week following this reading assignment, participants complete a writing assignment inspired by Lutsky (n.d.), in which they write an essay in which they identify a number that they believe well-educated people should know and present reasons for why the number is important. Some of the interesting numbers participants have chosen are the national debt, the median household income, the number of electors in the Electoral College, the distance from coast to coast across the United States, the golden ratio phi, and the relative size of one billion.

The second reading assignment is "The Role of Mathematics Courses in the Development of Quantitative Literacy," by Deborah Hughes Hallett (2003). The author examines quantitative reasoning as a habit of mind, explores the mathematical underpinnings of quantitative literacy, and suggests ways in which the curriculum and pedagogy of mathematics could better enhance quantitative literacy. In the week after reading this essay, participants write their own quantitative autobiography in which they describe their own experiences with the development of quantitative literacy, as a learner and as an educator.

The third reading assignment is "Mathematics for Literacy," by Jan de Lange (2003). In this essay, de Lange describes examples from the mathematics curriculum of the Netherlands. He discriminates between the concepts of numeracy, special literacy, quantitative literacy, and mathematical literacy, and he contrasts mathematical literacy with formal school mathematics. He lists eight competencies that form the heart of mathematical literacy and four phenomenological categories that describe what constitutes mathematics. The subsequent writing assignment asks participants to identify five ideas that they found important, interesting, or controversial in the essay and to describe how these ideas might impact their work.

The final reading assignment is "Practice 2: Reason Abstractly and Quantitatively" (Koestler et al. 2013), a chapter from the book *Connecting the NCTM Process Standards to the CCSSM Practices*. The chapter highlights the role of quantitative reasoning in the Common Core State Standard for Mathematics and explores connections to the National Council of Teachers of Mathematics standards for problem solving, representation, communication, connections, number and operations, and algebra. Participants then incorporate ideas from this reading into a lesson plan for their own classroom. The intention is

for participants to choose something that will fit into their existing curricula and provide opportunities for students to explore the ideas in new and interesting ways. Participants design their lesson plans around a problem situation — they may choose one from a collection of examples or create one of their own — and they use a Launch-Explore-Summarize teaching model (Umbeck 2011) to describe how students will engage in problem solving and class discussion. Participants are assigned to online groups in which they critique each other's lesson plans. When they submit their final lesson plans, they describe how they incorporated the feedback from their peers and what they learned from the experience.

Group Problem Solving. Because problem solving and student collaboration play important roles in the development of quantitative literacy (MAA 1994), this course for teachers provides three opportunities for participants to engage in these activities as learners. For each problem-solving assignment, participants are assigned to online groups. They first use the discussion board in their groups to choose a time to meet through Skype or some other face-to-face communication tool. Before the group meeting, participants work independently to begin solving the assigned problem. They are advised that it is okay if they do not completely solve the problem because they will have an opportunity to continue solving it with their group members. Participants post their individual work in the discussion board of their group before the group meeting. When participants meet with their group, they are advised to describe their solution processes to their group members, to listen to their descriptions, and to continue working collaboratively to solve the problem. When they have reached consensus on a solution, they discuss how they could verify the solution. Finally, each participant reflects on the problem-solving process and submits a report that includes the independent work, the group work, and a description of what was learned about mathematics and about learning mathematics.

The problems selected for these assignments tend to focus on two aspects of mathematics and quantitative reasoning: analyzing and describing patterns in numbers and making sense of quantities in challenging word problems. An attempt is made to select problems that are accessible to a wide range of K-12 teachers and prospective community college instructors.

News-of-the-Day Presentations. In this sequence of three assignments inspired by Boersma (n.d.), participants find articles that show the use or misuse of quantitative information in the current news media and they create presentations for their classmates. The first presentation focuses on comparisons of numbers or quantities, the second on accuracy of numerical information, and the third on the use of graphs to display quantitative information. Participants design Power Point

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presentations with voice-over narration and post them on a discussion board. They each view and respond to three of their classmates' presentations.

Designing Case Studies

The capstone assignment in this course for teachers is for participants to design a set of three case studies, based on news articles in the current media, to develop their students' quantitative reasoning. Participants may choose to have the case studies address different topics or the same topic, and they may use the articles they or their classmates used for news-of-the-day presentations or find new articles. The case studies they create are modeled after those in *Case Studies for Quantitative Reasoning* (Madison et al. 2012). That is, each case study must include: (1) learning goals, (2) a set of warm-up exercises, (3) A news article with identification of its source, and (4) a set of study questions.

The case studies created by participants have focused on mathematical concepts typically seen in the media: ratios, rates, unit rates, percentages, percent change, taxes, body mass index, probabilities, and various types of graphs. The topics of the articles vary according to the interests of participants and their students. Recent topics have included high school graduation rates, income changes over time, gas prices, maternal death rates, America's obesity and opioid epidemics, shark attacks, election probabilities, holiday travel, and the Chicago Cubs.

For this assignment, participants are assigned to small groups for peer critique. Group members are asked to review the case studies of their classmates and provide specific feedback in the form of questions and/or suggestions. In particular, they are asked to focus on the following: Are the learning goals clearly stated and appropriate for the case study? Are the warm-up exercises clearly stated and do they provide appropriate preparation for the case study? Are the study questions clearly stated and do they develop the ideas specified in the learning goals? Are there other issues that come to mind as you read the case studies? Participants then use the comments and questions suggested by their classmates to make final revisions.

Participant Learning

Participants are asked to describe what they have learned from the process of creating and revising their set of case studies. One middle school teacher reported,

> "I enjoyed this process a lot…. We are always encouraged to integrate different subject matter in our classrooms, and case studies incorporate current affairs and/or subject matter content, reading comprehension, and real-world math applications. Further, I liked the process of reviewing each other's case studies. Not only did I have access to new and well-written case studies that might be edited to fit my teaching settings, I also got to participate in a professional development activity. While I have participated in lesson

> planning with colleagues, it was never in this way, and I thought it was a productive process."

A prospective community college instructor said,

> "I learned a LOT from this assignment. I knew that the case studies we did all semester were really well done and the authors had analyzed a lot, but I hadn't realized quite how much work went into the whole process. Not only do you have to find great articles, but also those articles must have a mathematical theme you can pull out to ask pointed questions about. This assignment also reinforced for me how much is misleading in everyday articles. I am sure that most Americans don't think twice as they read them, just accepting whatever it is the author says, if they even understand it at all."

A current community college instructor said,

> "Writing these case studies was really a fun and educational experience. It has become more and more clear to me that devising thought-provoking questions is key to teaching. Maybe this in fact ties in to student-centered learning in ways that I haven't fully explored. If I can see the value in good questions, that might carry over to seeing the value in stepping out of the traditional 'sage on the stage' role, and entering the realm of guide/catalyst for discovery."

Conclusions

The quantitative reasoning course for teachers described here is one educator's attempt to engage participants in an exploration of foundational aspects of quantitative reasoning as students and as teachers. The course materials and assignments were carefully selected and designed to introduce participants to important ideas and new experiences. It is hoped that course participants gain insights into their own quantitative reasoning as well as form a vision for providing dynamic opportunities for their students to develop quantitative reasoning. It is also hoped that readers here appreciate this model of a course that was informed by a body of literature in quantitative reasoning and quantitative literacy and was designed specifically for teachers.

References

Boersma, Stuart. n.d. "Central Washington University Course Outline." Accessed March 6, 2017 (no longer available online, pers. comm. SB to SS, May 27, 2017). http://www.cwu.edu/~boersmas/QRCW/syllabi/cwu_outline.html (see Dingman and Madison 2010 for related reference).

de Lange, Jan. 2003. "Mathematics for Literacy." In *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges*, edited by Bernard L. Madison and Lynn Arthur Steen, 75–89. Princeton, NJ: National Council on Education and the Disciplines. Accessed May 15, 2017. http://www.maa.org/sites/default/files/pdf/QL/WhyNumeracyMatters.pdf

DOI: http://doi.org/10.5038/1936-4660.10.2.9

Dingman, S. W. and B. L. Madison 2010. “Quantitative Reasoning in the Contemporary World, 1: The Course and its Challenges.” *Numeracy* 3(2): Article 4. Accessed May 28, 2017. https://doi.org/10.5038/1936-4660.3.2.4

Hughes Hallett, Deborah. 2003. “The Role of Mathematics Courses in the Development of Quantitative Literacy.” In *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges*, edited by Bernard L. Madison and Lynn Arthur Steen, 91–98. Princeton, NJ: National Council on Education and the Disciplines. Accessed May 15, 2017. http://www.maa.org/sites/default/files/pdf/QL/WhyNumeracyMatters.pdf

Koestler, Courtney, Mathew Felton-Koestler, Kristen Bieda, and Samuel Otten. 2013. *Connecting the NCTM Process Standards to the CCSSM Practices*. Reston, VA: National Council of Teachers of Mathematics.

Lutsky, Neil. 2008. “Arguing with Numbers: Teaching Quantitative Reasoning through Argument and Writing.” In *Calculation vs. Context: Quantitative Literacy and Its Implications for Teacher Education*, edited by Bernard L. Madison and Lynn Arthur Steen, 59–74. Washington, DC: Mathematical Association of America. Accessed May 15, 2017. https://www.maa.org/external_archive/QL/cvc/CalcVsContext.pdf

———. 2012. **“**Measured Thinking: Reasoning with Numbers about World Events, Health, Science, and Social Issues.” Accessed May 15, 2017. http://www.acad.carleton.edu/curricular/PSYC/lutsky/IDSCsyll12.htm

———. n.d.“Writing about Numbers We Should Know.”Accessed May 15, 2017. https://serc.carleton.edu/sp/library/quantitative_writing/examples/numbers.html

Madison, Bernard. 2006. “Assessing QL: Double Trouble.” Presentation to the *Northeast Consortium on Quantitative Literacy*, Amherst College. Accessed May 15, 2017. http://slideplayer.com/slide/5148548/

Madison, Bernard. L., Stuart Boersma, Caren. L. Diefenderfer, and Shannon. W. Dingman. 2012. *Case Studies for Quantitative Reasoning: A Casebook of Media Articles* (3rd ed.). Boston: Pearson.

Mathematical Association of America. 1994. *Quantitative Reasoning for College Graduates: A Complement to the Standards*. Accessed May 15, 2017. http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/quantitative-literacy/quantitative-reasoning-college-graduates

Paulos, John Allen. 2001. *Innumeracy: Mathematical Illiteracy and Its Consequences*. New York: Hill and Wang.

Umbeck, Lindsay M. 2011. “Navigating Classroom Change.” *Mathematics Teaching in the Middle School* 17 (2): 89–95.