Numeracy
Advancing Education in Quantitative Literacy

Volume 4 | Issue 1 Article 3

2011

Calculus, Biology and Medicine: A Case Study in Quantitative Literacy for Science Students

Kim Rheinlander
Dartmouth College, kimr@math.dartmouth.edu
Dorothy Wallace
Dartmouth College, dorothy.wallace@dartmouth.edu

Recommended Citation
Rheinlander, Kim and Wallace, Dorothy (2011) "Calculus, Biology and Medicine: A Case Study in Quantitative Literacy for Science Students," *Numeracy*: Vol. 4 : Iss. 1 , Article 3.
DOI: http://dx.doi.org/10.5038/1936-4660.4.1.3
Available at: http://scholarcommons.usf.edu/numeracy/vol4/iss1/art3

Calculus, Biology and Medicine: A Case Study in Quantitative Literacy for Science Students

Abstract

This paper describes a course designed to enhance the numeracy of biology and pre-medical students. The course introduces students with the background of one semester of calculus to systems of nonlinear ordinary differential equations as they appear in the mathematical biology literature. Evaluation of the course showed increased enjoyment and confidence in doing mathematics, and an increased appreciation of the utility of mathematics to science. Students who complete this course are better able to read the research literature in mathematical biology and carry out research problems of their own.

Keywords

Calculus, biology, medicine, modeling, differetial equation, numeracy, curriculum, pedagogy

Cover Page Footnote

Kim Rheinlander is the Director of the Center for Mathematics and Quatitative Education at Dartmouth College. She has worked with faculty both locally and nationally creating and evaluating innovative pedagogical materials. She is a charter board member of the National Numeracy Network.

Dorothy Wallace is a professor of mathematics at Dartmouth. She was 2000 New Hampshire CASE Professor of the Year, and the lead PI of the seminal NSF project, Mathematics Across the Curriculum. She recently finished a text in mathematical biology for first year students, *Situated Complexity*. She was a charter board member of the National Numeracy Network and is now co-editor of this journal.

Introduction

Discussions of quantitative literacy often focus on skills relevant to citizenship and therefore useful to all students. Particular disciplines also require a kind of numeracy that is often lacking in students and not represented in courses aimed at a general audience. Mathematics instruction assumes that students will be able to apply abstract concepts in their future science courses, yet the problem of transfer is known to be a difficult one. Some researchers postulate that transfer requires local knowledge of the phenomenon and emerges in response to context (Salomon and Perkins 1989; Renkl et al. 1996). Even defining what numeracy might mean for a biology student (for example) is tricky. Richards takes the right first step in this direction, stating:

> The concept of quantitative literacy is rooted in the connection between mathematics and reason . . . When teaching mathematics is seen as a way of teaching people how to think, it can no longer be isolated. Its implications spread throughout the curriculum and it has a place in every class. (Richards 2001)

In grappling with what this statement implies for a biology or pre-medical student, one is forced to consider what aspects of mathematics “give reason” to biology. The ability to model change in biological quantities has been a productive force across all of the biological sciences and is naturally connected to the beginning calculus courses that most biology and pre-medical students take. Since 2003, Wallace has been teaching a course targeting this audience, designed to take advantage of the natural utility of modeling with systems of ordinary differential equations in a variety of biological contexts.

The course has been through many revisions, simultaneously strengthening the mathematics content and expanding the range of biological contexts approachable by the students. Steen points out the difficulty of embedding mathematics in a rich situation without losing focus.

> Connecting mathematics to authentic contexts demands delicate balance. On the one hand, contextual details camouflage broad patterns that are the essence of mathematics; on the other hand, these same details offer associations that are critically important for many students’ long term learning. (Steen 2001)

This case study offers an example of a course that seems to have succeeded in achieving this balance. Biology students use the course to strengthen not only their mathematics, but also their understanding of biology and medicine, although by doing so they avoid a second traditional calculus course. Schneider (2001) comments that this tradeoff is an acceptable necessity. Students seem to agree:

> In this class I've actually, for the first time in my life, found math useful and interesting, which is what I'll need if I want to pursue a career in science.

A course that explicitly creates the link between the abstract structure of mathematics and specific research questions in a scientific discipline gives students a roadmap they can follow to transfer their understanding of mathematics to a scientific context. Applications of Calculus to Medicine and Biology, is one example of such a course.

In the next section, we describes the course in detail, including its goals, the topics covered, timing, student work, and useful resources. Under "Analysis" we describe the evaluation methods and results according to the dimensions along which change could be measured. Then we describe the impact of the course on the instructor's ability to integrate research and teaching. We close with our conclusions and interpretations. Funding for this project was provided by the National Science Foundation (NSF DUE-0736749).

The Course

Instructor's Goals and Beliefs

The goals of the course have evolved over its duration. At the start, the object was to impress biology students with the utility of calculus to their field and get them to think about dynamical systems to the point where they could at the very least make sense of a phase portrait. Both of these goals were based on conversations with faculty in the biology department. It is worth noting that the biologists stated another goal that this course does not attempt to address: understanding eigenvalues of matrices. The reason for ignoring eigenvalues is just to keep the mathematics in one domain for the short nine-week winter quarter.

A third goal rapidly emerged: to make the material relevant to premedical students. Very few undergraduate courses address the career interests of these students, and it was clear that, by doing this, the course would become attractive to a larger population. One aspect of this goal is the prerequisite required—a single semester of calculus. Another aspect is the official name of the course: Math 4, Applications of Calculus to Medicine and Biology. The course counts as the "second calculus course" desired by many medical schools. As with all new courses, an important unspoken goal is to secure enrollments.

When constructing a new course with particular content goals, it is possible to win the battle while losing the larger war. If students do not emerge from the course with an increased appreciation of the utility of mathematics, confidence, and a willingness to engage with real world problems, then the course has not done much good on balance. In addition, students need to be willing to write about mathematics to succeed in this course. While assessment of content goals is the job of the instructor, these subtle attitude goals, described more fully under

"Analysis," were measured by the evaluator. Improved performance in subsequent biology courses is not an explicit goal of the course.

The instructor believes that learning should be hands-on, that doing mathematics is far more instructive than talking about it, that students should be given big open messy problems with multiple possible approaches (and therefore answers). She also believes that students, even with just one semester of calculus behind them, can do useful research. However the actual choice of curriculum and general approach to the course was greatly influenced by a single early conversation with Dartmouth Professor of Psychology Chris Jernstedt. The instructor's interpretation of this conversation is reproduced here.

Piaget gave the psychology of learning the notion of a "scheme" (Duckworth, 1996) or "schema" (Skemp, 1987). Loosely defined, a schema is a set of beliefs (sometimes expressed as metaphors) and behaviors of an individual learner, related to a particular phenomenon (the object of learning). Skemp (1987) attempted to refine and explain this notion in the context of learning mathematics. Harper (1987) and others used Skemp's explanation as a basis for studying student learning of particular parts of mathematics such as high school algebra (in Harper's work). Piaget is at pains to point out that a scheme for understanding something (such as mass, a favorite topic of Piaget) may only be adequate for understanding certain aspects of it. To grasp more complicated or subtle facts one may actually have to break one's existing schema and build a more comprehensive and often very different one. Skemp makes the point that this is the goal of almost every mathematics class: to build new schema in the student by confronting mathematical patterns for which their current understanding is inadequate. Mathematicians would recognize this as a pretty good psychological description of the process of abstraction.

As an example in dynamical systems, the process of thinking about the time series of a system is fairly straightforward for Math 4 students. Switching their visualization to a phase portrait takes quite a bit of effort for them. Bifurcations are even harder because they require the student to think about, not a single system, but an infinite family of them that are all related. This is both an example of increasing abstraction and also an example of building new schema in order to ask questions that cannot be addressed with an earlier framework. It is the essence of learning mathematics.

Chris Jernstedt stated that, in this emphasis, mathematics is very unusual. He pointed out that, in the sciences, a single schema is exploited across many domains and courses. Students stabilize a schema and become experts within it as they employ it in the service of many different inquiries. The idea of natural selection is a perfect example of what Jernstedt means. Once the concept of natural selection is introduced, the student uses it in many biology courses, in environmental studies courses, anthropology, geology, and even perhaps in medical school in the context of drug resistant strains or epidemiology.

In one short conversation, Jernstedt gave permission for a whole new kind of math course, in which a single schema is established early and used repeatedly in multiple contexts until it becomes a natural part of a student's conceptual framework. Jernstedt's observation fit well with the instructor's desire for her students to become really good at using basic (highly coupled nonlinear) dynamical systems rather than merely being exposed to a lot of interesting mathematics and biology.

At this point the goal of the course came into focus. Students should understand dynamical systems well enough to read the research literature that applies nonlinear highly coupled systems to specific questions in biology or medicine. This specific ability would be a measure of content mastery. Reading the research literature is how students are most likely to make use of their understanding beyond the end of the course.

To this end the instructor believes that in order to understand the literature properly one must have the experience of producing it. This would include generating one's own research question, developing a model to answer it, and running simulations. The course work would consist of three research papers developing these different skills in the student. Problems would become less well defined, messier, and more open as the quarter progresses. The type of research problems students have studied are described in more detail below under "Course Materials and Resources." "Integration of Research and Teaching" notes two papers that were accepted and presented at a professional meeting.

How the Course Works

The majority of students enrolled in Math 4 are planning a career in medicine. Many but not all of these are majoring in biology. Typically the class has 15-20 students. Only one term of calculus is required, so this course satisfies the "second calculus course" that the students believe they need for medical school. The choice of biology topics is explicitly geared to this population. About one third of the chapters in the text (Wallace, 2010) are medical topics and the rest concern ecology. However the assignments are reversed, with two out of three required papers usually addressing a medical question. The mathematics in the course is confined to systems of ordinary differential equations. Because most models in the literature are nonlinear, the students do not learn the classical linear systems. Rather, they use software (described under "Course Materials") to study complex systems, compute equilibrium values and check for stability. Between the mathematical theory and the biological question lies the crux of the course: the ability to build mathematical models based on understanding of the underlying biological phenomena, and to think critically about such models in the literature.

DOI: http://dx.doi.org/10.5038/1936-4660.4.1.3

Many of the students have not taken a math course for several years. Some protest that they have forgotten all of calculus. The course begins with a short reminder of what the derivative is and the simplest differential equation (exponential population growth). Students need to think of the derivative as a rate of change, not an algorithm for producing a function such as $f'(x)=3$ from another function such as $f(x)=3x$. From the example of exponential population growth it is an easy step to the pharmacokinetics of drugs given by intravenous bolus, which gives an exponentially decreasing solution. Progress at this point is deceptively slow, with a lot of attention to the biology and chemistry involved in the simple assumptions governing the differential equations. From there we move to logistic population growth and a two-box model of pharmacokinetics describing a drug passing from G.I. tract to blood and then being eliminated. Students begin their first paper in the second week of class. It is tightly prescribed, the equations are well known in the literature, the parameters are specific to a given disease and drug regimen, and the research problem is set by the instructor with only a few degrees of freedom. The students attempt to solve the problem and write a paper due two weeks later. They find it difficult.

As they are working on this first paper, the course proceeds through ever more demanding ecological models, including predator-prey, competition models, predator functional responses such as satiation, and larger highly coupled systems with these basic features. Students learn to find equilibrium values, check for stability, and read a phase portrait (in two dimensions). The text has examples for students to practice in class and in groups. They also begin to see how the various assumptions made about biology translate into the specific equations in the model. The system for doing this is to begin with a box model (see Figure 1 for an example) of inputs and outputs for various quantities to be modeled, then write equations in verbal form describing all inputs and outputs, and finally place the mathematical expressions in each equation.

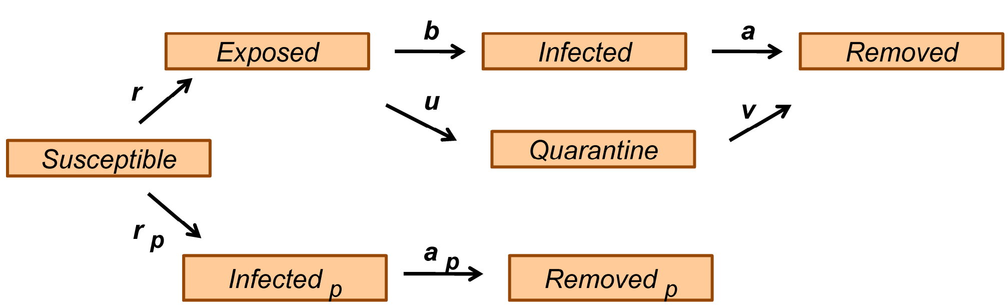

Figure 1. An example of a box model for a double epidemic of SARS, with quarantine added.

An example of a verbal equation corresponding to the topmost left box would be: *Change in exposed* = (*increase from susceptible*) – (*loss to infected*) – (*quarantined*).

This model is describing a strategy used for controlling a particular SARS epidemic in China, where individuals were quarantined if they were exposed, whether they got sick or not.

The discussion of a new problem is always from the biological system downward, rather than from the equations upward. Preparation for the second paper includes a group discussion of a particular research paper (Edwards and Brindley 1999), which is an excellent example of the construction and analysis of a predator-prey model. It also happens to be a beautiful piece of writing, and the first few sections of it serve as a prototype for the second paper.

For the next paper the students are asked to pick an interesting topic in ecology and model it, making use of the relationships they now understand. The choice is wide open; some students modify an existing paper by examining and altering the assumptions behind the model, and others build models from scratch. Setting up a model is the main point of this paper and where most of the work goes. Choosing the structure of the box model takes much thought and discussion. The exact form of the equations must be thoroughly justified in the paper. A large amount of time goes into estimating what the parameters are in their equations. Identifying parameters usually involves tackling a lot of literature, reworking units to be compatible with the model, and making quite a few estimates. This is where laboratory science and fieldwork intersect most strongly with the construction of models. Usually this paper takes three weeks, and very little time is given to analyzing the resulting model. Some examples of student projects were: “Modeling population dynamics of algae and silver carp in the Taihu Lake ecosystem,” “The predation of flatworms on mosquito larvae in rice fields as a biological control of mosquito disease transmission,” “An investigation of interspecific competition: The effects of Lion predation and competition with wild dogs on cheetah populations in the Serengeti.”

As they work on the second paper, the biology topics covered in class move into the realm of medicine. The text explicitly discusses epidemiology and the SIR (susceptible, infected, recovered) model, and ends with a chapter on malaria that merely outlines the problem, offering no specific models. The text is doing this on purpose, to force the students into the literature and away from reliance on predigested expository material. The mathematical tenor of the discussion changes from how a model is constructed to the types of behavior a model might exhibit, such as the appearance of new equilibrium states due to a pitchfork bifurcation or stable cycles arising from a Hopf bifurcation. A lot of attention is given to how one might ask a model a question, what features of the computer output really capture the answer to that question, and how to summarize the

DOI: http://dx.doi.org/10.5038/1936-4660.4.1.3

results of many computer experiments in a concise way. Students look at the Edwards and Brindley (1999) paper again, as well as others, often including an older paper on HIV by Nowak and May (1991) that is a useful example of how to ask a less obvious question of a model.

At the end of six weeks of class the entire text has been covered, two papers written, and at least one research paper read and discussed as a group. The students are in a good position for their third paper, fully equipped with all the tools they might be expected to use.

The third paper is on a medical topic chosen by the student, who is explicitly directed to find a paper containing a model of some interesting phenomenon. The paper must already have a good estimate of the constants (parameters) in the model. This is an easier assignment in the realm of medicine than ecology, where far more research and funding go into measuring such things. The student must then do original research based on the model they have found. Some alter an assumption and make a small change in the model, comparing their results with the original paper. Some use the model as it is, but ask different questions that the authors have not addressed. Some do a sensitivity analysis on the parameters of the model. The time frame for this assignment is short, but the students seem to be able to handle it. The last day of class is a poster session where everybody can see what each group has done. Some examples of student projects were: “Modeling HIV dynamics as applied to the pharmacokinetics of the anti-retroviral drug Atripla,” “Effects of acquisition of sexual partners and condom compliance on a model of HIV/AIDS transmission in Zimbabwe,” “Output comparison and sensitivity analysis of four West Nile virus models in relation to species specific population outputs and infection reproduction rates,” “Modeling gonorrhea rates across genders as a function of access to health care in the United States.”

During the last few weeks of the course, a lot of time is spent working directly on the papers in class. There are simply not enough hours in the day for the instructor to meet with every group outside of class. There might be a few short lectures on special topics. This is the place in the quarter where visitors may come and talk to the class about their research. In 2010 Miranda Teboh-Ewungkem spoke about her work on malaria models (Teboh-Ewungkem, 2009; Teboh-Ewungkem and Yuster, 2010) just as the second paper came due. Two groups of students used her work as a basis for their final papers.

To a mathematics instructor it may seem that too much time and effort are devoted to establishing the models and too little devoted to studying them. The result of such an approach turns the math professor into a storyteller. We argue that the emphasis of the course is well placed. These students are in the process of becoming scientists. The words that scientists use to describe the natural world should be the very basis of each term in a differential equation modeling that world. If the scientist cannot identify his or her stated assumptions or observations in the equations of a model and manipulate those assumptions at will, it matters little whether that model exhibits a bifurcation or predicts extinction. To make the mathematics meaningful we must be able justify every aspect of a model with our words.

Course Materials and Resources

There are many excellent mathematical biology texts such as Murray (1993), Edelstein-Keshet (2005), Yeargers et al. (1996), all of which provide the instructor with inspiration and useful examples. All of these texts expect the reader to know a fair amount of mathematics, and many of the examples are launched without much discussion of the underlying science. While an excellent choice for an advanced student in mathematics, these texts are too difficult for biology students whose mathematical background is one course in calculus and who have forgotten a lot of it. At the other extreme are standard calculus texts that purport to serve majors in biology. These certainly contain biology examples but in no way prepare a student to approach any subset of the mathematical biology literature. What was needed was a text that begins simply with easy discussion of systems leading to exponential increase and decrease, and proceeds to more complicated models that lie strictly within the realm of systems of ordinary differential equations. The text should give equal emphasis to the science and to the math, with a lot of discussion about how models arise. Wallace (2010) attempts to follow such a trajectory.

In addition the text should provide motivation. Biologists are interested in mathematics because of what it can do for biology. Premedical students are interested in biology because of its obvious connection to medicine. Often premedical students are interested in medicine because of their concern with socially important issues. Medicine always occurs in a social and cultural context, and an explicit connection to such a context makes both mathematics and biology more relevant for some students. However too much emphasis on context might detract from the actual goal of the course. The text attempts to solve this problem by picking one part of the world and relating every biological, medical or mathematical topic to this particular context. The author chose East Africa, specifically the region surrounding Lake Victoria, which is a rich source of examples of ecological issues and infectious diseases.

The text should be easy for undergraduates to read and understand. It is informal and is not meant to be a prototype for scientific writing or an encyclopedia on the subject. The point is to lead the student gently towards the literature. In Math 4 the text is completely done by the sixth week of a nine-week course, with the remainder of the work and discussion centering on student projects and papers from the literature. Edwards and Brindley (1999) serves as an ongoing prototype for clear writing about research, and other papers are selected for discussion depending on the interests and needs of the students.

We do not use any formal text on writing, nor is it mentioned in the textbook. We do look at how both good (as in the Edwards and Brindley paper) and bad papers are constructed, what makes them work, and what features must be present in order for the result to be "reproducible." A reader of a research paper containing mathematics should be able to reproduce the equations with the correct constants on their own computer and get the same results as the authors. Students become very critical of the literature that does not satisfy this criterion. Students also look at some examples of good writing from previous courses. We devote a portion of class time to discussing writing in mathematical biology.

The nature of the subject requires software for any serious investigation of a complex system. Again, many fine packages are available. However the focus of the course is not on learning a particular package. Most good packages are thick with syntax and require some time and effort to become sufficiently familiar to do even simple tasks. Math 4 needed software that did only a few things: numerically integrate a big system of ordinary differential equations, display graphical output as time series or in the phase plane, plot data points, read and store values of the output. A special applet (Reid, 2009) was built for the course and humorously named the Big Green Differential Equation Machine (after Dartmouth's nonexistent mascot, the Big Green). The students still suffer slightly from syntax issues but they are easily resolved and much calculus is learned during the course of using Big Green. In order to use the software a student must be clear about the difference between parameters and initial values, must be aware of the numerical dangers of very large derivatives, and must absolutely get the idea of step size in an algorithm. The software need not be a completely black box, as a short discussion of Euler's method (repeated as often as necessary and available in every standard calculus text and on Wikipedia) explains the main idea.

Analysis

Methods of Evaluation

This section describes the evaluation of the course goals concerning student attitude: a belief in the utility of mathematics, confidence, and willingness to approach real world problems. We relied on three approaches to assess student learning and overall response to the course: a quantitative attitudes survey, an open-ended post questionnaire and classroom observation. Students completed the pre-post Dartmouth Mathematics Survey, a 37-item, 5-point Likert-scaled instrument (where 1 = strongly disagree and 5 = strongly agree,) created for the course by the evaluation team, on the first and last class day. The survey is designed to measure changes in student attitudes about their abilities and interest in mathematics and its applicability to their lives. The core questions of the survey were drawn from the Mathematics Across the Curriculum Survey, which has been used widely at institutions throughout the country. The original Dartmouth Mathematics Survey was created as part of the NSF funded-Mathematics Across the Curriculum Project 1995–2000 (NSF DUE-9552462). Additional questions were developed to address specific course goals. Negatively phrased questions have been reverse-scored so that changes in the desirable direction are always positive.

At the end of the term, students completed an open-ended nine-item questionnaire by e-mail to provide in-depth information about their response to the course. Thirteen out of sixteen students responded with paragraph-length answers. Finally, the evaluator observed all but three or four classes and gathered informal feedback from students during the term. The responses in 2010 were similar to prior years but data were not considered in aggregate because the course itself was in flux and 2010 was the first year that all the pieces of the course were in place. Because the sample is small, findings from the survey are supported with data from student interviews and classroom observation.

The attitude survey showed strong improvement in student attitudes about math. Whether measured by changes in a broad survey index, comprehensive factors or individual questions, students met the course goals of greater interest in, and appreciation for the usefulness of mathematics. To give a broad-brush sense of attitude change we created a survey index by dividing each student's mean post-response by the mean pre-response; thus an index score greater than one indicates change in the desired direction. While this rough index obscures many fine points, it captures overall attitude movement. For the twelve 2010 Math 4 students for whom we have paired pre- and post-tests, the survey index is 1.11. By comparison, the 106 Dartmouth students in a first term calculus course enriched by five shorter, independent applications posted an index of 0.96 on a closely similar survey.

Although this paper focuses on year three of the evaluation, when the course was in its complete form, we used factor analysis on the pre-survey data from all three years of students in Math 4 to extract four factors with high reliability. These factors we termed, "Confidence" (Cronbach's alpha= .76), "Utililty" (alpha= .78), "Real Problems" (alpha= .85) and "Concept of Math" (alpha= .74). The first two factors "Confidence" and "Utility" showed statistically significant gains. The second two factors, "Real Problems" and "Concept of Math," while not statistically significant, are still important to consider as they indicate change in attitudes that reflect the course goals. Even in this small population students showed statistically significant gains in eight individual survey questions, all of them directly relevant to the course goals. We will examine each factor separately, along with individual questions related to each factor. Two questions, concerning reading the literature and writing research papers, are considered separately under "Writing and Reading Mathematics" below. One question, discussed under "Student Discomfort," showed decline.

Enjoyment and Confidence in Doing Mathematics

The first factor, "Confidence," comprises five survey items (1, 3, 8, 24, 26) all of which link confidence and pleasure in doing math with engagement. The class mean moved from 3.1 to 3.6 from the pre- to post-test for this factor. Students clearly came to appreciate that when they are involved and contribute to the process of mathematics they gain more confidence in their ability and ultimately pleasure in doing mathematics.

> I've learned how to build box models to create differential equations for many types of scenarios: We constructed box models to quench our own curiosity, to find out how many people would have gotten SARS had it hit the Beijing Olympics, to find out whether lion predation or wild dog competition will cause cheetahs to go extinct faster in the Serengheti. Most of all, I've learned that math is interesting, not just memorizing equations....it's actually fun, I actually tell people outside of class about the cool data my models portrayed in my papers.

Two questions in this factor showed particularly strong gains. Question 1, "In mathematics I can be creative and discover things for myself," showed one of the largest gains with the mean moving from 2.8 to 4.0. (p = .003). Students perceived that the structure of the course and the rich set of problems allowed them to draw on their background knowledge and experience to inform how they developed their solutions.

> This math course was different in the fact that we were supposed to come up with our own solutions by thinking critically before the answer was given to us. Also, we built upon the mathematical literature and used calculus to describe biological situations. This course did not just deal with numbers, but also with real-life situations.
>
> By writing a math research paper, I was able to demonstrate that not only did I understand the papers I have read, but I was able to synthesize novel information from them.

Thus, the experience of math in this course becomes similar to student's experiences in humanities courses where their own ideas are integral to their scholarship.

Confidence is further indicated by the strong gains in question 24, "After I've forgotten all the formulas, I'll still be able to use ideas I've learned in math," with the mean moving from 3.3 to 4.1 (p = .017). Students perceived that the concepts they learned in mathematics would have utility beyond the duration of the course.

> The skill of evaluating different papers and analyzing data will be useful in the future, I think. Even though I hope to go to law school in the future I think these skills can be

> easily applied.
>
> By paper 3, I believe I understood what qualifies a certain literature as excellent after reading more than a handful of papers. I cannot say I've mastered mathematical modeling but I can definitely put in my two cents if asked certain questions.

Utility of Mathematics to Science

The second factor, termed “Utility” (including items 5, 18, 30, 33, 35) showed a strong gain with the class mean moving from 3.3 to 3.9. This factor references students’ appreciation of the application of mathematics outside the classroom. This course, unlike other mathematics courses, explicitly links mathematics with doing science.

> In most math classes that I've taken, I've spent time working through problems given by the teacher and haven't spent time researching a biological phenomenon. Learning how to apply this research to a model of the phenomenon is certainly something I wouldn't have learned in any other class.

Students come to understand that math is not an instance of solving a given problem, but, as in science, they have to find and define the problem. Through reviewing the scientific data and developing their models, they come to understand the biological systems.

> One of the major benefits of writing the biological calculus research papers was that it helped you learn the biological system much better. Understanding the complex workings of the system and the factors that caused the most influence and change was instrumental to understanding and remembering the biological system better.

Survey questions 30, “Doing mathematics raises interesting new questions about the world” and 35, “Math helps me understand the world around me,” showed statistically significant gains with the mean moving from 3.3 to 4.2 (p = .005) on question 30 and from 3.0 to 3.7 (p = .013) on question 35. Student comments give a dynamic picture of how the connection to other science stimulates their interest and affinity to math.

> In this math course not only do you get to interact with the professor, but you feel that you can interact with the material as well. This class actually applies and gives reason to those abstract concepts we learned in Math 3 and elsewhere. From this class I actually learned HOW math is relevant to biology and medicine, and how it is used in our daily lives ... like for example how pharmacologists decide what drug dose to give us and how long in between doses.
>
> I learned how to use differential equations to account for nearly every aspect of biology. While no model is perfect and accounts for everything affecting a species/disease, equations hypothetically could account for anything affecting what you're studying.
>
> There are many people out there who abhor math, but they don't realize it's not because they don't like the material, it's because they don't like WebWork [an online homework

> system] and lecture classes. Thanks to Math 4, I am definitely more interested in math, and I even plan to take more higher-level math courses, something I never thought I would do.

Students also found manipulating data to be compelling. Question 5, "I like exploring problems using real data and computers," showed a large gain, the mean moving from 2.7 to 3.5 (p = .003). Students struggled in class to make their models work—at times with significant frustration—yet by the end of the course their attitudes had moved in the positive direction. In spite of their initial resistance to using Big Green and other tools, they came to appreciate and value how manipulating data with a computer can be extremely useful and even enjoyable.

> I learned a lot about the topics that I wrote my papers on, as well as different types of mathematical modeling, such as using Big Green and designing tornado diagrams for a sensitivity analysis in Excel.

As a whole, factor 2 highlights similarities of mathematics to science and indeed all problem solving where students must bring their own resources to the problem, make decisions about what is relevant and choose which of several viable approaches is best.

Problems of Scientific and Social Importance

The third factor, "Real Problems" (items 16 and 25) refers to student's preference for problems with scientific and social importance. This factor showed strong gains with the class mean moving from 4.0 to 4.3 from the pre- to post-test. Although students entered the course with a positive attitude about working with problems of this type, they still showed a positive gain in attitude.

> Now that we are older, I think it is critical to include "real world" applications into math. Using information from the WHO [World Health Organization] and other reliable sources helped increase my interest in the topics as opposed to focusing on hypothetical scenarios that have no relevance in my life.
>
> [Writing papers] allowed us to have more emphasis on applying the math to something real, instead of just doing a numerical problem.

The emphasis on Africa was of value to some students. In an informal setting two students expressed their appreciation for this aspect of the course.

Math as an Open-ended Endeavor

Factor 4, "Concept of Math" (including items 6, 10,11, 13,19, 28, 29, 31, 36) refers to the student's perception of math as a closed vs. open-ended system. The class showed positive gains with the mean moving from 3.6 to 3.8. While this is not a large change it still shows movement in the desired direction. Individual questions in this factor showed statistically significant changes and illuminate how student attitudes do change as a result of the course experience. Students come to appreciate that judgment calls and guessing have a place in mathematics. Question 11, "Guessing (conjecturing) is an important part of doing mathematics," showed a change in mean from 3.0 to 3.8 (p = .021) and for question 36, "There's no such thing as a judgment call in mathematics" (reverse scored), the mean moved from 3.3 to 4.1 (p = .043). This response indicates that students gain an understanding that math is an open system and a useful tool. When prompted, "Please tell me something you learned in this course." One student responded:

> I learned that mathematics is not just numbers with right and wrong answers. It can be much more complex, complete with ambiguity, and helpful in understanding the world around us.

Students made a distinction that this class was unique from other math classes in that it was not based around problem sets. It is interesting that the messy problems presented in this class are viewed more as research than as math.

> All other math courses have been very problem-based. Never have I had to do research for a comparable course. By the end of the term, I found the ability to do so extremely useful.

Writing and Reading Mathematics

Two questions capture some of the goals of the course not covered by the four factors. Survey question 17, "Writing about mathematics makes it easier to learn," showed one of the largest positive gains, with the mean moving from 2.9 to 4.2 (p = .001). While this question was not an element of factor 4, it is closely related. The process of writing papers changed the way students perceived mathematics. They moved from the concept of math as a series of problem sets to the broader idea of math as a process for connecting to science, as a basis for scholarly argument and as a tool for critical thinking.

> I've never written a math paper before. Writing a paper definitely changed what I learned. Instead of just playing with numbers, I had to think about the impact of those numbers in biologically relevant settings.

> I learned how to use mathematics to supplement scholarly papers. While I'm used to writing research papers, I had never used mathematics in this way as the basis for my argument. This skill will definitely be helpful in future papers.

> [Writing a research paper in biological mathematics] was the best part of the course because I was able to learn the material by doing it myself and trying to answer questions by myself.

Question 22, "I can read a research paper in mathematical biology and get something out of it," showed the strongest gain of any individual question. The mean moved from 2.9 to 4.3 ($p < .001$). This question is central to the stated goal that students be able to understand and evaluate papers in the mathematical biology literature.

> The biggest value is that I came to feel more comfortable with reading math papers by the end of the term. Before taking this class, I would not have imagined that I could've understood what the papers were talking about, let alone write my own papers based on them.
>
> At first it was a little difficult for me to figure out, but the Prof gave us sample papers, and went over how to find the thesis and information you are looking for. I can now easily look at a preexisting mathematical biology paper and understand the gist of it. This I have found will be very important for me as a future biologist. We also found that some of the papers were in fact wrong; often the data given was not correlated to the equations they gave, so I've definitely learned that you have to evaluate the modeling in papers because many times the constants and data are slightly fabricated.
>
> Though it was difficult, I think that reading original research was the most valuable part of the course, specifically for the first paper (since it was more related to pre-med stuff!)
>
> Extremely valuable; I now have confidence to read a lot of original research and at least parceling out the important information.

Student Discomfort

Although the format of Math 4 works very well for the majority of students, for a few students the uncertainties of not having a concrete answer and working with a big messy problem, even if fully acceptable in science, are not comfortable for them in math. For these students mathematics is about learning more math content and not how to apply the math they know in a creative, integrated and precise way. The following quotes are taken from all three years of the course because there were not that many negative comments in any given year.

> Writing papers is a handicap to the course. I can write, but explaining math concepts is hard —not like doing problems. Problem sets would be more helpful. Papers seem kind of arbitrary. You did the math to solve the question and explained with the graphs and the writing is just busywork to fill in around the graphs. Why write to explain the math? The professor knows what a differential equation is, so why spend all that time trying to write it out?
>
> The second paper was awful. Finding constants was a huge waste of time. It did not teach us anything at all. Ninety five percent of the time was wasted looking on the Internet and almost no math was done.

We think these comments touch upon the ongoing discussion in the professional community about what constitutes numeracy. Item 10 on the attitude survey, "I don't need a good understanding of math to achieve my career goals," went down significantly (4.4 to 3.8, p = .013, reverse scored), indicating that students felt understanding math was less necessary to their careers after taking the class. In view of all the positive comments and significant attitude changes in other measures, we wonder if this particular item reflects student beliefs about what mathematics is or ought to be, again highlighting the distinction between mathematics, as it is traditionally taught, and something far closer to numeracy. Student comments support this interpretation, as they may not perceive Math 4 as an actual mathematics course.

> This course had more emphasis on paper writing and less on problem sets. I feel like I learned more about research papers in mathematics, but not necessarily any different math.
>
> I don't think it had lots of calculus. I would barely call it a math course. There were no numbers to crunch or formulas to remember. It was more of just a modeling class. I would hardly call it a math class.

Integration of Research and Teaching

After some years of offering the course it became apparent that students were tackling problems of comparable complexity, and getting results of comparable value, to those in the literature. Also, as the instructor became more familiar with the territory, the possibility of guiding research to a formal, publishable conclusion became imaginable. The three-week time frame of any given paper in the course is too short to achieve this goal. However there are strategies that can get around such a constraint.

Students are now invited to write papers that improve upon (very good) papers that have been done by Math 4 students in prior years. Although only a few papers are worth building upon and only a few students choose to do so, a steady stream of research projects is evolving. In addition to Math 4, there is also a mathematical biology course (Math 27) for more advanced students, who have had linear algebra and often the standard differential equations course as well. Math 27 is co-listed and offered at the same time and place as Math 4. The few students in that course (1-4 typically) can also build on prior Math 4 papers. Like most departments, the mathematics department offers an independent study course that students can use to complete a project. Additionally there are various internships for which students may apply. In 2010, two papers from the 2009 offering of Math 4 were presented in class by the authors of those papers. These events had the predictable result that two groups of 2010 students chose to work further on those topics.

DOI: http://dx.doi.org/10.5038/1936-4660.4.1.3

In 2010 two papers that began as papers in Math 4 were accepted and presented at the Society for Mathematical Biology annual meeting. One is on the bioaccumulation of methylmercury in aquatic ecosystems (with data from Lake Erie). The initial model was developed by two Math 4 students in 2009 and brought to a convincing conclusion by two Math 27 students in 2010. The second is a study of the epidemiology of West Nile virus begun in 2009 by a Math 4 student who studied the interaction of the mosquito vector with the avian reservoir by comparing four published models implemented with the parameters studied in six different bird species. Two Math 4 students in 2010 took the best model found by that student, coupled it with the human population, and studied various strategies for using insecticide to reduce the severity of an outbreak. Two papers on the SARS epidemic from 2009 are the inspiration for an independent study project in 2010, requested by a student from the 2008 offering of Math 4 who wanted an interesting research project. One of the Math 27 students from 2009 is continuing a project she started in that class and extending it to a senior thesis in mathematics.

It seems that Math 4 is an ideal opportunity to integrate research and teaching. It is also an opportunity to create a research community persisting outside the classroom and spanning several years of students, with research problems that arise naturally from within that community. With a backlog of partially finished projects, interesting leads, and good starts, it is always possible to provide a research project to a Math 4 student, a Math 27 student, or someone seeking an internship, independent study, or senior thesis.

The instructor has extended this goal to her standard differential equations class by assigning a short research project (supplanting one of the two traditional midterm exams) and inviting interested students to pursue further work after the course ends. She hopes that the stronger math background of those in the standard differential equations class, frequently enhanced by familiarity with advanced software packages, will be a good complement to the more practical scientific perspective characteristic of Math 4 students. In 2010 two students (out of 53) from the standard differential equations class have asked for research projects.

Conclusions

It is possible to design a calculus sequel that makes calculus relevant to biology and pre-medical students. Posing one's own research question, framing it mathematically and solving it puts the student in the position of the working scientist who uses mathematics to address a question of interest. Interaction with the research literature makes the student a part of that research community if only for a short while. Writing and presenting results allows students to use mathematics to "give reason" to biology and builds confidence while doing so.

We concede that a small class is necessary for teaching Math 4 effectively. Fortunately this precondition is met in many small liberal arts colleges, community colleges, and some state institutions.

For a serious student of science, quantitative literacy goes far beyond measuring the carpet. Mathematical biology includes a wide variety of topics, but to give mathematical reason to biology requires a level of integration that few mathematics courses ever achieve. We believe that the course described and evaluated in this paper represents one successful example of numeracy for future scientists. We also agree with the student who said:

> Every biology major should take this course.

Acknowledgments

The authors wish to thank the students of math 4 for their willingness to answer endless questions and give their honest feedback; Jane Korey for her wise advice; and reviewers and editor for their suggestions for improvements to the manuscript. This work was made possible by NSF DUE-0736749, Collab Res: Six Short Weeks, a classroom strategy for promoting undergraduate research in mathematical biology.

References

Duckworth, E. 1996. *"The having of wonderful ideas" and other essays on teaching and learning*, 2nd ed. New York: Teacher's College Press.

Edwards, A. M., and J. Brindley. 1999. Zooplankton mortality and dynamical behavior of plankton population models. *Bulletin of Mathematical Biology* 61:303–309. http://dx.doi.org/10.1006/bulm.1998.0082

Edelstein-Keshet, L. 2005. *Mathematical models in biology*. Philadelphia: Society for Industrial and Applied Mathematics.

Harper, E. 1987. Ghosts of Diophantus. *Educational Studies in Mathematics* 18 (1): 75–90. http://dx.doi.org/10.1007/BF00367915

Murray, J. D. 1993. *Mathematical Biology: I. an Introduction*, 3rd ed. New York: Springer.

Nowak, M. A., and R. M. May. 1991. Mathematical biology of HIV infections: antigenic variation and diversity threshold. *Mathematical Biosciences* 6 (1): 1–21. http://dx.doi.org/10.1016/0025-5564(91)90037-J

Renkl, A., H. Mandl, and H. Gruber. 1996. Inert knowledge: Analyses and remedies. *Educational Psychologist* 31(2): 115–121. http://dx.doi.org/10.1207/s15326985ep3102_3

Reid, B. 2009. Big Green Differential Equation Machine (Version 2). Software. Available at: http://www.bpreid.com/bgdem.php (accessed October 24, 2010).

Richards, J. L. 2001. Connecting mathematics with reason. In Steen, 31–36.

Salomon, G., and D. N. Perkins.1989. Rocky roads to transfer: Rethinking mechanisms of a neglected phenomenon. *Educational Psychologist* 24 (2): 113–142. http://dx.doi.org/10.1207/s15326985ep2402_1

Schneider, C. G. 2001. Setting greater expectations for quantitative learning. In *Mathematics and democracy*, ed. L. A. Steen, 99–105. Princeton, NJ: National Council on Education and the Disciplines.

Skemp, R. R. 1987. *The psychology of learning mathematics*. Hillsdale NJ: Lawrence Erlbaum Associates, Inc.

Steen, L. A., ed. 2001. *Mathematics and democracy*. Princeton, NJ: National Council on Education and the Disciplines. Available at: http://www.maa.org/ql/mathanddemocracy.html (accessed Dec 28, 2010).

———. 2001. The case for quantitative literacy. In Steen 2001, 1–22.

Teboh-Ewungkem, M. I. 2009. Malaria control: The role of local communities as seen through a mathematical model in a changing population—Cameroon. In *Advances in disease Epidemiology*, ed. J.M. Tchuenche, Z. Mukandavire, 101–138. Hauppauge NY: Nova Science.

———, and T. Yuster. 2010. A within-vector mathematical model of *Plasmodium falciparum* and implications of incomplete fertilization on optimal gametocyte sex ratio. *Journal of Theoretical Biology* 264 (2): 273–286. http://dx.doi.org/10.1016/j.jtbi.2009.12.017

Wallace, D. I. 2010. *Situated complexity: Modeling change in non-linear biological systems with a focus on Africa*. Xlibris.

Yeargers, E. K., R. W. Shonkwiler, J. K Herod.1996. *An introduction to the mathematics of biology with computer algebra models*. Boston: Birkhäuser.